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Beamforming for Space Division Duplexing

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***Abstract*—Eigenmode transmission in multiple-input multiple-output (MIMO) systems is examined under space division duplexing (SDD). The antennas of each full-duplex node are partitioned to form two antenna banks – one for transmission, the other for reception. Self-interference is suppressed by utilizing the nullspace (or the left nullspace) of corresponding self-interference channel for transmission (or reception). Simulation results are provided on the error performance. Useful insights are obtained on how finite computational precision and quantization errors affect the feasibility of SDD.**

***Index Terms*—MIMO, space division duplexing, eigenmode transmission, null space**

I. Introduction

In this paper, the use of multiple antennas and spatial signal processing to make a wireless terminal full-duplex is considered. Full-duplex wireless communication may be achieved exploiting the degrees of freedom (DoFs) available in time-, frequency- or any suitable dimension. Frequency division duplexing (FDD) and time division duplexing (TDD) techniques are proven; and their applications are ubiquitous. Spectral efficiency being based on the resource utilization in time- and frequency- dimensions, the use of other independent dimensions to achieve duplexing has become attractive, despite the practical challenges. Space division duplexing (SDD) for single-antenna systems has been attempted [1], [2] in this respect, however, with non-spatial techniques for interference suppression. It is with multiple-input multiple-output (MIMO) technology, which supports system nodes with multiple spatial DoFs, that spatial interference suppression became possible.

Full-duplex MIMO repeaters [3] and relays [4], [5] are already receiving the attention, evidently because of the prospects relaying has on extending the coverage of existing/ emerging MIMO compliant cellular and wireless data networks. In a SDD configuration, a given antenna may not transmit and receive simultaneously over the same frequency band. Therefore, the antennas at a node are partitioned to form 2 banks - one dedicated for transmission, and the other, for reception (e.g. N_t transmit antennas vs. N_r receive antennas, in Fig. 1). Duplexing is achieved through signal processing techniques that suppress the self-interference the node's transmission causes on its own reception.

SDD in MIMO wireless channels resembles suppressing near-end crosstalk in digital subscriber lines (DSLs) [6]. However, the spatial channels in wireless systems are virtual, and arise as a result of transmit/ receive beamforming, whereas the wire-pairs of a DSL exist physically. This distinction makes SDD more challenging than crosstalk cancellation.

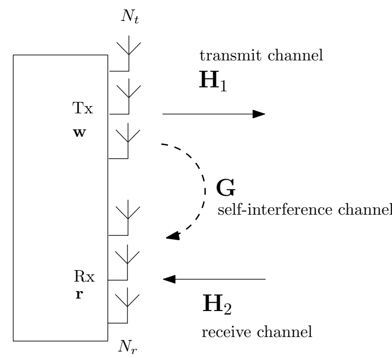

Fig. 1. A MIMO node transmitting and receiving over same frequency band.

SDD also pose significant practical challenges in the form of its high amplifier dynamic range requirement, and the high analog-to-digital converter (ADC) resolution requirement. Inspired by new experimental evidence [7] on achieving over 45 dB of spatial interference suppression, SDD techniques are investigated with a renewed interest.

The simplest, and perhaps, the most obvious approach for self-interference cancellation is temporal. It involves assessing and subtracting self-interference from the received signal [4, Sec. III]. Its variant for full-duplex relay nodes is regarding self-interference a 'feedback' (as in a control system), and optimizing the relay gain matrix for interference suppression [3], [5], [8].

Spatial interference mitigation is an alternative, in which the transmit precoding matrix ($\mathbf{w}$, in Fig. 1) and/ or receiver reconstruction matrix ($\mathbf{r}$) are chosen such that the self-interference, irrespective of the data being transmitted in either direction, has zero (or negligible) effect at the input of the detector. Such techniques are based on: (i) the additional spatial DoFs transmit (or receive) antennas of a node has [4], [7], [9], or (ii) the orthogonality of distinct spatial modes in the self-interference channel [10]. Joint optimization of transmitter-, relay- and receiver- processing for full-duplex relaying too has been considered [11].

