Power Allocation for Two-Way Amplify-Forward Relaying with Receive Channel Knowledge

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***Abstract*—The power allocation problem corresponding to the communication of two sources via a relay with amplify-forward capability is studied from the outage probability perspective. Analog network coding is considered for half-duplex nodes with perfect channel state information at the receiver side. Under a sum-power constraint on the transmit powers of the nodes, an optimal power allocation strategy that minimizes the high signal-to-noise ratio approximation of the outage probability is derived and shown to improve the performance significantly. As a reference for comparison, a cut-set type bound is also optimized for the setup.**

I. Introduction

Bidirectional relaying, in which two nodes exchange independent messages through a relay node while using the same physical channels, has attracted a great research attention. This is motivated by the capability of two-way relaying in recovering a significant portion of the spectral efficiency loss occurred as a result of the half-duplex operation of network nodes [1]. With the half-duplex constraint, each node cannot transmit and receive in the same time-frequency resource. Despite the increased spectral efficiency, bidirectional relaying suffers from reliability degradation in a fading environment [2]. The present paper aims to improve the reliability of this network by optimally distributing the total transmit power among network nodes.

Since the relay is not interested in knowing the messages, it makes sense for the relay to clean the received signal and forward it to the users instead of decoding the messages. Therefore, the unnecessary rate-loss associated with decoding at the relay (due to the multiple-access decoding constraints) is avoided [3], [4]. The cleaning can be done if the relay decodes the sum of its received signals using the idea of compute-forward based on lattices [5]. A simpler strategy that does not need lattices is to amplify the received signal, which is known as *analog network coding* (ANC) [6]. However, this comes at the price of performance loss. For instance, decode-forward strategy, has been shown to outperform ANC when either multiplexing gain or signal-to-noise ratio (SNR) is small enough [7]. Nevertheless, ANC is desirable due to its low-complexity structure.

The approximate outage probability of ANC has been investigated in previous studies [8]–[10]. In [8], an optimum power allocation (OPA) is obtained based on unidirectional link outage calculation, and under the equal power assumption for the sources. In [10] and [11], OPA strategies are derived based on the assumption of channel state information available at the transmitters (CSIT). To increase the reliability of the network, an opportunistic source selection protocol has been proposed in [2]. Using CSIT, the protocol supports one traffic flow at a time; therefore, provides improved reliability. Other relevant references concerning power allocation for two-way multi-user/relay configurations are [12]–[14].

Contributions and relation to previous work. In this paper, we consider the amplify-forward two-way relay channel (AF-TWRC) with an equal target rate for both users under a sum transmit power constraint[1]. We assume all nodes have perfect channel knowledge at their reception time (CSIR). We obtain an OPA for ANC protocol that minimizes the outage probability for a given target rate. The allocation turns out to be only a function of the ratio of the average fading powers (AFP). A similar observation has been made for the one-way relay channel with the same channel knowledge assumption [15]. In performance analysis, it is always interesting to compare the results against a bound. This motivates us to develop a cut-set type lower bound on the outage performance of the TWRC, and find the corresponding OPA. In [16], an OPA that maximizes the sum-capacity upper bound of the TWRC with CSIT assumption has been derived.

The rest of the paper is structured as follows. Section II introduces the system and channel model, and covers the preliminaries. Section III derives the optimum power vector that minimizes the outage probability for ANC protocol as well as for a cut-set type bound. Section IV concludes the paper. Some details and proofs are deferred to appendices[2].

II. System Model and Preliminaries

In this work, a dual-hop communication system, depicted in Fig. 1, is studied. The model consists of two sources S_1 and S_2 interested in exchanging their messages via relay R.

[1]Our results can be modified to account for per node maximum power constraints as well as for sources with different target rates.

[2]Throughout the paper, $i, j \in \{1, 2\}$, and $i \neq j$. Prob($\mathcal{E}$) denotes the probability of the event $\mathcal{E}$. A circularly symmetric complex Gaussian random variable (r.v.) z with mean m and variance v is represented by $z \sim \mathcal{CN}(m, v)$. Finally, the logarithmic base 2 is used unless otherwise stated.

