

# Spectrum Sensing in Low SNR: Diversity Combining and Cooperative Communications

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**Abstract**—In this paper, the detection performance of an energy detector used for cooperative spectrum sensing in cognitive radio networks is investigated under very low signal-to-noise ratio (SNR) levels. The analysis focuses on the derivation of closed-form expressions for the false-alarm and the average missed-detection probabilities for two cases over different fading channels: (i) with diversity combining; and (ii) with cooperative communications. The detection threshold is optimized by minimizing the total error rate. The analysis is validated by numerical and semi-analytical Monte-Carlo simulation results, which focus on the sensing requirements defined in IEEE 802.22.

**Index Terms**—Cognitive radio, cooperative communications, diversity combining, energy detection, spectrum sensing.

## I. INTRODUCTION

The concept of cognitive radio has been introduced to alleviate the spectrum under-utilization problem of wireless communications. One of the most challenging tasks in cognitive radio networks is spectrum sensing, which is required to opportunistically access the idle radio spectrum. Among several spectrum sensing techniques such as matched filter, cyclostationary feature detection and eigenvalue detection, the energy detection has gained renewed interests in recent research efforts due to the low complexity and low implementation cost [1]. The test statistic of an energy detector which is proportional to the received signal energy is compared with a threshold to make the decision on radio spectrum availability.

When the transmitted signal is assumed to be deterministic, a basic mathematical model of the test statistic and the performance of the energy detector have been analyzed in [2]–[6]. When the transmitted signal is assumed to be random, the decision statistic is modeled as a Gaussian process using the central limit theorem (CLT) [7], [8]. However, a theoretical performance analysis is not available. Thus, this paper is focused on the theoretical analysis for detection of random signal, using the basic results introduced in [9].

The IEEE 802.22 wireless regional area networks (WRANs) proposal recommends allowable false alarm probability and missed-detection probability be both less than 0.1 with the receiver sensitivity being -116dBm [10]–[13], and maximum sensing time be less than 2 seconds. These performance metrics reflect the overall efficiency and reliability of the cognitive network at very low SNR. However, an energy detector performs poorly at low SNR due to noise uncertainty. For low SNR spectrum sensing, a basic analytical model is given in [9] which provides theoretical analysis for performance of energy

detection at low SNR, and determines the optimal detection threshold so as to meet the stringent IEEE 802.22 WRAN requirements.

Using optimal detection threshold analysis in [9], it shows that the recommended error rate requirements cannot be achieved even at the optimal threshold value in traditional wireless networks. This is a main drawback in spectrum sensing in low SNR. In this paper, we address this problem using diversity combining and/or cooperative communications. The false alarm and average missed-detection probabilities are derived, and the optimal threshold selection is also discussed with numerical examples for low SNR region over different fading channels.

The rest of this paper is organized as follows. Section II discusses basics of energy detection. Spectrum sensing with diversity combining and cooperative communications are considered in Section III. Section IV presents numerical and simulation results, followed by concluding remarks in Section V.

## II. ENERGY DETECTION

The energy detector decides whether the primary signal is present or not from the received signal,  $\mathbf{y}(t) = \theta \mathbf{h} \mathbf{s}(t) + \mathbf{w}(t)$ , which follows a binary hypothesis:  $\mathcal{H}_0$  (signal absent,  $\theta = 0$ ) and  $\mathcal{H}_1$  (signal present,  $\theta = 1$ ). Here,  $\mathbf{h}$ ,  $\mathbf{s}(t)$  and  $\mathbf{w}(t)$  denote the wireless channel gain, the primary signal and the additive white Gaussian noise (AWGN). The output of the integrator of the energy detector is the test statistic which is proportional to the received signal energy, given as

$$\Lambda(\mathbf{y}) = \sum_{n=1}^N |\mathbf{y}(n)|^2 \quad (1)$$

where  $N$  is the number of samples and  $\mathbf{y}(n)$  is the  $n$ th sample. The test statistic is then compared with a threshold  $\lambda$ , and the detection decision is that the primary signal is present if  $\Lambda(\mathbf{y}) > \lambda$ , or the primary signal is absent otherwise.