The aim of this paper is exploring spatial self-interference mitigation techniques usable for MIMO SDD eigenmode transmission. The paper is organized as follows: Section II presents the mathematical framework. Numerical results on the performance of selected MIMO SDD configurations are provided in Section III. The conclusion follows, highlighting certain limitations that need to be overcome to realize SDD.

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Notation: Given a matrix $\mathbf{A}$, its transpose, conjugate transpose, rank, nullity, nullspace, and left nullspace are denoted: $\mathbf{A}^T$, $\mathbf{A}^H$, $\text{rank}(\mathbf{A})$, $\text{nullity}(\mathbf{A})$, $\text{Null}(\mathbf{A})$, $\text{Null}(\mathbf{A}^T)$ respectively [12]. $\{\mathbf{A}\}_{\mathcal{C}(n)}$ and $\{\mathbf{A}\}_{\mathcal{R}(m)}$ give the sub-matrices of $\mathbf{A}$ formed with its first n columns, and first m rows, respectively. $\{\mathbf{A}\}_{\mathcal{C}(m:n)}$ is the sub-matrix of $\mathbf{A}$ formed with its columns m through n. Main diagonal of $\mathbf{A}$ is given by $\text{diag}(\mathbf{A})$. $\mathbf{A} \in \mathbb{C}^{m\times n}$ denotes that $\mathbf{A}$ is an $m \times n$ matrix. The notation $[\mathbf{A}_1,\ \mathbf{A}_2]$ represents the concatenation of matrices $\mathbf{A}_1$ and $\mathbf{A}_2$.

II. Mathematical Framework

A. SDD through nullspace & left nullspace projection

The singular value decomposition (SVD) of a matrix $\mathbf{G} \in \mathbb{C}^{m\times n}$ is of the form $\mathbf{G} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$, where (i) $\mathbf{\Sigma} \in \mathbb{C}^{m\times n}$ is nonnegative real rectangular diagonal; and (ii) $\mathbf{U} \in \mathbb{C}^{m\times m}, \mathbf{V} \in \mathbb{C}^{n\times n}$ are unitary.

Suppose $\mathbf{G}$ does not have full column-rank (i.e. $r = \text{rank}(\mathbf{G}) < n$). The columns of $\mathbf{V}^{(0)} = \{\mathbf{V}\}_{\mathcal{C}(r+1:n)}$ span $\text{Null}(\mathbf{G})$, such that $\mathbf{G}\mathbf{V}^{(0)} = \mathbf{0} \in \mathbb{C}^{m\times(n-r)}$. Similarly, $\text{Null}(\mathbf{G}^T)$ is spanned by the columns of $\mathbf{U}^{(0)} = \{\mathbf{U}\}_{\mathcal{C}(r+1:m)}$ such that $\left(\mathbf{U}^{(0)}\right)^H\mathbf{G} = \mathbf{0} \in \mathbb{C}^{(m-r)\times n}$, whenever $\mathbf{G}$ does not have full row-rank (i.e. $r < m$). The nullspace and the left nullspace exist simultaneously iff $\mathbf{G}$ is rank deficient (i.e. $r < \min(m, n)$).

Suppose $\mathbf{G}$ corresponds to the self-interference channel of the MIMO capable node shown in Fig. 1. Given $\mathbf{x}$, the symbols to be transmitted, self-interference component at the detector input is given by the term $\mathbf{rGwx}$. The interference can be nullified irrespective of $\mathbf{x}$, if either of the constraints:

$$\mathbf{G}\mathbf{w} = \mathbf{0}, \tag{1a}$$

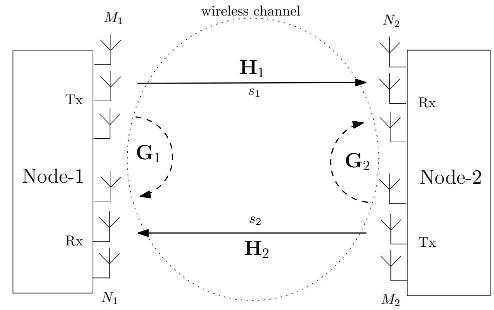
$$\mathbf{r}\mathbf{G} = \mathbf{0}, \tag{1b}$$

can be enforced. The constraints (1a) and (1b) provide three possibilities for implementing SDD at a node.

