Two-way Amplify-and-Forward MIMO Relay Networks with Antenna Selection

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***Abstract*—A novel transmit/receive (Tx/Rx) antenna selection strategy is proposed and analyzed for two-way multiple-input multiple-output (MIMO) amplify-and-forward (AF) relay networks. This strategy involves choosing the best transmit and receive antennas at the two sources and the relay based on the minimization of the overall outage probability. The performance of the proposed selection strategy is quantified by deriving the overall outage probability and its high SNR approximation. Specifically, the diversity order is derived to obtain valuable insights into practical system designing. In particular, our results are extended to cater the multiple relay scenario, and thereby, a joint relay and Tx/Rx antenna selection strategy is proposed and analyzed. To this end, the overall outage probability, its high SNR approximation and diversity order are derived. Our numerical results show that the proposed selection strategies achieve the full diversity order. All the analyses are validated through Monte-Carlo simulations.**

I. Introduction

Two-way relaying have been emerging as a promising spectral efficient transmission protocol for wireless networks with half-duplex terminals [1]–[4]. Specifically, two-way relay networks (TWRNs) avoid the pre-log factor of one-half in capacity expressions, and thus, are as twice spectrally efficient as the conventional one-way relay networks [1], [2]. The performance of TWRNs can be further improved by integrating multiple-input multiple-output (MIMO) transmission technology [5], [6]. However, the main drawback of any MIMO system is the increased system complexity, and hence, the additional cost for enabling multiple transmit and receive radio frequency (RF) chains[1] [7]. Antenna selection for single-hop MIMO systems has been widely studied to circumvent these drawbacks [7]. In particular, antenna selection reduces the complexity and the power requirements of the MIMO transmitter much more than most other transmit diversity schemes such as beamforming [8]. In this paper, a new transmit/receive (Tx/Rx) transmit strategy is proposed for MIMO amplify-and-forward (AF) TWRNs[2].

Prior related research: In the wide body of the relay literature, there appear only two references, [9] and [10], dealing with the issue of antenna selection for TWRNs. In [9], upper bounds for the average symbol error rate of network-coded TWRNs having two single-antenna sources and a dual-antenna relay are studied. In [9], during the first time-slot, two independent symbols are transmitted simultaneously by both sources to the relay. At the relay, these two symbols are decoded separately and in the second time-slot, a physical layer network-coded symbol (XOR of the two symbols) is transmitted back to two sources by the relay by using Alamouti coding or antenna selection. Furthermore, [10] extends the results of [9] by using either max-min antenna selection or maximal ratio transmission in the second time-slot. In particular, the transmission strategy in [10] achieves a diversity gain in the order of number of antennas at the relay.

In addition to the above studies, [5], [11] investigate the designing of optimal transmit precoders and receiver filters for MIMO TWRNs with the availability of perfect channel state information (CSI). Moreover, [6] studies the effects of channel estimation errors on the receivers of MIMO AF TWRNs.

For the sake of completeness, the prior related research on single-antenna TWRNs is also summarized. References [3], [12] provide rigorous analyses on practical physical layer network coding for TWRNs and thereby quantify the outage probability, sum-rate and corresponding high SNR approximations. In [4], [13], relay selection for TWRNs are studied.

Motivation and our contribution: References [9], [10] investigate the antenna selection only for DF TWRNs, where individual symbols from the two sources are first decoded separately and then a network-coded symbol is broadcast back to two sources. In particular, the system models in both [9][3] and [10] employ multiple antennas at the relay only, and each source is equipped with a single antenna. Furthermore, in [9], [10], the transmit antenna selection is considered in the second time-slot (broadcast phase) only.

Therefore, to the best of our knowledge, both Tx/Rx antenna selection for single-relay MIMO AF TWRNs and joint relay and antenna selection for multi-relay MIMO AF TWRNs have not yet been studied.

This paper fills this gap by proposing a new Tx/Rx antenna selection strategy for MIMO AF TWRNs. The key design criterion is the minimization of the overall outage probability while retaining the full diversity order available in the system. Specifically, each terminal in our system model is equipped with multiple antennas and the proposed transmission strategy jointly selects the best single Tx/Rx antennas at the two sources and the relay. In particular, our results are extended to cater the multi-relay scenario by proposing a joint relay and Tx/Rx antenna selection strategy.

The performance of the proposed transmission strategies is studied for MIMO AF TWRNs over frequency-flat Rayleigh

[1]Passive antenna elements and additional digital signal processing are increasingly becoming cheaper, however, RF elements are still expensive and do not follow Moore's law [7].

