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Spectrum Sensing via Energy Detector in Low SNR

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***Abstract*—As required in the IEEE 802.22 proposal, spectrum sensing techniques should be capable enough to sense the primary signal with very low receiver sensitivity such as at -116 dBm. In this paper, the detection performance of an energy detector used for spectrum sensing in cognitive radio networks is investigated under such very low signal-to-noise ratio (SNR) levels. The analysis focuses on the derivation of a closed-form expression for the average missed-detection probability over Rayleigh fading and Nakagami-m fading channels. Subsequently, the detection threshold is optimized for minimizing the total error rate. The analysis is validated by numerical and simulation results. The sensing requirements defined in IEEE 802.22 are also discussed with numerical examples.**

***Index Terms*—Cognitive radio, energy detection, spectrum sensing, threshold selection.**

I. Introduction

One of the most challenging tasks in cognitive radio networks is spectrum sensing. In the IEEE 802.22 wireless regional area networks (WRAN) proposal, no specific spectrum sensing technique is given. So designers have freedom to select any spectrum sensing technique to meet the specified sensing requirements [1]. Among the available spectrum sensing techniques such as matched filter, cyclostationary feature detection and eigenvalue detection, *energy detection* has gained renewed interests in recent research efforts due to its low complexity. The conventional energy detector measures the energy associated with the received signal over a specified time period and a bandwidth. The decision of an energy detector is feasible even when little prior knowledge of the transmitted signal is available. The decision statistic of an energy detector is a measure of the received signal energy after proper filtering, sampling, squaring and integration.

Assuming a deterministic signal is transmitted over a flat band-limited Gaussian noise channel, a basic mathematical model of the decision statistic is given in [2] in order to calculate the detection probability (P_d) and false alarm probability (P_f). Subsequently, the performance of an energy detector in terms of the average detection probability, the receiver operating characteristic (ROC), and the area under the ROC curve (AUC) over different fading channels, diversity techniques and cooperative relay networks has been analyzed in [3]–[10]. In spectrum sensing of cognitive radio networks, the secondary user has no *a priori* knowledge of the primary signal. The information bearing signal can have different possible waveforms with random data sequences. Therefore, it is appropriate to treat the received signal samples as a random process. When both signal and noise follow Gaussian processes, the decision statistic is modeled with a Gaussian distribution using the central limit theorem (CLT) [11]–[13].

The Gaussian model is popular in the parameter optimization problems, e.g., optimizing the operating threshold or the power allocation so as to achieve the maximal throughput or minimal error rate. This model often gives a more convenient cost function which may result in a convex optimization problem. However, based on the Gaussian model, the analysis of the average detection performance of an energy detector over different fading scenarios is not available in the open literature because of the involved mathematical complexity. The existing analytical results are limited to the additive white Gaussian noise (AWGN) channel, and performance over other fading scenarios are obtained only by simulations.

In this paper, we derive the average missed-detection probability of an energy detector in low signal-to-noise ratio (SNR) region over Rayleigh fading and Nakagami-m fading channels. The low SNR assumption is fairly reasonable because, as in IEEE 802.22 WRAN, the spectrum sensing technique should be able to detect the primary signal with the missed-detection and the false alarm probabilities less than 0.1 and the receiver sensitivity being -116 dBm [1], [14], [15]. More importantly, we determine the optimal detection threshold of the energy detector to minimize the total error rate. Based on the analytical results, some numerical examples are given to meet the IEEE 802.22 WRAN requirements.

The rest of this paper is organized as follows. Section II briefly discusses energy detection and its low SNR model. Section III gives the average missed-detection probability. Sections IV is devoted to the analysis of the optimal detection threshold. Section V presents numerical and simulation results, followed by concluding remarks in Section VI.

II. Energy Detection and Low SNR Approximation

The spectrum sensing in cognitive radio networks follows a binary hypothesis testing problem: hypothesis $\mathcal{H}_0$ (signal absent) and hypothesis $\mathcal{H}_1$ (signal present). The received signal for the binary hypothesis can be given as

$$\mathbf{y}(t) = \begin{cases} \mathbf{w}(t) & : \mathcal{H}_0 \\ \mathbf{h}\mathbf{x}(t) + \mathbf{w}(t) & : \mathcal{H}_1 \end{cases}$$