The AWGN samples  $\mathbf{w}(n)$  are assumed to be independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian (CSCG) random variables with mean zero and variance  $\mathbb{E}[|\mathbf{w}(n)|^2] = \sigma_w^2$  where  $\mathbb{E}[\cdot]$  stands for expectation. The signal samples  $\mathbf{s}(n)$  are from an i.i.d. random process with mean zero and variance  $\mathbb{E}[|\mathbf{s}(n)|^2] = \sigma_s^2$ . For a sufficiently large number of samples  $N$ , the statistics of  $\Lambda(\mathbf{y})$  can be obtained using CLT. Thus, the probability density function (PDF) of  $\Lambda(\mathbf{y})$  under  $\mathcal{H}_0$ ,  $f_{\Lambda|\mathcal{H}_0}(x)$ , is a normal distribution with mean  $N\sigma_w^2$  and variance  $N\sigma_w^4$ . For given  $|\mathbf{h}|$ , the PDF of  $\Lambda(\mathbf{y})$  under  $\mathcal{H}_1$ ,  $f_{\Lambda|\mathcal{H}_1,|\mathbf{h}|}(x)$ , is a normal distribution with

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mean  $N\sigma_w^2(1+\gamma)$  and variance  $N\sigma_w^4(1+2\gamma)$  for a complex-valued phase-shift keying (PSK) signal where  $\gamma = \frac{|h|^2\sigma_s^2}{\sigma_w^2}$  is the instantaneous receive SNR. Therefore, the false alarm probability  $P_f$ , and the missed-detection probability  $P_{\text{md}}(\gamma)$ , can be evaluated as [7, eqs. (7) and (10)].

#### A. Low SNR Model

Under the low SNR assumption (i.e.,  $\sigma_s^2 \approx \sigma_w^2$ ), the signal has a little impact on the variance of the test statistic under  $\mathcal{H}_1$ . Thus, the PDF  $f_{\Lambda|\mathcal{H}_1,|h|}(x)$  can be approximated as a normal distribution with mean  $N\sigma_w^2(1+\gamma)$  and variance  $N\sigma_w^4$ . This approximation is used in the rest of this paper. Therefore,  $P_f$  and  $P_{\text{md}}(\gamma)$  can be derived as [9]

$$P_f = \frac{1}{2} \text{Erfc} \left( \frac{\lambda - N\sigma_w^2}{\sqrt{2N}\sigma_w^2} \right) \quad (2)$$

$$P_{\text{md}}(\gamma) = 1 - \frac{1}{2} \text{Erfc} \left( \frac{\lambda - N\sigma_w^2(1+\gamma)}{\sqrt{2N}\sigma_w^2} \right), \quad (3)$$

respectively, where  $\text{Erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-t^2} dt$  is the complementary error function [14]. In [9], the accuracy of the approximation is confirmed by comparing approximated and exact cumulative distribution function (CDF) and receiver operating characteristic (ROC) curves.

#### B. Average Missed-Detection Probability

The average missed-detection probability over AWGN channel is in the form of (3) when  $\gamma$  is replaced by the average SNR,  $\bar{\gamma}$ , where  $\bar{\gamma} = \mathbb{E}[\gamma]$ . However, the received signal is affected by the fluctuations of the propagation channel due to path loss, large-scale fading and small-scale fading. When the SNR distribution is  $f_\gamma(\gamma)$ , the average missed-detection probability,  $\bar{P}_{\text{md}}$ , can be calculated as

$$\bar{P}_{\text{md}} = \int_0^\infty P_{\text{md}}(x) f_\gamma(x) dx.$$

The Rayleigh and Nakagami- $m$  fading channels, which can model a variety of fading effects, are considered in [9], in which the average missed-detection probability over Rayleigh fading channel,  $\bar{P}_{\text{md}}^{\text{Ray}}$ , is derived as

