Performance Analysis of Zero-Forcing for Two-Way MIMO AF Relay Networks

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***Abstract*—Transmit/receive zero-forcing (ZF) is studied for multiple-input multiple-output (MIMO) amplify-and-forward (AF) two-way relay networks (TWRNs). Specifically, two sources employ transmit and receive ZF during two consecutive time-slots for transmission and reception, respectively, while the relay performs analog network coding. Each source then requires only the instantaneous respective source-to-relay channel knowledge, and hence, the complexity of practical implementation is significantly reduced. The performance of this system set-up is studied by deriving the upper and lower bounds of the overall outage probability, their high signal-to-noise ratio approximations and diversity order. To obtain valuable insights into practical MIMO TWRN implementation, the diversity-multiplexing trade-off is also quantified.**

***Index Terms*—Two-way relay networks, MIMO, zero forcing.**

I. Introduction

TWO-WAY relay networks (TWRNs) promise spectral efficiency improvements for wireless networks with half-duplex terminals [1]–[5]. Specifically, conventional one-way relay networks (OWRNs) require four orthogonal channel-uses to exchange two messages between two sources via a relay, whereas TWRNs require just two orthogonal channel-uses. Thus, TWRNs avoid the pre-log factor of one-half in capacity expressions, and hence, are twice as spectral efficient as OWRNs [1], [2]. Moreover, multiple-input multiple-output (MIMO) technologies can further enhance the performance of single-antenna TWRNs [3]–[5]. Consequently, amplify-and-forward (AF) MIMO TWRNs can use zero-forcing (ZF) transmit beamforming and ZF receive equalization strategies.

Prior related research: In [3]–[5], precoder/decoder designs for MIMO AF TWRNs are studied with varying degrees of complexity and performance. Specifically, in [3], the optimal relay beamforming structure is derived and used to study the achievable capacity regions. Moreover, [4] develops the optimal relay precoder for MIMO TWRNs based on the minimization of the mean square error (MSE) between the transmitted and received signals with a total power constraint. In [5], the relay processing is optimized based on both ZF and minimum MSE criteria under relay power constraints for multiuser MIMO TWRNs. Specifically, [5] designs optimized transceivers at the relay to mitigate both co-channel interference and self-interference to cater a multiuser scenario by using steered beams through multiple antennas at the relay. All the aforementioned studies [3]–[5] involve convex optimization techniques for designing MIMO precoders/decoders.

For the sake of completeness, we also mention the prior related research on OWRNs with ZF precoder/decoder designs. In [6], ZF is studied for four different system set-ups of multiuser OWRNs by deriving the outage probability of an arbitrary data substream.

Motivation and our contribution: Although [3]–[5] derive transceiver structures for MIMO TWRNs, they do not facilitate deriving important performance metrics such as the outage probability and achievable diversity/multiplexing gains in closed-form due to the complicated MIMO precoder/decoder designs. Moreover, in all the above studies, the relay complexity is high and consequently undermines one of the main trade-offs of deploying relay networks; i.e., the implementation complexity versus performance. In particular, to implement the source/relay transceiver structures [3]–[5], sources/relay require global channel state information (CSI)[1], and thus, increasing the additional feedback/overhead and lowering the spectral efficiencies.

In this letter, a suboptimal yet simple transmission strategy, which circumvents the complex precoder designs of [3]–[5], while achieving comparable performance gains, is developed and analyzed. Specifically, we consider a half-duplex MIMO AF TWRN consisting of two sources and one relay. In the first time-slot, both sources simply employ transmit-ZF, while the relay receives a superimposed-signal without using a specific receiver reconstruction filtering. In the second time-slot, relay performs a simple AF operation with a specific gain that is designed to constraint the long-term total transmission power at the relay. The two sources receive this amplified superimposed-signal by employing receive-ZF. One practical significance of our system set-up is that each source requires only the respective source-to-relay CSI knowledge as opposed to the global CSI requirement of [3]–[5], while the relay requires only the long-term channel statistics. This letter aims to establish basic performance metrics of this system set-up.

The performance of the aforementioned system set-up is quantified by first deriving the effective end-to-end signal-to-noise ratio (e2e SNR) of each source, and then, deriving the upper and lower bounds for the overall outage probability, their asymptotically exact high SNR approximations and diversity order. Moreover, useful insights into practical MIMO TWRN implementation are obtained by quantifying the diversity-multiplexing trade-off.