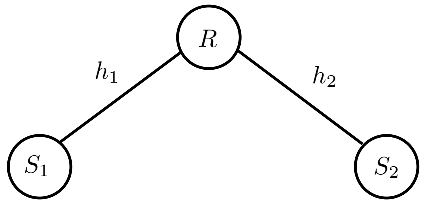

Fig. 1. Two-way relay channel (TWRC) model.

A. Assumptions

The following assumptions are made through the rest of the paper. First, there is no direct link between the sources (for example due to shadowing or large distance between the sources). Second, all terminals operate in half-duplex mode. Third, the channel noise on each link is an independent and additive r.v. with $\mathcal{CN}(0,1)$. The channel gain between S_i and R is denoted by $h_i \sim \mathcal{CN}(0,\Omega_i)$. Without loss of generality, it is assumed that $\Omega_2 \leq \Omega_1$. The channels are independent, frequency-flat, and constant over the signaling duration. Moreover, the uplink and downlink channels are reciprocal. In addition, the channel realization is perfectly known by the receiving end of each transmission. Also, it is assumed that the realization of h_j is known by user i at the end of each transmission block. This task can be accomplished by broadcasting (at a small rate from R) the quantized version of both channel coefficients to the users with arbitrarily small error [17]. Finally, an equal target rate $\frac{R_t}{2}$ is considered for users, and the total transmit power is assumed to be P_T.

B. Definitions

The multiplexing gain r is defined as $r \triangleq \frac{R_t}{\mathcal{C}(P_T)}$, where $\mathcal{C}(P) \triangleq \log(1+P)$. In addition, $g_i \triangleq |h_i|^2$, and $\omega \triangleq \frac{\Omega_2}{\Omega_1}$.

C. ANC Protocol

In ANC protocol, S_i transmits a unit-power signal x_i to R during the first time slot. In the second time slot, the relay forwards a scaled version of its received signal to both users. The received signals by R, and S_i are

$$\begin{aligned} y_r &= h_1\sqrt{P_1}x_1 + h_2\sqrt{P_2}x_2 + n_r \\ y_i &= h_i\sqrt{P_r}x_r + n_i, \end{aligned} \tag{1}$$

where n represents the noise signal at the corresponding receiver, P_r and P_i are the transmit powers for R and S_i, respectively, and

$$x_r = \frac{1}{\sqrt{g_1P_1 + g_2P_2 + 1}}\ y_r. \tag{2}$$

In this scheme, each user receives a copy of its own signal as interference. After removing the known interference, the instantaneous SNR at destination i is

$$\tilde{\gamma}_i = \frac{g_1g_2P_rP_j}{g_i(P_r+P_i) + g_jP_j + 1} \overset{(a)}{\leq} P_r \min\{g_i, \frac{g_jP_j}{P_r+P_i}\}, \tag{3}$$

where (a) is obtained by neglecting 1 in the denominator (high SNR approximation, see for example [10], [15], [18]), and using $\frac{xy}{x+y} \leq \min\{x,y\}$ [10][3]. The latter approximation is referred to as harmonic-to-min approximation (HMA) in this paper. Therefore, assuming Gaussian input signals, the outage probability becomes

$$\begin{aligned} P_{\text{outage}}(R_t) &= \text{Prob}(\min\{\tilde{\gamma}_1,\tilde{\gamma}_2\} < \Gamma \triangleq 2^{R_t}-1) \\ &\approx \text{Prob}(P_r \min\{g_1, \frac{g_2P_2}{P_r+P_1},\ g_2, \frac{g_1P_1}{P_r+P_2}\} < \Gamma). \end{aligned} \tag{4}$$

D. Lower Bound on the Outage Probability

A lower bound on the outage probability of all two phase schemes (comprising the multiple-access (MAC) and broadcast (BC) phases) is provided here. It is assumed the MAC and BC phases take β and $1-\beta$ fractions of time, respectively. We use cut-set bounds and consider the relay in two formats: part of transmitter or receiver i for each flow. This leads to:

$$R_i^{\text{up}} \leq \min\{(1-\beta)\mathcal{C}(g_i\frac{P_r}{1-\beta}), \beta\mathcal{C}(g_j\frac{P_j}{\beta})\}. \tag{5}$$

