New Performance Approximations for Multi-Hop Fixed-gain AF Relay Networks

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***Abstract*—A novel approximation for the end-to-end signal-to-noise ratio (e2e SNR) of multi-hop ($N \geq 2$) fixed-gain amplify-and-forward (FG-AF) relay networks over independent and non-identically distributed Nakagami-m fading channels is proposed. Two types of FG-AF relays; (i) blind-AF, and (ii) semi-blind-AF are treated. The cumulative distribution function and the moment generating function of the proposed e2e SNR approximation are derived in closed-form and used to derive the outage probability, the average symbol error rate, and the generalized SNR moments. The resulting performance metrics for the blind-AF relay case are asymptotically exact and thus, the asymptotic outage probability, the asymptotic average SER, the diversity order, and the coding gain are derived. Numerical and simulation results are presented to verify the comparative performance against the exact performance metrics and existing bounds. Our results reveal that the proposed performance approximations outperform the existing bounds in most of the cases.**

I. Introduction

Multi-hop relay networks have numerous advantages such as broader coverage and enhanced-throughput over single-hop networks, and have thus resulted a furry of research activities and wireless standards [1]–[6]. Multi-hop relay networks can broadly be divided into two groups: (i) channel-assisted amplify-and-forward (CA-AF) relays, and (ii) fixed-gain amplify-and-forward (FG-AF) relays [3], [4]. In this paper, we focus on the multi-hop FG-AF relay networks.

Prior related research: The dual-hop ($N = 2$) FG-AF relay networks over independent and non-identically distributed (i.n.i.d.) Rayleigh fading channels are studied in [3]. In [6], a comprehensive performance analysis framework is proposed for the system set-up in [3] over generalized fading channels. Specifically, the analytical framework in [6] provides either exact analysis or accurate bounding techniques to derive the moment generating function (MGF) of the end-to-end signal-to-noise ratio (e2e SNR).

Although the exact closed-form performance metrics for the dual-hop FG-AF relay networks are available, the exact analysis for a number of hops $N \geq 3$ appears to be intractable. Thus, the performance of the multi-hop FG-AF relay networks have been studied by using a particular bounding technique on the e2e SNR [4], [5]. Specifically, in [4], multi-hop e2e SNR is upper bounded by using the well-known inequality between the harmonic mean and the geometric mean. By using this bounding technique, lower bounds for the outage probability and the average symbol error rate (SER) are derived over i.n.i.d. Nakagami-m fading. Reference [5] extends the analysis in [4] by studying the performance of the system set-up in [4] over generalized fading channels. In particular, [5] provides the closed-form expressions for the generalized SNR moments over various fading channels, and used to obtain accurate approximations for the outage probability and the average SER by using the moments-based approach in [7].

For the sake of completeness, we briefly discuss the prior related research on multi-hop CA-AF relay networks. The exact performance of dual-hop networks is studied in [8], [9]. Similar to the FG-AF case, the exact analysis of CA-AF relays for a number of hops $N \geq 3$ appear to be intractable. Thus, the prior related studies provide bounds on the e2e SNR [4], [10] or asymptotic approximations and numerical methods [11], [12]. Recently, [13] proposed asymptotically exact performance bounds for multi-hop CA-AF relay networks and showed that the proposed bounds outperform both the bounds in [4] and [10] at medium-to-high SNRs.

Motivation and our contribution: Although the elegant SNR bounding technique in [4], [5] results in tight performance bounds in low SNRs and for less severe fading ($m \geq 5$, where m is the Nakagami-m fading parameter), they weaken for high SNRs and for severe fading environments such as Rayleigh fading. The bounds in [4], [5] may thus not provide an accurate assessment of system performance. Specifically, the important system-design parameters such as the diversity order and the coding gains derived by using the bounds in [4], [5] deviate significantly from the exact ones. Further, the multi-hop networks having FG-AF relays have gained much attention than their counterparts, CA-AF relays, because of the reduced system complexity of the former. Thus, having an accurate analytical framework for the multi-hop FG-AF relay networks as well is important. The above mentioned gaps in the performance analysis of multi-hop FG-AF relay networks, arising mainly due to the intractability of the problem, motivated us to develop new performance approximations.