1) Transmit SDD: Forming $\mathbf{w}$ with columns of $\mathbf{V}^{(0)}$ enforces (1a). It makes transmitted signal $\mathbf{wx}$ to be orthogonal to $\mathbf{G}$. This approach requires $\mathbf{G}$ to not have full-column rank, a sufficient condition for which is allotting more antennas for transmission than for reception.
2) Receive SDD: Forming $\mathbf{r}$ using rows of $\left(\mathbf{U}^{(0)}\right)^H$ enforces (1b). The desired received signal component is forced to be orthogonal to the row space of $\mathbf{G}$. This approach requires $\mathbf{G}$ to not have full-row rank, guaranteed if the majority of antennas are set aside for reception.
3) Joint Transmit and Receive SDD: Simultaneously enforcing (1a) and (1b) as in reference [4], requires[1] $\mathbf{G}$ to be rank deficient. This may only be achieved through proper antenna design and placement (e.g. by arranging a key-hole channel to exist between the antenna banks).

Since $\mathbf{G}$ is not bidirectional, 'Joint Transmit and Receive SDD' appears redundant. Moreover, it complicates beamforming where two nodes implementing SDD communicate. Hence, we focus only on 'Transmit SDD' and 'Receive SDD'.

[1]Reference [10] examines another possibility based on the orthogonality of eigenvectors. It holds if numbers of transmit and receive antennas are equal.



	Node-1	Node-2
# transmit antennas	M_1	M_2
# receive antennas	N_1	N_2
transmit precoding matrix	$\mathbf{w}_1$	$\mathbf{w}_2$
receiver reconstruction matrix	$\mathbf{r}_1$	$\mathbf{r}_2$
transmitted signal	$\mathbf{x}_1$	$\mathbf{x}_2$
additive noise at reception	$\mathbf{n}_1$	$\mathbf{n}_2$

Fig. 2. System model: eigenmode transmission over a MIMO SDD system.

B. Eigenmode transmission with SDD

Consider two MIMO capable nodes: Node-i, $i \in \{1, 2\}$ (see Fig. 2), each having a subset of M_i antennas set aside for transmission, and the remaining N_i antennas dedicated for reception. The transmit (or receive) antennas of a given node need not be physically adjacent.

Suppose the forward MIMO channel from Node-i is $\mathbf{H}_i \in \mathbb{C}^{N_j\times M_i}$ for $i, j \in \{1, 2\}, i \neq j$. Its self-interference MIMO channel $\mathbf{G}_i \in \mathbb{C}^{N_i\times M_i}$ may or may not be rank deficient[2]. $\mathbf{w}_i \in \mathbb{C}^{M_i\times M_i}$ and $\mathbf{r}_i \in \mathbb{C}^{N_i\times N_i}$ are the transmit precoding and receiver reconstruction matrices. $\mathbf{x}_i \in \mathbb{C}^{M_i\times 1}$ denotes the signal transmitted by Node-i, while $\mathbf{y}_i \in \mathbb{C}^{N_i\times 1}$ is the signal it receives. $\mathbf{n}_i \in \mathbb{C}^{N_i\times 1}$ is the additive noise component at reception. The received signal at the detector input of each Node-i is then given by

$$\mathbf{y}_i = \mathbf{r}_i\left(\mathbf{H}_j\mathbf{w}_j\mathbf{x}_j + \mathbf{G}_i\mathbf{w}_i\mathbf{x}_i + \mathbf{n}_i\right). \tag{2}$$

Suppose s_i spatial modes need to be facilitated from each Node-i to the other. This requires

$$\text{rank}(\mathbf{H}_i) \geq s_i, \tag{3}$$

and, either of

$$\text{nullity}(\mathbf{G}_i) \geq s_i, \quad \text{or} \tag{4a}$$

$$\text{nullity}\left({\mathbf{G}_i}^T\right) \geq s_j, \tag{4b}$$

to be satisfied for $i, j \in \{1, 2\}, i \neq j$.

1) Case: Transmit SDD implemented at both nodes:
Design requirements: A necessary, but not sufficient condition for (3) is having $N_j \geq s_i$. The requirement (4a) can be met, irrespective of $\text{rank}(\mathbf{G}_i)$, by ensuring that $(M_i - N_i) \geq s_i$. Where $\mathbf{H}_i$s are not rank-deficient, the requirements are satisfied for $(M_i - s_i) \geq N_i \geq s_j$.