The high SNR approximation of the bound on outage becomes

$$\begin{aligned} P_{\text{outage}}^{\text{LB}}(R_t) &= \text{Prob}(\min\{R_1^{\text{up}}, R_2^{\text{up}}\} < \frac{R_t}{2}) = 1- \\ &\quad \text{Prob}(g_1 \geq \max\{\frac{\Gamma_1}{P_r}, \frac{\Gamma_2}{P_1}\}, g_2 \geq \max\{\frac{\Gamma_1}{P_r}, \frac{\Gamma_2}{P_2}\}) \\ &\approx \frac{\max\{\frac{\Gamma_1}{P_r}, \frac{\Gamma_2}{P_1}\}}{\Omega_1} + \frac{\max\{\frac{\Gamma_1}{P_r}, \frac{\Gamma_2}{P_2}\}}{\Omega_2}, \end{aligned} \tag{6}$$

where $\Gamma_1 \triangleq (2^{\frac{R_t}{2(1-\beta)}}-1)(1-\beta)$, and $\Gamma_2 \triangleq (2^{\frac{R_t}{2\beta}}-1)\beta$. It is remarked that for equal transmission power, i.e., $P_1 = P_2 = P_r$, the optimal $\beta^* = \frac{1}{2}$.

III. Optimal Power Allocation for TWRC with Sum-Power Constraint

In this section, the optimal transmit power vector is derived, as a function of the statistical properties of the channel, that minimizes the high SNR approximation of the outage probabilities (4) and (6), subject to a sum-power constraint. In particular, defining $\vec{P} \triangleq \{P_1, P_2, P_r\}$, we have

$$\begin{aligned} \min_{\vec{P}(\Omega_1,\Omega_2,P_T)} \quad & P_{\text{outage}} \\ s.t. \quad & P_1 + P_2 + P_r \leq P_T. \end{aligned} \tag{7}$$

A. OPA for ANC

First, it is noted that for $\Omega_2 \leq \Omega_1$, a suitable power allocation satisfies $P_1 \leq P_2$. The reason is to balance (on average) the second and fourth terms in Eq. (4) as much as possible. The formal proof is straightforward, and hence, omitted. The arguments of the $\min\{\cdot\}$ operator in (4) can then be simplified to either of the following:

$$\begin{cases} \text{case I:} \quad \frac{g_2P_2}{P_r+P_1}, \frac{g_1P_1}{P_r+P_2}, & \text{if } P_2 \leq \frac{P_T}{2}; \\ \text{case II:} \quad g_2, \frac{g_1P_1}{P_r+P_2}, & \text{if } \frac{P_T}{2} \leq P_2. \end{cases} \tag{8}$$

[3]It is remarked that since $\frac{1}{2}\min\{x,y\} \leq \frac{xy}{x+y} \leq \min\{x,y\}$, the approximation in Eq. (3) is different than the actual value by at most 3 dB. However, as mentioned in [10] and also shown in Fig. 4, the approximation is quite tight. Appendix A investigates the accuracy of the approximation in more detail.

For case I, (4) can be written as

$$P_{\text{outage}}^{\text{Case I}}(R_t) \approx 1 - e^{-\frac{\Omega_1'+\Omega_2'}{\Omega_1'\Omega_2'}\Gamma} \approx \frac{\Omega_1'+\Omega_2'}{\Omega_1'\Omega_2'}\Gamma, \tag{9}$$

where $\Omega_i' \triangleq \frac{P_i P_r}{P_r+P_j}\Omega_i$. It can be shown that the approximate outage probability (9) is convex. Therefore, the optimal $\vec{P}$ is obtained by forming the following Lagrange cost function with parameter λ

$$\mathcal{J}(\vec{P},\lambda) = \frac{\Omega_1'+\Omega_2'}{\Omega_1'\Omega_2'} + \lambda(P_1+P_2+P_r-P_T), \tag{10}$$

which leads to

$$P_2 = \frac{P_T}{1+\sqrt{\omega}+\sqrt[4]{4\omega}}, \quad P_1 = \sqrt{\omega}P_2. \tag{11}$$

It is remarked that the above solution is desirable for the range of ω that satisfies $P_2 \le \frac{P_T}{2}$.

