

Spatial Multipath Resolution for MIMO Systems

Damith Senaratne and Chintha Tellambura, *Fellow, IEEE*

Abstract—Wireless multiple-input multiple-output (MIMO) terminals with a large number of antennas are becoming a reality, increasing the spatial degree of freedoms (DoFs) available at the terminals. This paper proposes exploiting the excess spatial DoFs available at the receiver for spatial multipath resolution. A rake-receiver structure, whose fingers are implemented through spatial signal processing, is introduced for that. The joint computation of beamforming matrices for the transmitter and each receiver finger to achieve single-carrier eigenmode transmission over a multipath MIMO channel is developed. Numerical results on the error performance are provided for a practical multipath MIMO channel based on the IEEE 802.15.3c NLOS (CM4) model.

Index Terms—MIMO, frequency selective fading, resolving multipaths, rake receiver.

I. INTRODUCTION

EVER increasing demand for higher data rates, and potential diversity and/or multiplexing gains, have made multiple-input multiple-output (MIMO) wireless technology commercially feasible. The use of higher carrier frequencies, as well as the advances of electronics, is making MIMO transceivers with many antennas (e.g. 16 element antenna-on-package design [1] for millimeter wave systems) technically and economically viable. The transition towards higher frequency bands is evident through successive generations of cellular systems (e.g. 900MHz \rightarrow 1.9 GHz \rightarrow 2.1 GHz), and more prominently, with respect to local area data network standards (e.g. Wi-Fi: 2.4GHz \rightarrow 5GHz \rightarrow Wi-Gig: 60GHz). Correspondingly diminishing wavelengths enable antenna element sizes, as well as the antenna correlation in antenna-arrays given a certain antenna separation, to reduce. As a result, such transceivers have ample spatial degree of freedoms (DoFs)¹. Conventionally, these DoFs are exploited to increase the diversity and/or multiplexing gains, for interference cancellation [3], and others. DoFs available at a wireless terminal could be more than what required to assure the desired quality of service, for instance, to support the target data rate and the uncoded error rate. For example, it can be shown through simulation that a 3×4 MIMO system under a rich-scattering channel achieves a 10^{-3} average uncoded bit error rate at 0 dB average signal-to-noise ratio (SNR), for binary phase shift keying modulation scheme. A 5×4 MIMO system thus would have 2 antennas in excess, if achieving the same error rate is the target. In such cases, the *excess DoFs* can be utilized for other purposes.

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The authors are with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6G 2V4, Canada (e-mail: {damith, chintha}@ece.ualberta.ca).

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¹A wireless terminal has as many DoFs [2] as the maximum number of independent data streams it may send and/or receive. Spatial DoFs, i.e. the DoFs in the space-dimension alone, are governed by the effective MIMO channel at the terminal, and crudely, by the number of antennas.

At high data rates, frequency selective fading and inter-symbol interference (ISI) limit the reliability. Such fading is typically mitigated by orthogonal frequency division multiplexing (OFDM) [4], in which the bandwidth is split into a number of narrowband subcarriers, each carrying data at a lower rate. Several emerging wireless standards, including 3GPP LTE and WiGig, use OFDM.

Spatial multipath resolution (SMR), that uses the excess spatial DoFs to combat such fading, is proposed in this letter for the first time. It yields a *flattened* effective MIMO channel, on top of which, any MIMO signal processing technique can be applied. Eigenmode transmission is considered for illustration. Space being an orthogonal dimension to time and frequency, SMR may also be used in hybrid with MIMO OFDM, to reduce the number of subcarriers required, making SMR promising and worth investigating.