[2]Amplify-and-forward two-way relaying is also known as analog network coding [2], [5].

[3]The system model in [9] is restricted to dual-antenna relay terminal.

fading. To this end, the overall outage probability and its high SNR approximation are derived. Specifically, the diversity order is quantified to show that our proposed transmission strategy is optimal in the sense of diversity order. Moreover, numerical results are provided to show the performance gains and our analysis is validated through Monte-Carlo simulations.

II. System model

We consider a MIMO AF TWRN consisting of two source nodes (S_1 and S_2), and one relay node (R). Specifically, S_1, S_2 and R are equipped with N_1, N_2 and N_R antennas, respectively. All nodes are assumed to be half-duplex and all channel amplitudes are assumed to be independently distributed frequency-flat Rayleigh fading. The channel matrix from $S_i|_{i=1}^2$ to R is denoted by $H_{S_iR}|_{i=1}^2$. All the channel coefficients are assumed to be fixed over two consecutive time-slots [1]. Thus, the channels matrix from R to $S_i|_{i=1}^2$ can be denoted as $(H_{S_iR})^T\Big|_{i=1}^2$. Further, the (k,l)-th element[4] of H_{S_iR} is denoted by $h_{S_iR}^{(k,l)}$ and modeled as $h_{S_iR}^{(k,l)} \sim \mathcal{CN}(0,\zeta_i)$. Here, $\zeta_i|_{i=1}^2$ accounts for the path-loss effect and modeled as $\zeta_i \propto (d_{S_iR})^{-\varpi}$, where $d_{S_iR}|_{i=1}^2$ is the distance between S_i and R, and ϖ is the path-loss exponent. The additive noise at all the receivers is modeled as complex zero mean white Gaussian noise. The direct channel between S_1 and S_2 is assumed unavailable due to heavy path-loss and shadowing.

In this protocol, S_1 and S_2 exchange their information-bearing symbols[5], $\mathcal{X}_1$ and $\mathcal{X}_2$, respectively, in two time-slots. In the first time-slot, both S_1 and S_2 transmit $\mathcal{X}_1$ and $\mathcal{X}_2$ simultaneously by selecting the j-th and l-th transmit antennas, respectively, to R over a multiple access channel. Then R receives the superimposed-signal[6] by selecting the m-th receive antenna as follows:

$$Y_R = \sqrt{\mathcal{P}_{S_1}} h_{S_1R}^{(m,j)} \mathcal{X}_1 + \sqrt{\mathcal{P}_{S_2}} h_{S_2R}^{(m,l)} \mathcal{X}_2 + n_R, \tag{1}$$

where $\mathcal{P}_{S_i}|_{i=1}^2$ is the transmit power of S_i and n_R is the additive white Gaussian noise (AWGN) at R having mean zero and variance σ_R^2. In the second time slot, R amplifies Y_R with a gain $G = \sqrt{\frac{\mathcal{P}_R}{\mathcal{P}_{S_1}\left|h_{S_1R}^{(m,j)}\right|^2 + \mathcal{P}_{S_2}\left|h_{S_2R}^{(m,l)}\right|^2 + \sigma_R^2}}$ and then broadcasts it again by using the m-th transmit antenna to $S_i|_{i=1}^2$ over the broadcast channel. Here, $\mathcal{P}_R$ is the transmit power at R. Then, S_1 and S_2 receive the signal by again using the j-th and l-th receive antennas, respectively, as follows:

$$Y_{S_1} = G h_{S_1R}^{(m,j)} Y_r + n_1 \quad \text{and} \quad Y_{S_2} = G h_{S_2R}^{(m,l)} Y_r + n_2, \tag{2}$$

where $n_i|_{i=1}^2$ is the AWGN at S_i having mean zero and variance σ_i^2. By substituting (1) into (2) and removing the self-interference[7] [1], the end-to-end signal-to-noise ratio (e2e SNR) at $S_i|_{i=1}^2$ can be derived as

$$\gamma_{S_1}^{(j,l,m)} = \frac{\left(\frac{\mathcal{P}_R|h_{S_1R}^{(m,j)}|^2}{\sigma_1^2}\right)\left(\frac{\mathcal{P}_{S_2}|h_{S_2R}^{(m,l)}|^2}{\sigma_R^2}\right)}{\left(\frac{\mathcal{P}_R}{\sigma_1^2} + \frac{\mathcal{P}_{S_1}}{\sigma_R^2}\right)|h_{S_1R}^{(m,j)}|^2 + \frac{\mathcal{P}_{S_2}|h_{S_2R}^{(m,l)}|^2}{\sigma_R^2} + 1} \tag{3a}$$