In this paper, a simple and accurate approximation is proposed for the e2e SNR of the N-hop ($N \geq 2$) FG-AF relay networks over i.n.i.d. Nakagami-m fading. By using the proposed SNR approximation, the cumulative distribution function (CDF) and the MGF are derived and used to obtain the outage probability, the average symbol error rate (SER), and the generalized SNR moments. Two types of FG-AF relays are treated: (i) blind-AF relays, and (ii) semi-blind-AF relays. The different between these two types is that the former uses an arbitrary fixed gain whereas the latter uses a gain having the knowledge of statistical CSI, in particular, the average SNR of the previous hop. The resulting performance metrics for

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the blind-AF relays are asymptotically exact, however, the same does not hold for the semi-blind-AF relays. Thus, to obtain valuable system-design insights, the asymptotic outage probability, the average SNR, the diversity order and the coding gain are derived for the blind-AF case. Numerical results are provided to compare the proposed approximations with the exact metrics and the existing bounds [4], [5].

Notations: $\mathcal{K}_\nu(z)$ is the Modified Bessel function of the second kind of order ν [14, Eq. (8.407.1)]. $\mathcal{W}_{\nu,\mu}(z)$ is the Whittaker-W function [14, Eq. (9.222.1)]. $\mathcal{E}_\Lambda\{.\}$ denotes the expected value over the random variable Λ.

II. System And Channel Models

Consider a multi-hop relay network with N hops, source (S), destination (D) and $N-1$ FG-AF relays. The gain of the n-th FG-AF relay is given by $G_n|_{n=1}^{N-1} = \sqrt{\frac{1}{C_n N_{0,n}}}$ [3], [4], where C_n is a constant, and $N_{0,n}$ is the variance of the zero-mean additive white Gaussian noise at the n-th relay's receiver. The e2e SNR γ_{e2e} of a N-hop FG-AF relay network is given by [4], [5]

$$\gamma_{\text{e2e}} = \left[\sum_{n=1}^{N} \frac{1}{\beta_n}\right]^{-1}, \tag{1}$$

where $\beta_n = \prod_{j=1}^{n} \frac{\gamma_j}{C_{j-1}}$ with $C_0 = 1$, and $\gamma_n = \mathcal{P}_n|h_n|^2/N_{0,n}$ is the n-th hop SNR. Further, h_n is the i.n.i.d. Nakagami-m fading channel gain of the n-th hop.

In practice, multi-hop FG-AF relay networks can be divided into two distinct groups; (i) blind-AF relay networks and (ii) semi-blind-AF relay networks [3], [4]. In blind-AF relay networks, each relay does not have access to any channel state information (CSI) and they just amplify the previous hop signal by an arbitrary constant gain regardless of the previous hop's fading state. On the other hand, the semi-blind relays have the knowledge of the statistical (long-term) CSI of the previous hop's fading state, in particular the average SNR. The gain of the n-th semi-blind-AF relay over i.n.i.d. Nakagami-m fading is given by $G_n^2 = \mathcal{E}\left\{\frac{1}{\mathcal{P}_n|h_n|^2+N_{o,n}}\right\} = \frac{m_n^{m_n}}{N_{0,n}\Gamma(m_n)(\bar{\gamma}_n)^{m_n}} \mathrm{e}^{\frac{m_n}{\bar{\gamma}_n}} \Gamma(m_n)\Gamma(1-m_n, \frac{m_n}{\bar{\gamma}_n})$ [5, Eq. (19)], where m_n and $\bar{\gamma}_n$ are the Nakagami-m fading parameter and the average SNR of the n-th hop.

The probability distribution of (1) is not mathematically tractable, particularly for $N \geq 3$. The exact closed-form performance metrics of multi-hop FG-AF relay networks having more than three relays are not yet available in the literature. In order to develop a more accurate and simpler performance analysis framework, we propose a new approximation for (1) as follows:

$$\gamma_{\text{e2e}} \approx \gamma_{\text{e2e}}^{\text{ap}} = \frac{\Gamma_1\Gamma_2}{\Gamma_2 + C_{eq}}, \tag{2}$$

where $\Gamma_1 = \gamma_1$, and $\Gamma_2 = \min_{2\leq n\leq N}(\gamma_n)$. Further, for blind-AF relays, C_{eq} is given by $C_{eq} = \frac{1}{N}\sum_{n=1}^{N} C_n$ and for semi-blind-AF relays, $C_{eq} = \frac{1}{N^2}\sum_{n=1}^{N} m_n \sum_{n=1}^{N} C_n$, where $m_n|_{n=1}^{N}$ are the Nakagami-m fading parameters.

Interestingly, when $N = 2$, (2) reduces to the exact e2e SNR of dual-hop dual-hop FG-AF relay networks given in [3, Eq. (6)].