Example 1: Having $M_i = 4$ and $N_i = 2$, for instance, guarantees 2 spatial modes in each direction, provided $\mathbf{H}_i, i \in \{1, 2\}$ are not keyhole channels. If communications were only from Node-1 to Node-2, each node would have had 6 DoFs; but SDD yields only 4 spatial modes.

[2]Rank deficiencies in $\mathbf{G}_i$s would lessen the spatial DoFs SDD costs.

Beamforming matrices : Suppose the SVDs: $\mathbf{G}_i = \mathbf{U}_i \mathbf{\Sigma}_i {\mathbf{V}_i}^H$ hold for $i \in \{1, 2\}$. The columns of each $\mathbf{V}_i^{(0)} = \{\mathbf{V}_i\}_{\mathcal{C}(\mathrm{rank}(\mathbf{G}_i)+1:M_i)}$ span $\mathrm{Null}(\mathbf{G}_i)$. Define[3] $\hat{\mathbf{H}}_i = \mathbf{H}_i \mathbf{V}_i^{(0)}$ for $i \in \{1, 2\}$, and let their SVDs be $\hat{\mathbf{H}}_i = \mathbf{Q}_i \mathbf{\Lambda}_i {\mathbf{W}_i}^H$.

The choice of $\mathbf{w}_i = \mathbf{V}_i^{(0)} \{\mathbf{W}_i\}_{\mathcal{C}(s_i)}$ and $\mathbf{r}_j = \left\{ {\mathbf{Q}_i}^H \right\}_{\mathcal{R}(s_i)}$ therefore produces the spatial modes in both directions.

Remarks:

- The effective MIMO channel $\hat{\mathbf{H}}_i$ is $N_j \times \mathrm{nullity}(\mathbf{G}_i)$, and no longer $N_j \times M_i$. This implies reduced diversity orders. Since $\mathrm{rank}\left(\hat{\mathbf{H}}_i\right) \leq \min\left(\mathrm{rank}\left(\mathbf{H}_i\right), \mathrm{nullity}(\mathbf{G}_i)\right)$ a loss of multiplexing gain too is apparent.
- Under Rayleigh fading, each $\mathbf{H}_i$ would be a complex Gaussian random matrix; $\mathbf{H}_i \mathbf{V}_i$ would have the same distribution since $\mathbf{V}_i$ is unitary. Therefore, $\hat{\mathbf{H}}_i$ would also be complex Gaussian irrespective of the distribution of $\mathbf{G}_i$s. This premise makes performance analysis of MIMO SDD under Rayleigh fading straightforward.
- Channel estimation may be easily performed, for example, by TDD the pilot signals, and estimating each $\mathbf{G}_i$, $\mathbf{H}_j$ pair while Node-i transmits the pilots, for $i, j \in \{1, 2\}, i \neq j$.
- Transmit SDD requires that each Node-i (i) receives channel state information (CSI) for the forward channel $\mathbf{H}_i$ from Node-j; (ii) computes $\mathbf{w}_i, \mathbf{r}_j$ as outlined above; and (iii) conveys $\mathbf{r}_j$ and the gains $\mathrm{diag}\left(\mathbf{\Lambda}_i\right)$ back to Node-j.

2) Case: Receive SDD implemented at both nodes:

Design requirements: Where $\mathbf{H}_i$s are not rank-deficient, the requirements (3), (4b) are satisfied for $(N_i - s_j) \geq M_i \geq s_i$.

Beamforming matrices : Suppose the SVDs: $\mathbf{G}_i = \mathbf{U}_i \mathbf{\Sigma}_i {\mathbf{V}_i}^H$ hold for $i \in \{1, 2\}$. Each $\mathbf{U}_i^{(0)} = \{\mathbf{U}_i\}_{\mathcal{C}(\mathrm{rank}(\mathbf{G})+1:N_i)}$ would span $\mathrm{Null}\left({\mathbf{G}_i}^T\right)$. Define $\hat{\mathbf{H}}_i = \left(\mathbf{U}_j^{(0)}\right)^H \mathbf{H}_i$ for $i, j \in \{1, 2\}, i \neq j$; and let their SVDs be $\hat{\mathbf{H}}_i = \mathbf{Q}_i \mathbf{\Lambda}_i {\mathbf{W}_i}^H$.