A jointly computed transmit beamforming matrix, and a set of receiver beamforming matrices are proposed here such that: (i) applying each receiver beamforming matrix separates a distinct multipath component of the received signal, and (ii) appropriately delaying and combining the separated components flatten the channel, and in combination with transmit beamforming, result in ISI-free *single-carrier eigenmode transmission*. The proposed multipath resolver has a *rake receiver* structure, but uses the *space-dimension* at its fingers to extract the multipath components, instead of the code-dimension used by direct-sequence code division multiple access (DS-CDMA) receivers [5]. As in spatial interference cancellation, signal processing at each finger exploits the left null space of the interferer channels, in this case, the channel matrix taps of interfering multipaths, to extract the desired signal. Therefore, for perfect SMR, the receiver requires more DoFs than all the interferers' combined, i.e. it requires possibly many antennas as the number of transmitter antennas times the channel length. Partial SMR is possible with lesser number of receiver antennas.

The main contributions of this work are as follows.

- Spatial multipath resolution is proposed.
- Joint computation of beamforming matrices for the transmitter and the rake receiver fingers for ISI-free eigenmode transmission is outlined.
- Symbol error rate (SER) simulations for the scheme are given, highlighting how the performance compares with the best hypothetical SER. Insights on the number-of-antenna requirement for perfect SMR, and the improvement achievable through random permutation of combiner weight matrices are obtained.

The paper is organized as follows. Section II details the system model, outlining how beamforming matrices for the transmitter and each finger of the rake-receiver can be computed jointly. Numerical results highlighting the error performance of the scheme are given in Section III. A practical channel

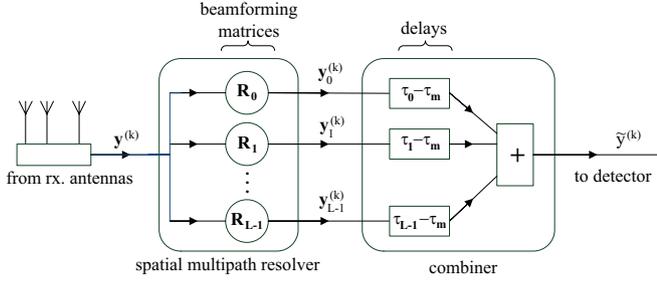


Fig. 1. A rake receiver with L fingers: Each receiver beamforming matrix \mathbf{R}_l extracts the multipath signal component $\mathbf{y}_l^{(k)}$ of the incoming signal $\mathbf{y}^{(k)}$. The combiner adds together all components corresponding to an input signal $\mathbf{x}^{(k)}$, and outputs $\tilde{\mathbf{y}}^{(k)}$.

based on the IEEE 802.15.3c NLOS (CM4) model [6, p.16] is assumed for simulation purposes.

Notation: $\mathbf{A} \in \mathbb{C}^{m \times n}$ is an $m \times n$ matrix. $\{\mathbf{A}\}_{\mathcal{C}(m:n)}$ is the sub-matrix of \mathbf{A} formed with its columns m through n ; $\{\mathbf{A}\}_{\mathcal{D}(1:n)}$ is a square-diagonal matrix formed of the first n main-diagonal elements of \mathbf{A} . The conjugate transpose of \mathbf{A} is \mathbf{A}^H , and $\|\mathbf{A}\|_F$ is its Frobenius norm [7, p.291]. The rank [7, p.12] and the inverse of a square matrix \mathbf{A} are given by $\text{rank}(\mathbf{A})$ and \mathbf{A}^{-1} . Complex Gaussian $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma})$ has mean zero and covariance matrix $\mathbf{\Sigma}$. \mathbf{I}_n is the rank- n identity matrix, and $\mathcal{E}\{\cdot\}$, the expectation operator. An $N_r \times N_t$ MIMO system has N_t transmit antennas and N_r receiver antennas.

Assumptions: Since a transmit beamforming matrix is calculated, perfect channel state information (CSI) is assumed to be available at both the transmit and receive terminals. Block fading is assumed, since neither channel estimation nor beamforming is practicable if the channel varies faster.