III. Performance Analysis

This section presents the performance analysis of multi-hop FG-AF relay networks by using (2). First, the CDF and the MGF of the proposed e2e SNR are derived and then used to obtain the outage probability, average SER and ergodic capacity approximations.

A. Statistical characterization of the SNR

The CDF of the proposed e2e SNR (2) of multi-hop FG-AF relay networks over i.n.i.d. Nakagami-m (integer m) fading channels is given by (see Appendix for the proof)

$$F_{\gamma_{e2e}^{ap}}(x) = 1 - \sum_{k_1,k_2\cdots k_N} 2\prod_{n=1}^{N}\left[\frac{1}{(k_n)!}\left(\frac{m_n}{\bar{\gamma}_n}\right)^{k_n}\right]\binom{m_1-1}{k_1} \times C_{eq}^{\frac{2\sum_{n=2}^{N}k_n+\psi}{2}}\left[\left(\frac{\bar{\gamma}_1}{m_1}\right)^2\omega\right]^{\frac{\psi}{2}} x^{\eta}\mathrm{e}^{-\frac{m_1 x}{\bar{\gamma}_1}}\mathcal{K}_\psi\Big(2\sqrt{\omega C_{eq}x}\Big), \tag{3}$$

where $\psi = 1 + k_1 - \sum_{n=2}^{N} k_n$, $\omega = \frac{m_1}{\bar{\gamma}_1}\sum_{n=2}^{N}\frac{m_n}{\bar{\gamma}_n}$, $\eta = m_1 - \frac{\psi}{2}$, and $\sum_{k_1,k_2\cdots k_N} = \sum_{k_1=0}^{m_1-1}\sum_{k_2=0}^{m_2-1}\cdots\sum_{k_N=0}^{m_N-1}$.

The MGF of γ_{e2e}^{ap} can be derived by substituting (3) into $\mathcal{M}_{\gamma_{e2e}^{ap}}(s) = 1 - \int_0^\infty s\left(1 - F_{\gamma_{\text{eq}}}(x)\right)\mathrm{e}^{-sx}\mathrm{d}x$ and evaluating the integral by using [14, Eq. (6.643.3)] as follows:

$$\mathcal{M}_{\gamma_{e2e}^{ap}}(s) = 1 - \sum_{k_1,k_2\cdots k_N} 2\prod_{n=1}^{N}\left[\frac{1}{(k_n)!}\left(\frac{m_n}{\bar{\gamma}_n}\right)^{k_n}\right]\binom{m_1-1}{k_1} \times C_{eq}^{\frac{2\sum_{n=2}^{N}k_n+\psi}{2}}\left[\left(\frac{\bar{\gamma}_1}{m_1}\right)^2\omega\right]^{\frac{\psi}{2}} s\,\mathbb{J}(\mu,\nu,\alpha,\beta), \tag{4}$$

where

$$\mathbb{J}(\mu,\nu,\alpha,\beta) = \frac{\Gamma\left(\mu+\nu+\frac{1}{2}\right)\Gamma\left(\mu-\nu+\frac{1}{2}\right)\psi^{-\mu}\mathrm{e}^{\frac{\omega^2}{2\psi}}}{2\omega}\mathcal{W}_{-\mu,\nu}\left(\frac{\omega^2}{\psi}\right). \tag{5}$$

Here, $\mu = 2m_1 - k_1 + \sum_{n=2}^{N} k_n$, $\nu = \frac{1}{2}\left(k_1 - \sum_{n=2}^{N} k_n + 1\right)$, $\alpha = s + \frac{m_1}{\bar{\gamma}_1}$, and $\beta = \sqrt{\frac{m_1 C_{eq}}{\bar{\gamma}_1}\sum_{n=2}^{N}\frac{m_n}{\bar{\gamma}_n}}$.

