# Joint Relay and Antenna Selection for Dual-Hop Amplify-and-Forward MIMO Relay Networks

Gayan Amarasuriya, *Student Member, IEEE*, Chintha Tellambura, *Fellow, IEEE*, and Masoud Ardakani, *Senior Member, IEEE*

***Abstract*—Four joint relay and antenna selection strategies for dual-hop amplify-and-forward (AF) multiple-input multiple-output relay networks are studied. Two of them require full channel state information (CSI) whereas the other two require only partial CSI. The relays are either channel-assisted AF or fixed-gain AF type. The first joint selection strategy involves choosing the best relay and the best single transmit antennas at the source and the relay. The second strategy jointly involves choosing the best relay and the best single transmit/receive antenna pairs at the source-to-relay and relay-to-destination channels. Moreover, two partial selection strategies, which can be used when the global CSI is not available, are also proposed and analyzed. In order to quantify the system performance analytically, the exact outage probability of all selection strategies is derived in closed-form. Direct insights into the system-design are obtained by deriving the asymptotic outage probability, asymptotic average symbol error rate, diversity order and array gain.**

***Index Terms*—Relay networks, MIMO, antenna and relay selection.**

## I. Introduction

RELAY selection strategies for dual-hop multi-relay networks have been actively investigated due to the potential advantages of transmit power savings and higher spectral efficiencies [1]–[3]. The performance of single-antenna single-relay networks can be further improved by integrating multiple-input multiple-output (MIMO) transmission technology [4]–[6]. However, MIMO systems have increased system complexity due to the additional cost for enabling multiple transmit and receive radio frequency (RF) chains[1] [7]–[9]. Thus, there is considerable incentive for low-complexity and low-cost MIMO techniques with comparable performance benefits. One such technique is antenna selection, which has been widely studied to circumvent aforementioned drawbacks in the context of single-hop MIMO networks [7]–[9]. Specifically, MIMO antenna selection reduces the complexity and the power requirements of the MIMO transmitter much more than most other transmit diversity schemes such as beamforming [10]. In this letter, joint relay and antenna selection strategies for MIMO amplify-and-forward (AF) multi-relay networks are developed and analyzed.

Best relay selection (BRS) for dual-hop cooperative networks has been widely studied [2], [11], [12]. In BRS, a single relay with maximum end-to-end (e2e) signal-to-noise ratio (SNR) is selected for relaying. This scheme achieves the full diversity while maintaining a higher throughput than the repetition-based relaying [11]. However, in [2], [11], [12] and many others, the selection of a relay is considered, but no antenna selection is considered.

Nevertheless, for MIMO multi-relay networks, both relays and antennas can be selected jointly. In the wide body of relay literature, there appear only three references, [13], [14], and [15], dealing with the issue of joint selection. In [13], joint antenna and relay selection is studied for MIMO decode-and-forward (DF) relay networks. References [14] and [15] investigate the joint antenna and relay selection to maximize the channel capacity. Specifically, [14] uses the transmit antenna selection algorithm from [16] with instantaneous channel state information (CSI), while [15] extends [14] for statistical CSI.

Therefore, to the best of our knowledge, joint relay and antenna selection to maximize the diversity gains for dual-hop MIMO AF relay networks has not yet been studied.

This letter fills this gap by proposing four joint antenna and relay selection strategies which are optimal in the sense of the diversity order, and hence, in the outage probability. Two of them require global CSI whereas the other two require only partial CSI. The two selection strategies, which require global CSI, are referred to as joint relay and transmit antenna selection (R-TAS) and joint relay and antenna pair selection (R-APS). Specifically, R-TAS implements the joint selection of the best single transmit antenna at the source, the best single relay and the best single transmit antenna at the relay. Similarly, the R-APS strategy jointly selects the best single relay, the best single transmit and receive (Tx/Rx) antenna pairs at the source-to-relay and relay-to-destination channels. Furthermore, two partial selection strategies, a highly useful option when the global CSI is not available, are proposed and analyzed. In the sequel, they are referred to as partial R-TAS and partial R-APS, and only assume the availability of CSI of source-to-relay channels.

The performance of these four selection strategies with dual-hop MIMO AF relay networks over Nakagami-$m$ fading channels is studied. To this end, the exact outage probability is derived in closed-form for both the CA-AF (channel-assisted AF) and FG-AF (fixed-gain AF) relays. In order to obtain direct insights into the system-design, the asymptotic outage probability and the asymptotic average symbol error rate (ASER), which are exact at high SNRs, are derived and used to obtain the diversity order and array gain. The impact of outdated CSI on the system performance is studied as well. Furthermore, numerical results are provided to show the performance gains of the joint relay and antenna selection, and

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The authors are with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB, Canada T6G 2V4 (e-mail: {amarasur, chintha, ardakani}@ece.ualberta.ca)

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[1] In particular, passive antenna elements and additional digital signal processing are becoming increasingly cheaper; however, RF elements are still expensive and do not follow Moore's law [7]–[9].

our analysis is validated through Monte-Carlo simulations.

**Notations:** $\mathcal{K}_\nu(z)$ is the Modified Bessel function of the second kind of order $\nu$ [17, Eq. (8.407.1)]. $||\mathbf{y}||$ and $|z|$ denote the $\mathcal{L}$-2 norm and magnitude of the vector $\mathbf{y}$ and scalar $z$, respectively. $\mathcal{Q}(z)$ denotes the Gaussian Q-function. $\mathcal{E}_\Lambda\{z\}$ is the expected value of $z$ over $\Lambda$.

## II. System Model and Problem Formulation

We consider a dual-hop AF MIMO multi-relay network having a single source ($S$), $Q$ relays $\left(R_q|_{q=1}^{Q}\right)$ and a single destination ($D$). Specifically, $S$, $D$ and $R_q$ are half-duplex [18], [19], and are equipped with $N_S$, $N_D$ and $N_{R_q}$ antennas, respectively. All channel amplitudes are assumed to be independently distributed Nakagami-$m$ fading, where $m \in \mathbb{Z}^+$. The feedbacks for relay and antenna selection are assumed to be perfect unless otherwise stated[2]. The channel matrix from terminal $X$ to terminal $Y$, where $X \in \{S, R\}, Y \in \{R, D\}$, and $X \neq Y$, is denoted by $\mathbf{H}_{XY}$. The elements of $\mathbf{H}_{XY}$ are denoted by $h_{XY}^{(i,j)}$. The channel vector from the $j$-th transmit antenna at $X$ to $Y$ is denoted by $\mathbf{h}_{XY}^{(j)}$. The additive noise is modeled as complex zero mean white Gaussian noise. The gains of $q$-th CA-AF and FG-AF relays are given by [18]

$$\begin{aligned} G_{\text{CA}-\text{AF}} &= \sqrt{P_{R_q}/(P_S|h_{SR_q}|^2+\sigma_{R_q}^2)} \quad \text{and} \\ G_{\text{FG}-\text{AF}} &= \sqrt{P_{R_q}/(P_S\mathcal{E}|h_{SR_q}|^2+\sigma_{R_q}^2)}, \end{aligned} \tag{1}$$

respectively. In (1), $P_S$ and $P_{R_q}$ are the transmit powers at $S$ and $R_q$. Here, $|h_{SR_q}|$ is given for R-TAS and R-APS by $|h_{SR_q}| = \max\limits_{1\leq i\leq N_S} ||h_{SR_q}^{(i)}||$ and $|h_{SR_q}| = \max\limits_{1\leq i\leq N_S, 1\leq i\leq N_{r_q},} |h_{SR_q}^{(i,j)}|$, respectively. Moreover, $\sigma_{R_q}^2$ is the noise variance at the $q$-th relay.