II. SYSTEM MODEL

A. Multipath MIMO Channel:

Consider an L -tap $N_r \times N_t$ multipath MIMO channel. Let $\mathbf{H}_l \in \mathbb{C}^{N_r \times N_t}$, $l \in \{0, \dots, L-1\}$, represent the channel matrix tap of the l^{th} strongest multipath component, such that $\|\mathbf{H}_j\|_F \geq \|\mathbf{H}_l\|_F$ for $j < l$. Let τ_l be corresponding discretized delay in *time-units*, each equal to a symbol duration, and define $m = \arg \min_l (\tau_l)$. The effective discrete MIMO channel is given by $\mathbf{H}[\tau] = \sum_{l=0}^{L-1} \mathbf{H}_l \delta[\tau - \tau_l]$, where $\delta[\cdot]$ is the Kronecker delta function.

Let $\mathbf{x}^{(k)} \in \mathbb{C}^{n \times 1}$, $k \in \{1, 2, \dots\}$, where $n \leq N_t$ is the number of spatial modes², denote the signal transmitted at each time instance k ; and $\mathbf{W} \in \mathbb{C}^{N_t \times n}$ be the transmit precoding matrix. The received signal, at the point of the reception of $\mathbf{x}^{(k)}$ over the first tap, is given by

$$\mathbf{y}^{(k)} = \sum_{l=0}^{L-1} \mathbf{H}_l \mathbf{W} \mathbf{x}^{(k-\tau_l+\tau_m)} + \mathbf{n}^{(k)}, \quad (1)$$

where $\mathbf{n}^{(k)}$ denotes additive noise at the receiver. Note that $\mathbf{y}^{(k)}$ occurs τ_m time steps after the transmission of $\mathbf{x}^{(k)}$. Conventionally, the term $l = 0$ of the summation is deemed the desired signal, and the others, the source of ISI.

²More specifically $n \leq \text{rank}(\mathbf{H}_{\text{eff}})$, and \mathbf{H}_{eff} is defined in (5).

B. Receiver Design:

Fig. 1 shows the proposed rake receiver structure. The multipath resolver employs linear spatial signal processing to resolve the taps. Corresponding receiver beamforming matrices $\mathbf{R}_l \in \mathbb{C}^{n \times N_r}$, $l \in \{0, \dots, L-1\}$ need to be computed for each rake receiver finger such that

$$\mathbf{R}_l \mathbf{H}_l \neq \mathbf{0}, \text{ and } \mathbf{R}_l \mathbf{H}_j = \mathbf{0}, \forall j \in \{0, \dots, L-1\} - \{l\}. \quad (2)$$

The conditions (2) can be achieved exploiting the left null space [7, p.5] of the channel matrix taps, provided that a sufficient number of receiver antennas is at avail. This is guaranteed when $N_r > (L-1)N_t + n$. Any \mathbf{H}_l not having full column-rank would relax this requirement further.

Each \mathbf{R}_l seeks to extract the l^{th} tap while nullifying the others. Thus, the corresponding l^{th} path signal component $\mathbf{y}_l^{(k)}$, which is subsequently fed to the combiner, is given by

$$\mathbf{y}_l^{(k)} = \mathbf{R}_l \mathbf{y}^{(k)} = \mathbf{R}_l \left(\mathbf{H}_l \mathbf{W} \mathbf{x}^{(k-\tau_l+\tau_m)} + \mathbf{n}^{(k)} \right). \quad (3)$$

The combiner appropriately delays its inputs so as to add the signals $\mathbf{y}_l^{(k+\tau_l-\tau_m)} \in \mathbb{C}^{n \times 1}$, $l \in \{0, \dots, L-1\}$. For transmit symbol vector $\mathbf{x}^{(k)}$, the combiner output $\tilde{\mathbf{y}}^{(k)}$ is given by

$$\tilde{\mathbf{y}}^{(k)} = \sum_{l=0}^{L-1} \mathbf{y}_l^{(k+\tau_l-\tau_m)} = \sum_{l=0}^{L-1} \mathbf{R}_l \mathbf{H}_l \mathbf{W} \mathbf{x}^{(k)} + \tilde{\mathbf{n}}^{(k)}, \quad (4)$$

where the total noise vector $\tilde{\mathbf{n}}^{(k)} = \sum_{l=0}^{L-1} \mathbf{R}_l \mathbf{n}^{(k+\tau_l-\tau_m)}$.