$$\gamma_{S_2}^{(j,l,m)} = \frac{\left(\frac{\mathcal{P}_{S_1}|h_{S_1R}^{(m,j)}|^2}{\sigma_2^2}\right)\left(\frac{\mathcal{P}_R|h_{S_2R}^{(m,l)}|^2}{\sigma_R^2}\right)}{\frac{\mathcal{P}_{S_1}|h_{S_1R}^{(m,j)}|^2}{\sigma_R^2} + \left(\frac{\mathcal{P}_R}{\sigma_2^2} + \frac{\mathcal{P}_{S_2}}{\sigma_R^2}\right)|h_{S_2R}^{(m,l)}|^2 + 1}. \tag{3b}$$

In the next section, the optimal selection of antenna indices (j, l, and m) by using (3a) and (3b) is described in detail.

III. Problem formulation

In this section, a novel antenna selection strategy is proposed for MIMO AF TWRNs. The key design criterion is the joint selection of best single transmit and receive antennas at S_1, S_2 and R to minimize the overall outage probability.

The overall performance of multiuser systems is governed by the performance of the weakest user [14]. Thus, our system is in outage if either S_1 or S_2 is in outage. This motivates our antenna selection criterion; the joint maximization of the e2e SNR of the weakest user. To this end, the antenna indices at S_1, S_2 and R are selected to minimize the overall system outage probability as follows:

$$\begin{aligned}\{J,L,M\} &= \underset{\substack{1\le j\le N_1, 1\le l\le N_2\\ 1\le m\le N_R}}{\operatorname{argmin}} \left\{\Pr\left[\min\left(\gamma_{S_1}^{(j,l,m)}, \gamma_{S_2}^{(j,l,m)}\right) \le \gamma_{th}\right]\right\} \\ &= \underset{\substack{1\le j\le N_1, 1\le l\le N_2\\ 1\le m\le N_R}}{\operatorname{argmax}} \left[\min\left(\gamma_{S_1}^{(j,l,m)}, \gamma_{S_2}^{(j,l,m)}\right)\right],\end{aligned} \tag{4}$$

where J, L, and M are best antenna indices at S_1, S_2 and R, respectively[8], which minimize the overall outage probability of the two-way MIMO relay network.

IV. Outage Probability Analysis

In this section, the overall outage probability and its high SNR approximation are derived to obtain valuable insights about the system-design parameters such as the diversity order and the array gain.

A. Overall outage probability

The overall outage probability, P_{out}, is the probability that the instantaneous e2e SNR of the weakest source node falls below a preset threshold γ_{th}[9]. Thus, P_{out} is given by

$$P_{out} = \Pr\left[Z = \max_{\substack{1\le j\le N_1, 1\le l\le N_2\\ 1\le m\le N_R}} \left\{\min\left(\gamma_{S_1}^{(j,l,m)}, \gamma_{S_2}^{(j,l,m)}\right)\right\} \le \gamma_{th}\right]. \tag{5}$$

[4] Here, $h_{S_iR}^{(k,l)}$ is the channel coefficient from the l-th transmit antenna of S_i to the k-th receive antenna of R.

[5] The information-bearing symbols have unit symbol energies, i.e., $\mathcal{E}\{|\mathcal{X}_1|^2\} = 1$ and $\mathcal{E}\{|\mathcal{X}_2|^2\} = 1$.

[6] This superimposed-signal is also known as the analog network code in the two-way relay networks [2], [5].

[7] It is assumed that S_i knows its own information-bearing symbol $\mathcal{X}_i$ and all the channel coefficients.

[8] Since the channel matrices, H_{S_1R} and H_{S_2R}, remain static over the two time-slots, S_1, S_2 and R can use the J-th, L-th and M-th antennas, respectively, for both transmission and reception.

[9] Similarly, the information outage probability can be defined as follows: $P_{out} = \Pr\left[\log(1+z) \le R_{th}\right] = \Pr\left[z \le 2^{R_{th}} - 1\right]$, where $z = \min\left(\gamma_{S_1R}^{(J,L,M)}, \gamma_{S_2R}^{(J,L,M)}\right)$.