where $\mathbf{x}(t)$ is the transmitted signal, $\mathbf{h}$ is the wireless channel gain, and $\mathbf{w}(t)$ is the additive white Gaussian noise (AWGN) which is assumed to be a circularly symmetric complex Gaussian (CSCG) random variable with mean zero and variance σ_w^2. The conventional analog energy detector consists of a pre-filter followed by a square-law device and a finite time integrator. The output of the integrator is called the decision statistic. It may be proportional to the received signal energy and can be given as $\Lambda(\mathbf{y}) = \sum_{n=1}^{N} |\mathbf{y}(n)|^2$ where N is the number of samples. When the signal has an unknown form, it is appropriate to treat the signal samples as a random process. Thus, the sample of the transmitted signal $\mathbf{x}(n)$ follows an independent and identically distributed (i.i.d.) random process with mean zero and variance σ_s^2. Then, the received SNR at the detector is $\gamma = \frac{|\mathbf{h}|^2\sigma_s^2}{\sigma_w^2}$ for the given channel $\mathbf{h}$. When the sample number N is large enough, using CLT, the probability density function (PDF) of $\Lambda(\mathbf{y})$ under $\mathcal{H}_0$, $f_\Lambda(\mathbf{y}, \mathcal{H}_0)$, is a normal distribution with $N\sigma_w^2$ mean and $N\sigma_w^4$ variance. Similarly, the PDF of $\Lambda(\mathbf{y})$ under $\mathcal{H}_1$, $f_\Lambda(\mathbf{y}, \mathcal{H}_1)$, is a normal distribution with $N\sigma_w^2(1+\gamma)$ mean and $N\sigma_w^4(1+2\gamma)$ variance for a complex-valued phase-shift keying (PSK) modulated signal. The false alarm probability, P_f, and the detection probability for given $\mathbf{h}$, $P_{d|\mathbf{h}}(\gamma)$, are given as

$$P_f = Q\left(\left(\frac{\lambda}{N\sigma_w^2} - 1\right)\sqrt{N}\right) \tag{1}$$

$$P_{d|\mathbf{h}}(\gamma) = Q\left(\left(\frac{\lambda}{N\sigma_w^2} - 1 - \gamma\right)\sqrt{\frac{N}{(1+2\gamma)}}\right), \tag{2}$$

respectively, where λ is the threshold and $Q(\cdot)$ is the standard Q-function.

Since IEEE 802.22 WRAN standard is interested in the spectrum sensing in very low SNR region (<-20 dB), we consider a low SNR energy detection model. Under the low SNR assumption (i.e., $\sigma_s^2 \approx \sigma_w^2$), the signal has a little impact on the variance of the decision statistic under $\mathcal{H}_1$. Therefore, PDF $f_\Lambda(\mathbf{y}, \mathcal{H}_1)$ is Gaussian distributed with $N\sigma_w^2(1+\gamma)$ mean and $N\sigma_w^4$ variance. Fig. 1 shows cumulative distribution function (CDF) of the exact and its low SNR approximation for three different low SNR values when $\sigma_w = 1$ and $N = 2000$. The approximated and the exact CDFs are very close. Therefore, for low SNR, P_f and $P_{d|\mathbf{h}}(\gamma)$ can be given in alternative forms as

$$P_f = \frac{1}{2}\mathrm{Erfc}\left(\frac{\lambda - N\sigma_w^2}{\sqrt{2N}\sigma_w^2}\right)$$

$$P_{d|\mathbf{h}}(\gamma) = \frac{1}{2}\mathrm{Erfc}\left(\frac{\lambda - N\sigma_w^2(1+\gamma)}{\sqrt{2N}\sigma_w^2}\right), \tag{3}$$

respectively, where $\mathrm{Erfc}(\cdot)$ is the complementary error function which is defined as $\mathrm{Erfc}(z) = \frac{2}{\sqrt{\pi}}\int_z^\infty e^{-t^2}dt$ and we have $Q(z) = \frac{1}{2}\mathrm{Erfc}\left(\frac{z}{\sqrt{2}}\right)$.