Choosing $\mathbf{w}_i = \{\mathbf{W}_i\}_{\mathcal{C}(s_i)}$, and $\mathbf{r}_j = \left\{ \left(\mathbf{U}_j^{(0)} \mathbf{Q}_i\right)^H \right\}_{\mathcal{R}(s_i)}$ would yield the desired spatial modes.

Remarks:

- The effective channel $\hat{\mathbf{H}}_i$ is $\mathrm{nullity}\left({\mathbf{G}_j}^T\right) \times M_i$. Moreover, $\mathrm{rank}\left(\hat{\mathbf{H}}_i\right) \leq \min\left(\mathrm{rank}\left(\mathbf{H}_i\right), \mathrm{nullity}\left({\mathbf{G}_j}^T\right)\right)$. A loss of diversity and multiplexing gains results in.
- Swapping the transmit/ receive role of each antenna should convert a given Receive SDD configuration to a Transmit SDD configuration exhibiting equivalent error performance, and vice versa. Receive SDD appears simpler in practice, since it requires only the $\mathbf{w}_i$s to be exchanged over the channel as an overhead.

3) Case: Transmit SDD implemented at one node, and Receive SDD at the other:

Without a loss of generality, suppose that Node-1 implements Transmit SDD, while Node-2 implements Receive SDD.

The requirements (3) and (4) are met if $(M_1 - s_1) \geq N_1 \geq s_2$ and $(N_2 - s_1) \geq M_2 \geq s_2$. The effective channel for eigenmode transmission from Node-1 to Node-2 would be $\hat{\mathbf{H}}_1 = \left(\mathbf{U}_2^{(0)}\right)^H \mathbf{H}_1 \mathbf{V}_1^{(0)}$. The channel $\mathbf{H}_2$ can be used as is for the other direction.

[3]Defining $\hat{\mathbf{H}}_i = \mathbf{H}_i \left\{\mathbf{V}_i^{(0)}\right\}_{\mathcal{C}(s_i)}$, using s_i columns from $\mathbf{V}_i^{(0)}$ too is possible here. It would however yield lower diversity orders.

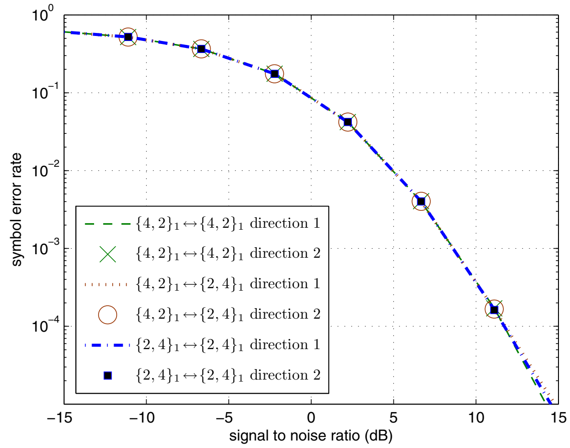


Fig. 3. SNR vs. average SER curves in either direction of $\{M_1, N_1\}_1 \leftrightarrow \{M_2, N_2\}_1$ MIMO SDD configurations.

Remark:

- $\hat{\mathbf{H}}_1$ becomes $\mathrm{nullity}\left({\mathbf{G}_2}^T\right) \times \mathrm{nullity}(\mathbf{G}_1)$. But $\hat{\mathbf{H}}_2 = \mathbf{H_2}$ remains unchanged as $N_1 \times M_2$ for the opposite direction. Notably, the excess spatial DoFs can help achieve SDD, even if available only in one direction.

III. Numerical Results

Let us denote by $\boxed{\{M_1, N_1\}_{s_1} \leftrightarrow \{M_2, N_2\}_{s_2}}$ the MIMO SDD configuration having M_i transmit antennas and N_i receive antennas at Node-i; and s_i spatial modes in-use from Node-i to Node-j, where $i, j \in \{1, 2\}, i \neq j$.