For case II, the outage probability is

$$P_{\text{outage}}^{\text{Case II}}(R_t) \approx 1 - e^{-(\frac{1}{\Omega_1 P_1}+\frac{P_2}{\Omega_1 P_1 P_r}+\frac{1}{\Omega_2 P_r})\Gamma}$$
$$\approx (\frac{1}{\Omega_1 P_1} + \frac{P_2}{\Omega_1 P_1 P_r} + \frac{1}{\Omega_2 P_r})\Gamma. \tag{12}$$

Since $\frac{P_T}{2} \le P_2$ for this case, then it is clear that choosing $P_2 = \frac{P_T}{2}$ minimizes the outage expression. Hence, solving the corresponding optimization problem provides

$$P_1 = \frac{\sqrt{1+\frac{1}{2}(\frac{1}{\omega}-1)}-1}{\frac{1}{\omega}-1}P_T, \quad P_2 = \frac{P_T}{2}. \tag{13}$$

For both cases, $P_r = P_T - P_1 - P_2$. Since the allocation is only a function of ω, i.e., the ratio of average fading powers, it can be seen that for $\omega \le \theta$, we have $P_2 = \frac{P_T}{2}$, where θ is the switching value between the two cases, and $\theta = (\sqrt{1.5}-\sqrt{0.5})^4 \approx 0.07$.

Fig. 2 shows the normalized (to P_T) share of power for each node. The average power shares in the case of having CSIT (cf. [10] for the corresponding PA) are also plotted. An interesting observation is that the behavior of CSIT and CSIR curves is quite similar for each of the source nodes. In the case of CSIT, the relay always gets half of the power, whereas with CSIR, the relay's share varies between 0.36 and 0.5. It is worth mentioning that an exhaustive search to minimize the original outage probability (without HMA) yields similar power shares for outage probabilities less than 0.01.

B. OPA for the Cut-set Type Lower Bound

The optimization problem associated with the outage probability (6) can be formulated as

$$\min_{\vec{P},\beta} \frac{\max\{\frac{\Gamma_1}{P_r},\frac{\Gamma_2}{P_1}\}}{\Omega_1} + \frac{\max\{\frac{\Gamma_1}{P_r},\frac{\Gamma_2}{P_2}\}}{\Omega_2}$$
$$s.t.\ P_1+P_2+P_r \le P_T,\ 0 \le \beta \le 1 \tag{14}$$

We first solve the above problem assuming a fixed β. One can find the optimal β by performing an exhaustive search. It is noted that the objective function is convex and the constraints

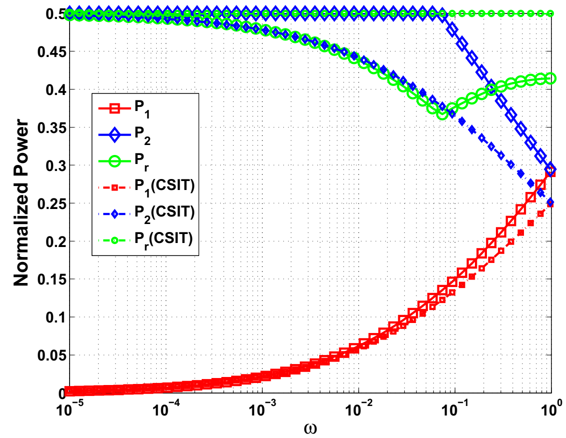

Fig. 2. Optimum share of power for each node. For the case of CSIT, average share of power is plotted.

form a convex set. Rewriting the above optimization problem in the form of Lagrangian, we have

$$\mathcal{J} = \frac{R_1}{\Omega_1} + \frac{R_2}{\Omega_2} + \lambda(P_1+P_2+P_r-P_T) + \mu_1(\frac{\Gamma_1}{P_r}-R_1)$$
$$+\mu_2(\frac{\Gamma_2}{P_1}-R_1) + \mu_3(\frac{\Gamma_1}{P_r}-R_2) + \mu_4(\frac{\Gamma_2}{P_2}-R_2), \tag{15}$$

where R_1 and R_2 are dummy variables. Appendix B shows that based on the value of $\eta \triangleq \frac{\Gamma_2}{\Gamma_1}$, the optimal allocation is

1) $\frac{\eta}{\eta+1} < \omega$:

$$P_1 = P_2 = \eta P_r = \frac{\eta}{2\eta+1}P_T. \tag{16}$$

2) $\omega \le \frac{\eta}{\eta+1}$:

$$P_1 = \frac{P_T}{1+\sqrt{\frac{\eta}{\omega(\eta+1)}}+\sqrt{\frac{1}{\omega\eta(1+\eta)}}},$$
$$P_2 = \eta P_r = \frac{P_T}{\sqrt{\frac{\omega(1+\eta)}{\eta}}+1+\frac{1}{\eta}}. \tag{17}$$

For $\omega = 1$, it can be seen that $P_1 = P_2$.