Remark II.1: The CDF and MGF of γ_{e2e}^{ap} given in (3) and (4) can readily be simplified for i.n.i.d Rayleigh and i.i.d. Nakagami-m fading as follows:

1) The CDF and MGF of γ_{e2e}^{ap} over i.n.i.d. Rayleigh fading: The CDF of γ_{e2e}^{ap} over i.n.i.d Rayleigh fading is given by

$$F_{\gamma_{e2e}^{ap}}(x) = 1 - 2\sqrt{\lambda}x^{\frac{1}{2}}\mathrm{e}^{-\frac{x}{\bar{\gamma}_1}}\mathcal{K}_1\left(2\sqrt{\lambda x}\right), \tag{6}$$

where $\lambda = \frac{C_{eq}}{\bar{\gamma}_1}\sum_{n=2}^{N}\frac{1}{\bar{\gamma}_n}$. Similarly, the MGF of γ_{e2e}^{ap} over i.n.i.d Rayleigh fading is given by

$$\mathcal{M}_{\gamma_{e2e}^{ap}}(s) = 1 - 2\sqrt{\lambda}\,s\,\mathbb{J}\left(1, \frac{1}{2}, s + \frac{1}{\bar{\gamma}_1}, \sqrt{\lambda}\right). \tag{7}$$

2) The CDF and MGF of γ_{e2e}^{ap} over i.i.d.Nakagami-m fading: The CDF of γ_{e2e}^{ap} over i.i.d.Nakagami-m fading is given by

$$F_{\gamma_{e2e}^{ap}}(x) = 1 - \sum_{i=0}^{(N-1)(m-1)} \sum_{j=0}^{m-1} \frac{2\phi_{i,N-1} C_{eq}^{\frac{i+j+1}{2}} (N-1)^{\frac{i-j+1}{2}} \binom{m-1}{j}}{\Gamma(m)} \times \left(\frac{m}{\bar{\gamma}}\right)^{m+i} x^{\frac{i-j+2m-1}{2}} \mathrm{e}^{-\frac{mx}{\bar{\gamma}}} \mathcal{K}_{j-i+1}\left(2\beta\sqrt{x}\right), \quad (8)$$

where $\beta = \frac{m}{\bar{\gamma}}\sqrt{C_{eq}(N-1)}$ and $\phi_{k,N} = \sum_{i=k-m+1}^{k} \frac{\phi_{i,N-1}}{(k-i)!} I_{[0,(N-1)(m-1)]}(i)$, $\phi_{0,0} = \phi_{0,N} = 1$, $\phi_{k,1} = 1/\,k!$, $\phi_{1,N} = N$, and $I_{[a,c]}(b) = \begin{cases} 1, & a \le b \le c \\ 0, & \text{otherwise.} \end{cases}$

The MGF of γ_{e2e}^{ap} over i.i.d.Nakagami-m fading is given by

$$\mathcal{M}_{\gamma_{e2e}^{ap}}(s) = 1 - \sum_{i=0}^{(N-1)(m-1)} \sum_{j=0}^{m-1} \frac{2\phi_{i,N-1} C_{eq}^{\frac{i+j+1}{2}} (N-1)^{\frac{i-j+1}{2}}}{\Gamma(m)} \times \binom{m-1}{j} \left(\frac{m}{\bar{\gamma}}\right)^{m+i} s\, \mathbb{J}(\mu,\nu,\alpha,\beta), \quad (9)$$

where $\mu = \frac{i-j+2m}{2}$, $\nu = \frac{j-i+1}{2}$, $\alpha = s + \frac{m}{\bar{\gamma}}$, and β is defined in (4).

The PDF of γ_{e2e}^{ap} can readily be derived by differentiating the CDF by using [14, 8.486.12]. However, the PDF results are omitted, for the sake of brevity.

B. Outage probability

The outage is the probability that the instantaneous SNR γ_{eq} falls below a pre-defined threshold SNR γ_{th}. Thus, the tight approximations for the outage probability P_{out} can immediately be obtained by using the CDF results given in (3), (6) and (8) as: $P_{out} = \Pr(\gamma_{\text{e2e}}^{\text{ap}} \le \gamma_{th}) = F_{\gamma_{e2e}^{ap}}(\gamma_{th})$.

C. Average symbol error rate

The average SER rate ($\bar{P}_e$) can be derived by averaging the conditional error probability (CEP) $P_e|\gamma$ over the PDF of the e2e SNR. For a wide range of modulation schemes, the CEP can be expressed as $P_e|\gamma = a\mathcal{Q}(\sqrt{b\gamma})$, where a and b are modulation-dependent constants. The average SER can be simplified by using integration by parts as $\bar{P}_e = \frac{a}{2} - \frac{a}{2}\sqrt{\frac{b}{2\pi}} \int_0^\infty x^{-\frac{1}{2}} \exp\left(-\frac{bx}{2}\right)\left(1 - F_{\gamma_{e2e}^{ap}}(x)\right) \mathrm{d}x$. By substituting (3) into $\bar{P}_e$ and by using [14, Eq. (6.643.3)], the average SER over i.n.i.d. Nakagami-m can be derived as