Fig. 3 depicts the signal-to-noise ratio (SNR) vs. average symbol error rate (SER) curves corresponding to the first spatial mode in either direction, for MIMO SDD configurations: (a) $\{4, 2\}_1 \leftrightarrow \{4, 2\}_1$, (b) $\{4, 2\}_1 \leftrightarrow \{2, 4\}_1$, and (c) $\{2, 4\}_1 \leftrightarrow \{2, 4\}_1$. 10^6-point Monte-Carlo simulation is used.

> Assumptions: Block fading (with 10 symbols, per spatial mode, per channel realization), quadrature phase shift keying (QPSK) modulation scheme, independent and identically distributed (i.i.d.) Rayleigh faded $\mathbf{H}_i$s, and i.i.d. Rayleigh faded $\mathbf{G}_i$s are assumed. Elements of $\mathbf{G}_i$s have $100\,\mathrm{dB}$ $(= 10^{10})$ greater variance than those of $\mathbf{H}_i$s.

All three configurations show identical performance, which is expected since the effective MIMO channel $\hat{\mathbf{H}}_i$ in either direction is 2×2 complex Gaussian, for all three cases.

Fig. 4 illustrates more clearly the diversity and multiplexing gain reduction owing to SDD, considering the $\{7, 4\}_3 \leftrightarrow \{5, 3\}_2$ MIMO SDD configuration. The assumptions are as highlighted with respect to Fig. 3. Spatial modes in 'direction 1' (i.e. from Node-1 to Node-2) exhibit error performance identical to that of a 3×3 MIMO channel; while a 4×2 MIMO channel is resembled in the opposite direction. This observation confirms our premise that each $\hat{\mathbf{H}}_i$, although of reduced dimensionality: $N_j \times \mathrm{nullity}(G_i)$, represents i.i.d.

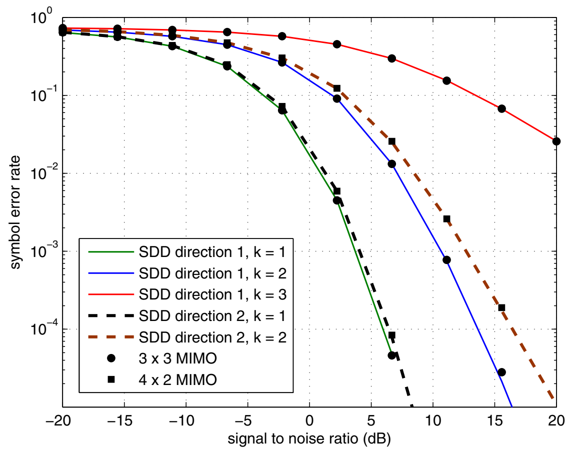


Fig. 4. SNR vs. average SER curves for each spatial mode k in either direction of $\{7,4\}_3 \leftrightarrow \{5,3\}_2$ MIMO SDD configuration. Average SER performance for eigenmode transmission over 3×3 MIMO (●), and 4×2 MIMO (■) channels included for comparison.

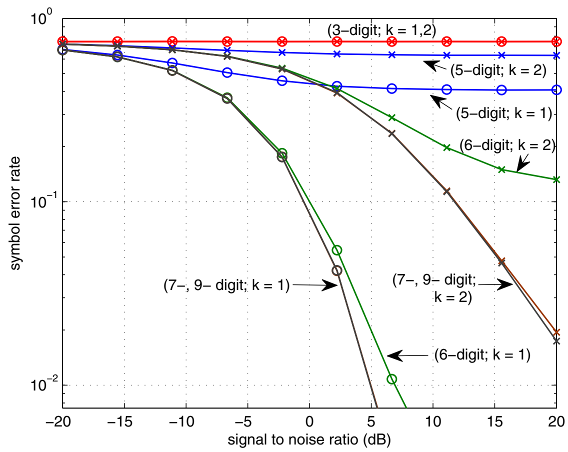


Fig. 5. SNR vs. average SER curves for each spatial mode k in Node-1 to Node-2 direction of $\{4,2\}_2 \leftrightarrow \{4,2\}_2$ MIMO SDD configuration, assuming: 3-, 5-, 6-, 7-, and 9- digit computational precision.