### A. R-TAS strategy

During the first time-slot, $S$ transmits its signal to the best relay $R_{\hat{Q}}$ by selecting its best transmit antenna, and $R_{\hat{Q}}$ combines the signal by using maximal ratio combining (MRC). In the second time-slot, the best relay amplifies the received signal and forwards it again by using the best transit antenna to $R$, where MRC is again used. The best transmit antenna indexes at $S$ and $D$, and the best relay index are denoted by $I$, $K$, and $\hat{Q}$, respectively and given by

$$\{I, K, \hat{Q}\} = \underset{1\leq i\leq N_S, 1\leq k\leq N_D, 1\leq q\leq Q}{\operatorname{argmax}} \left(\gamma_{\text{e2e}}^{(i,k,q)}\right), \tag{2}$$

where the e2e SNR, $\gamma_{\text{e2e}}^{(i,k,q)}$, is given by [5], [6]

$$\gamma_{\text{e2e}}^{(i,k,q)} = \gamma_{SR_q}^{(i)}\gamma_{R_qD}^{(k)}/(\eta_q\gamma_{SR_q}^{(i)}+\gamma_{R_qD}^{(k)}+\zeta_q). \tag{3}$$

In (3), $\gamma_{SR_q}^{(i)} = \bar{\gamma}_{SR_q}||\mathbf{h}_{SR_q}^{(i)}||^2$ and $\gamma_{R_qD}^{(k)} = \bar{\gamma}_{R_qD}||\mathbf{h}_{R_qD}^{(k)}||^2$ are the equivalent instantaneous SNRs, and $\bar{\gamma}_{SR_q}$, and $\bar{\gamma}_{R_qD}$ are the average SNRs of the $S \to R_q$, and $R_q \to D$ channels, respectively. Moreover, $\gamma_{SR_q}^{(i)}$ and $\gamma_{R_qD}^{(k)}$ are independent Gamma distributed random variables; $\gamma_{SR_q}^{(i)} \sim \mathcal{G}\left(m_{SR_q}N_{R_q}, \beta_{SR_q}\right)$ and $\gamma_{R_qD}^{(k)} \sim \mathcal{G}\left(m_{R_qD}N_D, \beta_{R_qD}\right)$, where $\beta_{SR_q} = \bar{\gamma}_{SR_q}/m_{SR_q}$ and $\beta_{R_qD} = \bar{\gamma}_{R_qD}/m_{R_qD}$. Further, $m_{SR_q}$ and $m_{R_qD}$ are the integer severity of the fading parameters of the Nakagami fading in the $S \to R$ and $R \to D$ channels. In particular, in (3), the tuples $\{\eta_q = 1, \zeta_q = 1\}$ and $\{\eta_q = 0, \zeta_q \neq 0\}$ stand for CA-AF and FG-AF relays, respectively.

### B. R-APS strategy

In the first time-slot, $S$ transmits its signal to the best relay $R_{\hat{Q}}$ by selecting the best Tx/Rx antenna pair of the $S \to R_{\hat{Q}}$ channel. In the second time-slot, $R_{\hat{Q}}$ amplifies and forward its received signal again by selecting the best Tx/Rx antenna pair of the $R_Q \to D$ channel. The best antenna pair indexes of the $S \to R_{\hat{Q}}$ and the $R_{\hat{Q}} \to D$ channels, and the best relay index are denoted by $(I, J)$, $(K, L)$ and $\hat{Q}$, and given by

$$\{(I,J),(K,L),\hat{Q}\} = \underset{\substack{1\leq i\leq N_S, 1\leq j,k\leq N_{R_q}, 1\leq l\leq N_D \\ 1\leq q\leq Q}}{\operatorname{argmax}} \left(\gamma_{\text{e2e}}^{(i,j,k,l,q)}\right), \tag{4}$$

where the e2e SNR, $\gamma_{\text{e2e}}^{(i,j,k,l,q)}$ is given by

$$\gamma_{\text{e2e}}^{(i,j,k,l,q)} = \gamma_{SR_q}^{(i,j)}\gamma_{R_qD}^{(k,l)}/(\eta_q\gamma_{SR_q}^{(i,j)}+\gamma_{R_qD}^{(k,l)}+\zeta_q), \tag{5}$$

where $\gamma_{SR_q}^{(i,j)} = \bar{\gamma}_{SR_q}\left|\mathbf{h}_{SR_q}^{(i,j)}\right|^2$ and $\gamma_{R_qD}^{(k,l)} = \bar{\gamma}_{R_qD}\left|\mathbf{h}_{R_qD}^{(k,l)}\right|^2$ are the equivalent instantaneous SNRs. Similar to (3), $\gamma_{SR_q}^{(i,j)}$ and $\gamma_{R_qD}^{(k,l)}$ in (5) are independent Gamma distributed random variables; $\gamma_{SR_q}^{(i,k)} \sim \mathcal{G}\left(m_{SR_q}, \beta_{SR_q}\right)$ and $\gamma_{R_qD}^{(k,l)} \sim \mathcal{G}\left(m_{R_qD}N_D, \beta_{R_qD}\right)$.

## III. Performance Analysis

The outage probability[3] is the probability that the instantaneous e2e SNR falls below a threshold, $\gamma_{th}$, and is given by $P_{\text{out}} = \Pr(\gamma_{\text{e2e}} \leq \gamma_{th})$ [22]. In this section, $P_{\text{out}}$ of R-TAS and R-APS is derived and used to obtain valuable system-design parameters such as diversity and array gains.

### A. Exact outage probability of R-TAS strategy

The outage probability of R-TAS strategy can be derived as

$$\begin{aligned} P_{\text{out}}^{\text{R-TAS}} &= \Pr\left(\max_{\substack{1\leq i\leq N_S, 1\leq k\leq N_{R_q} \\ 1\leq q\leq Q}} \frac{\gamma_{SR_q}^{(i)}\gamma_{R_qD}^{(k)}}{\eta_q\gamma_{SR_q}^{(i)}+\gamma_{R_qD}^{(k)}+\zeta_q} \leq \gamma_{th}\right) \\ &= \Pr\left(\max_{1\leq q\leq Q}\left[\max_{\substack{1\leq i\leq N_S \\ 1\leq k\leq N_{R_q}}} \frac{\gamma_{SR_q}^{(i)}\gamma_{R_qD}^{(k)}}{\eta_q\gamma_{SR_q}^{(i)}+\gamma_{R_qD}^{(k)}+\zeta_q}\right] \leq \gamma_{th}\right). \end{aligned} \tag{6}$$

The second equality of (6) yields from the mutual independence of $\gamma_{SR_q}^{(i)}\Big|_{q=1}^{Q}$ and $\gamma_{R_qD}^{(k)}\Big|_{q=1}^{Q}$. Next, (6) can be further

[2]However, in practical MIMO systems, the estimated channel coefficient matrices are generally perturbed by Gaussian errors due to channel estimation errors. Moreover, the transmit antennas could be selected by using the outdated CSI due to feedback delays [20], [21]. Thus, the impact of imperfect CSI on the performance of our system model is studied in Section III-F.