$$\bar{P}_e = \frac{a}{2} - \frac{a}{2}\sqrt{\frac{b}{2\pi}} \sum_{k_1,k_2\cdots k_N} 2\prod_{n=1}^{N}\left[\frac{1}{(k_n)!}\left(\frac{m_n}{\bar{\gamma}_n}\right)^{k_n}\right]\binom{m-1}{j} \times \binom{m_1-1}{k_1} C_{eq}^{\frac{2\sum_{n=2}^{N} k_n + \psi}{2}} \left[\left(\frac{\bar{\gamma}_1}{m_1}\right)^2 \omega\right]^{\frac{\psi}{2}} \mathbb{J}(\mu,\nu,\alpha,\beta), \quad (10)$$

where $\mu = m_1 - \frac{1}{2}(1 + k_1 - \sum_{n=2}^{N} k_n)$, $\nu = \frac{1}{2}(1 + k_1 - \sum_{n=2}^{N} k_n)$, $\alpha = \frac{b}{2} + \frac{m_1}{\bar{\gamma}_1}$, and β is defined in (4).

As per Remark II.1, the average SER over i.n.i.d. Rayleigh fading is given by

$$\bar{P}_e = \frac{a}{2} - \frac{a}{2}\sqrt{\frac{b\lambda}{2\pi}} \mathbb{J}\left(\frac{1}{2}, \frac{1}{2}, \frac{b}{2} + \frac{1}{\bar{\gamma}_1}, \sqrt{\lambda}\right), \quad (11)$$

where λ is defined in (6). Similarly, the average SER over i.i.d. Nakagami-m fading is given by

$$\bar{P}_e = \frac{a}{2} - a\sqrt{\frac{b}{2\pi}} \sum_{i=0}^{(N-1)(m-1)} \sum_{j=0}^{m-1} \frac{2\phi_{i,N-1} C_{eq}^{\frac{i+j+1}{2}} (N-1)^{\frac{i-j+1}{2}}}{\Gamma(m)} \times \binom{m-1}{j} \left(\frac{m}{\bar{\gamma}}\right)^{m+i} \mathbb{J}(\mu,\nu,\alpha,\beta), \quad (12)$$

where $\mu = \frac{1}{2}(i - j + 2m - 1)$, $\nu = \frac{1}{2}(j - i + 1)$, $\alpha = \frac{b}{2} + \frac{m}{\bar{\gamma}}$, and $\beta = \frac{m}{\bar{\gamma}}\sqrt{((N-1)C_{eq})}$.

D. Generalized SNR moments

Generalized SNR moments ($\overline{\gamma_{e2e}^l}$) are useful statistics and can be used to obtain performance metrics such as the amount of fading and ergodic capacity. By substituting (3) into $\overline{\gamma_{e2e}^l} = \int_0^\infty l x^{l-1}\left(1 - F_{\gamma_{\text{e2e}}}(x)\right) \mathrm{d}x$ and by using [14, Eq. (6.643.3)], $\overline{\gamma_{e2e}^l}$ can be derived as

$$\overline{\gamma_{e2e}^l} = \sum_{k_1,k_2\cdots k_N} 2\prod_{n=1}^{N}\left[\frac{1}{(k_n)!}\left(\frac{m_n}{\bar{\gamma}_n}\right)^{k_n}\right]\binom{m_1-1}{k_1} C_{eq}^{\frac{2\sum_{n=2}^{N} k_n + \psi}{2}} \times \left[\left(\frac{\bar{\gamma}_1}{m_1}\right)^2 \omega\right]^{\frac{\psi}{2}} l\, \mathbb{J}(\mu,\nu,\alpha,\beta), \quad (13)$$

where $\mu = l + m_1 - 1 - \frac{1}{2}(k_1 - \sum_{n=2}^{N} k_n)$, $\nu = \frac{1}{2}(1 + k_1 - \sum_{n=2}^{N} k_n)$, $\alpha = \frac{m_1}{\bar{\gamma}_1}$, and β is defined in (4). As per Remark II.1, by using (13), simplified expressions for $\overline{\gamma_{e2e}^l}$ over i.n.i.d. Rayleigh and i.i.d. Nakagami-m fading can be obtained. However, for the sake of brevity, these results are omitted. In order to demonstrate the usefulness of the moment analysis, (13) can readily be used to obtain the capacity approximation as [15, Eq. (9)]

$$C \approx \frac{1}{N\ln(2)}\left(\ln(1 + \bar{\gamma}_{e2e}) - \frac{\overline{\gamma_{e2e}^2} - (\bar{\gamma}_{e2e})^2}{2\left(1 + \bar{\gamma}_{e2e}\right)^2}\right). \quad (14)$$

Remark II.2: The outage probability and the average SER for multi-hop blind-AF relay networks derived by using the proposed SNR approximation (2) are asymptotically exact. However, the corresponding performance metrics for the multi-hop semi-blind relay networks do not hold this asymptotically exactness.