$$\begin{aligned} \bar{P}_{\text{md}}^{\text{Ray}} = & \frac{1}{2} \left[ \text{Erfc} \left( \frac{N\sigma_w^2 - \lambda}{\sqrt{2N}\sigma_w^2} \right) - e^{\frac{\frac{1}{\bar{\gamma}^2} + \frac{4}{\bar{\gamma}} \left( \frac{N\sigma_w^2 - \lambda}{\sqrt{2N}\sigma_w^2} \right) \sqrt{\frac{N}{2}}}} \right. \\ & \left. \times \text{Erfc} \left( \frac{N\sigma_w^2 - \lambda}{\sqrt{2N}\sigma_w^2} + \frac{1}{\bar{\gamma}\sqrt{2N}} \right) \right], \end{aligned} \quad (4)$$

and the average missed-detection probability over Nakagami- $m$  fading channel (for integer  $m$ ),  $\bar{P}_{\text{md}}^{\text{Nak}}$ , is derived as

$$\bar{P}_{\text{md}}^{\text{Nak}} = \left( \frac{m}{\bar{\gamma}} \right)^m \mathcal{I} \left( m-1, p, \sqrt{\frac{N}{2}}, \frac{N\sigma_w^2 - \lambda}{\sqrt{2N}\sigma_w^2} \right) \Bigg|_{p=\frac{m}{\bar{\gamma}}} \quad (5)$$

where  $\mathcal{I}(n, p, a, b) \triangleq (-1)^n \frac{\partial^n}{\partial p^n} \left[ \frac{\text{Erfc}(b) - e^{\frac{p^2 + 4pab}{4a^2}} \text{Erfc}(b + \frac{p}{2a})}{p} \right]$ .

In the following section, we use these results to derive the false alarm and the average missed-detection probabilities for diversity receptions and cooperative communications scenarios.

### III. PERFORMANCE ANALYSIS WITH DIVERSITY COMBINING AND COOPERATIVE COMMUNICATIONS

As shown in [9], it is very difficult to meet the sensing requirements of IEEE 802.22 WRANs proposal in low-SNR environment. Therefore, diversity combining and/or cooperative communications can be used to improve the detection performance, since the two techniques can improve end-to-end SNR and the effective number of test statistics at the detector at the cost of processing delay and implementation complexity. In this section, detection capability of these techniques is analyzed assuming low SNR environments.

#### A. Using Diversity Reception Techniques

In traditional diversity techniques such as maximal ratio combining (MRC), selection combining (SC), etc., the energy detector follows the combiner. For MRC or SC, the energy detector processes the samples of combined signal of  $L$  diversity branches as its test statistics. Thus, the effective number of samples for a test statistic,  $N$ , is independent of  $L$ . Further, these diversity techniques do not guarantee a significant SNR improvement with a moderate number of diversity branches in low SNR such -20dB. And also, a coherent combining technique such as MRC needs channel state information in non-coherent energy detection, which increases the design complexity. Due to these drawbacks of aforementioned diversity techniques, energy-law combining techniques such as square-law combining (SLC) and square-law selection (SLS) are more attractive for energy detection in cognitive radio. The energy-law combining techniques combine the energy of each branch to get the final test statistic. Therefore, the effective number of samples depends on  $L$  [4]. In what follows, the average missed-detection probabilities of SLC and SLS techniques are derived in low SNR.

1) *Square-Law Combining (SLC)*: Each diversity branch has a square-law device which performs the square-and-integrate operation. The combiner is implemented following the square-law operation. The decision statistic has  $LN$  effective samples from  $L$  diversity branches (each branch has  $N$  samples) which is given with a form of (1) as

$$\Lambda_{\text{SLC}}(\mathbf{y}) = \sum_{i=1}^L \Lambda(\mathbf{y}_i)$$

where  $\Lambda(\mathbf{y}_i)$  is the test statistic of the  $i$ th branch. With CLT, the PDF of  $\Lambda_{\text{SLC}}(\mathbf{y})$  under  $\mathcal{H}_0$  is a normal distribution with mean  $LN\sigma_w^2$  and variance  $LN\sigma_w^4$ , and the PDF of  $\Lambda_{\text{SLC}}(\mathbf{y})$  under  $\mathcal{H}_1$  is a normal distribution with mean  $LN\sigma_w^2(1+\gamma)$  and variance  $LN\sigma_w^4(1+2\gamma)$ . For low SNR, the false alarm probability, and the missed-detection probability under AWGN channels can be evaluated as (2) and (3), respectively, with  $N$  being replaced by  $LN$ . With the aid of (5), the average missed-detection probability,  $\bar{P}_{\text{md}}^{\text{SLC}}$ , over Nakagami- $m$  fading channels can be derived as