Rayleigh fading (just as corresponding $\mathbf{H}_i$ does). The loss of diversity gains is implicit. Since only 5 spatial modes are facilitated with 11 antennas at Node-1, and 8 antennas at Node-2, a loss of 3 spatial DoFs is also apparent. These losses represent the cost of SDD; the benefit is, obviously, the duplexing capability.

From a mathematical point of view, the SDD techniques we have examined suppress the self-interference perfectly. That is not so in practice, when finite computational precision (in transmitter- and receiver- signal processing) and/ or quantization errors (at analog-to-digital conversion) are in effect.

Fig. 5 depicts approximately[4], how the number of significant digits of computation affects the average SER, using the $\{4,2\}_2 \leftrightarrow \{4,2\}_2$ MIMO SDD configuration. 10^5-point Monte-Carlo simulation has been used; other assumptions are as same as before. The SER floors hint the presence of un-

[4]Approximate, because the internal precision of MATLAB's 'svd' routine was not restricted. Inputs and outputs of the routine were nevertheless truncated to have the desired number of significant digits.

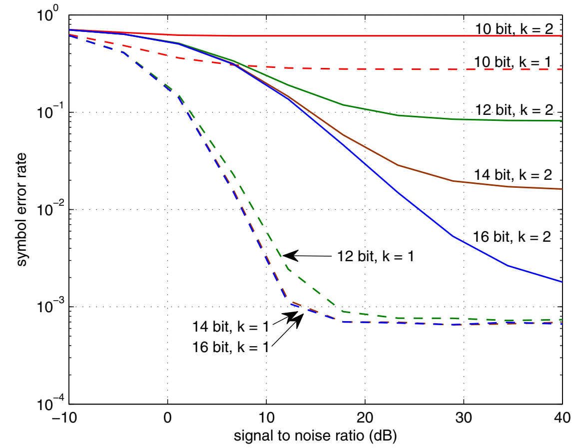


Fig. 6. SNR vs. average SER curves for each spatial mode k in Node-1 to Node-2 direction of $\{4,2\}_2 \leftrightarrow \{4,2\}_2$ MIMO SDD configuration, assuming 10-, 12-, 14-, and 16- bit ADCs.

mitigated interference. Apparently, self-interference does not get suppressed for precisions less than 6-digits. The effect of truncation errors is evident even at 6-digit precision. However, the error performance improves rapidly as the number of significant digits of computation increases beyond a threshold, that depends on the ratio of transmit and receive signal strengths (note: $\log_{10}\left(\sqrt{100\,\mathrm{dB}}\right) = \frac{1}{2}\log_{10}\left(10^{10}\right) = 5$).

Low resolution of the ADC is another concern. It gives rise to quantization errors, and hence, to increased average SERs.

Example 2: If each element of $\mathbf{G}_i$s is zero mean complex Gaussian with $2\sigma^2$ variance, each of the real and imaginary components of the self-interference may lie in the $(-8\sigma, 8\sigma)$ range, at $\mathrm{erf}\left(8/\sqrt{2}\right) = 0.9999999999999988$ (i.e. practically 1) probability. An n-bit linear quantizer for that range would be able to resolve only up to $\Delta = \frac{16\sigma}{2^n} = \frac{\sigma}{2^{n-4}}$. Corresponding quantization error $\frac{\Delta}{2}$ needs to be insignificant with respect to both the desired signal and the self-interference.

The effect of quantization is severe than that of finite computational precision, because linear quantization at a wide dynamic range is required for SDD to function properly. Self-interference dominates the received signal; hence, the dynamic range depends on the $\mathbf{G}_i$s. Linear quantization is required since the self-interference is additive.

Fig. 6 illustrates the effect the quantization errors the 10-, 12-, 14- and 16-bit ADCs introduce have on the average SER.

Assumptions: Elements of $\mathbf{H}_i$s have unit variance, while those of $\mathbf{G}_i$s have $40\,\mathrm{dB}$ variance[5]. Midtread quantization at a dynamic range of 16σ is considered, where $\sigma = \sqrt{10^4/2}$. 10 data symbols, per spatial mode, per channel realization are assumed; along with 10 pilot symbols, per transmit antenna, per channel realization. Least square method is used for channel estimation.