Notations: $\mathcal{E}_\Lambda\{z\}$ is the expected value of z over Λ. $\mathbf{Z}^H$, $\mathbf{Z}^T$, $[\mathbf{Z}]_{k,l}$ and $\lambda_k(\mathbf{Z})$ denote the conjugate-transpose, transpose, the (k,l)th diagonal element and the kth eigenvalue of the square matrix, $\mathbf{Z}$, respectively. The operator $\otimes$ denotes the Kronecker product. $\mathbf{I}_M$ and $\mathbf{O}_{M\times N}$ are the $M\times M$ Identity matrix and $M\times N$ matrix of all zeros, respectively. $f(x) = o(g(x))$, $g(x)>0$ states that $f(x)/g(x)\to 0$ as $x\to 0$.

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[1]Here, instantaneous global CSI refers to instantaneous full channel knowledge of both hops, i.e., $S_1\to R$ and $S_2\to R$.

II. System Model

We consider a MIMO AF TWRN consisting of two source nodes (S_1 and S_2), and one relay node (R), where each of them is equipped with N_1, N_2 and N_R antennas, respectively[2]. All nodes are assumed to be half-duplex, and all channel amplitudes are assumed to be independently distributed frequency-flat Rayleigh fading. The channel matrix from S_i to R is denoted by $\mathbf{H}_{i,R} \sim \mathcal{CN}_{N_R \times N_i}\left(\mathbf{0}_{N_R \times N_i}, \mathbf{I}_{N_R} \otimes \mathbf{I}_{N_i}\right)$, where $i \in \{1,2\}$. All the channel coefficients are assumed to be fixed over two consecutive time-slots [1], [2], and hence, the channel matrix from R to S_i, $\mathbf{H}_{R,i}$, can be denoted as $\mathbf{H}_{i,R}^T$ for $i \in \{1,2\}$. The additive noise at all the receivers is modeled as complex zero mean additive white Gaussian (AWGN) noise. The direct channel between S_1 and S_2 is assumed to be unavailable due to heavy path-loss and shadowing [1], [2].

In this protocol, S_1 and S_2 exchange their information-bearing vectors, $\mathbf{x}_1$ and $\mathbf{x}_2$, satisfying $\mathcal{E}\left[\mathbf{x}_i \mathbf{x}_i^H\right] = \mathbf{I}_{N_i}$, $i \in \{1,2\}$, respectively, during two time-slots. In the first time-slot, both S_1 and S_2 transmit $\mathbf{x}_1$ and $\mathbf{x}_2$ simultaneously by employing transmit-ZF precoding to R over a multiple access channel. The received superimposed-signal vector[3] at R is given by

$$\mathbf{y}_R = \Psi_1 \mathbf{H}_{1,R} \mathbf{U}_1 \mathbf{x}_1 + \Psi_2 \mathbf{H}_{2,R} \mathbf{U}_2 \mathbf{x}_2 + \mathbf{n}_R, \quad (1)$$

where $\mathbf{n}_R$ is the $N_R \times 1$ zero mean AWGN vector at R satisfying $\mathcal{E}\left(\mathbf{n}_R \mathbf{n}_R^H\right) = \mathbf{I}_{N_R} \sigma_R^2$, and $\mathbf{U}_i$ is the transmit-ZF precoding matrix at S_i, and is given by [7]

$$\mathbf{U}_i = \mathbf{H}_{i,R}^H \left(\mathbf{H}_{i,R} \mathbf{H}_{i,R}^H\right)^{-1} \mathbf{\Pi}_i \quad \text{for} \quad i \in \{1,2\}, \quad (2)$$

where $\mathbf{\Pi}_i$ is the $N_R \times N_i$ permutation matrix[4], which ensures only N_R data streams are transmitted by S_i for $i \in \{1,2\}$. In (2), Ψ_i, $i \in \{1,2\}$, is the power normalizing factor, which constraints the long-term total power at S_i, and is given by

$$\Psi_i = \sqrt{\frac{\mathcal{P}_i}{\text{Tr}\left(\mathcal{E}\left[\mathbf{U}_i \mathbf{U}_i^H\right]\right)}} = \sqrt{\frac{\mathcal{P}_i}{\mathcal{T}_i}}, \quad \text{for} \quad i \in \{1,2\}, \quad (3)$$

where $\mathcal{P}_i$ is the transmit power at S_i and $\mathcal{T}_i \triangleq \text{Tr}\left(\mathcal{E}\left[\mathbf{U}_i \mathbf{U}_i^H\right]\right) = \frac{N_R}{N_i - N_R}$ [6].