$$\bar{P}_{\text{md}}^{\text{SLC}} = \left( \frac{m}{\bar{\gamma}} \right)^m \mathcal{I} \left( m-1, p, \sqrt{\frac{LN}{2}}, \frac{LN\sigma_w^2 - \lambda}{\sqrt{2LN}\sigma_w^2} \right) \Bigg|_{p=\frac{m}{\bar{\gamma}}} \quad (6)$$

2) *Square-Law Selection (SLS)*: Similar to SLC, each diversity branch has a square-law operation before selection combining. In SLS, the branch with the maximum decision statistic is to be selected such as  $\mathbf{y}_{\text{SLS}} = \max\{\mathbf{y}_1, \dots, \mathbf{y}_L\}$  [4]. The CDF of  $\mathbf{y}_{\text{SLS}}$  can be written for independent decision statistics as

$$F_{\mathbf{y}_{\text{SLS}}}(x) = P(\mathbf{y}_{\text{SLS}} \leq x) = P(\mathbf{y}_1 \leq x, \dots, \mathbf{y}_L \leq x) \\ = \prod_{i=1}^L F_{\mathbf{y}_i}(x) \quad (7)$$

where  $F_{\mathbf{y}_i}(x)$  is the CDF of  $\mathbf{y}_i$ .

The false alarm probability is

$$P_f^{\text{SLS}} = P(\mathbf{y}_{\text{SLS}} \geq \lambda | \mathcal{H}_0) = 1 - P(\mathbf{y}_{\text{SLS}} \leq \lambda | \mathcal{H}_0) \\ = 1 - \prod_{i=1}^L F_{\mathbf{y}_i}(\lambda | \mathcal{H}_0) \\ = 1 - \prod_{i=1}^L (1 - P(\mathbf{y}_i \geq \lambda | \mathcal{H}_0)) \\ = 1 - (1 - P_f)^L \quad (8)$$

where  $P_f$  is given in (2). Similarly, under  $\mathcal{H}_1$ , the missed-detection probability can be written as

$$P_{\text{md}}^{\text{SLS}} = \prod_{i=1}^L F_{\mathbf{y}_i}(\lambda | \mathcal{H}_1) = \prod_{i=1}^L P(\mathbf{y}_i \leq \lambda | \mathcal{H}_1) = \prod_{i=1}^L P_{\text{md}}(\gamma_i)$$

where  $P_{\text{md}}(\gamma_i)$  is given in (3) with  $\gamma_i$  being the instantaneous SNR at the  $i$ th branch. If the SNR distribution of the  $i$ th branch is  $f_{\gamma_i}(\gamma_i)$ , the average missed-detection probability can be evaluated as  $\overline{P_{\text{md}}^{\text{SLS}}} = \prod_{i=1}^L \int_0^\infty P_{\text{md}}(x) f_{\gamma_i}(x) dx$ . For i.i.d. Nakagami- $m$  fading branches, i.e., each branch has the same average SNR, the average missed-detection probability,  $\overline{P_{\text{md}}^{\text{SLS}}}$  can be given as

$$\overline{P_{\text{md}}^{\text{SLS}}} = \left( \overline{P_{\text{md}}^{\text{Nak}}} \right)^L \quad (9)$$

where  $\overline{P_{\text{md}}^{\text{Nak}}}$  is given in (5).