- **Remark 1:** Note that the combining process introduces a delay of $\max_l (\tau_l) - \tau_m$ time steps, on top of the quickest path's delay τ_m . The receiver should function till the arrival of the last symbol over all the paths, if (4) is to be used unmodified for SMR of all the symbols.

Provided the conditions in (2) for orthogonal reception are satisfied, (4) yields a flattened effective channel $\sum_{l=0}^{L-1} \mathbf{R}_l \mathbf{H}_l \mathbf{W}$. It has to be *diagonal* to achieve eigenmode transmission.

The case: $N_r \leq (L-1)N_t + n$, where interference gets only partially suppressed, is realistic when L is large. *Partial SMR* through disregarding paths $\lfloor (N_r - n)/N_t \rfloor + 1$ through $L-1$ is a practical solution. For simplicity, the following discussion assumes $N_r > (L-1)N_t + n$. The numerical results in Section III, however, consider both the possibilities.

Define $\tilde{\mathbf{H}}_l = [\mathbf{H}_1 \cdots \mathbf{H}_{l-1} \mathbf{H}_{l+1} \cdots \mathbf{H}_L]$ for each $l \in \{0, \dots, L-1\}$. Let $\tilde{\mathbf{H}}_l = \mathbf{U}_l \mathbf{\Sigma}_l \mathbf{V}_l^H$ be the singular value decomposition (SVD) [7, Thm. 7.3.5], and $m_l = \text{rank}(\tilde{\mathbf{H}}_l)$.

Define $\tilde{\mathbf{U}}_l = \left[\{\mathbf{U}_l\}_{\mathcal{C}(m_l+1:N_r)} \mathbf{0}_l \right]$, where $\mathbf{0}_l \in \mathbb{C}^{N_r \times m_l}$ is a zero matrix, and compute

$$\mathbf{H}_{\text{eff}} = \sum_{l=0}^{L-1} \mathbf{C}_l \left(\tilde{\mathbf{U}}_l \right)^H \mathbf{H}_l, \quad (5)$$

where matrices $\mathbf{C}_l \in \mathbb{C}^{N_r \times N_r}$, $l \in \{0, \dots, L-1\}$ represent the *combiner weights*.

Let $\mathbf{H}_{\text{eff}} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^H$ be the SVD, which makes

$$\begin{aligned} \mathbf{R}_l &= \{\mathbf{\Sigma}\}_{\mathcal{D}(1:n)}^{-1} \left(\{\mathbf{U}\}_{\mathcal{C}(1:n)} \right)^H \mathbf{C}_l \left(\tilde{\mathbf{U}}_l \right)^H, \\ \mathbf{W} &= \{\mathbf{V}\}_{\mathcal{C}(1:n)}, \end{aligned} \quad (6)$$

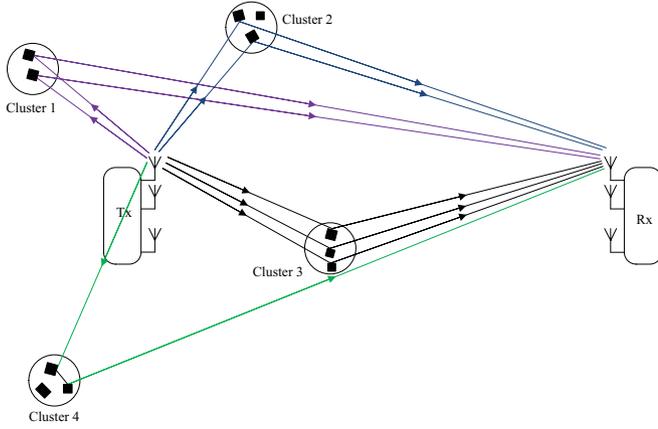


Fig. 2. Scattered rays from the 1st transmit antenna to the 1st receiver antenna, during a channel realization assuming 4 clusters, and, for this specific antenna pair, 2, 2, 3, and 1 rays scattering via clusters 1 through 4 respectively.

an appropriate choice for beamforming matrices. Amplitude gains corresponding to the spatial modes are given by the diagonal elements of $\{\Sigma\}_{\mathcal{D}(1:n)}$.