In the second time slot, R amplifies $\mathbf{y}_R$ with a gain[5] $G = \sqrt{\frac{\mathcal{P}_R}{\Psi_1^2 + \Psi_2^2 + \sigma_R^2}}$ and broadcasts to both sources over the broadcast channel. Here, $\mathcal{P}_R$ is the transmit power at R. Then, each source, S_i, receives the $N_R \times 1$ signal vector by employing the receive-ZF as follows:

$$\mathbf{y}_{S_i} = \mathbf{V}_i \left(G \mathbf{H}_{R,i} \mathbf{y}_R + \mathbf{n}_i\right), \quad \text{for} \quad i \in \{1,2\}, \quad (4)$$

where $\mathbf{H}_{R,i} = \mathbf{H}_{i,R}^T$ and $\mathbf{n}_i$ is the $N_i \times 1$ zero mean AWGN at S_i satisfying $\mathcal{E}\left(\mathbf{n}_i \mathbf{n}_i^H\right) = \mathbf{I}_{N_i} \sigma_i^2$, where $i \in \{1,2\}$. Furthermore, in (4), $\mathbf{V}_i$, $i \in \{1,2\}$, is the receive-ZF matrix at S_i, and given by [7]

$$\mathbf{V}_i = \left(\mathbf{H}_{R,i}^H \mathbf{H}_{R,i}\right)^{-1} \mathbf{H}_{R,i}^H, \quad \text{for} \quad i \in \{1,2\}. \quad (5)$$

By substituting (1) and (5) into (4), and by removing the self-interference[6] [1], the post-processing e2e SNR of the kth, $k \in \{1 \cdots N_R\}$, data substream at S_i can be derived as

$$\gamma_{S_i^{(k)}} = \frac{\mathcal{T}_i \bar{\gamma}_{R,i} \bar{\gamma}_{i',R}}{\mathcal{T}_i \mathcal{T}_{i'} \bar{\gamma}_{R,i} + \left(\mathcal{T}_i \bar{\gamma}_{i',R} + \mathcal{T}_{i'} \bar{\gamma}_{i,R} + \mathcal{T}_i \mathcal{T}_{i'}\right) \left[\mathbf{H}_{R,i}^H \mathbf{H}_{R,i}\right]_{k,k}^{-1}}, \quad (6)$$

where $\bar{\gamma}_{i,R} \triangleq \frac{\mathcal{P}_i}{\sigma_R^2}$, $\bar{\gamma}_{R,i} \triangleq \frac{\mathcal{P}_R}{\sigma_i^2}$, $i \in \{1,2\}$, $i' \in \{1,2\}$ and $i \neq i'$. It is worth noticing the statistical independence of $\gamma_{S_1^{(k)}}$ and $\gamma_{S_2^{(k)}}$ of (6) for a given k. However, post-processing SNRs of multiple substreams belong to a given source are correlated.

III. Performance Analysis

In this section, the performance of transmit/receive ZF for MIMO AF TWRNs is studied. To this end, closed-form upper and lower bounds for the overall outage probability are derived. To obtain further insights into practical system designing, the diversity-multiplexing trade-off is also quantified.

A. Overall outage probability

The overall performance of multi-source systems is governed by the performance of the weakest source [8]. Thus, the overall outage probability of our system set-up is defined as

$$P_{\text{out}} = \Pr\left[\min\left(\min_{k \in \{1 \cdots N_R\}} \gamma_{S_1^{(k)}}, \min_{k \in \{1 \cdots N_R\}} \gamma_{S_2^{(k)}}\right) \leq \gamma_{th}\right], \quad (7)$$

where γ_{th} is the threshold SNR[7]. Direct computation of (7) appears complicated due to the correlation of $\gamma_{S_i^{(k)}}$ for $k \in \{1 \cdots N_R\}$. Thus, simple closed-form upper and lower bounds for the overall outage probability are derived.