In order to evaluate P_{out} in closed-form, the cumulative distribution function (CDF) of the random variable Z should be derived and then evaluate it at γ_{th}. Thus, the CDF of Z is given by (see the Appendix for the proof)

$$F_Z(z) = \left[F\left(z, \frac{p+1}{\zeta_1}, \frac{q+1}{\zeta_2}, \frac{q}{\zeta_2}, \frac{N_1}{\zeta_1}, N_1-1, N_2\right) + F\left(z, \frac{q+1}{\zeta_2}, \frac{p+1}{\zeta_1}, \frac{p}{\zeta_1}, \frac{N_2}{\zeta_2}, N_1, N_2-1\right)\right]^{N_R}, \quad (6)$$

where $F(x, a, b, c, d, u, v)$ is given by (7) in the top of the next page. In (7), $\alpha = \frac{\bar{\gamma}_S+\bar{\gamma}_R}{\bar{\gamma}_R\bar{\gamma}_T}$, $\beta = \frac{1}{\bar{\gamma}_R}$, $\phi(z) = \frac{1}{\bar{\gamma}_S\bar{\gamma}_R}\sqrt{(\bar{\gamma}_S^2+\bar{\gamma}_S\bar{\gamma}_R+\bar{\gamma}_R^2/2)z^2+\bar{\gamma}_S\bar{\gamma}_R z}+\frac{z}{2\bar{\gamma}_S}$. Further, $\mathcal{J}(z)$ in (7) can be derived in two forms as follows: (i) By using Gauss-Laguerre quadrature (GLQ) [15, Eq. (25.4.45)], $\mathcal{J}(z)$ can be evaluated as $\mathcal{J}(z) = \sum_{t=1}^{T_g} w_t \mathrm{e}^{-\frac{acz(\alpha\beta z+\eta)}{x_t+a\phi(z)}} + \mathcal{R}_{T_g}$, where $\eta = \frac{1}{\bar{\gamma}_S\bar{\gamma}_S}$. Here, $x_t|_{t=1}^{T_g}$ and $w_t|_{t=1}^{T_g}$ are the abscissas and weights of the GLQ, respectively, and can be efficiently computed by using the classical algorithm in [16]. Moreover, T_g is the number of terms used for the GLQ and $\mathcal{R}_{T_g}$ is the remainder term, which diminishes as T_g approaches as small as 10 [16]. (ii) Alternatively, by using Taylor series expansion, $\mathcal{J}(z)$ can be derived as $\mathcal{J}(z) = \sum_{i=0}^{\infty}\frac{(-1)^i}{i!}(acz(\alpha\beta z+\eta))^i \mathrm{e}^{a\phi(z)}\Gamma(1-i, a\phi(z))$.

Next, the overall outage probability can be derived readily by evaluating the CDF of Z in (6) at the threshold, γ_{th}, as follows: $P_{out} = F_Z(\gamma_{th})$.

B. High SNR approximation of the overall outage probability

To obtain direct insights, the high SNR approximation of the overall outage probability is derived and thereby the diversity order is quantified.

The high SNR approximation of the overall outage probability is given by

$$P_{out}^{\infty} = \Omega\left(\frac{\gamma_{th}}{\bar{\gamma}}\right)^{G_d} + o\left(\bar{\gamma}^{-(G_d+1)}\right), \quad (8)$$

where G_d is the diversity order and given by

$$G_d = N_R \min(N_1, N_2). \quad (9)$$

Furthermore, the system-dependent parameter Ω is given by

$$\Omega = \begin{cases} \left(\frac{C_S+C_R}{\zeta_1 C_S C_R}\right)^{N_1 N_R}, & N_1 < N_2 \\ \left(\frac{1}{\zeta_1^N}+\frac{1}{\zeta_2^N}\right)^{N_R}\left(\frac{C_S+C_R}{C_S C_R}\right)^{N N_R}, & N_1 = N_2 = N \\ \left(\frac{C_S+C_R}{\zeta_2 C_S C_R}\right)^{N_2 N_R}, & N_1 > N_2, \end{cases} \quad (10)$$

where C_S and C_R are the ratios of the source and relay average transmit SNR to the reference average transmit SNR ($\bar{\gamma}$), respectively, i.e., $C_S = \frac{\bar{\gamma}_S}{\bar{\gamma}}$ and $C_R = \frac{\bar{\gamma}_R}{\bar{\gamma}}$.