E. High SNR analysis

In order to obtain direct insights from our analysis, the asymptotic performance metrics for the multi-hop blind-AF relay networks are derived. Specifically, valuable system-design parameters such as the diversity order and the coding gain are derived.

1) Asymptotic outage probability: At high SNRs, the outage probability of multi-hop blind-AF relay networks over i.n.i.d. Nakagami-m fading can be approximated as

$$P_{out}^\infty \approx \frac{m_1}{\Gamma(m_1+1)k_1^{m_1}}\left(\frac{\gamma_{th}}{\bar{\gamma}}\right)^{m_1} + o\left(\bar{\gamma}^{-(m_1+1)}\right), \quad (15)$$

where $\bar{\gamma}_n|_{n=1}^{N} = k_n\bar{\gamma}$.

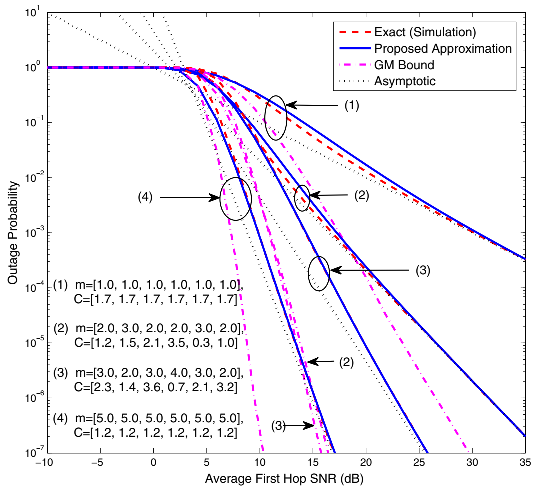

Fig. 1. The outage probability of multi-hop blind-AF relay networks over i.n.i.d. Nakagami-m fading. The number of hops $N = 6$, $\mathbf{k} = [0.2, 0.1, 0.2, 0.3, 0.1, 0.1]$, and the path-loss exponent $\varphi = 2.5$.

2) Asymptotic average SER: As $\bar{\gamma}_n \to \infty$, the asymptotic average SER ($\bar{P}_e^{\infty}$) can be approximated by $\bar{P}_e^{\infty} = (G_c\bar{\gamma})^{-G_d}$, where G_d and G_c are referred to as the diversity order and the array gain [12]. Then, $\bar{P}_e^{\infty}$ can be derived as

$$\bar{P}_e^{\infty} \approx \frac{a m_1^{m_1} 2^{m_1-1}\Gamma(m_1+\frac{1}{2})}{\sqrt{\pi}\Gamma(m_1+1)(bk_1\bar{\gamma})^{m_1}} + o\left(\bar{\gamma}^{-(m_1+1)}\right). \quad (16)$$

By using (16), the diversity order and the coding gain can be obtained as $G_d = m_1$ and $G_c = \left(\frac{a m_1^{m_1} 2^{m_1-1}\Gamma(m_1+\frac{1}{2})}{\sqrt{\pi}\Gamma(m_1+1)(bk_1)^{m_1}}\right)^{-\frac{1}{m_1}}$, respectively.

Interestingly, both the asymptotic outage probability (15) and average SER (16) of multi-hop blind-AF relay networks depend solely on the first hop fading parameters.

IV. Numerical Results

This section provides the numerical and simulation results to show the tightness of the proposed performance approximations, in particular, they are compared against those in [4][1]. To capture the network geometry, the average SNR of the n-th hop is modeled by $\bar{\gamma}_n = k_n\bar{\gamma}$, where $k_n = \left(\frac{d_{SD}}{d_n}\right)^{\varphi}$, $\bar{\gamma}$ and φ are the average transmit SNR and path-loss exponent. The distance between the source-destination pair is d_{SD} and that between R_n and R_{n+1} is d_n. Further, the coefficients $k_n|_{n=1}^N$, the Nakagami-m fading parameters $m_n|_{n=1}^N$ and the gain-related parameter $C_n|_{n=1}^N = \frac{1}{G_n^2 N_{0,n}}$ are denoted by $1 \times N$ vectors $\mathbf{k}$, $\mathbf{m}$, and $\mathbf{C}$, respectively.