[3]The information outage probability is defined as the probability that the instantaneous mutual information $\mathcal{I}$ falls below the target rate $\mathcal{R}_{th}$; $\Pr\left(\mathcal{I} = \frac{1}{2}\log\left(1+\gamma_{\text{e2e}}\right) \leq \mathcal{R}_{th}\right) = F_{\gamma_{\text{e2e}}}(\gamma_{th})$, where $\gamma_{th} = 2^{2\mathcal{R}_{th}} - 1$.

simplified by solving the inner maximization problem by using [5] as

$$P_{\text{out}}^{\text{R-TAS}} = \Pr\left(\max_{1\le q\le Q} \quad \frac{\gamma_{SR_q}^{(I)}\gamma_{R_qD}^{(K)}}{\eta_q\gamma_{SR_q}^{(I)}+\gamma_{R_qD}^{(K)}+\zeta_q}\le\gamma_{th}\right), \quad (7)$$

where $\gamma_{SR_q}^{(I)} = \max_{1\le i\le N_S}\left(\gamma_{SR_q}^{(i)}\right)$ and $\gamma_{R_qD}^{(K)} = \max_{1\le k\le N_{R_q}}\left(\gamma_{R_qD}^{(k)}\right)$. Just as in (2), the tuples $\{\eta_q=1,\zeta_q=1\}$ and $\{\eta_q=0,\zeta_q\neq 0\}$ stand for CA-AF and FG-AF relays, respectively, in (6) and (7) as well.

Next, (7) can further be simplified as

$$P_{\text{out}}^{\text{R-TAS}} = \prod_{q=1}^{Q}\Bigg[F_{\gamma_{SR_q}^{(I)}}(\gamma_{th}) + \int_{\gamma_{th}}^{\infty}\Pr\left(\gamma_{R_qD}^{(K)}\le\frac{(\eta_q y+\zeta_q)\gamma_{th}}{y-\gamma_{th}}\right) f_{\gamma_{SR_q}^{(I)}}(y)\,\mathrm{d}y\Bigg]. \quad (8)$$

By using a variable change, $z=y-x$, $P_{\text{out}}^{\text{R-TAS}}$ can be expressed in a compact single-integral form as

$$P_{\text{out}}^{\text{R-TAS}} = \prod_{q=1}^{Q}\Bigg[1-\int_0^{\infty}\bar{F}_{\gamma_{R_qD}^{(K)}}\left((\eta_q(z+\gamma_{th})+\zeta_q)\gamma_{th}/z\right) \times f_{\gamma_{SR_q}^{(I)}}(z+\gamma_{th})\,\mathrm{d}z\Bigg], \quad (9)$$

where $\bar{F}_{\gamma_{R_qD}^{(K)}}(x)$ is complementary cumulative distribution function (CCDF) of $\gamma_{R_qD}^{(K)}$ and given by [8]

$$\bar{F}_{\gamma_{R_qD}^{(K)}}(x) = 1-\left(1-\mathrm{e}^{-\frac{x}{\beta_{R_qD}}}\sum_{t=0}^{m_{R_qD}N_D-1}\frac{1}{t!}\left(\frac{x}{\beta_{R_qD}}\right)^t\right)^{N_{R_q}} = \sum_{p=1}^{N_{R_q}}\sum_{l=0}^{p(m_{R_qD}N_D-1)}\frac{(-1)^{p+1}\binom{N_{R_q}}{p}\phi_{l,p,m_{R_qD}N_D}}{\beta_{R_qD}^l}x^l\mathrm{e}^{-\frac{px}{\beta_{R_qD}}}. \quad (10)$$

Similarly, in (9), $f_{\gamma_{SR_q}^{(I)}}(x)$ is the probability density function (PDF) of $\gamma_{SR_q}^{(I)}$ and given by [8]

$$\begin{aligned} f_{\gamma_{SR_q}^{(I)}}(x) &= \frac{N_S x^{m_{SR_q}N_{R_q}-1}\mathrm{e}^{-\frac{x}{\beta_{SR_q}}}}{\Gamma(m_{SR_q}N_{R_q})(\beta_{SR_q})^{m_{SR_q}N_{R_q}}} \times\left(1-\mathrm{e}^{-\frac{x}{\beta_{SR_q}}}\sum_{t=0}^{m_{SR_q}N_{R_q}-1}\frac{1}{t!}\left(\frac{x}{\beta_{SR_q}}\right)^t\right)^{N_S-1}\\ &= \sum_{a=0}^{N_S-1}\sum_{b=0}^{a(m_{SR_q}N_{R_q}-1)}\frac{(-1)^a N_S\binom{N_S-1}{a}\phi_{b,a,m_{SR_q}N_{R_q}}}{\Gamma(m_{SR_q}N_{R_q})\beta_{SR_q}^{b+m_{SR_q}N_{R_q}}} \times x^{b+m_{SR_q}N_{R_q}-1}\mathrm{e}^{-\frac{(a+1)x}{\beta_{SR_q}}}. \end{aligned} \quad (11)$$

In (10) and (11), $\phi_{k,N,L}$ is the coefficient of the expansion of

$$\left[\sum_{u=0}^{L-1}\frac{1}{u!}\left(\frac{x}{\bar{\gamma}}\right)^u\right]^N = \sum_{k=0}^{N(L-1)}\phi_{k,N,L}\left(\frac{x}{\bar{\gamma}}\right)^k, \quad \text{where}$$

$$\phi_{k,N,L} = \sum_{i=k-L+1}^{k}\frac{\phi_{i,N-1,L}}{(k-i)!}I_{[0,(N-1)(L-1)]}(i). \quad (12)$$

In (12), $\phi_{0,0,L}=\phi_{0,N,L}=1$, $\phi_{k,1,L}=1/\,k!$, $\phi_{1,N,L}=N$ and, $I_{[a,c]}(b)=1$ for $a\le b\le c$ and $I_{[a,c]}(b)=0$ otherwise.