III. Average Missed-Detection Probability

The instantaneous detection probability given in (3) is equivalent to the average detection probability over AWGN channel when γ is replaced by the average SNR $\bar{\gamma}$. Here, we are interested in the missed-detection probability (as it is one requirement in IEEE 802.22 WRAN), $P_{md|\mathbf{h}}(\gamma)$, which is given as $P_{md|\mathbf{h}}(\gamma) = 1 - P_{d|\mathbf{h}}(\gamma)$. When the SNR distribution is $f_\gamma(x)$, the average missed-detection probability, $\overline{P_{md}}$, is $\overline{P_{md}} = \int_0^\infty P_{md|\mathbf{h}}(x) f_\gamma(x)dx$. We consider Rayleigh

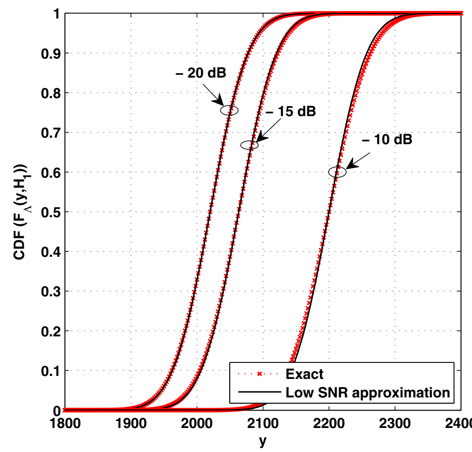

Fig. 1. CDF of the decision statistic under hypothesis $\mathcal{H}_1$.

fading and Nakagami-m fading channels, which can model a variety of fading effects. If the signal amplitude follows a Nakagami-m distribution, the SNR distribution is given as $f_{\gamma^{Nak}}(x) = \frac{\left(\frac{m}{\bar{\gamma}}\right)^m}{\Gamma(m)} x^{m-1} e^{-\frac{m}{\bar{\gamma}}x}$ [16] where m is the fading parameter, and it follows Rayleigh fading when $m = 1$. With the aid of the identity $\mathrm{Erfc}(-x) = 2 - \mathrm{Erfc}(x)$, and after some algebraic manipulations, the average missed-detection probability over Nakagami-m fading channel, $\overline{P_{md}^{Nak}}$, can be given as

$$\overline{P_{md}^{Nak}} = \frac{\left(\frac{m}{\bar{\gamma}}\right)^m}{2\Gamma(m)} \int_0^\infty x^{m-1} e^{-\frac{mx}{\bar{\gamma}}} \mathrm{Erfc}\left(\sqrt{\frac{N}{2}}x + \frac{N\sigma^2 - \lambda}{\sqrt{2N}\sigma^2}\right)dx. \tag{4}$$

In the following, we define an integral expression, $\mathcal{I}(n, p, a, b)$, as [17, eq. (2.8.9.1)]

$$\mathcal{I}(n,p,a,b) \triangleq \int_0^\infty x^n e^{-px}\mathrm{Erfc}(ax+b)dx$$
$$= (-1)^n \frac{\partial^n \left[\frac{\mathrm{Erfc}(b) - e^{\frac{p^2+4pab}{4a^2}}\mathrm{Erfc}\left(b+\frac{p}{2a}\right)}{p}\right]}{\partial p^n}$$

where n is a positive integer, $\mathrm{Re}[p] > 0$, $c > 0$, and $\frac{\partial^n}{\partial p^n}[\cdot]$ is the nth order partial derivative with respect to p. For integer m, $\overline{P_{md}^{Nak}}$ given in (4) can be evaluated as

$$\overline{P_{md}^{Nak}} = \frac{\left(\frac{m}{\bar{\gamma}}\right)^m}{2\Gamma(m)} \mathcal{I}\left(m-1, p, \sqrt{\frac{N}{2}}, \frac{N\sigma^2 - \lambda}{\sqrt{2N}\sigma^2}\right)\Bigg|_{p=\frac{m}{\bar{\gamma}}}. \tag{5}$$

When $m = 1$, the average missed-detection probability over Rayleigh fading channel, $\overline{P_{md}^{Ray}}$, is

$$\overline{P_{md}^{Ray}} = \frac{1}{2}\Bigg[\mathrm{Erfc}\left(\frac{N\sigma^2 - \lambda}{\sqrt{2N}\sigma^2}\right) - e^{\frac{\frac{1}{\bar{\gamma}^2} + \frac{4}{\bar{\gamma}}\left(\frac{N\sigma^2-\lambda}{\sqrt{2N}\sigma^2}\right)\sqrt{\frac{N}{2}}}{2N}}$$
$$\times\, \mathrm{Erfc}\left(\frac{N\sigma^2 - \lambda}{\sqrt{2N}\sigma^2} + \frac{1}{\bar{\gamma}\sqrt{2N}}\right)\Bigg]. \tag{6}$$

When a random signal is present, the results in (5) and (6) are novel closed-form expressions for the average missed-detection probability.