Fig. 1 shows the outage probability of six-hop blind-AF relay network over i.n.i.d. Nakagami-m fading channels. The exact outage curves are plotted by using Monte-Carlo simulations. Specifically, the proposed outage approximation (3) is compared against the existing outage bound obtained by using geometric mean bounding technique in [4, Eq. (21)]. For the sake of notational simplicity, we name the bound in [4] as "GM" bound. Fig. 1 shows clearly that the proposed outage approximation outperforms the GM bound in entire

[1]Reference [5] uses the same geometric mean bounding technique of [4].

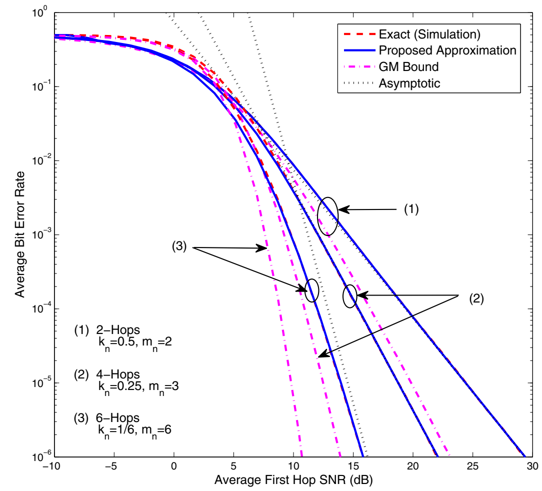

Fig. 2. The average BER of multi-hop blind-AF relay networks over i.i.d. Nakagami-m fading channels. The path-loss exponent $\varphi = 2.5$ and $C_n|_{n=1}^N = 1.5$.

SNR regime; specifically, our outage approximation is asymptotically exact whereas the GM bound deviates significantly compared to the exact outage. The asymptotic outage curves plotted by using (15) also confirm this observation.

In Fig. 2, the average bit error rate (BER) of binary phase shift keying (BPSK) is plotted for multi-hop blind-AF relay networks over i.i.d. Nakagami-m fading. Three networks having (i) 2-hops, (ii) 4-hops, and (iii) 6-hops are treated. Again, the average BER obtained by using the GM bound of [4, Eq. (25)] is plotted for comparison purposes. Fig. 2 reveals that the proposed BER approximation outperforms the GM bound in low-to-high SNR regime. In fact, our BER approximation is asymptotically exact and on the contrary, the GM bound deviates approximately 6 dB from the exact BER at a BER of 10^{-6}. Thus, the GM bound can not be used to obtain valuable system-design insights such as the diversity order and the coding gain. Furthermore, Fig. 2 verifies that the proposed BER approximation is exact for dual-hop ($N = 2$) blind-AF relay networks. The asymptotic BER curves reveal that the diversity order of the system is governed solely by the first hop fading parameters.

Fig. 3 shows the average BER of BPSK for 4-hop semi-blind-AF relay network over i.n.i.d. Nakagami-m fading. The proposed BER approximation for semi-blind-AF relays outperforms the GM bound at low-to-high SNRs for severe fading environments ($m \leq 4$). However, GM bound is tighter than our approximation in low-to-moderate SNRs for less severe fading ($m \geq 5$). It is also important to note that the asymptotically exactness of our approximations does not hold for semi-blind-AF relays. Still, our BER approximation is useful as a benchmark for practical multi-hop relay networks.

In Fig. 4, the ergodic capacity of five-hop FG-AF relay network over i.i.d. Rayleigh fading is plotted. Both the semi-blind-AF and blind-AF relay networks are treated. The exact ergodic capacity is obtained by using Monte-Carlo simulations and the GM bound is obtained by using the geometric mean

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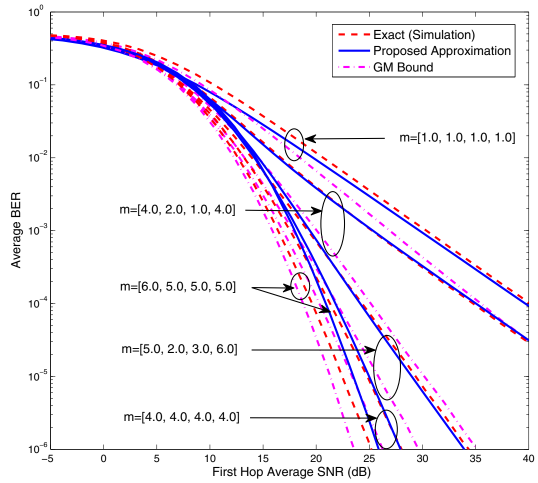