By substituting $\bar{F}_{\gamma_{R_qD}^{(K)}}(x)$ and $f_{\gamma_{SR_q}^{(I)}}(x)$ given in (10) and (11) into (9), the single-integral expression for $P_{\text{out}}^{\text{R-TAS}}$ is derived as

$$\begin{aligned} P_{\text{out}}^{\text{R-TAS}} = \prod_{q=1}^{Q}\Bigg[1-&\sum_{a=0}^{N_S-1}\sum_{b=0}^{a(m_{SR_q}N_S-1)}\sum_{p=1}^{N_{R_q}}\sum_{l=0}^{p(m_{R_q}N_D-1)}\frac{(-1)^{a+p+1}}{\beta_{R_qD}^l}\\ &\times\frac{N_S\binom{N_S-1}{a}\binom{N_{R_q}}{p}\phi_{b,a,m_{SR_q}N_{R_q}}\phi_{l,p,m_{R_qD}N_D}}{\Gamma(m_{SR_q}N_{R_q})\beta_{SR_q}^{b+m_{SR_q}N_{R_q}}}\\ &\times\gamma_{th}^l\mathrm{e}^{-\gamma_{th}\left(\frac{a+1}{\beta_{SR_q}}+\frac{p\eta_q}{\beta_{R_qD}}\right)}\mathbb{J}_{a,b,p,l}^{m_{SR_q}N_{R_q},q}(\gamma_{th})\Bigg], \end{aligned} \quad (13)$$

where the integral $\mathbb{J}_{c,d,e,f}^{M,N}(x)$ is given by

$$\mathbb{J}_{c,d,e,f}^{M,q}(x) = \int_0^{\infty}z^{-f}(z+x)^{M+d-1}(\eta_q z+\eta_q x+\zeta_q)^f \times \mathrm{e}^{-\left(\frac{ex(\eta_q x+\zeta_q)}{\beta_{R_qD}}+\frac{((c+1)z)}{\beta_{SR_q}}\right)}\mathrm{d}z. \quad (14)$$

By first employing binomial theorem and then using [17, Eq. (3.471.9)], $\mathbb{J}_{c,d,e,f}^{M,q}(x)$ in (14) can be solved in closed-form as

$$\mathbb{J}_{c,d,e,f}^{M,q}(x) = \sum_{u=0}^{f}\sum_{v=0}^{M+d-1}\frac{2\eta_q^u(e\beta_{SR_q})^{\frac{u+v-f+1}{2}}\binom{f}{u}\binom{M+d-1}{v}}{((c+1)\beta_{R_qD})^{\frac{u+v-f+1}{2}}} \times\frac{x^{\frac{2M+2d+f+u-v-1}{2}}}{(\eta_q x+\zeta_q)^{\frac{u-v-f-1}{2}}}\mathcal{K}_{u+v-f+1}\left(2\sqrt{\frac{e(c+1)x(\eta_q x+\zeta_q)}{\beta_{SR_q}\beta_{R_qD}}}\right). \quad (15)$$

Now, by substituting (15) into (13), $P_{\text{out}}^{\text{R-TAS}}$ can be derived in closed-form as shown in (19) on the top of the next page.

### B. Exact outage probability of the R-APS strategy

By using similar techniques to those in Section III-A, the outage probability of the R-APS strategy can be derived as follows:

$$P_{\text{out}}^{\text{R-APS}} = \Pr\left(\max_{1\le q\le Q} \quad \frac{\gamma_{SR_q}^{(I,J)}\gamma_{R_qD}^{(K,L)}}{\eta_q\gamma_{SR_q}^{(I,J)}+\gamma_{R_qD}^{(K,L)}+\zeta_q}\le\gamma_{th}\right), \quad (16)$$

where $\gamma_{SR_q}^{(I,J)} = \max_{1\le i\le N_S, 1\le j\le N_{R_q},}\left(\gamma_{SR_q}^{(i,j)}\right)$ and $\gamma_{R_qD}^{(K,L)} = \max_{1\le k\le N_{r_q}, 1\le l\le N_D}\left(\gamma_{R_qD}^{(k,l)}\right)$. The outage probability of the R-APS strategy can readily be derived by replacing $\bar{F}_{\gamma_{R_qD}^{(K)}}(x)$ and $f_{\gamma_{SR_q}^{(I)}}(x)$ of (9) with the corresponding $\bar{F}_{\gamma_{R_qD}^{(K,L)}}(x)$ and $f_{\gamma_{SR_q}^{(I,J)}}(x)$. They are given by

$$\bar{F}_{\gamma_{R_qD}^{(K,L)}}(x) = 1-\left(1-\mathrm{e}^{-\frac{x}{\beta_{R_qD}}}\sum_{t=0}^{m_{R_qD}-1}\frac{1}{t!}\left(\frac{x}{\beta_{R_qD}}\right)^t\right)^{N_{R_q}N_D} \quad \text{and} \quad (17)$$

$$f_{\gamma_{SR_q}^{(I,J)}}(x) = \frac{N_S N_{R_q} x^{m_{SR_q}-1}\mathrm{e}^{-\frac{x}{\beta_{SR_1}}}}{\Gamma(m_{SR_q})(\beta_{SR_q})^{m_{SR_q}}} \times\left(1-\mathrm{e}^{-\frac{x}{\beta_{SR_1}}}\sum_{t=0}^{m_{SR_q}-1}\frac{1}{t!}\left(\frac{x}{\beta_{SR_q}}\right)^t\right)^{N_S N_{R_q}-1}. \quad (18)$$

$$P_{\text{out}}^{\text{R-TAS/R-APS}} = \prod_{q=1}^{Q} \left[ 1 - \sum_{a=0}^{\mu_q-1} \sum_{b=0}^{a(\lambda_q-1)} \sum_{p=1}^{\psi_q} \sum_{l=0}^{p(\xi_q-1)} \sum_{u=0}^{l} \sum_{v=0}^{\lambda_q+b-1} \frac{2(-1)^{a+p+1}\mu_q \eta_q^u \binom{\psi_q}{p}\binom{\mu_q-1}{a}\binom{l}{u}\binom{\lambda_q+b-1}{v} p^{\frac{u+v-l+1}{2}} \phi_{b,a,\lambda_q}\phi_{q,p,\xi_q}}{\Gamma(\lambda_q)(a+1)^{\frac{u+v-l+1}{2}} (\beta_{SRq})^{\frac{2\lambda_q+2b+l-u-v-1}{2}} \left(\beta_{R_qD}\right)^{\frac{u+v+l+1}{2}}} \right.$$
$$\left. \times \quad \gamma_{th}^{\frac{2\lambda_q+2b+l+u-v-1}{2}} (\eta_q\gamma_{th}+\zeta_q)^{\frac{l+v-u+1}{2}} \mathrm{e}^{-\gamma_{th}\left(\frac{a+1}{\beta_{SR_q}}+\frac{p\eta_q}{\beta_{R_qD}}\right)} \mathcal{K}_{u+v-l+1}\left(2\sqrt{\frac{p(a+1)\gamma_{th}(\eta_q\gamma_{th}+\zeta_q)}{\beta_{SR_q}\beta_{R_qD}}}\right)\right], \quad (19)$$

By substituting $\bar{F}_{\gamma_{R_qD}^{(K,L)}}(x)$ and $f_{\gamma_{SR_q}^{(I,J)}}(x)$ given in (17) and (18) into (9) and evaluating the integral again by using [17, Eq. (3.471.9)] as in (13), outage probability of R-APS can be derived as in (19).

Moreover, in (9), the tuples $\{\lambda_q = m_{SR_q}N_{R_q},\ \mu_q = N_S,\ \xi_q = m_{R_qD}N_D,\ \psi_q = N_{R_q}\}$ and $\{\lambda_q = m_{SR_q},\ \mu_q = N_SN_{R_q},\ \xi_q = m_{R_qD},\ \psi_q = N_{R_q}N_D\}$ stand for R-TAS and R-APS strategies, respectively.