IV. Threshold Selection

The threshold, λ, which varies form 0 to ∞ is a common parameter for the false alarm, the detection and the missed-detection probabilities which are denoted as $P_f(\lambda)$, $P_d(\lambda)$ and $P_{md}(\lambda)$, respectively. The common practice of setting the threshold is based on the false alarm probability. For given N, σ_w, and the constant false alarm probability (CFAP=$\bar{P}_f$), the selected threshold is $\lambda = \left(\sqrt{2}\mathrm{Erfc}^{-1}(2\bar{P}_f) + \sqrt{N}\right)\sqrt{N}\sigma_w^2$. Although achieving a high $P_d(\lambda)$ while keeping $P_f(\lambda)$ low is preferable (e.g., in IEEE 802.22 WRAN recommendations, $\bar{P}_f \leq 0.1$ and $\bar{P}_d \geq 0.9$), such a threshold selection is not always possible in practice. Therefore, the threshold selection can be viewed as an optimization problem. Some research work has been done for this problem based on different objectives [11], [18]–[23]. The minimization of the total error rate which is defined as $P_e(\lambda) \triangleq P_f(\lambda) + P_{md}(\lambda)$ is a possible way of selecting λ [18]. We consider different fading scenarios in the following subsections, and the optimal threshold, λ^*, is derived such that the total error is minimized, i.e., $\lambda^* = \arg\min\limits_{\lambda} P_e(\lambda)$.

A. AWGN Channel

For an AWGN fading channel, the optimal threshold is given using (1) and (2) as

$$\lambda^* = \arg\min_{\lambda}\left(1+\frac{1}{2}\mathrm{Erfc}\left(\frac{\lambda - N\sigma_w^2}{\sqrt{2N}\sigma_w^2}\right) - \frac{1}{2}\mathrm{Erfc}\left(\frac{\lambda - N\sigma_w^2(1+\gamma)}{\sqrt{2N(1+2\gamma)}\sigma_w^2}\right)\right).$$

Therefore, the optimal threshold for any SNR value can be derived as (see the Appendix)

$$\lambda^* = \frac{N\sigma_w^2}{2}\left(1+\sqrt{1+2\gamma\left(1+\frac{(1+2\gamma)\ln(1+2\gamma)}{N\gamma^2}\right)}\,\right). \quad (7)$$

In low SNR, i.e., $\gamma \ll 1$, thus $1+2\gamma \approx 1$, the optimal threshold can be well-approximated as

$$\lambda^* \approx \frac{N\sigma_w^2}{2}\left(1+\sqrt{1+2\gamma}\,\right) \approx N\sigma_w^2. \quad (8)$$

B. Rayleigh Channel

For a Rayleigh fading channel, the total error is $P_e = (P_f + \overline{P_{md}^{Ray}})$ where $\overline{P_{md}^{Ray}}$ is given in (6). When $\frac{\partial P_e}{\partial \lambda} = 0$, it can be simplified as[1]

$$e^{\left(\frac{1}{\sqrt{2N}\bar{\gamma}} - \frac{\lambda - N\sigma_w^2}{\sqrt{2N}\sigma_w^2}\right)^2}\mathrm{Erfc}\left(\frac{1}{\sqrt{2N}\bar{\gamma}} - \frac{\lambda - N\sigma_w^2}{\sqrt{2N}\sigma_w^2}\right) = \sqrt{\frac{2N}{\pi}}\bar{\gamma}. \quad (9)$$

It is complicated to derive the exact solution for λ^* with this non-linear equation. The solution can be obtained numerically. However, with the assumption of very low SNR and the observation made in the AWGN channel, we can find an approximated optimal value. When $\bar{\gamma} \ll 1$, the right hand side of (9) approaches a very small value. We take $\alpha = \frac{1}{\sqrt{2N}\bar{\gamma}} - \frac{\lambda - N\sigma_w^2}{\sqrt{2N}\sigma_w^2}$. When $\bar{\gamma} \ll 1$ and $\lambda \to N\sigma_w^2$, the left hand side of (9) also approaches a very small value due to $\lim\limits_{\alpha\to 0} e^{\alpha^2}\mathrm{Erfc}(\alpha) \to 0$. Thus, we can say that the equation (9) is satisfied for λ around $N\sigma_w^2$. Further, it can be shown that the second order derivative of $P_e(\lambda)$, $\frac{\partial^2 P_e(\lambda)}{\partial \lambda^2} > 0$ when $\lambda \approx N\sigma_w^2$. Therefore, $P_e = (P_f + \overline{P_{md}^{Ray}})$ has a minimum around $\lambda \approx N\sigma_w^2$. This observation is also concluded with the numerical results given in Section V.