[5]An order of separation above $40\,\mathrm{dB}$ is not achievable with the ADC resolutions considered. Additional $K\,\mathrm{dB}$ separation would approximately require extra $\frac{1}{2}\log_2\left(10^{0.1K}\right)$ bit precision at the ADC.

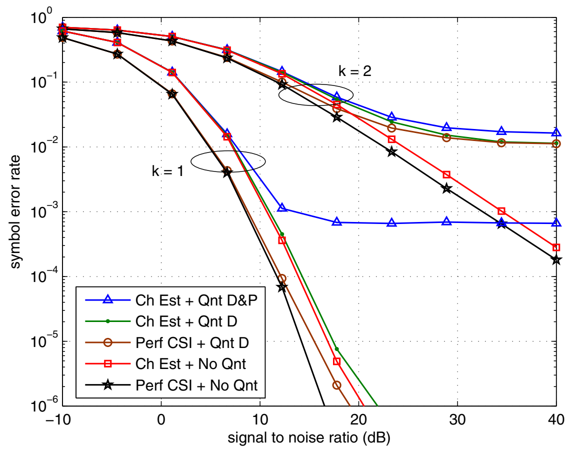


Fig. 7. SNR vs. average SER curves for each spatial mode k in Node 1 to Node 2 direction of $\{4,2\}_2 \leftrightarrow \{4,2\}_2$ MIMO SDD configuration, assuming 14-bit ADC. Five cases reflecting realistic to idealistic assumptions on quantization and channel estimation errors are compared.

10^6-point Monte-Carlo simulation has been used; and other assumptions are as with Fig. 3. Error rates improve with the number of bits the ADCs output per sample. An abrupt degradation of error performance can be seen in the first spatial mode (i.e. $k = 1$) as the precision reduces from 12-bits to 10-bit. A likely reason for it is having $\log_2(16\sigma) = 10.1439$. Moreover, an error floor, which is common with systems affected by SNR invariant errors, can be seen in the average SER curves.

Quantization of the pilot symbols gives rise to channel estimation errors, which significantly influence the error rates. Fig. 7 confirms the fact for a 14-bit ADC, and the same MIMO SDD configuration and assumptions as with Fig. 6, by comparing the error performance for the following cases.

i) Ch Est + Qnt D&P: both the data and pilots (used for channel estimation) quantized;
ii) Ch Est + Qnt D: data quantized, but not the pilots;
iii) Perf CSI + Qnt D: data quantized, perfect CSI assumed;
iv) Ch Est + No Qnt: neither data nor the pilots quantized;
v) Perf CSI + No Qnt: with perfect CSI and no quantization.

10^6-point Monte-Carlo simulation has been used. The case i) is realistic; while the cases ii) through v) depict increasingly idealistic scenarios.

The curves corresponding to cases i) and ii) highlight the degradation of the performance quantization of pilots induces. Quantization induced channel estimation errors set an error floor in both the spatial modes. Quantization of data appears to have a less significant effect; an error floor is apparent only with $k = 1$. The cases iv) and v) let the effect of channel estimation errors be assessed in isolation. An error floor does not appear, evidently because the least square method of estimation improves with the SNR. To sum, the non-availability of perfect CSI appears to be the main contributing factor for errors when ADC resolution is coarse.

IV. Conclusion

Beamforming for eigenmode transmission over MIMO space division duplexing was examined. Associated loss of diversity and multiplexing benefits was highlighted. Further insights were obtained on the adverse effects of finite computational precision and quantization errors on the error rate.

General purpose ADCs operating above 10^7 samples per second do not currently have resolutions beyond 16-bits [13], [14]. Improving both the sampling rate and the resolution appears to be challenging due to high data rates, and other factors such as synchronization and jitter. This, along with non-linearities in the amplifiers makes suppressing self-interference greater than $40\,\text{dB}$ challenging at present. Amount of self-interference suppression required could be manageable for short-range links; it may be reduced further by using directional antennas. But SDD, as was discussed here, may not be feasible in general until the hardware limitations are overcome.

Acknowledgment

This work is supported in part by the Alberta Ingenuity Fund through the iCORE ICT Graduate Student Award.

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