### B. Using Cooperative Communications

Cooperative communications can improve the signal detection capability as sharing and combining the information of intermediate cognitive nodes (called cooperative nodes). As many previous work efforts are focused on the medium or high SNR region [15]–[17], the impact of cooperative communications in low SNR is not obvious. To investigate this case, we consider the decision fusion strategy in which each cooperative node makes a decision on the primary user activity, and the individual 1-bit decisions are reported to the fusion center. If there are  $K$  cooperative nodes and the fusion center has  $k$ -out-of- $K$  fusion rule (i.e., the fusion center decides the presence of primary activity if there are  $k$  or more cooperative nodes that individually decide the presence of primary activity), the false alarm and the average missed-detection probabilities of cooperative communications over Nakagami- $m$  fading channels are

$$P_f^{\text{Coop}} = \sum_{i=k}^K \binom{K}{i} (P_f)^i (1 - P_f)^{K-i}, \quad (10)$$

$$\overline{P_{\text{md}}^{\text{Coop}}} = 1 - \sum_{i=k}^K \binom{K}{i} \left( \overline{P_{\text{md},i}^{\text{Nak}}} \right)^i \left( 1 - \overline{P_{\text{md},i}^{\text{Nak}}} \right)^{K-i}, \quad (11)$$

respectively, where  $\overline{P_{\text{md},i}^{\text{Nak}}}$  is the average missed-detection probability at the  $i$ th cooperative node, which is equal to (5) with  $\bar{\gamma}$  being replaced by the  $i$ th channel average SNR  $\bar{\gamma}_i$ .

As shown in previous studies [17]–[19], since cooperative communications provide a higher diversity advantage, the detection capability and communication reliability are improved. Such a setup includes the reporting channels that are mutually orthogonal to avoid the inter-channel interference and data collision at the fusion center. Orthogonal channels are realized either by using frequency division (FDMA) or time division (TDMA) multiple access techniques. As FDMA requires larger frequency bandwidth, it is not a good solution for the frequency scarcity problem. In TDMA, each reporting channel requires different time slot which has duration  $\tau_r$ . If we neglect other processing delays at the cooperative nodes and the fusion center, the number of cooperative nodes that can participate in the cooperation is  $K \leq \left( \tau - \frac{N}{f_s} \right) \frac{1}{\tau_r}$  where  $\tau$  is the maximum allowable sensing time and  $f_s$  is the sampling frequency of the energy detector. Therefore,  $K$  is limited by the sensing time.

### C. Threshold Selection

The threshold,  $\lambda$ , which varies from 0 to  $\infty$  is a common parameter for the false alarm, the detection and the missed-detection probabilities which are denoted as  $P_f(\lambda)$ ,  $P_d(\lambda)$  and  $P_{\text{md}}(\lambda)$ , respectively. The traditional way of setting the threshold is based on the false alarm probability. Such a threshold selection does not always guarantee the requirements of IEEE 802.22 WRANs proposal in practice. In this paper, the optimal threshold  $\lambda^*$  is selected such that the total error rate defined as  $P_e(\lambda) \triangleq P_f(\lambda) + P_{\text{md}}(\lambda)$  is minimized. This is a possible way of selecting the threshold to satisfy both false alarm and missed-detection probability requirements [20].

As discussed in [9], the required error rates ( $\overline{P_f} \leq 0.1$  and  $\overline{P_{\text{md}}} \leq 0.1$ ) may not be achieved at the optimal threshold value for given  $N$  and  $\sigma_w$  in low SNR, but they can be achieved by increasing  $N$ . However, sensing time,  $\tau$ , is also increased when  $N$  is increased because  $N \approx \tau f_s$ . Therefore, a possible solution is to use a diversity combining or cooperative communications technique. For example, the optimal threshold in SLC can be derived as [9, eq. (8)] with  $N$  being replaced by  $LN$ , and it can be further approximated for low SNR as

$$\lambda^* \approx \frac{LN\sigma_w^2}{2} \left( 1 + \sqrt{1 + 2\gamma} \right) \approx LN\sigma_w^2$$

which implies that the sensing time can be reduced when the number of diversity branches increases. More numerical examples will be given in the next section.