Note that the optimal threshold selection with a Nakagami-m fading channel is analytically complicated because $P_e = (P_f + \overline{P_{md}^{Nak}})$ has highly non-linear behavior. Since Nakagami-m fading (when $1 < m < \infty$) is varying between the Rayleigh fading and the Gaussian fading, we can also claim that the optimal threshold is around $\lambda \approx N\sigma_w^2$. However, the exact solution can be obtained numerically using mathematical software packages such as MATHEMATICA and MATLAB.

It is possible that we cannot achieve the recommended error rate requirements ($\overline{P}_f \leq 0.1$ and $\overline{P}_{md} \leq 0.1$) even at the optimal threshold value. One possible way of achieving the requirements is by increasing the number of samples N. Since $N \approx \tau f_s$ where τ is the sensing time and f_s is the sampling frequency, the sensing time also increases when N increases. However, there is a limitation for the allowable sensing time, i.e., $\tau \leq 2$ seconds in accordance with IEEE 802.22 WRAN [1], [15]. This is a main drawback in spectrum sensing with energy detection in low SNR. This issue is discussed with numerical examples in the following section.

V. Numerical/Simulation Results and Discussion

This section provides numerical and simulation results. We defined the *normalized threshold* as $\hat{\lambda} \triangleq \frac{\lambda}{N}$, i.e., the threshold is normalized by the number of samples.[2] The noise variance is set to $\sigma_w^2 = 1$, unless specified otherwise. We denote $P_e^* = P_e(\lambda^*)$, $P_f^* = P_f(\lambda^*)$, and $P_{md}^* = P_{md}(\lambda^*)$.

One of the main contributions of this research is to derive a closed-form expression for the average missed-detection probability over Nakagami-m fading channel in low SNR. Fig. 2 shows ROC curves (i.e., P_d versus P_f) for three different fading scenarios such as AWGN, Nakagami-4 and Rayleigh fading channels. The numerical results which are based on expressions (5) and (6) are represented by curves, while simulation results are represented by discrete marks. The ROC curves are plotted by varying $\hat{\lambda}$ from 0.95 to 1.50 for the average SNR $\bar{\gamma}$=-20 dB and $\bar{\gamma}$=-15 dB when $N = 2000$. The numerical results closely match with the simulation results for all three fading scenarios at -15 dB and -20 dB, which confirms the accuracy of the approximation. The multipath fading parameter m has a negligible impact on the energy detection at -20 dB, because the faded replicas of different

[1]Due to space limitation, the detailed derivation is omitted.

[2]This is the threshold if the decision statistic is selected as $\Lambda(\mathbf{y}) = \frac{1}{N}\sum_{n=1}^{N}|\mathbf{y}(n)|^2$.

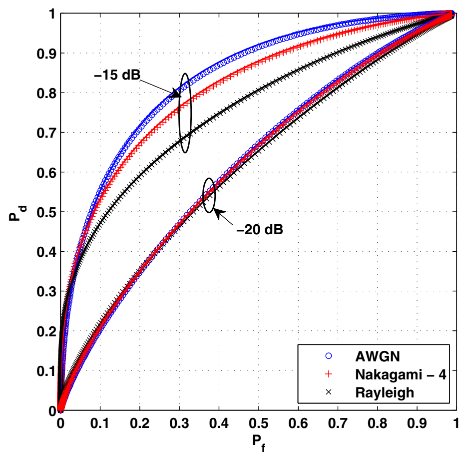

Fig. 2. ROC curves of an energy detector over fading channels.

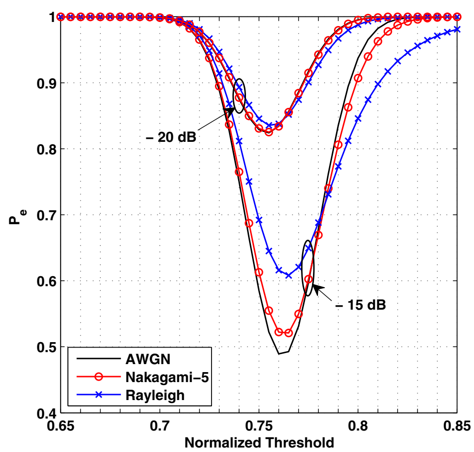

Fig. 3. Total error rate versus normalized threshold.