Fig. 3, demonstrates the gain of the OPA (Eqs. (11) and (13)) with respect to (w.r.t.) equal power allocation (EPA), in which $P_1 = P_2 = P_r = \frac{P_T}{3}$. The maximum gain is 4.77 dB, which is similar to the gain of the EPA scenario with total power $3P_T$. We note that for $\omega = 1$, the optimum power vector is $\vec{P}^* \approx \{0.29, 0.29, 0.42\}$. In the figure, the gain (w.r.t. ANC with EPA) of power optimized cut-set bound with equal timing between MAC and BC phases is also plotted. It can be seen that as ω decreases, optimized ANC approaches the best of two-way relaying protocols with $\beta = \frac{1}{2}$.

Fig. 4 evaluates the exact outage expressions for ANC and for the cut-set bound under equal as well as optimum power allocations. The figure considers $\omega = 0.05$ and $\omega = 0.25$ cases with the same multiplexing gain of 0.25. The approximate outage expressions for ANC with EPA and OPA are also plotted.

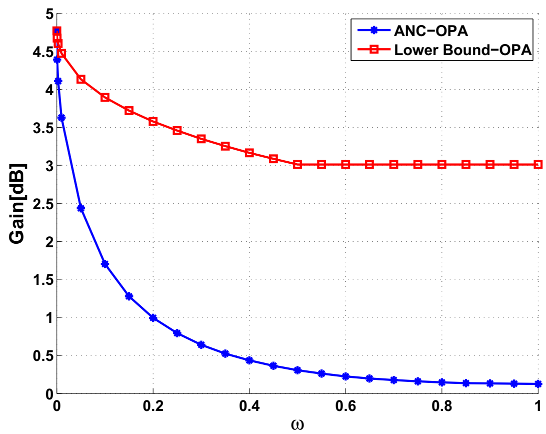

Fig. 3. Power allocation gain with respect to ANC with a fixed target rate.

From the plots, we infer that the high SNR approximations are fairly accurate. Furthermore, it is interesting to note that the gap between ANC curves is larger than that of the lower bound in this figure. The outage probability, when OPA is performed based on CSIT [10], is also provided to appreciate the gain of having CSIT.

IV. Conclusion

We have presented optimal transmit power allocation for ANC protocol with perfect channel knowledge at the receiver side. The optimal allocation is only a function of the ratio of average fading powers, and in particular, not the target rate. In highly asymmetric TWRC, OPA can bring impressive gains (up to 4.77 dB) w.r.t. EPA. A cut-set type lower bound on the outage probability is also optimized for such a setup. The optimal allocation for the bound is however a function of the target rate in addition to the ratio of average fading powers.

Appendix A
Accuracy of Harmonic-to-Min Approximation

Here, we analyze the accuracy of HMA method used in Eq. (3). In the following, we use a slightly different but simpler notation. Let γ_1 and γ_2 represent two independent exponentially distributed r.v.s with means $\bar{\gamma}_1$ and $\bar{\gamma}_2$, respectively. The outage probabilities ($\text{Prob}(\text{SNR} \le \Gamma)$) corresponding to the following SNRs, are given in Eq. (19) (see [19] for P_{Out_1})

$$\begin{aligned}\text{SNR}_1 &= \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1}\\ \text{SNR}_2 &= \min\{\gamma_1,\gamma_2\}.\end{aligned} \tag{18}$$

$$\begin{aligned}P_{\text{Out}_1} &= 1-\frac{2\sqrt{\Gamma^2+\Gamma}}{\sqrt{\bar{\gamma}_1\bar{\gamma}_2}}K_1\Big(\frac{2\sqrt{\Gamma^2+\Gamma}}{\sqrt{\bar{\gamma}_1\bar{\gamma}_2}}\Big)e^{-\Gamma(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2})}\\ P_{\text{Out}_2} &= 1-e^{-\Gamma(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2})},\end{aligned} \tag{19}$$

where $K_1(.)$ is the first order modified Bessel function of the second kind. It can be seen that for $\frac{\sqrt{\Gamma^2+\Gamma}}{\sqrt{\bar{\gamma}_1\bar{\gamma}_2}} \ll 1$ (which is true for high SNRs and practical target rates with small outage probabilities), $P_{\text{Out}_1} \approx P_{\text{Out}_2}$. Therefore, the approximation is quite tight for practical scenarios. It is remarked that due to the $\min\{\cdot\}$ operator used in $\min\{\tilde{\gamma}_1,\tilde{\gamma}_2\}$ in Eq. (4), the exact outage calculation appears to be difficult, making the HMA a viable approach to follow.