- **Remark 2:** Notably, $\left(\left(\{\mathbf{U}\}_{\mathcal{C}(1:n)}\right)^H \mathbf{C}_l \tilde{\mathbf{U}}_l^H\right) \times \left(\left(\{\mathbf{U}\}_{\mathcal{C}(1:n)}\right)^H \mathbf{C}_l \tilde{\mathbf{U}}_l^H\right)^H \neq \mathbf{I}_n$, even when \mathbf{C}_l is unitary. Correlated noise arises, since each product $\mathcal{E}\left\{\mathbf{R}_l \mathbf{n}^{(k)} \left(\mathbf{R}_l \mathbf{n}^{(k)}\right)^H\right\}$ is not diagonal even for $\mathbf{n}^{(k)} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_{N_r})$.
- **Remark 3:** Choosing any *permutation matrix* [7, p.25] as \mathbf{C}_l s amounts to a form of *equal-gain combining*. However, the performance varies with the exact choices. The trivial case of having $\mathbf{C}_l = \mathbf{I}_{N_r}, \forall l$, for example, causes the last $\min_l(m_l)$ rows of \mathbf{H}_{eff} to be zero all the time - i.e. makes the effective flattened channel $(N_r - \min_l(m_l)) \times N_t$, reducing the achievable diversity orders. The mere modification of randomly permuting the rows of \mathbf{C}_l s, for instance, would cause the resulting channel have more than $(N_r - \min_l(m_l))$ non-zero rows, and produce higher diversity orders.

III. NUMERICAL RESULTS

Monte-Carlo simulation of the SER of the SMR scheme is reported here. 10^7 channel realizations are simulated assuming quasi-static fading, and additive Gaussian noise. And 100 quadrature phase shift keying (QPSK) modulated symbols are transmitted per spatial mode for each channel realization.

A. Modeling the Multipath MIMO Channel:

The ‘Omni-Tx, Rx-15°, NICTA’ scenario of the IEEE 802.15.3c NLOS (CM4) multipath signal-input single-output (SISO) model [6, Sec. 6.2.2] is used. The model assumes multiple clusters of scatterers surrounding the transmitter and forming a small angle ($< 15^\circ$) at the receiver, and rays arriving at the receiver after being scattered at the clusters. Parameters including the number of clusters, number of rays per each cluster, and inter-cluster/inter-ray arrival times and decay rates of the gains, are modeled as random variables.

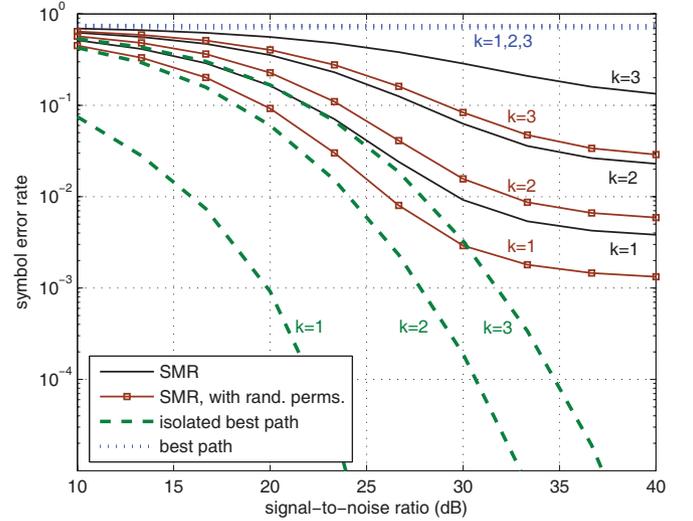


Fig. 3. SER performance of each eigenmode $k \in \{1, 2, 3\}$ for $N_t = 3, N_r = 10$, and $n = 3$. SER vs. transmit SNR curves are shown for the cases: (i) SMR, (ii) SMR with random permutation of \mathbf{C}_l s, (iii) isolated best path, and (iv) best path.