1) Upper bound of P_{out}: The maximum diagonal element of $\left[\mathbf{H}_{R,i}^H \mathbf{H}_{R,i}\right]^{-1}$ can be upper bounded as [7]

$$\max_{k \in \{1 \cdots N_R\}} \left[\mathbf{H}_{R,i}^H \mathbf{H}_{R,i}\right]_{k,k}^{-1} \leq \lambda_{\min}^{-1}\left(\mathbf{H}_{R,i}^H \mathbf{H}_{R,i}\right). \quad (8)$$

By substituting (8) into (6), the smallest substream SNR of S_i, $i \in \{1,2\}$, can be lower bounded as follows:

$$\gamma_{S_{i,\min}} = \min_{k \in \{1 \cdots N_R\}} \gamma_{S_i^{(k)}} \geq \gamma_{S_{i,\min}}^{\text{lb}} = \frac{\eta_i}{\zeta_i + \mu \lambda_{\min}^{-1}\left(\mathbf{H}_{R,i}^H \mathbf{H}_{R,i}\right)}, \quad (9)$$

where $\mu = \mathcal{T}_1 \bar{\gamma}_{2,R} + \mathcal{T}_2 \bar{\gamma}_{1,R} + \mathcal{T}_1 \mathcal{T}_2$, $\eta_i = \mathcal{T}_i \bar{\gamma}_{R,i} \bar{\gamma}_{i',R}$, and $\zeta_i = \mathcal{T}_i \mathcal{T}_{i'} \bar{\gamma}_{R,i}$, where $i \in \{1,2\}$, $i' \in \{1,2\}$ and $i \neq i'$.

By substituting (9) into (7), P_{out} can be upper bounded as

$$P_{\text{out}} \leq P_{\text{out}}^{\text{ub}} = \Pr\left[\min\left(\gamma_{S_{1,\min}}^{\text{lb}}, \gamma_{S_{2,\min}}^{\text{lb}}\right) \leq \gamma_{th}\right]. \quad (10)$$

The upper bound for the overall outage probability in (10) can be derived in closed-form as (see Appendix I for the proof)

$$P_{\text{out}}^{\text{ub}} = \sum_{i=1}^{2} F_{\gamma_{S_{i,\min}}^{\text{lb}}}\left(\gamma_{th}\right) - \prod_{i=1}^{2} F_{\gamma_{S_{i,\min}}^{\text{lb}}}\left(\gamma_{th}\right), \quad (11)$$

where $F_{\gamma_{S_{i,\min}}^{\text{lb}}}(x)$ is the cumulative distribution function (CDF) of $\gamma_{S_{i,\min}}^{\text{lb}}$ and is given by

$$F_{\gamma_{S_{i,\min}}^{\text{lb}}}(x) = \begin{cases} 1 - \frac{\det\left[\mathbf{Q}_i\left(\frac{\mu x}{\eta_i - \zeta_i x}\right)\right]}{\prod_{j=1}^{N_R}\left[\Gamma(N_i - j + 1)\Gamma(N_R - j + 1)\right]}, & 0 < x < \frac{\eta_i}{\zeta_i} \\ 1, & x \geq \frac{\eta_i}{\zeta_i}. \end{cases} \quad (12)$$

[2]Specifically, we restrict N_1, N_2 and N_R to satisfy the constraint $N_R < \min(N_1, N_2)$ in the sequel. This constraint allows us to employ joint transmit/receive ZF for the same antenna configuration and renders mathematical tractability. Consequently, the maximum number of end-to-end data substreams is constrained to N_R.

[3]This superimposed-signal can also be identified as an analog network code vector in the two-way relay networks [3].

[4]The permutation matrix, $\mathbf{\Pi}_i$, $i \in \{1,2\}$, can be constructed by horizontally concatenating a $N_R \times N_R$ permutation matrix and a $N_R \times (N_i - N_R)$ zero matrix.

[5]Here, the amplification gain, G, is designed as a normalizing constant to constraint the long-term total power at R.

[6]It is assumed that S_i knows its own information-bearing symbol vector, $\mathbf{x}_i$, CSI of $\mathbf{H}_{i,R}$, and G which requires Ψ_i, where $i \in \{1,2\}$.

[7]This threshold SNR, γ_{th}, can be determined to satisfy the minimum service-rate constraint; $\gamma_{th} = 2^{\mathcal{R}_{th}} - 1$, where $\mathcal{R}_{th}$ is the target rate [8].