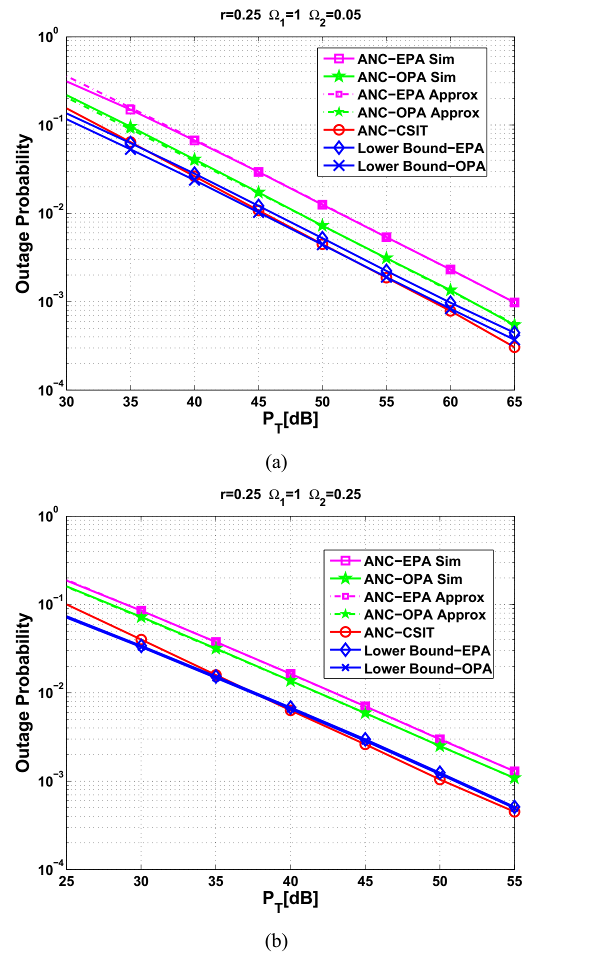

Fig. 4. The role of power allocation on the performance of ANC and the relative lower bound. The transmit power of ANC curves is doubled to compensate for the power scaling in Eq. (5), and hence, to provide a fair comparison with the corresponding lower bound plots.

Another measure of accuracy with the advantage of being independent of the target SNR Γ, is the average difference D between the SNRs, defined as[4]

$$D \triangleq \mathbb{E}[10\log_{10}(\frac{\text{SNR}_1}{\text{SNR}_2})] = \int_1^{\infty} 10\log_{10}(\frac{\alpha}{\alpha+1})f(\alpha)d\alpha,$$

where, $\alpha \triangleq \frac{\max\{\gamma_1,\gamma_2\}}{\min\{\gamma_1,\gamma_2\}}$, and $f(\alpha)$ represents the probability density function of α. To calculate D, we first obtain $F(\alpha)$,

[4]Assuming high average SNR, we neglect 1 in the denominator of SNR_1 hereafter.

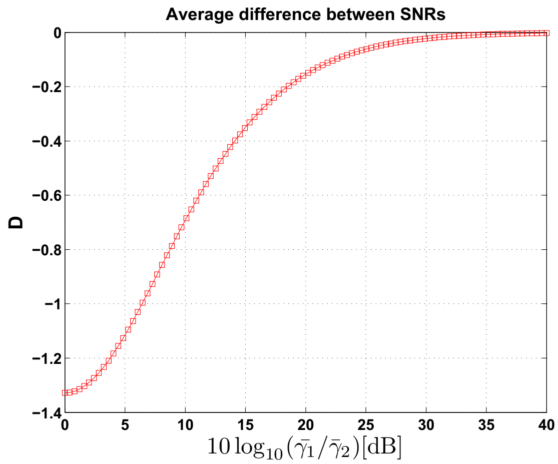

Fig. 5. Average difference between the actual SNR and the approximate SNR obtained by HMA (see Eq. (18)) as a function of mean SNR ratio.