The model is extended here for MIMO as follows (see Fig. 2). For a given channel realization, the scatterer clusters are assumed common for all transmit-receive antenna pairs. Hence, the *inter-cluster parameters* [6, 4.3] are assumed common. The rays passing through each cluster, on the other hand, are assumed independent for each transmit-receive antenna pair. Thus, the *intra-cluster parameters* are independently instantiated for different transmit-receive antenna pairs. The resulting discrete multipath MIMO channel is normalized such that $\sum_{l=0}^L \mathcal{E}\left\{\|H_l\|_F^2\right\} = 1$. This extension should hold where inter-antenna separation within the antenna arrays is significantly smaller than the distances between the transceivers and the scatterers. Note that L , the number of taps of the multipath MIMO channel, varies between channel realizations.

Simulation parameters are those given under ‘Omni-Tx, Rx-15°, NICTA’ in [6, Table 4]. Shadowing effect, and hence, the parameters σ_c and σ_r are disregarded. Average numbers of 6 clusters, and 6 rays per cluster are assumed. Unit receiver antenna gains are assumed irrespective of the angle of arrival, and hence, the angle spread σ_ϕ in the model is immaterial for simulation. A 25 ns symbol duration, corresponding to a 40M symbol/s rate, or 20 MHz signal bandwidth³ is assumed.

- **Remark 4:** Our choice of the above channel model is arbitrary. SMR is feasible with any multipath MIMO channel, provided the receiver has sufficient spatial DoFs.

B. Symbol Error Rate Performance:

Fig. 3 depicts the average SNR vs. the SER of a 10×3 MIMO configuration, supporting $n = 3$ spatial modes $k \in \{1, 2, 3\}$. Eigenmode transmission over the best (i.e. the 0th) path, corresponding to dotted SER curves, becomes futile (e.g. SER=0.49, at 40 dB average SNR), and shows no improvement of the SER even at high SNRs. Dashed lines depict

³20 MHz is typical for subcarrier bandwidth of a modern OFDM system. Thus, a signal of such bandwidth is naturally expected to undergo flat fading. However, under the NLOS (CM4) model, the channel appears frequency selective. This observation is not an anomaly, because the model assumes an indoor office environment.

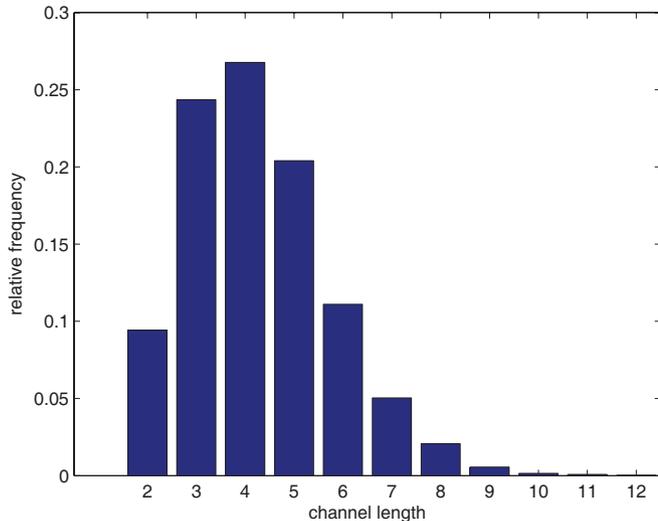


Fig. 4. Histogram of the number of taps, for 10^4 channel realizations of the 10×3 multipath MIMO channel corresponding to Fig. 3.

selection of the best path, assuming no multipath interference exists! They provide a hypothetical unachievable lower bound on the SERs, for comparison purpose. Solid SER curves corresponding to SMR improve initially with SNR, but level off at high SNR, indicating that perfect multipath interference cancellation is impossible. This observation is as expected, since an average channel length of 4.27 could be observed, with more than the resolvable $\lfloor (N_r - n)/N_t \rfloor + 1 = 3$ channel taps existing 66.21% of the times (see: Fig. 4 for a histogram on the channel length). Moreover, as emphasized in Remark 3, having random permutation matrices as \mathbf{C}_l s outperforms having all $\mathbf{C}_l = \mathbf{I}_{N_r}$, even though both are forms of equal-gain combining. An exhaustive search over possible permutations would have improved the performance further. Non equal-gain forms (e.g. maximal-ratio combining) are also a possibility.