In (12), $\mathbf{Q}(x)$ is an $N_R \times N_R$ matrix, where the (u,v)th element is given by [9, Eq. (2.73)]

$$[\mathbf{Q}_i(x)]_{u,v} = \Gamma(N_i - N_R + u + v - 1, x). \quad (13)$$

2) Lower bound of P_{out}: The maximum diagonal element of $\left[\mathbf{H}_{R,i}^H \mathbf{H}_{R,i}\right]^{-1}$ can be lower bounded by an arbitrary kth diagonal element, $\left[\mathbf{H}_{R,i}^H \mathbf{H}_{R,i}\right]_{k,k}^{-1}$. Thus, the smallest post-processing substream SNR of S_i, $i \in \{1,2\}$, can be upper bounded as

$$\gamma_{S_{i,\min}} = \min_{k\in\{1\cdots N_R\}} \gamma_{S_i^{(k)}} \leq \gamma_{S_{i,\min}}^{\text{ub}} = \frac{\eta_i}{\zeta_i + \mu\left[\mathbf{H}_{R,i}^H \mathbf{H}_{R,i}\right]_{k,k}^{-1}}. \quad (14)$$

By substituting (14) into (7), P_{out} can be lower bounded as

$$P_{\text{out}} \geq P_{\text{out}}^{\text{lb}} = \Pr\left[\min\left(\gamma_{S_{1,\min}}^{\text{ub}}, \gamma_{S_{2,\min}}^{\text{ub}}\right) \leq \gamma_{th}\right]. \quad (15)$$

By using similar steps to those in Appendix I, the lower bound for the overall outage probability (15) can be derived as

$$P_{\text{out}}^{\text{lb}} = \sum_{i=1}^{2} F_{\gamma_{S_{i,\min}}^{\text{ub}}}(\gamma_{th}) - \prod_{i=1}^{2} F_{\gamma_{S_{i,\min}}^{\text{ub}}}(\gamma_{th}), \quad (16)$$

where $F_{\gamma_{S_{i,\min}}^{\text{ub}}}(x)$ is the CDF of $\gamma_{S_{i,\min}}^{\text{ub}}$ and is derived by using similar steps to those in [6] as

$$F_{\gamma_{S_{i,\min}}^{\text{ub}}}(x) = \begin{cases} 1 - \frac{\Gamma\left(N_i - N_R + 1, \frac{\mu x}{\eta_i - \zeta_i x}\right)}{\Gamma(N_i - N_R + 1)}, & 0 < x < \frac{\eta_i}{\zeta_i} \\ 1, & x \geq \frac{\eta_i}{\zeta_i}. \end{cases} \quad (17)$$

B. Diversity order

In this subsection, the diversity order of transmit/receive ZF for MIMO AF TWRNs is derived by using the upper and lower bounds for the overall outage probability.

1) Diversity order by using the upper bound of P_{out}: An asymptotic high SNR approximation for the upper bound of the overall outage probability (11) can be derived as (see Appendix II for the proof)

$$P_{\text{out}}^{\text{ub},\infty} = \Omega_{\text{ub}} \left(\frac{\gamma_{th}}{\bar{\gamma}_{S,R}}\right)^{G_d^{\text{ub}}} + o\left(\bar{\gamma}_{S,R}^{-(G_d^{\text{ub}}+1)}\right), \quad (18)$$

where the upper bound of the diversity order is given by

$$G_d^{\text{ub}} = \min(N_1, N_2) - N_R + 1. \quad (19)$$

In (18), the system dependent constant, Ω_{ub}, is given by

$$\Omega_{\text{ub}} = \begin{cases} \frac{\nu_1 \phi_1^{N_1 - N_R + 1}}{(N_1 - N_R + 1)\beta^{N_1 - N_R + 1}}, & N_1 < N_2 \\ \frac{\nu_2 \phi_2^{N_2 - N_R + 1}}{(N_2 - N_R + 1)\beta^{N_2 - N_R + 1}}, & N_2 < N_1 \\ \frac{\left(\nu_1 \phi_1^{N - N_R + 1} + \nu_2 \phi_2^{N - N_R + 1}\right)}{(N - N_R + 1)\beta^{N - N_R + 1}}, & N_1 = N_2 = N, \end{cases} \quad (20)$$

where $\bar{\gamma}_{1,R} = \bar{\gamma}_{2,R} = \bar{\gamma}_{S,R}$, $\bar{\gamma}_{R,1} = \bar{\gamma}_{R,2} = \bar{\gamma}_{R,S}$, $\bar{\gamma}_{R,S} = \beta\bar{\gamma}_{S,R}$, $\phi_1 = \frac{\mathcal{T}_1 + \mathcal{T}_2}{\mathcal{T}_1}$, and $\phi_2 = \frac{\mathcal{T}_1 + \mathcal{T}_2}{\mathcal{T}_2}$.