the cumulative distribution function of α.

$$\begin{aligned}F(\alpha) &= \text{Prob}(\frac{\max\{\gamma_1,\gamma_2\}}{\min\{\gamma_1,\gamma_2\}} \le \alpha)\\ &= \text{Prob}(\frac{\gamma_1+\gamma_2}{\min\{\gamma_1,\gamma_2\}} \le \alpha+1)\\ &= \text{Prob}(\frac{1}{\alpha} \le \frac{\gamma_1}{\gamma_2} \le \alpha)\\ &\overset{(a)}{=} \frac{\bar{\gamma}_1\bar{\gamma}_2(\alpha-\frac{1}{\alpha})}{\bar{\gamma}_1^2+\bar{\gamma}_2^2+\bar{\gamma}_1\bar{\gamma}_2(\alpha+\frac{1}{\alpha})}\end{aligned}$$

where (a) is obtained by employing Lemma 1 in [18]. Using integral by parts,

$$D = -\frac{10}{\ln(10)}\int_1^\infty \frac{\bar{\gamma}_1\bar{\gamma}_2(\alpha-\frac{1}{\alpha})}{(\bar{\gamma}_1^2+\bar{\gamma}_2^2+\bar{\gamma}_1\bar{\gamma}_2(\alpha+\frac{1}{\alpha}))\alpha(\alpha+1)}d\alpha.$$

Fig. 5 shows D w.r.t. the ratio of mean values. It can be seen that the maximum difference is about 1.3 dB and occurs when $\bar{\gamma}_1 = \bar{\gamma}_2$.

Appendix B
OPA for the Cut-Set Type Lower Bound

By setting the partial derivatives w.r.t. the variables in (15) to zero, we have

$$\begin{aligned}&\mu_1+\mu_2 = \frac{1}{\Omega_1}, \quad \mu_3+\mu_4 = \frac{1}{\Omega_2}\\ &\lambda = \mu_2\frac{\Gamma_2}{P_1^2} = \mu_4\frac{\Gamma_2}{P_2^2} = (\mu_1+\mu_3)\frac{\Gamma_1}{P_r^2}.\end{aligned} \tag{20}$$

Assuming $\mu_1 = 0$, we get

$$\begin{aligned}&\mu_2 = \frac{1}{\Omega_1}, \quad \mu_3+\mu_4 = \frac{1}{\Omega_2}, \quad P_1 = \sqrt{\frac{\Gamma_2}{\Omega_1\lambda}}\\ &\frac{P_2}{P_r} = \frac{\Gamma_2}{\Gamma_1} = \sqrt{\frac{\mu_4\Gamma_2}{\mu_3\Gamma_1}} \Rightarrow \frac{\mu_4}{\mu_3} = \frac{\Gamma_2}{\Gamma_1}\\ &P_T = \frac{1}{\sqrt{\lambda}}(\sqrt{\Gamma_2}(\sqrt{\mu_2}+\sqrt{\mu_4})+\sqrt{\Gamma_1}\sqrt{\mu_1+\mu_3}),\end{aligned} \tag{21}$$

which leads to

$$\begin{aligned}&\mu_3 = \frac{\Gamma_1}{\Omega_2(\Gamma_2+\Gamma_1)}, \quad \mu_4 = \frac{\Gamma_2}{\Omega_2(\Gamma_2+\Gamma_1)}\\ &\frac{1}{\sqrt{\lambda}} = \frac{P_T}{\sqrt{\frac{\Gamma_2}{\Omega_1}}+\sqrt{\frac{\Gamma_2}{\Omega_2(1+\frac{\Gamma_1}{\Gamma_2})}}+\sqrt{\frac{\Gamma_1}{\Omega_2(1+\frac{\Gamma_2}{\Gamma_1})}}},\end{aligned} \tag{22}$$

and (17). For this case, the condition is true when $\frac{\omega}{1-\omega} \le \eta$. It can be seen that the case of $\mu_3 = 0$ never happens since $\omega \le 1$. Therefore, for $\eta < \frac{\omega}{1-\omega}$, we achieve (16).

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