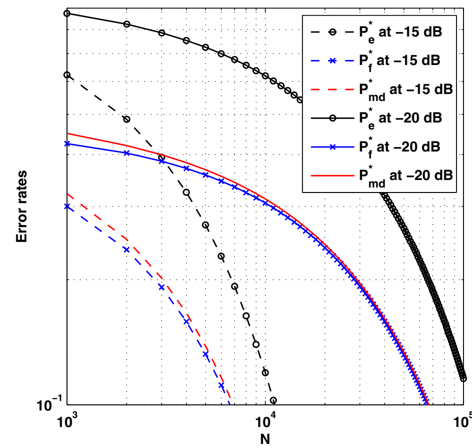

Fig. 4. Minimum error rates versus number of samples in AWGN channel.

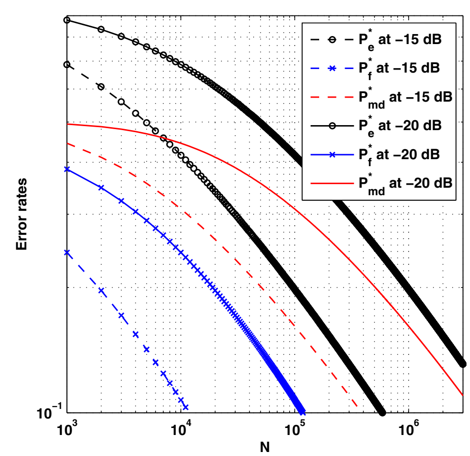

Fig. 5. Minimum error rates versus number of samples in Rayleigh channel.

multipaths have no significant contribution to increase the effective SNR at very low SNR.

We show that the total error rate $P_e = P_f + P_{md}$ can be minimized at the optimal threshold value λ^*. The total error rate versus normalized threshold for AWGN, Nakagami-5 and Rayleigh fading channels is shown in Fig. 3 at $\bar{\gamma} = -15$ dB and $\bar{\gamma} = -20$ dB when $N = 2000$. It shows that $\hat{\lambda}^* \approx 0.755$ at $\bar{\gamma}$= -20 dB, and $\hat{\lambda}^* \approx 0.760$ at $\bar{\gamma}$= -15 dB when $\sigma_w^2 = 0.75$ for all three different fading scenarios. As in expression (8), the optimal normalized threshold is given as $\hat{\lambda}^* = \frac{\lambda}{N}^* \approx \frac{\sigma_w^2}{2}\left(1+\sqrt{1+2\gamma}\;\right) \approx \sigma_w^2$ for AWGN, and it is also valid for Rayleigh and Nakagami-m fading channels. So the analytical results in Section IV is confirmed. Further, we can see that one possible way of minimizing the total error rate is by increasing the average SNR, e.g., P_e^* can be reduced from 0.82 to 0.48 by increasing SNR from -20 dB to -15 dB in AWGN channel.

Since our main focus is on low SNR region, another possible way of minimizing the total error rate is by increasing the number of samples. In the spectrum sensing, IEEE 802.22 WRAN expects $P_f \leq 0.1$ and $P_{md} \leq 0.1$, and channel detection time (CDT) $\tau \leq 2$ seconds under any detection technique. Since $N \approx \tau f_s$ where f_s is the sampling frequency which may depend on the sampling rate of the analog to digital converter and FFT (fast Fourier transform) bin resolution, we cannot increase N beyond τf_s. We take $f_s = 62.5$ kHz which is a typical FFT bin resolution of an experimental energy detection implementation [24]. We consider the minimum error rate requirements as $P_f \leq 0.1$ and $P_{md} \leq 0.1$. Fig. 4 and Fig. 5 show minimum error rates (P_e^*, P_f^*, and P_{md}^*) versus N for AWGN and Rayleigh channels, respectively. For -15 dB, the requirements can be achieved when $N \geq 7000$ with $P_e^* \approx 0.193$ in AWGN channel. If we neglect other processing delays, the minimum CDT is 0.112 seconds. For -20 dB, the requirements can be achieved when $N \geq 67000$ with $P_e^* \approx 0.197$ within 1.072 seconds in AWGN channel.