## IV. NUMERICAL/SIMULATION RESULTS AND DISCUSSION

This section provides numerical and semi-analytical Monte-Carlo simulation results. We define the *normalized threshold* as  $\hat{\lambda} \triangleq \frac{\lambda}{N}$ , i.e., the threshold is normalized by the number of samples. We denote  $P_e^* = P_e(\lambda^*)$ ,  $P_f^* = P_f(\lambda^*)$ , and  $P_{\text{md}}^* = P_{\text{md}}(\lambda^*)$  where  $\lambda^*$  is the optimal threshold.

Fig. 1 shows the ROC curves of (i) SLC, SLS combining techniques, and (ii) a cooperative spectrum sensing network.

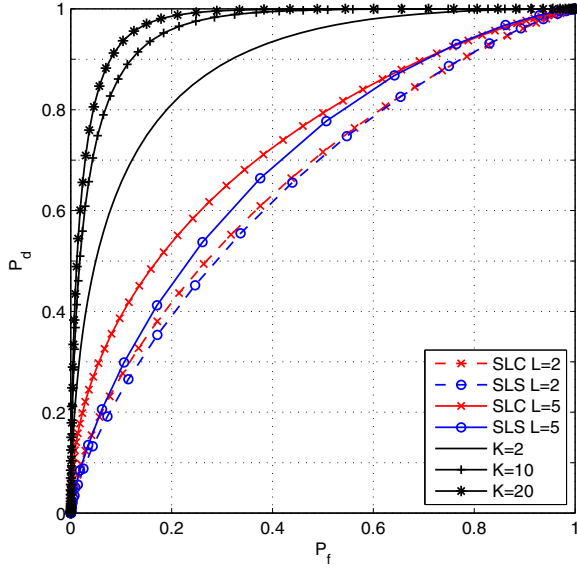


Fig. 1: ROC curves for SLC and SLS diversity techniques and cooperative communications.

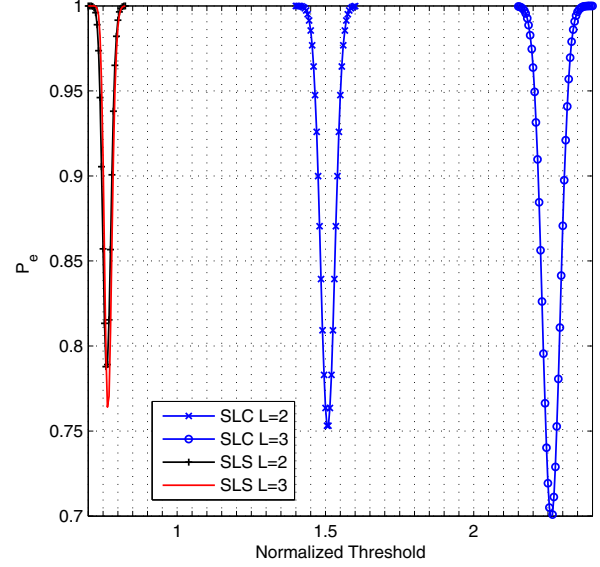


Fig. 3: Total error rate versus normalized threshold for diversity techniques (SLS and SLC).

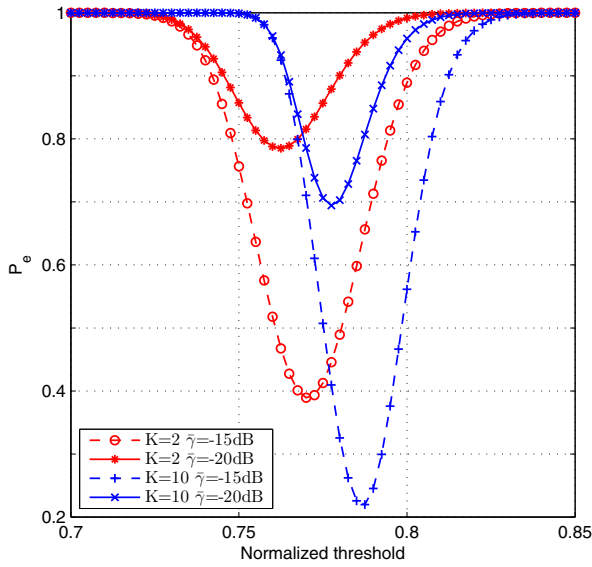
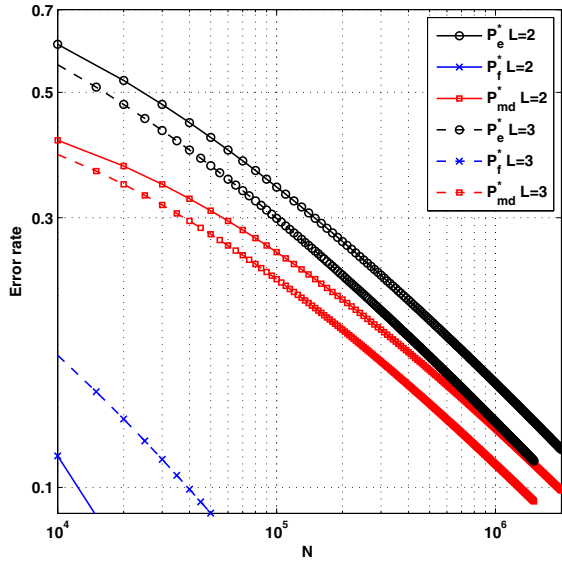