V. Joint Relay and Antenna Selection

In this section, our proposed antenna selection strategy is extended to two-way MIMO AF multi-relay networks. Here, we consider a MIMO AF TWRN having two source nodes, S_1 and S_2, and K number of potential relays ($R_k|_{k=1}^{K}$), each equipped with N_1, N_2 and $N_{R_k}|_{k=1}^{K}$ antennas, respectively. The key design criterion is the joint selection of the best relay (R_{K^*}), and the best antenna indices, J, K, and M_{K^*} of S_1, S_2 and R_{K^*}, respectively, to minimize the overall outage probability. Thus, this selection criterion is given by

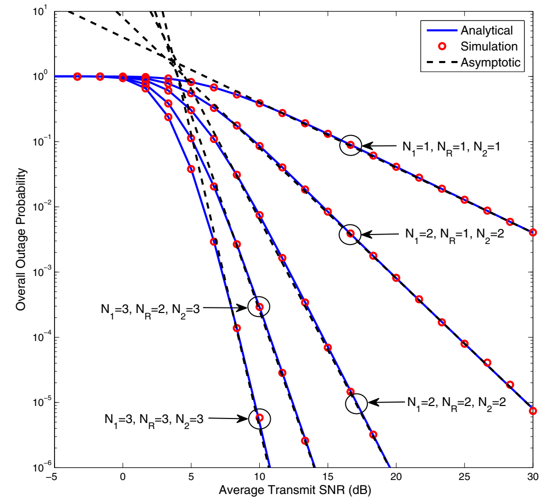

Fig. 1. The overall outage probability of Tx/Rx antenna selection for a MIMO AF TWRN with a single relay. The hop distances are $d_{S_1R} = 2d_{S_2R}$ and the path-loss exponent is $\varpi = 3.5$.

$$\{J, L, K^*, M_{K^*}\} = \underset{\substack{1\le j\le N_1, 1\le l\le N_2 \\ 1\le k\le K, 1\le m_k\le N_{R_k}}}{\operatorname{argmax}}\left[\min\left(\gamma_{S_1}^{(j,l,m_k)}, \gamma_{S_2}^{(j,l,m_k)}\right)\right]. \quad (11)$$

Next, the overall outage probability of the joint relay and antenna selection for MIMO AF TWRNs can be readily derived by using (6) as follows:

$$P_{out} = \prod_{k=1}^{K}\left\{\left[F\left(\gamma_{th}, \frac{p+1}{\zeta_{1,k}}, \frac{q+1}{\zeta_{2,k}}, \frac{q}{\zeta_{2,k}}, \frac{N_1}{\zeta_{1,k}}, N_1-1, N_2\right) + F\left(\gamma_{th}, \frac{q+1}{\zeta_{2,k}}, \frac{p+1}{\zeta_{1,k}}, \frac{p}{\zeta_{1,k}}, \frac{N_2}{\zeta_{2,k}}, N_1, N_2-1\right)\right]^{N_{R_k}}\right\}, \quad (12)$$

where $F(\cdot,\cdot,\cdot,\cdot,\cdot)$ is given by (7) after replacing ζ_1, ζ_2, N_R, α, β, and $\phi(z)$ with $\zeta_{1,k}$, $\zeta_{2,k}$, N_{R_k}, $\alpha_k = \frac{\bar{\gamma}_S+\bar{\gamma}_{R_k}}{\bar{\gamma}_S\bar{\gamma}_{R_k}}$, $\beta_k = \frac{1}{\bar{\gamma}_{R_k}}$ and $\phi_k(z) = \frac{1}{\bar{\gamma}_S\bar{\gamma}_{R_k}}\sqrt{(\bar{\gamma}_S^2+\bar{\gamma}_S\bar{\gamma}_{R_k}+\bar{\gamma}_{R_k}^2)z^2+\bar{\gamma}_S\bar{\gamma}_{R_k}z}+\frac{z}{2\bar{\gamma}_S}$, respectively.

The asymptotic outage probability at high SNRs for the joint antenna and relay selection can be derived as follows:

$$P_{out}^{\infty} = \left(\prod_{k=1}^{K}\Omega_k\right)\left(\frac{\gamma_{th}}{\bar{\gamma}}\right)^{\sum_{k=1}^{K}G_{d_k}} + o\left(\bar{\gamma}^{-(\sum_{k=1}^{K}G_{d_k}+1)}\right), \quad (13)$$

where the diversity order G_d is given by

$$G_d = \sum_{k=1}^{K}G_{d_k} = \sum_{k=1}^{K}N_{R_k}\min(N_1, N_2). \quad (14)$$

Here, Ω_k can be obtained again by replacing ζ_1, ζ_2 and C_R of (10) with $\zeta_{1,k}$, $\zeta_{2,k}$ and $C_{R_k} = \frac{\bar{\gamma}_{R_k}}{\bar{\gamma}}$, respectively.