**Remark III.1:** Eq. (19) is valid for both the CA-AF and FG-AF relays. Specifically, for the CA-AF case, $\eta_q = 1$ and $\zeta_q = 1$. For the FG-AF case, $\eta_q = 0$ and $\zeta_q = \frac{P_{R_q}}{G_{FG-AF}^2\sigma_{R_q}^2}$. Furthermore, this $\zeta_q$ can be derived by substituting (1) and evaluating the integral by using [17] as

$$\zeta_q = 1 + \sum_{a=0}^{\mu_q-1}\sum_{b=0}^{a(\lambda_q-1)} \frac{(-1)^a\mu_q\binom{-1}{a}\phi_{b,a,\lambda_q}\beta_{SR_q}\Gamma(\lambda_q+b+1)}{\mu_q^{-1}\Gamma(\lambda_q)(a+1)^{\lambda_q+b+1}}, \quad (20)$$

where the tuples $\{\lambda_q = m_{SR_q}N_{R_q}, \mu_q = N_S\}$ and $\{\lambda_q = m_{SR_q}, \mu_q{=}N_SN_{R_q}\}$ stand for R-TAS and R-APS.

### C. Asymptotic outage probability analysis

In order to obtain direct insights, the asymptotic outage probability at high SNRs is derived for both the R-TAS and R-APS strategies. For the sake of brevity, only the CA-AF relays are treated.

The behavior of $P_{\text{out}}$ for a large average transmit SNR ($\bar{\gamma}$) is equivalent to the behavior of $P_{\text{out}}$ around $\gamma_{th} = 0$. By substituting $\bar{\gamma}_{SR_q} = u_q\bar{\gamma}$, $\bar{\gamma}_{R_qD} = v_q\bar{\gamma}$ and $\gamma_{th} = \bar{\gamma}y$ into (19), and by expressing the exponential function and Bessel function in terms of their Taylor series expansion around $y = 0$ [17, Eq. (1.211) and Eq. (8.446)], $P_{\text{out}}$ at high SNR for both R-TAS and R-APS can be derived as

$$P_{\text{out}}^{\infty} = \prod_{q=1}^{Q}\Omega_q\left(\frac{\gamma_{th}}{\bar{\gamma}}\right)^{G_d} + o\left(\bar{\gamma}^{-(G_d+1)}\right), \quad (21)$$

where $\Omega_q$ is given by

$$\Omega_q = \begin{cases} \frac{(m_{SR_q}/u_q)^{m_{SR_q}N_SN_{R_q}}}{(\ \lambda_q!)^{\mu_q}}, & m_{SR_q}N_S < m_{R_qD}N_D \\ \left[\frac{(m_{SR_q}/u_q)^{m_{SR_q}N_SN_{R_q}}}{(\ (\lambda_q)!)^{\mu_q}} \right. & \\ \left. + \frac{(m_{R_qD}/v_q)^{m_{R_qD}N_DN_{R_q}}}{(\ \xi_q!)^{\psi_q}}\right], & m_{SR_q}N_S = m_{R_qD}N_D \\ \frac{(m_{R_qD}/v_q)^{m_{R_qD}N_{R_q}N_D}}{(\ \xi_q!)^{\psi_q}}, & m_{SR_q}N_S > m_{R_qD}N_D. \end{cases} \quad (22)$$

In (22), the tuples $\{\lambda_q = m_{SR_q}N_{R_q},\ \mu_q = N_S,\ \xi_q = m_{R_qD}N_D,\ \psi_q = N_{R_q}\}$ and $\{\lambda_q = m_{SR_q},\ \mu_q = N_SN_{R_q},\ \xi_q = m_{R_qD},\ \psi_q = N_{R_q}N_D\}$ stand for R-TAS and R-APS strategies, respectively. For both R-TAS and R-APS strategies, the diversity order is given by

$$G_d = \sum_{q=1}^{Q} N_{R_q}\min\left(m_{SR_q}N_S, m_{R_qD}N_D\right). \quad (23)$$

### D. Asymptotic average symbol error rate analysis

The high SNR ASER for CA-AF relays can be derived by using (21) and

$$\bar{P}_e^{\infty} = \frac{\alpha}{2}\sqrt{\frac{\varphi}{2\pi}}\int_0^{\infty} x^{-\frac{1}{2}}\mathrm{e}^{-\frac{\varphi x}{2}}F_{\gamma_{e2e}^{\infty}}(x)\,\mathrm{d}x, \quad (24)$$

where $\alpha$ and $\varphi$ are modulation-dependent constants of the conditional error probability $P_e|\gamma = \alpha\mathcal{Q}(\sqrt{\varphi\gamma})$ [22]. Further, $F_{\gamma_{e2e}^{\infty}}(x)$ is the asymptotic CDF of the e2e SNR and can readily be obtained by replacing $\gamma_{th}$ in (21) by $x$. The asymptotic ASER is given by

$$P_e^{\infty} = \frac{\alpha\left(\prod_{q=1}^{Q}\Omega_q\right)2^{G_d-1}\Gamma\left(G_d+\frac{1}{2}\right)}{\sqrt{\pi}(\varphi\bar{\gamma})^{G_d}} + o\left(\bar{\gamma}^{-(G_d+1)}\right), \quad (25)$$

where $\Omega_q$ for R-TAS and R-APS is given by (22). The diversity order is given in (23) and the array gain can be obtained as

$$G_a = \left(\frac{\alpha}{\sqrt{\pi}(\varphi)^{G_d}}\left(\prod_{q=1}^{Q}\Omega_q\right)2^{G_d-1}\Gamma(G_d+\frac{1}{2})\right)^{-\frac{1}{G_d}}. \quad (26)$$

It is important to note that both the R-TAS and R-APS achieve the full diversity order (23) for the given system set-ups.

### E. Joint antenna and relay selection under partial CSI

The implementation of both R-TAS and R-APS strategies requires global CSI, i.e., the CSI of the $S \rightarrow R_q|_{q=1}^{Q}$ and $R_q|_{q=1}^{Q} \rightarrow D$. However, in practice, the realization of global CSI can be difficult. Thus, in this subsection, two partial joint relay and antenna selection schemes, where the best relay is selected by only considering the $S \rightarrow R_q|_{q=1}^{Q}$ channels are proposed and analyzed.

*1) Partial R-TAS strategy:* In this strategy, the single transmit antenna index ($I$) at the source and the single best relay ($R_{\hat{Q}}$) is jointly selected by considering only the $S \rightarrow R_q|_{q=1}^{Q}$ channels as follows:

$$\{I, \hat{Q}\} = \underset{1\le i\le N_S, 1\le q\le Q}{\operatorname{argmax}}\left(\gamma_{SR_q}^{(i)}\right), \quad (27)$$

where $\gamma_{SR_q}^{(i)} = \bar{\gamma}_{SR_q}||\mathbf{h}_{SR_q}^{(i)}||^2$. Then the relay $R_{\hat{Q}}$ forwards an amplified version of its signal to the destination by selecting the best single transmit antenna as $\{K\} = \underset{1\le k\le N_{r_{\hat{Q}}}}{\operatorname{argmax}}\left(\gamma_{R_{\hat{Q}}D}^{(k)}\right)$, where $\gamma_{R_{\hat{Q}}D}^{(k)} = \bar{\gamma}_{R_{\hat{Q}}D}||\mathbf{h}_{R_{\hat{Q}}D}^{(k)}||^2$. Thus, the e2e SNR of the partial R-TAS is given by $\gamma_{e2e} = \gamma_{SR_{\hat{Q}}}^{(I)}\gamma_{R_{\hat{Q}}D}^{(K)}/(\eta_{\hat{Q}}\gamma_{SR_{\hat{Q}}}^{(I)} + \gamma_{R_{\hat{Q}}D}^{(K)} + \zeta_{\hat{Q}})$, where the tuples $\{\eta_{\hat{Q}} = 1, \zeta_{\hat{Q}} = 1\}$ and $\{\eta_{\hat{Q}} = 0, \zeta_{\hat{Q}} \neq 0\}$ stand for CA-AF and FG-AF relay types, respectively. The outage probability of partial R-TAS is thus given by