Fig. 2: Total error rate versus normalized threshold for cooperative communications.

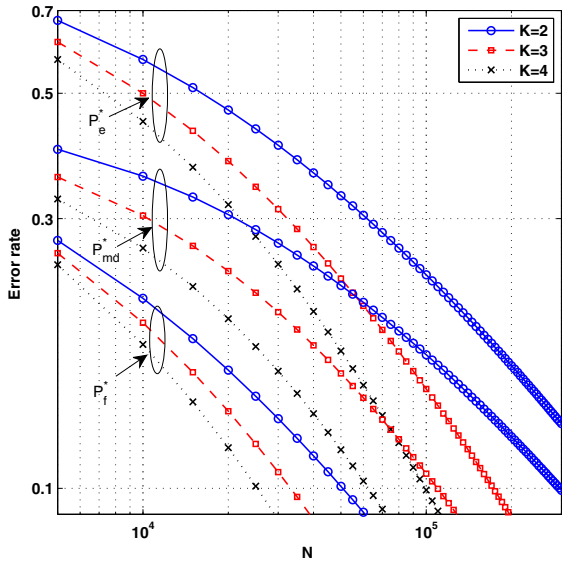
For SLC and SLS combining techniques with  $\bar{\gamma} = -20\text{dB}$ ,  $\sigma_w^2 = 1$ , and  $N = 2 \times 10^3$  per branch over Nakagami-2 fading, the detection capability is significantly increased with  $L$  due to the effect of diversity advantage. The detection performance difference between SLC and SLS is not clear-cut because two ROC curves of SLC and SLS cross each other for a given  $L$ . The impacts of the number of cooperative nodes ( $K$ ) on the detection capability are shown when  $\bar{\gamma} = -20\text{dB}$ ,  $\sigma_w^2 = 1$ , and  $N = 2 \times 10^4$  over AWGN channels. It clearly shows that larger  $K$  improves detection performance.

The total error rate versus normalized threshold for a cooperative sensing network over AWGN channel is shown in Fig. 2 when  $\bar{\gamma} = -15\text{dB}$ ,  $-20\text{dB}$ ,  $\sigma_w^2 = 0.75$  and  $N = 2 \times 10^3$ . The optimal threshold is  $\hat{\lambda}^* \approx 0.76$ ,  $0.77$  when  $K = 2$  for  $\bar{\gamma} = -20\text{dB}$ ,  $-15\text{dB}$ , respectively. Thus, the approximation  $\hat{\lambda}^* \approx \sigma_w^2$  is valid for the cooperative case with small  $K$ . However, when  $K$  is increased, this approximation is no longer valid tightly because the effective SNR is increased considerably with  $K$ , and therefore, it has a significant contribution for  $\hat{\lambda}^*$ , e.g.,  $\hat{\lambda}^* \approx 0.78$  for  $K = 10$ . The total error rate versus normalized threshold for SLC and SLS diversity reception techniques over AWGN channel is shown in Fig. 3 when  $\bar{\gamma} = -20\text{dB}$ ,  $L = 2, 3$ ,  $\sigma_w^2 = 0.75$  and  $N = 2 \times 10^3$ . The optimal threshold values for SLS are  $\hat{\lambda}^* \approx 0.762$  for  $L = 2$  and  $\hat{\lambda}^* \approx 0.767$  for  $L = 3$ . In this case, the approximation  $\hat{\lambda}^* \approx \sigma_w^2$  is also valid. On the other hand, for SLC, the optimal threshold values are  $\hat{\lambda}^* \approx 1.507$  for  $L = 2$  and  $\hat{\lambda}^* \approx 2.261$  for  $L = 3$  which confirms that  $\hat{\lambda}^* \approx L\sigma_w^2$ .