VI. Numerical Results

Fig. 1 shows the overall outage probability of the proposed Tx/Rx antenna selection strategy for a MIMO AF TWRN

$$F(z,a,b,c,d,u,v) = \sum_{p=0}^{N_1-1}\sum_{q=0}^{N_2-1} \frac{N_1 N_2 \binom{N_1-1}{p}\binom{N_2-1}{q}(-1)^{p+q}}{b\zeta_1\zeta_2}\left[\frac{1-\mathrm{e}^{-a\beta z}}{a} - \frac{1-\mathrm{e}^{-(a+b)\beta z}}{a+c}\right] + \sum_{p=0}^{u}\sum_{q=0}^{v} d\binom{u}{p}\binom{v}{q}(-1)^{p+q}\left[\frac{\mathrm{e}^{-(a+c)\beta z}\left(1-\mathrm{e}^{-(a+c)\phi(z)}\right)}{a+c} - \frac{\mathrm{e}^{-(a\phi(z)+(a\beta+c\alpha)z)}}{a}\mathcal{J}(z)\right], \tag{7}$$

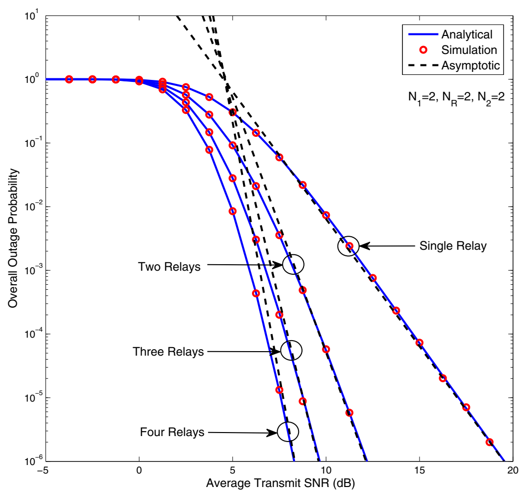

Fig. 2. The overall outage probability of joint relay and Tx/Rx antenna selection for MIMO AF TWRN. The hop distances are $d_{S_1R} = d_{S_2R}$ and the path-loss exponent is $\varpi = 3.5$.

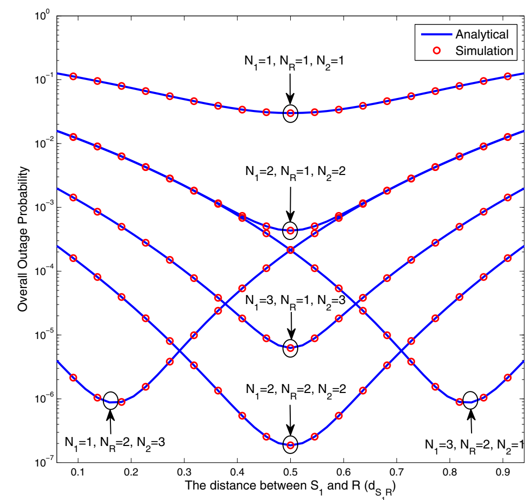

Fig. 3. The overall outage probability verses the relay location. Here, ζ_1 and ζ_2 are modeled as $\zeta_1 = (d_{S_1R})^{-\varpi}$ and $\zeta_2 = (d_{S_2R})^{-\varpi}$, where $\varpi = 3.5$. The transmit SNRs at each terminal is 10.79 dB.

having a single relay. The analytical outage curves are plotted for several antenna set-ups at the two sources and the relay by using (6) and (8). In particular, the outage curve corresponding to single antenna terminals is plotted for comparison purposes in order to show the performance gains obtained by the proposed antenna selection strategy. The asymptotic outage curves, which are exact at high SNRs, clearly reveal the diversity order of the system and provide insights into practical two-way relay system designing. The exact agreement between the Monte-Carlo simulations and the analytical curves verifies the accuracy of our derivations.

In Fig. 2, the overall outage probability of MIMO AF TWRNs having multiple relays is plotted for several relay set-ups. The analytical curves are plotted by using (12) and (13). In order to show the performance gains of joint relay and antenna selection, a fixed antenna set-up ($N_1 = N_R = N_2 = 2$) is considered. Fig. 2 clearly illustrates the performance gains of joint relay and Tx/Rx antenna selection of multi-relay TWRNs over their single-relay counterpart. Furthermore, the asymptotic outage curves verify our diversity order analysis. The Monte-Carlo simulations agree exactly with the analytical outage curves validating our analysis.