$$P_{\text{out, partial}}^{\text{R-TAS}} = \Pr\left(\frac{\gamma_{SR_{\hat{Q}}}^{(I)}\gamma_{R_{\hat{Q}}D}^{(K)}}{\eta_{\hat{Q}}\gamma_{SR_{\hat{Q}}}^{(I)} + \gamma_{R_{\hat{Q}}D}^{(K)} + \zeta_{\hat{Q}}} \leq \gamma_{th}\right). \quad (28)$$

Now, (28) can be simplified by using similar steps to those in (8) as

$$P_{\text{out, partial}}^{\text{R-TAS}} = 1 - \int_0^\infty \bar{F}_{\gamma_{SR_{\hat{Q}}}^{(I)}}\left((\eta_q(z+\gamma_{th}) + \zeta_q)\gamma_{th}/z\right) \times f_{\gamma_{R_{\hat{Q}}D}^{(K)}}(z+\gamma_{th})\, \mathrm{d}z. \quad (29)$$

For the sake of mathematical tractability, the $S \to R_q|_{q=1}^{Q}$ channels are assumed to be independent and identically distributed Nakagami-$m$ fading, i.e., $\beta_{SR_q}|_{q=1}^{Q} = \beta_{SR}$ and $m_{SR_q}|_{q=1}^{Q} = m_{SR}$. Further, we assume that each relay has the same number of antenna terminals ($N_{R_q}|_{q=1}^{Q} = N_R$). Thus, the CCDF of $\gamma_{SR_{\hat{Q}}}^{(I)}$ can be derived as

$$\bar{F}_{\gamma_{SR_{\hat{Q}}}^{(I)}}(x) = 1 - \left(1 - \mathrm{e}^{-\frac{x}{\beta_{SR}}}\sum_{t=0}^{m_{SR}N_R-1}\frac{1}{t!}\left(\frac{x}{\beta_{SR}}\right)^t\right)^{QN_S}. \quad (30)$$

Similarly, the PDF of $f_{\gamma_{R_{\hat{Q}}D}^{(K)}}(x)$ is given by

$$f_{\gamma_{R_{\hat{Q}}D}^{(K)}}(x) = \frac{N_R x^{m_{R_{\hat{Q}}D}N_D-1}\mathrm{e}^{-\frac{x}{\beta_{R_{\hat{Q}}D}}}}{\Gamma(m_{R_{\hat{Q}}D}N_D)(\beta_{R_{\hat{Q}}D})^{m_{R_{\hat{Q}}D}N_D}} \times \left(1 - \mathrm{e}^{-\frac{x}{\beta_{R_{\hat{Q}}D}}}\sum_{t=0}^{m_{R_{\hat{Q}}D}N_D-1}\frac{1}{t!}\left(\frac{x}{\beta_{R_{\hat{Q}}D}}\right)^t\right)^{N_R-1}. \quad (31)$$

By substituting (30) and (31) into (29), and evaluating the resulting integral by using [17, Eq. (3.471.9)] as in (13), the outage probability of partial R-TAS strategy can be derived as given in (33).

*2) Partial R-APS strategy:* In this strategy, the best single relay ($R_{\hat{Q}}$) and the best Tx/Rx antenna pair at the source and $R_{\hat{Q}}$ by only using $S \to R_q|_{q=1}^{Q}$ as follows:

$$\{(I,J), \hat{Q}\} = \underset{\substack{1\leq i\leq N_S, 1\leq j\leq N_{R_q}\\ 1\leq q\leq Q}}{\operatorname{argmax}}\left(\gamma_{SR_q}^{(i,j)}\right), \quad (32)$$

where $\gamma_{SR_q}^{(i,j)} = \bar{\gamma}_{SR_q}\left|\mathbf{h}_{SR_q}^{(i,j)}\right|^2$. In the second hop, $R_{\hat{Q}}$ and $D$ select their best pair of Tx/Rx antennas as $\{(K,L)\} = \underset{1\leq k\leq N_{R_{\hat{Q}}}, 1\leq l\leq N_D}{\operatorname{argmax}}\left(\gamma_{R_{\hat{Q}}D}^{(k,l)}\right)$, where $\gamma_{R_{\hat{Q}}D}^{(k,l)} = \bar{\gamma}_{R_{\hat{Q}}D}\left|\mathbf{h}_{R_{\hat{Q}}D}^{(k,l)}\right|^2$ are the equivalent instantaneous SNRs. The e2e SNR is given by $\gamma_{e2e} = \gamma_{SR_{\hat{Q}}}^{(I,J)}\gamma_{R_{\hat{Q}}D}^{(K,L)}/(\eta_q\gamma_{SR_{\hat{Q}}}^{(I,J)} + \gamma_{R_{\hat{Q}}D}^{(K,L)} + \zeta_{\hat{Q}})$. By following similar steps to that of Section III-A, the outage probability of partial R-APS can be derived as in (33) on the top of the next page. In (33), the tuples $\{\lambda = m_{SR}N_R$, $\mu = N_S$, $\xi_{\hat{Q}} = m_{R_{\hat{Q}}D}N_D$, $\psi = N_R\}$ and $\{\lambda = m_{SR}$, $\mu = N_SN_R$, $\xi_{\hat{Q}} = m_{R_{\hat{Q}}D}$, $\psi = N_RN_D\}$ stand for partial R-TAS and partial R-APS strategies, respectively.

TABLE I
DIVERSITY ORDERS OF THE FOUR JOINT SELECTION STRATEGIES.

| Selection Strategy | Diversity Order | |
|---|---|---|
| | Perfect CSI | Outdated CSI |
| R-TAS | Eq. (23) | $\min(m_{SR}N_R, m_{RD}N_D)$ |
| R-APS | Eq. (23) | $\min(m_{SR}, m_{RD})$ |
| Partial R-TAS | Eq. (36) | $\min(m_{SR}N_R, m_{RD}N_D)$ |
| Partial R-APS | Eq. (36) | $\min(m_{SR}, m_{RD})$ |

As per remark III.1, (33) holds for both CA-AF ($\eta_{\hat{Q}} = 1, \zeta_{\hat{Q}} = 1$) and FG-AF ($\eta_{\hat{Q}} = 0, \zeta_{\hat{Q}} \neq 0$) relays. Specifically, $\zeta_{\hat{Q}}$ for FG-AF relay is derived by using similar techniques to those in (20) as

$$\zeta_{\hat{Q}} = 1 + \sum_{a=1}^{Q\mu}\sum_{b=0}^{a(\lambda-1)}\frac{(-1)^a\mu\binom{\lambda-1}{a}\phi_{b,a,\lambda}\beta_{SR}\Gamma(b+1)}{\Gamma(\lambda)(a+1)^{\lambda+b+1}}. \quad (34)$$