Moreover, in (20), ν_i for $i \in \{1,2\}$ is given by

$$\nu_i = \begin{cases} \frac{\det(\mathbf{M}_i)}{\prod_{j=1}^{N_R}[\Gamma(N_i - j + 1)\Gamma(N_R - j + 1)]}, & N_R \neq 1 \\ \frac{1}{\Gamma(N_i)}, & N_R = 1, \end{cases} \quad (21)$$

where $\mathbf{M}_i$, $i \in \{1,2\}$, is an $(N_R - 1) \times (N_R - 1)$ matrix, where the (u,v)th element is given by $[\mathbf{M}_i]_{u,v} = \Gamma(N_i - N_R + u + v + 1)$.

2) Diversity order by using the lower bound of P_{out}: An asymptotic high SNR approximation for the lower bound of the overall outage probability (15) can be derived as[8]

$$P_{\text{out}}^{\text{lb},\infty} = \Omega_{\text{lb}} \left(\frac{\gamma_{th}}{\bar{\gamma}_{S,R}}\right)^{G_d^{\text{lb}}} + o\left(\bar{\gamma}_{S,R}^{-(G_d^{\text{lb}}+1)}\right), \quad (22)$$

where the lower bound of diversity order is given by

$$G_d^{\text{lb}} = \min(N_1, N_2) - N_R + 1. \quad (23)$$

In (22), the system dependent constant, Ω_{lb}, is defined as

$$\Omega_{\text{lb}} = \begin{cases} \frac{\phi_1^{N_1 - N_R + 1}}{\Gamma(N_1 - N_R + 2)\beta^{N_1 - N_R + 1}}, & N_1 < N_2 \\ \frac{\phi_2^{N_2 - N_R + 1}}{\Gamma(N_2 - N_R + 2)\beta^{N_2 - N_R + 1}}, & N_2 < N_1 \\ \frac{\phi_1^{N - N_R + 1} + \phi_2^{N - N_R + 1}}{\Gamma(N - N_R + 2)\beta^{N - N_R + 1}}, & N_1 = N_2 = N, \end{cases} \quad (24)$$

where $\bar{\gamma}_{S,R}$, ϕ_1 and ϕ_2 are defined in (20).

Remark III.1: Interestingly, the upper and lower bounds of the diversity order in (19) and (23) coincide, and consequently, the achievable diversity order can then be quantified as

$$G_d = \min(N_1, N_2) - N_R + 1. \quad (25)$$

C. Diversity-multiplexing trade-off

It is worth noticing that the diversity order reduces as the number of antennas at the relay (N_R) increases, and consequently, the multiplexing gain increases. This behavior is resulted from the fundamental diversity-multiplexing trade-off [8], and can be quantified as follows:

The effective mutual information can be upper bounded as

$$\begin{aligned} \mathcal{I}_{\text{eff}} &\leq \frac{N_R}{2}\left[\sum_{i=1}^{2} \log\left(1 + \gamma_{S_i,\min}^{\text{ub}}\right)\right] \\ &\approx N_R \log\left(1 + \min\left(\gamma_{S_1,\min}^{\text{ub}}, \gamma_{S_2,\min}^{\text{ub}}\right)\right). \end{aligned} \quad (26)$$

Consequently, the information outage probability can be lower bounded as

$$P_{\text{out}} \lesssim \Pr\left(\min\left(\gamma_{S_1,\min}^{\text{ub}}, \gamma_{S_2,\min}^{\text{ub}}\right) \leq 2^{\frac{\mathcal{R}_{th}}{N_R}} - 1\right), \quad (27)$$

where $\mathcal{R}_{th}$ is the target rate, and is defined as $\mathcal{R}_{th} = r\log\left(1 + \bar{\gamma}_{S,R}\right)$ [8]. By using (22), P_{out} in (27) can be approximated when $\bar{\gamma}_{S,R} \to \infty$ as

$$P_{\text{out}}^{\bar{\gamma}_{S,R}\to\infty} \approx \bar{\gamma}_{S,R}^{-(\min(N_1,N_2) - N_R + 1)\left(1 - \frac{r}{N_R}\right)}. \quad (28)$$