The Rayleigh fading represents the effect of heavily built-up urban environments on radio signals. As in Fig. 5, neither at -15 dB nor at -20 dB can the requirements be met within 2 seconds (or equivalently at $N \leq 125000$), e.g., $P_{md}^* \approx 0.293$ when $N = 125000$ at -20 dB. It needs more than 3×10^6 samples to achieve $P_{md}^* \leq 0.1$ at -20 dB. Therefore, one possible way of detecting the signal within 2 seconds is by increasing the sampling frequency f_s beyond 1.5 MHz. Another possible way is to use a diversity technique or cooperative spectrum sensing topology to increase the number of effective decision statistics while maintaining a small number of samples per node or per branch.

VI. Conclusion

Detection performance of the energy detector is studied in the low SNR regime. The average missed-detection probability is derived in a closed-form over Rayleigh and Nakagami-m fading channels. For minimizing the total error rate, the optimal detection threshold is exactly derived for AWGN channel, and it is also approximated for Rayleigh and Nakagami-m fading channels. The false alarm and the missed-detection probabilities may not be satisfied even at the optimal detection threshold. Increasing the number of samples until the maximum allowable sensing time is achieved, and increasing the sampling frequency are two possible ways to meet the requirements of the false-alarm and the missed-detection probabilities. The research findings help to design an energy detector in an implementable way as fulfilling fundamental sensing requirements proposed in IEEE 802.22 WRAN (channel detection time ≤ 2 seconds, missed-detection and false alarm probabilities ≤ 0.1).

Appendix

The optimal threshold, λ^*, is achieved when $\frac{\partial P_e(\lambda)}{\partial \lambda} = 0$. With the aid of (1), (2), $Q(x) = \frac{1}{2}\text{Erfc}\left(\frac{x}{\sqrt{2}}\right)$, and $\frac{\partial}{\partial x}\text{Erfc}\left(\frac{x-a}{b}\right) = -\frac{2e^{-\frac{(x-a)^2}{b^2}}}{b\sqrt{\pi}}$, we have

$$\frac{\partial P_e(\lambda)}{\partial \lambda} = \frac{e^{-\frac{(\lambda - N(1+\gamma)\sigma_w^2)^2}{2N(1+2\gamma)\sigma_w^4}}}{\sqrt{2\pi N(1+2\gamma)}\sigma_w^2} - \frac{e^{-\frac{(\lambda - N\sigma_w^2)^2}{2N\sigma_w^4}}}{\sqrt{2\pi N}\sigma_w^2} = 0. \quad (10)$$

After some algebraic manipulations and taking the logarithm, (10) can be simplified into a quadratic equation of λ as

$$\lambda^2 - N\sigma_w^2\lambda - \frac{N\sigma_w^4}{2}\left(N\gamma + \frac{(1+2\gamma)\ln(1+2\gamma)}{\gamma}\right) = 0.$$

Thus, the solution for λ is

$$\lambda = \frac{N\sigma_w^2}{2}\left(1 \pm \sqrt{1 + 2\gamma\left(1 + \frac{(1+2\gamma)\ln(1+2\gamma)}{N\gamma^2}\right)}\right).$$

Since $\lambda \geq 0$, λ^* can be selected as (7). In low SNR, i.e., $\gamma \ll 1$, $1 + 2\gamma \approx 1$ and thus $\ln(1 + 2\gamma) \approx 0$, the optimal threshold can be well approximated as (8). Now we can consider the second order derivative which is given as

$$\frac{\partial^2 P_e(\lambda)}{\partial \lambda^2} = \frac{(\lambda - N\sigma_w^2)e^{-\frac{(\lambda - N\sigma_w^2)^2}{2N\sigma_w^4}}}{\sqrt{2\pi}N^{3/2}\sigma_w^6} - \frac{(\lambda - N(1+\gamma)\sigma_w^2)e^{-\frac{(\lambda - N(1+\gamma)\sigma_w^2)^2}{2N(1+2\gamma)\sigma_w^4}}}{\sqrt{2\pi}N^{3/2}\sigma_w^6(1+2\gamma)^{3/2}}.$$

Using (7), it is easy to show that $\frac{\partial^2 P_e(\lambda^*)}{\partial \lambda^2} > 0$, and therefore, there is a global minimum at $\lambda = \lambda^*$.

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