The asymptotic outage probability and ASER of partial R-TAS and partial R-APS strategies for CA-AF relays are given by $P_{\text{out}}^{\infty} = \Omega\left(\frac{\gamma_{th}}{\bar{\gamma}}\right)^{G_d} + o\left(\bar{\gamma}^{-(G_d+1)}\right)$ and , $P_e^{\infty} = \frac{\alpha\Omega 2^{G_d-1}\Gamma\left(G_d+\frac{1}{2}\right)}{\sqrt{\pi}(\varphi\bar{\gamma})^{G_d}} + o\left(\bar{\gamma}^{-(G_d+1)}\right)$, respectively. Here, $\Omega$ is defined as

$$\Omega = \begin{cases} \left(\frac{(m_{SR}/u)^{m_{SR}N_SN_R}}{(\lambda!)^{\mu}}\right)^{Q}, & m_{SR}N_S < m_{R_{\hat{Q}}D}N_D \\ \left[\left(\frac{(m_{SR}/u)^{m_{SR}N_SN_R}}{((\lambda)!)^{\mu}}\right)^{Q} + \frac{(m_{R_{\hat{Q}}D}/v_{\hat{Q}})^{m_{R_{\hat{Q}}D}N_DN_R}}{(\xi_{\hat{Q}}!)^{\psi}}\right], & m_{SR}N_S = m_{R_{\hat{Q}}D}N_D \\ \frac{(m_{R_{\hat{Q}}D}/v_{\hat{Q}})^{m_{R_{\hat{Q}}D}N_RN_D}}{(\xi_{\hat{Q}}!)^{\psi}}, & m_{SR}N_S > m_{R_{\hat{Q}}D}N_D, \end{cases} \quad (35)$$

where $u = \bar{\gamma}_{SR}/\bar{\gamma}$ and $v_q = \bar{\gamma}_{R_qD}/\bar{\gamma}$. Further, the diversity order for both partial selection strategies is given by

$$G_d = N_R\min(Qm_{SR}N_S, m_{R_{\hat{Q}}D}N_D). \quad (36)$$

### F. Impact of imperfect CSI

In practical MIMO systems, the estimated channel matrices are generally perturbed by the addition of Gaussian errors due to channel estimation errors. Further, the transmit antennas could be selected by using outdated CSI matrices due to feedback delays. The channel matrices having these two practical transmission impairments can be modeled as follows [20], [21], [23], [24]:

$$\mathbf{H}_l(t)|_{l=1}^{2} = \rho_l\hat{\mathbf{H}}_l(t-\tau_l) + \mathbf{E}_{e,l} + \mathbf{E}_{d,l}, \quad (37)$$

where $\hat{\mathbf{H}}_l(t-\tau_l)\Big|_{l=1}^{2}$ is the $\tau_l$-delayed estimated channel matrix having mean zero and variance $(1-\sigma_{e,l}^2)$ Gaussian entries, and $\rho_l$ is the normalized correlation coefficient for the $\tau_l$-delayed feedback channel given by $\rho_l = \frac{\mathcal{E}\{\hat{h}_l^{i,j}(t)\hat{h}_l^{i,j}(t-\tau_l)\}}{1-\sigma_{e,l}^2}$. For Clarke's fading model, $\rho_l = \mathcal{J}_0(2\pi f_l\tau_l)$, where $f_l$ is the Doppler frequency. Further, $\mathbf{E}_{e,l} = \mathbf{H}_l(t) - \hat{\mathbf{H}}_l(t)$ is the channel estimation error matrix, independent with both $\hat{\mathbf{H}}_l(t)$ and $\mathbf{E}_{d,l}$, having mean zero and variance $\sigma_{e,l}^2$ Gaussian entries. The additional channel estimation errors perturbed by the feedback delay are modeled by $\mathbf{E}_{d,l} = \hat{\mathbf{H}}_l(t) - \rho_l\hat{\mathbf{H}}_l(t-\tau_l)$ having mean zero and variance $(1-\sigma_{e,l}^2)(1-\rho_l^2)$ Gaussian entries. The performance degradation due to imperfect CSI is studied by using Monte-Carlo simulations in Section IV.

$$P_{\text{out, partial}}^{\text{R-TAS/R-APS}} = 1 - \sum_{a=1}^{Q\mu} \sum_{b=0}^{a(\lambda-1)} \sum_{p=1}^{\psi-1} \sum_{l=0}^{p(\xi_{\hat{Q}}-1)} \sum_{u=0}^{b} \sum_{v=0}^{\xi_{\hat{Q}}+l-1} \frac{2(-1)^{a+p+1}\psi\eta_{\hat{Q}}^{u}\binom{N_R-1}{p}\binom{Q\mu}{a}\binom{b}{u}\binom{\xi_{\hat{Q}}+l-1}{v} a^{\frac{u+v-b+1}{2}}\phi_{b,a,\lambda}\phi_{l,p,\xi_{\hat{Q}}}}{\Gamma(\xi_{\hat{Q}})(p+1)^{\frac{u+v-b+1}{2}}\left(\beta_{R_{\hat{Q}}D}\right)^{\frac{2\xi_{\hat{Q}}+2l+b-u-v-1}{2}}(\beta_{SR})^{\frac{u+v+b+1}{2}}}$$
$$\times\ \gamma_{th}^{\frac{2\xi_{\hat{Q}}+2l+b+u-v-1}{2}}(\eta_{\hat{Q}}\gamma_{th}+\zeta_{\hat{Q}})^{\frac{l+v-u+1}{2}} \mathrm{e}^{-\gamma_{th}\left(\frac{\eta_{\hat{Q}}a}{\beta_{SR}}+\frac{p+1}{\beta_{R_{\hat{Q}}D}}\right)} \mathcal{K}_{u+v-b+1}\left(2\sqrt{\frac{a(p+1)\gamma_{th}(\eta_{\hat{Q}}\gamma_{th}+\zeta_{\hat{Q}})}{\beta_{SR}\beta_{R_{\hat{Q}}D}}}\right). \quad (33)$$

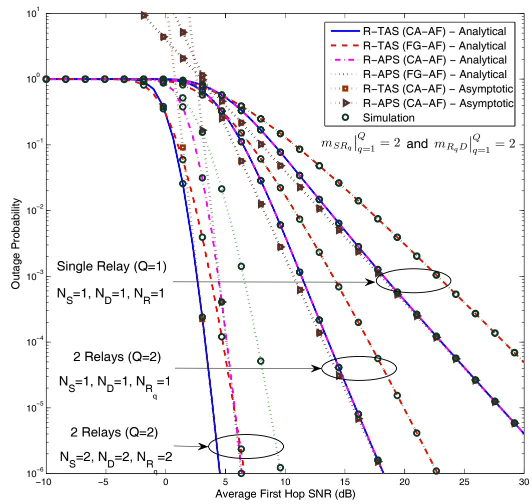


Fig. 1. The outage probability of R-TAS and R-APS strategies.