The diversity-multiplexing trade-off can then be quantified by using its definition [8] as[9]

$$d(r) = (\min(N_1, N_2) - N_R + 1)\left(1 - \frac{r}{N_R}\right). \quad (29)$$

IV. Numerical Results

In Fig. 1, the exact, upper and lower bounds ((11) and (15)), and asymptotically exact high SNR approximations ((18) and (22)) of the overall outage probability is plotted. Fig. 1 clearly reveals that the outage probability improves significantly as the number of relay antennas decreases. For instance, at 10^{-5} outage probability, single-antenna relay results in a 16 dB SNR gain over the triple-antenna relay. However, it is worth noticing that the single-antenna set-up achieves this outage gain over the latter at the expense of a drastic spatial multiplexing loss as quantified in (29). The asymptotic high SNR outage

[8]The proof of $P_{\text{out}}^{\text{lb},\infty}$ follows similar steps to those of $P_{\text{out}}^{\text{ub},\infty}$, and hence, is omitted for the sake of brevity.

[9]It is worth noticing that (29) can also be derived by using the outage upper bound; $P_{\text{out}} \gtrsim \Pr(\min(\gamma_{S_1,\min}^{\text{lb}}, \gamma_{S_2,\min}^{\text{lb}}) \leq 2^{\frac{\mathcal{R}_{th}}{N_R}} - 1)$.

bounds clearly reveal the achievable diversity order of the system, and provide insights into practical two-way relay system designing. In particular, our outage bounds reduce to exact outage for single-antenna relays. This behavior is not surprising as $N_R = 1$, $\mathbf{H}_{R,i}^H \mathbf{H}_{R,i}$ becomes a rank one matrix.

V. Conclusion

Transmit/receive ZF was studied for MIMO AF TWRNs. The performance of this system set-up was studied by deriving the upper and lower bounds of the overall outage probability and diversity-multiplexing trade-off in closed-form. Specifically, our outage bounds reduce to exact outage for single-antenna relays, and hence, serve as benchmarks for practical AF MIMO TWRNs. In particular, transmit/receive ZF strategy requires each source to know only its channel to the relay, and thus, eliminates the requirement of the global CSI for each source. We also studied the impact of number of relay antennas. Counter intuitively, increasing the number of relay antennas reduces the diversity gains but increases the multiplexing gains. Relay selection can improve this trade-off and will be investigated in the future.

Appendix I: Proof of $P_{\text{out}}^{\text{ub}}$

In this appendix, the proof of the CDF of $\gamma_{S_i,\min}^{\text{lb}}$, for $i \in \{1,2\}$, (9) is sketched as follows:

$$F_{\gamma_{S_i,\min}^{\text{lb}}}(x) = \Pr\left(\gamma_{S_i,\min}^{\text{lb}} = \frac{\eta_i}{\zeta_i + \mu \lambda_{\min}^{-1}\left(\mathbf{H}_{R,i}^H \mathbf{H}_{R,i}\right)} \le x\right). \quad (30)$$

Whenever $x \geq \frac{\eta_i}{\zeta_i}$, $F_{\gamma_{S_i,\min}^{\text{lb}}}(x) = 1$. On the contrary, for $x < \frac{\eta_i}{\zeta_i}$, (30) can be simplified as

$$F_{\gamma_{S_i,\min}^{\text{lb}}}(x) = F_{\lambda_{i,\min}}\left(\frac{\mu x}{\eta_i - \zeta_i x}\right), \quad (31)$$

where $\lambda_{i,\min} = \lambda_{\min}\left(\mathbf{H}_{R,i}^H \mathbf{H}_{R,i}\right)$ and $F_{\lambda_{i,\min}}(x)$ are the minimum eigenvalue of the central Wishart matrix, $\mathbf{H}_{R,i}^H \mathbf{H}_{R,i}$, and its CDF, respectively [9, Eq. (2.73)]. Next, by observing the statistical independence of $\gamma_{S_1,\min}^{\text{lb}}$ and $\gamma_{S_2,\min}^{\text{lb}}$, the CDF of $\gamma_{\text{eff}}^{\text{ub}} = \min\left(\gamma_{S_1,\min}^{\text{lb}}, \gamma_{S_2,\min}^{\text{lb}}\right)$ can be derived as $F_{\gamma_{\text{eff}}^{\text{ub}}}(x) = \sum_{i=1}^{2} F_{\gamma_{S_i,\min}^{\text{lb}}}(x) - \prod_{i=1}^{2} F_{\gamma_{S_i,\min}^{\text{lb}}}(x)$. The desired result (11) can then be obtained by substituting (31) into $F_{\gamma_{\text{eff}}^{\text{ub}}}(x)$, and evaluating at γ_{th}.