Fig. 4 shows error rates at the optimal threshold ( $P_e^*$ ,  $P_f^*$ , and  $P_{\text{md}}^*$ ) versus  $N$ . Fig. 4a is for SLC with  $L = 2, 3$  over Rayleigh fading channels at  $\bar{\gamma} = -20\text{dB}$ . We chose Rayleigh channel because Nakagami- $m$  ( $m > 1$ ) and AWGN channel models require a smaller number of samples than in the Rayleigh channel. As in Fig. 4a,  $P_f$  requirement ( $P_f \leq 0.1$ ) can be achieved when  $N \geq 1.2 \times 10^4$  and  $N \geq 4.0 \times 10^4$ , and  $P_{\text{md}}$  requirement ( $P_{\text{md}} \leq 0.1$ ) can be achieved when  $N \geq 1.9 \times 10^6$  and  $N \geq 1.3 \times 10^6$  for  $L = 2$  and  $L = 3$ , respectively. Therefore, to meet both  $P_f$  and  $P_{\text{md}}$  requirements, we should have  $N \geq 1.9 \times 10^6$  and  $N \geq 1.3 \times 10^6$  for  $L = 2$  and  $L = 3$ , respectively. Thus the required sampling rates ( $f_s = \frac{N}{\tau}$  with  $\tau \leq 2$  sec) are at least 970kHz and 647.5kHz for  $L = 2$  and  $L = 3$ , respectively. Fig. 4b is for cooperative communications with  $K = 2, 3$  and 4 over Rayleigh fading channels at  $\bar{\gamma} = -20\text{dB}$ . The  $P_f$  requirement can be achieved when  $N \geq 5.2 \times 10^4$ ,



(a)



(b)

Fig. 4: Error rates at the optimal threshold value (total error:  $P_e^*$ ; false alarm:  $P_f^*$ ; and missed-detection:  $P_{md}^*$ ) versus number of samples  $N$  for (a) SLC diversity technique; (b) cooperative spectrum sensing.

$N \geq 3.5 \times 10^4$  and  $N \geq 2.7 \times 10^4$ , and  $P_{md}$  requirement can be achieved when  $N \geq 3.0 \times 10^5$ ,  $N \geq 1.1 \times 10^5$  and  $N \geq 6.3 \times 10^4$  for  $K=2, 3$  and  $4$ , respectively. To meet both  $P_f$  and  $P_{md}$  requirements, we should have  $N \geq 3.0 \times 10^5$ ,  $N \geq 1.1 \times 10^5$  and  $N \geq 6.3 \times 10^4$  for  $K=2, 3$  and  $4$ .

## V. CONCLUSION

The low SNR detection performance of the energy detector is studied for spectrum sensing in cognitive radio networks where diversity combining and cooperative communications

are considered. The false-alarm and the average missed-detection probabilities are derived over fading channels. As minimizing the total error rate, the optimal detection threshold is also determined using numerical analysis. It is shown that increasing the number of samples by increasing the diversity order (through diversity combining or cooperative communications) is a possible way to meet the fundamental sensing requirements specified in IEEE 802.22 WRANs proposal, which are on channel detection time ( $\leq 2$  seconds), false alarm probability ( $\leq 0.1$ ) and missed-detection probability ( $\leq 0.1$ ) at low SNR (-20dB).

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