For Fig. 5, the same simulation parameters are assumed, however, with the maximum number of paths restricted to 2. As a result, the DoFs available at the receiver is always sufficient for perfect SMR. Maximal ratio combining of the paths assuming isolated reception (more specifically, eigenmode transmission over an effective channel $\sum_{l=0}^{L-1} \mathbf{H}_l^H \mathbf{H}_l$, through setting \mathbf{H}_l^H the only distinct factor of each \mathbf{R}_l), is considered as an unachievable lower bound on the error rates. SMR schemes are seen to perform within 5 to 10 dB of the bound. Even here, having random permutation matrices for combiner weights \mathbf{C}_l s yields better performance and higher diversity orders, over choosing identity matrices. Notably, the achievable diversity orders are less than those of the hypothetical MRC scheme. This reduction is owing to spending some of the receiver's spatial DoFs for assuring the orthogonality of the multipath components. Corresponding SER and capacity degradation is the cost of SMR.

IV. CONCLUSION

The use of excess spatial DoFs at a MIMO receiver for spatial multipath resolution (SMR) was proposed and investigated. Here, the spatial DoFs available in excess to what required to achieve the desired quality of service, are used to combat frequency selective fading and enable single-carrier

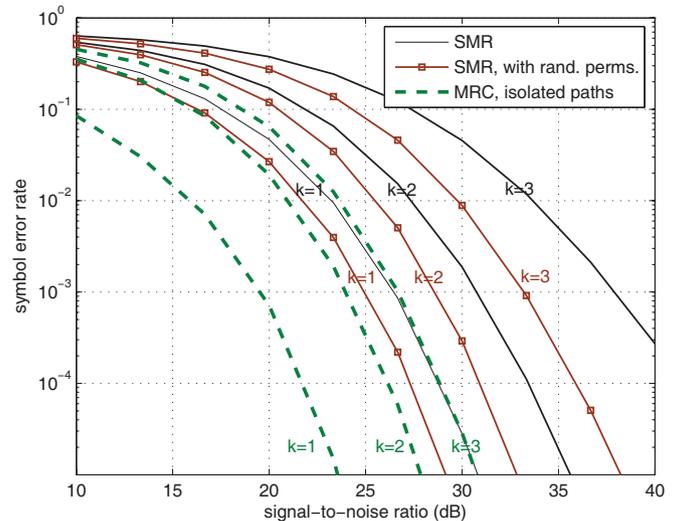


Fig. 5. SER performance of each eigenmode $k \in \{1, 2, 3\}$ for $N_t = 3$, $N_r = 10$, and $n = 3$. The maximum number of multipaths is forced to be 2. SER vs. transmit SNR curves are shown for the cases: (i) SMR, (ii) SMR with random permutation of \mathbf{C}_l s, (iii) MRC of isolated paths.

transmission. SMR may also be used in hybrid with MIMO OFDM, to reduce the number of subcarriers required. This is a very promising approach, which remains to be investigated.

Numerical results, were presented demonstrating that eigenmode transmission with SMR fares within 10 dB of the hypothetical case of receiving the multipath signal components ISI-free. This performance requires that the receiver has sufficient number of antennas (e.g. a length 3, 10×3 MIMO channel). Even when the number of receiver antennas is insufficient, partial suppression of multipath interference is feasible. In contrast, without SMR, eigenmode transmission completely fails. A realistic multipath MIMO channel based on the IEEE 802.15.3c NLOS (CM4) model was used for the simulation.

While multipath resolution yields multipath diversity, using spatial DoFs for SMR causes MIMO diversity to reduce. The tradeoff needs to be examined. The optimal choice of combiner weights \mathbf{C}_l s too needs to be investigated. Capacity comparison with multi-carrier transmission is also open for research.

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