Appendix II: Proof of $P_{\text{out}}^{\text{ub},\infty}$

In this appendix, the proof of the upper bound for the diversity order is sketched. To this end, the probability density function of $\lambda_{\min}\left(\mathbf{H}_{R,i}^H \mathbf{H}_{R,i}\right)$ is given as [10, Theorem 5.4]

$$f_{\lambda_{i,\min}}(x) = C_{N_R,N_i} x^{N_i - N_R} \mathrm{e}^{-\frac{x N_i}{2}} \mathbb{P}_{N_R,N_i}(x), \quad (32)$$

where C_{N_R,N_i} is a constant, and $\mathbb{P}_{N_R,N_i}(x)$ is a polynomial of degree $(N_i - N_R)(N_R - 1)$; $\mathbb{P}_{N_R,N_i} = \sum_{j=0}^{(N_i-N_R)(N_R-1)} \alpha_{N_R,N_i,j}\, x^j$. By using (31), the PDF of $\gamma_{S_i,\min}^{\text{lb}}$, $i \in \{1,2\}$, can be derived as

$$f_{\gamma_{S_i,\min}^{\text{lb}}}(x) = \frac{\eta_i \mu}{(\eta_i - \zeta_i x)^2} f_{\lambda_{i,\min}}\left(\frac{\mu x}{\eta_i - \zeta_i x}\right), \quad 0 \le x < \frac{\eta_i}{\zeta_i}. \quad (33)$$

The behavior of $f_{\gamma_{S_i,\min}^{\text{lb}}}(x)$ as $x \to 0$ is governed by the Maclaurin series expansion of (33) [11]. After some steps, the first order expansion of $f_{\gamma_{S_i,\min}^{\text{lb}}}(x)$ can be derived as

$$f_{\gamma_{S_i,\min}^{\text{lb}}}^{x\to 0}(x) = \nu_i \left(\frac{\phi_i}{\beta \bar{\gamma}_{S,R}}\right)^{N_i - N_R + 1} x^{N_i - N_R} + o\left(x^{N_i - N_R + 1}\right), \quad (34)$$

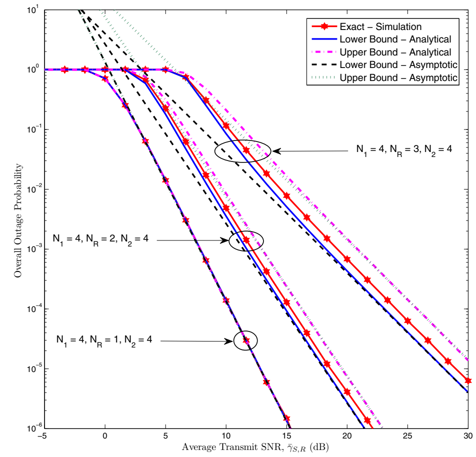

Fig. 1. The overall outage probability for the SNR threshold $\gamma_{th} = 6.02$ dB.

where $C_{N_R,N_i} \alpha_{N_R,N_i,0} = \nu_i$, $i \in \{1,2\}$, and can be derived by using [12, Eq. (10)] and [9] as given in (21). In (34), $\bar{\gamma}_{S,R}$, β, and ϕ_i are defined in (20). Now, by using (34) and adopting a similar approach of that of [11], the first order expansion of $F_{\gamma_{S_i,\min}^{\text{lb}}}(x)$, $i \in \{1,2\}$, as $x \to 0$ can be derived as

$$F_{\gamma_{S_i,\min}^{\text{lb}}}^{x\to 0}(x) = \frac{\nu_i \left(\frac{\phi_i x}{\beta \bar{\gamma}_{S,R}}\right)^{N_i - N_R + 1}}{(N_i - N_R + 1)} + o\left(x^{N_i - N_R + 2}\right), \quad (35)$$

By substituting (35) into (11), and obtaining its first order expansion, the high SNR approximation for the upper bound of the overall outage probability can be derived as in (18).

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