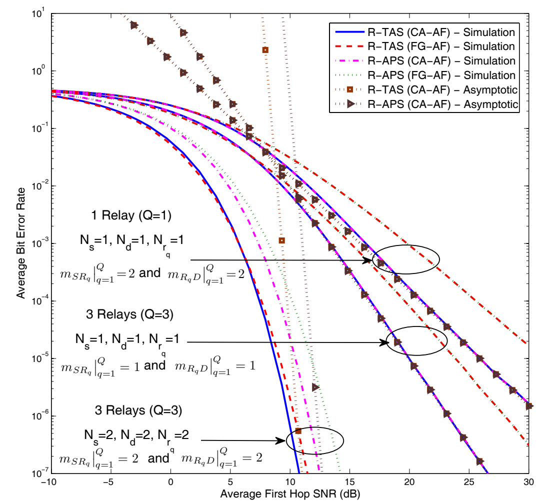


Fig. 2. The average bit error rate of BSPK of R-TAS and R-APS strategies.

**Remark III.2:** It is worth noticing that in fast-fading environments, the channel coefficients have to be estimated more frequently [25], [26], and hence, the selected antenna indices have to be fed back to $S$ and $R$ accordingly. This results in fast antenna switching, which may result in performance degradation due to antenna switching delays. Moreover, higher feedback rates significantly degrade the spectral efficiency. However, to reduce channel load caused by feedback, and thereby, to mitigate the antenna switching error, various channel-prediction algorithms can be adopted [25], [26].

## IV. Numerical Results

Fig. 1 shows the exact outage probability of the R-TAS and R-APS strategies for several system set-ups over Nakagami-$m$ fading channels ($m_{SR_q}\big|_{q=1}^{Q} = 2$ and $m_{R_qD}\big|_{q=1}^{Q} = 2$). Both the CA-AF and FG-AF relays are treated. First, the outage probability of a dual-relay ($Q = 2$) network having dual-antenna terminals is plotted. In order to depict the diversity order clearly, the asymptotic outage curves for CA-AF relays are plotted by using (21). In particular, the outage probability of a dual-relay network having single-antenna terminals is also plotted by using (19) with $N_S = 1$, $N_D = 1$ and $N_{R_q}\big|_{q=1}^{2} = 1$, for comparison purposes. Fig. 1 shows clearly that the dual-relay network with joint relay and antenna selection outperforms the dual-relay network having singe-antenna terminals. Further, a dual-hop single-relay network is also treated as a reference set-up to show the performance gains obtained by using relay and/or antenna selection strategies. Fig. 1 reveals that the FG-AF relays achieve the full diversity order just as the CA-AF relays, however, the CA-AF relays outperform FG-AF relays significantly in terms of array gain.

In Fig. 2, the average bit error rate (BER) of the binary phase shift keying (BPSK) for the R-TAS and R-APS strategies is plotted for several system set-ups. The exact average BER is plotted by using Monte-Carlo simulations, and the asymptotic average BER is plotted by using (25). The average BER curves of the R-TAS and R-APS strategies behave just as they do in terms of the outage probability.

Fig. 1 and Fig. 2 reveal that the R-TAS strategy always outperforms the R-APS strategy whenever the terminals have multiple antennas. This result is not surprising because the former uses MRC at $D$ whereas the latter uses one form of selection combining. The R-TAS strategy obtains this performance gain at the expense of additional hardware cost at the receivers at $R$ and $D$; in particular, the R-TAS requires a separate RF receiver chain for each receive antenna at $R$ and $D$ whereas the R-APS requires a single RF receiver chain at each receiving terminal[4]. Whenever the system set-up consists of all single-antenna terminals, both R-TAS and R-APS are equivalent.

Fig. 3 compares the outage probability of full R-TAS/R-APS strategies and partial R-TAS/R-APS strategies. This figure reveals clearly that the diversity order degrades due to the partial selection (see Table I for more details). For instance, at $10^{-5}$ outage probability, full R-TAS for a dual-antenna dual-relay network achieves about 5.6 dB gain over partial R-TAS. However, the former achieves this gain at the expense of global CSI compared to the latter's partial CSI requirement.

Fig. 4 shows the impact of outdated CSI on the average BER of BPSK of R-TAS and R-APS strategies. This figure clearly reveals that the joint relay and antenna selection based on the outdated CSI has a severe decremental effect on the BER performance (see Table I for more details). For example, at

[4]Both R-TAS and R-APS require only a single RF transmitter chain at $S$ and $R$.

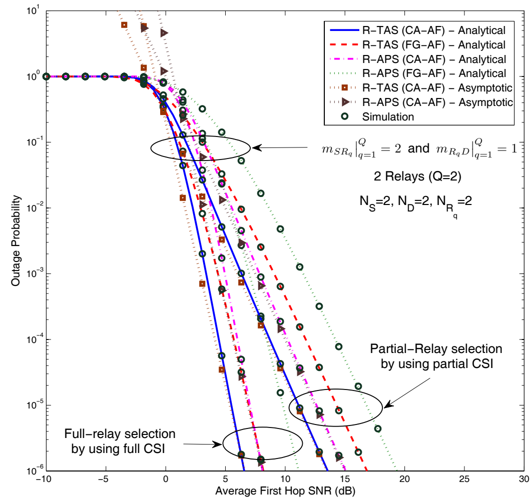


Fig. 3. The outage probability comparison of R-TAS and R-APS strategies with full and partial CSI.

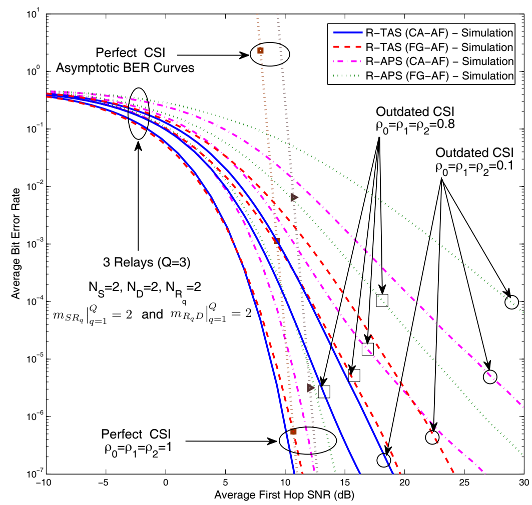


Fig. 4. The impact of outdated CSI on the average BER of BPSK. The $S \to R$ and $R \to D$ are modeled by using (37) with $\mathbf{E}_{e,l} = 0$.

$10^{-6}$ BER, outdated CSI with $\rho_1 = \rho_2 = 0.8$ results in 4.5 dB loss compared to perfect CSI case.

## V. Conclusion

Four joint relay and antenna selection strategies for dual-hop MIMO AF relay networks were proposed and analyzed. Two of them require global CSI while the rest are highly useful whenever partial CSI is available. The exact outage probability was derived for all selection strategies and used to obtain high SNR approximations for the outage and ASER, diversity order and array gain. Our analysis reveals that both R-TAS and R-APS, which require global CSI, achieve the full diversity order compared to suboptimal diversity gains provided by their partial selection strategies. The detrimental impact of imperfect CSI on the system performance was studied as well. Our results and analysis were verified through Monte-Carlo simulations. These results will spur further research on joint relay and antenna selection.

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