In Fig. 3, the effect of relay location on the overall outage probability is studied. Specifically, the outage probability is plotted against the distance between S_1 and R by modeling the path-loss dependent parameters ζ_1 and ζ_2 in (6) and (12) as $\zeta_1 = (d_{S_1R})^{-\varpi}$ and $\zeta_2 = (d_{S_2R})^{-\varpi}$ where $\varpi = 3.5$ is the path-loss exponent. Here, d_{S_1R} and d_{S_2R} are the distances between $S_1 \rightarrow R$ and $S_2 \rightarrow R$, respectively, and satisfy $d_{S_1R} + d_{S_1R} = 1$. In particular, Fig. 3 provides the following valuable insights: (i) when the antenna configuration at S_1, R and S_2 is symmetric (i.e., $N_1 = N$, $N_R = M$, $N_2 = N$), the optimal relay location, which minimizes the overall outage probability, is the half-way between S_1 and S_2, and (ii) when the antenna configuration at each terminal is asymmetric, this optimal relay location shifts toward the source, which has the lower number of antennas.

VII. Conclusion

A new Tx/Rx antenna selection strategy was proposed and analyzed for MIMO AF TWRNs based on the minimization of the overall outage probability. The performance of this transmission strategy was quantified by deriving the overall outage probability. The diversity order was derived by using the high SNR approximation of the outage probability. In particular, our results were extended for multi-relay MIMO AF TWRNs by proposing a joint relay and Tx/Rx antenna selection strategy. Our proposed selection strategies are optimal in the sense of outage probability and thus, in the sense of diversity order as well. Numerical results were provided to show the system performance and thereby to obtain valuable insights into practical two-way MIMO relay system designing.

VIII. Appendix

A. The proof of the CDF of the effective e2e SNR

$$\begin{aligned} F_Z(z) &= \Pr\left[Z = \max_{\substack{1\le j\le N_1, 1\le l\le N_2 \\ 1\le m\le N_R}}\left\{\min\left(\gamma_{S_1}^{(j,l,m)}, \gamma_{S_2}^{(j,l,m)}\right)\right\} \le z\right] \\ &= \Pr\left[\max_{1\le m\le N_R}\left\{\min\left(\gamma_{S_1}^{(J,L,m)}, \gamma_{S_2}^{(J,L,m)}\right)\right\} \le z\right], \end{aligned} \tag{16}$$

where $\gamma_{S_1}^{(j,l,m)}$ and $\gamma_{S_2}^{(j,l,m)}$ are defined in (3a) and (3b).

$$\begin{aligned} P_1(z) &= \Pr\left[\{X_m \le \beta z\} \cap \{Y_m \le X_m\}\right] + \Pr\left[\left\{Y_m \le \frac{z(\alpha X_m + \eta)}{X_m - \beta z}\right\} \cap \{Y_m \le X_m\}\right] \\ &= \int_{x=0}^{\beta z}\int_{y=0}^{x} f_{X_m}(x)\, f_{Y_m}(y)\,\mathrm{d}y\mathrm{d}x + \int_{t=0}^{\infty} \Pr\left[Y_m \le \min\left(\frac{z(\alpha(t+\beta z)+\eta)}{t}, t+\beta z\right)\right] f_{X_m}(t+\beta z)\,\mathrm{d}t \\ &= \underbrace{\int_{x=0}^{\beta z}\int_{y=0}^{x} f_{X_m}(x)\, f_{Y_m}(y)\,\mathrm{d}y\mathrm{d}x}_{\mathcal{I}_1(z)} + \underbrace{\int_{t=0}^{\phi(z)} F_{Y_m}(t+\beta z)\, f_{X_m}(t+\beta z)\,\mathrm{d}t}_{\mathcal{I}_2(z)} + \underbrace{\int_{t=\phi(z)}^{\infty} F_{Y_m}\left(\frac{z(\alpha(t+\beta z)+\eta)}{t}\right) f_{X_m}(t+\beta z)\,\mathrm{d}t}_{\mathcal{I}_3(z)} \end{aligned} \quad (15)$$