Fig. 3. The average BER of 4-hop ($N = 4$) semi-blind-AF relay networks over i.n.i.d. Nakagami-m fading channels. The number of hops $N = 4$, the path-loss exponent $\varphi = 2.5$ and $k_n|_{n=1}^{N} = 0.25$.

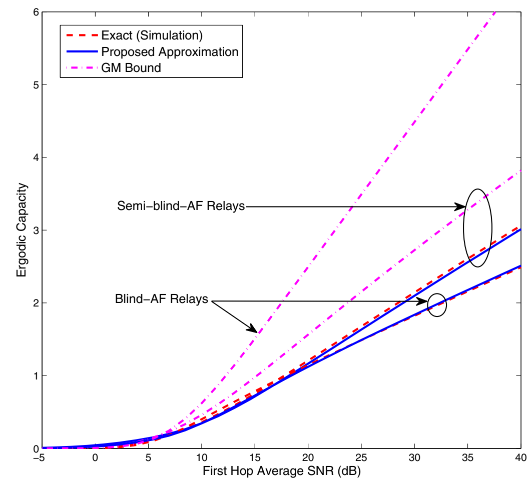

Fig. 4. The ergodic capacity of 5-hop ($N = 5$) FG-AF relay network over i.i.d. Rayleigh fading. The coefficients $k_n|_{n=1}^{N} = 0.2$, $C_n|_{n=1}^{N} = 2.6$, and the path-loss exponent $\varphi = 2.5$.

bound in [4, Eq. (19)]. Further, the ergodic capacity approximation is obtained by using (13) and (14). Fig. 4 reveals that our capacity approximations for both the blind-AF and semi-blind-AF cases are significantly tighter than the GM capacity bounds. Moreover, the GM bound is looser for blind-AF relays than that for semi-blind-AF relays. Thus, our performance metrics can be used to obtain accurate system-design insights.

V. Conclusion

A new e2e SNR approximation for the multi-hop FG-AF relay networks over i.n.i.d Nakagami-m fading channels was proposed and used to derive performance metrics. First, the CDF and the MGF of the e2e SNR approximation were derived and used to derive the outage probability, average SER and generalized SNR moments. In order to obtain valuable system-designing insights, the high SNR approximations; the asymptotic outage probability, average SER, diversity order and the coding gain were derived for the multi-hop blind-AF relay networks. The tightness of the proposed performance approximations are compared against the exact metrics and the existing bounds in [4]. The proposed approximations can be used as benchmarks for practical system designing.

VI. Appendix

The CDF of the e2e SNR approximation γ_{e2e}^{ap} can be expressed as

$$F_{\gamma_{e2e}^{ap}}(x) = 1 - \int_0^{\infty} f_{\Gamma_1}(x+z)\left[1 - F_{\Gamma_2}\left(\frac{C_{eq}x}{z}\right)\right] \mathrm{d}z, \quad (17)$$

where $\Gamma_1 = \gamma_1$, and $\Gamma_2 = \min_{2 \le n \le N}(\gamma_n)$. Now, by substituting the PDF of Γ_1, $f_{\Gamma_1}(x) = \frac{m_1^{m_1}}{\Gamma(m_1)(\bar{\gamma}_1)^{m_1}} x^{m_1-1} \mathrm{e}^{-\frac{m_1 x}{\bar{\gamma}_1}}$, and the CDF of Γ_2, $F_{\Gamma_2}(x) = \sum_{k_2=0}^{m_2-1} \sum_{k_3=0}^{m_3-1} \cdots \sum_{k_N=0}^{m_N-1} \prod_{n=2}^{N} \left[\left(\frac{m_n}{\bar{\gamma}_n}\right)^{k_n} \frac{1}{(k_n)!}\right] x^{\sum_{n=2}^{N} k_n} \times \mathrm{e}^{-(\sum_{n=2}^{N} \frac{m_n}{\bar{\gamma}_n})x}$, into (17), and evaluating the resulting integral by using [14, 3.471.9], the desired result given in (3) can be derived.

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