Further, $\gamma_{S_2}^{(J,L,m)}$ and $\gamma_{S_2}^{(J,L,m)}$ are given by

$$\gamma_{S_1}^{(J,L,m)} = \max_{1\le j\le N_1, 1\le l\le N_2}\left(\gamma_{S_1}^{(j,l,m)}\right) = \frac{X_m Y_m}{\alpha X_m + \beta Y_m + \eta} \quad \text{and}$$
$$\gamma_{S_2}^{(J,L,m)} = \max_{1\le j\le N_1, 1\le l\le N_2}\left(\gamma_{S_2}^{(j,l,m)}\right) = \frac{X_m Y_m}{\beta X_m + \alpha Y_m + \eta}, \quad (17)$$

where $X_m = |h_{S_1R}^{(m,J)}|^2 = \max_{1\le j\le N_1}\left(|h_{S_1R}^{(m,j)}|^2\right)$, and $Y_m = |h_{S_2R}^{(m,L)}|^2 = \max_{1\le l\le N_2}\left(|h_{S_2R}^{(m,L)}|^2\right)$. Here, $\alpha = \frac{\bar{\gamma}_S+\bar{\gamma}_R}{\bar{\gamma}_S\bar{\gamma}_R}$, $\beta = \frac{1}{\bar{\gamma}_R}$ and $\eta = \frac{1}{\bar{\gamma}_S\bar{\gamma}_R}$, where $\bar{\gamma}_S = \frac{\mathcal{P}_S}{\sigma_S^2}$ and $\bar{\gamma}_R = \frac{\mathcal{P}_R}{\sigma_R^2}$ are the average transmit SNRs at the source nodes and the relay[10].

Now, we define $Z_m = \min\left(\gamma_{S_1}^{(J,L,m)}, \gamma_{S_2}^{(J,L,m)}\right)$ and simplify it as follows[11]:

$$Z_m = \begin{cases} \gamma_{S_1}^{(J,L,m)}, & Y_m \le X_m \\ \gamma_{S_2}^{(J,L,m)}, & Y_m > X_m. \end{cases} \quad (18)$$

Next, the CDF of Z_m can be derived as follows:

$$F_{Z_m}(z) = \Pr\left[Z_m \le z\right] = P_1(z) + P_2(z), \quad \text{where} \quad (19a)$$

$$P_1(z) = \Pr\left[\left\{\gamma_{S_1}^{(J,L,m)} \le z\right\} \cap \{Y_m \le X_m\}\right] \quad \text{and} \quad (19b)$$

$$P_2(z) = \Pr\left[\left\{\gamma_{S_2}^{(J,L,m)} \le z\right\} \cap \{X_m < Y_m\}\right]. \quad (19c)$$

After some manipulations, the probability $P_1(z)$ can be expressed in a more mathematically tractable form as shown in (15) in the top of this page.

In (15), $F_{X_m}(x)$ and $F_{Y_m}(y)$ are the CDFs of X_m and Y_m (17), respectively, and given by

$$F_{X_m}(x) = \left[F_{|h^{(m,j)}|^2}(x)\right]^{N_1} = \sum_{p=0}^{N_1}\binom{N_1}{p}(-1)^p \mathrm{e}^{-\frac{px}{\zeta_1}} \quad \text{and}$$

$$F_{Y_m}(y) = \left[F_{|h^{(m,l)}|^2}(y)\right]^{N_2} = \sum_{q=0}^{N_2}\binom{N_2}{q}(-1)^q \mathrm{e}^{-\frac{qy}{\zeta_2}}. \quad (20)$$

In (15), $f_{X_m}(x)$ and $f_{Y_m}(y)$ denote the PDFs of X_m and Y_m, respectively, and derived readily by differentiating (20).

[10]Without loss of generality, the transmit powers and the AWGN noise variances at the both S_1 and S_2 is assumed to be identical, i.e., $\mathcal{P}_{S_1} = \mathcal{P}_{S_2} = \mathcal{P}_S$ and $\sigma_{S_1}^2 = \sigma_{S_2}^2 = \sigma_S^2$.

[11]It is important to note that the random variables $\gamma_{S_1}^{(J,L,m)}$ and $\gamma_{S_2}^{(J,L,m)}$ are not statistically independent.

By substituting (20), $f_{X_m}(x)$ and $f_{Y_m}(y)$ into (15), $\mathcal{I}_1(z)$ and $\mathcal{I}_2(z)$ can be evaluated exactly in closed-form as given in first and second terms of (7). Specifically, $\mathcal{I}_3(z)$ is mathematically intractable to be exactly solved. However, it can be evaluated approximately by using either the Gauss Laguerre quadrature (GLQ) rule [16] or infinite series expansion as given in (6).

Now, by following similar steps to those of P_1, P_2 in (19c) can be evaluated readily. Then the CDF of Z_m can be derived as $F_{Z_m}(z) = P_1 + P_2$, which yields the desired result $F_Z(z) = (F_{Z_m}(z))^{N_R}$ (6).

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