Complexity-Efficient Detection for MIMO Relay Networks

Shuangshuang Han and Chintha Tellambura
Department of Electrical and Computer Engineering
University of Alberta, Edmonton, Canada
Email: {shan7, chintha}@ece.ualberta.ca

***Abstract*—This paper provides the equivalent maximum likelihood (ML) detector at the destination of multi-branch dual-hop multiple-input multiple-output (MIMO) relay networks. Complexity-efficient detections by extending both the complexity-efficient sphere decoder (CSD) and the fixed complexity sphere decoder are proposed. Comparing to the direct link and the cooperative partial detection, our detection method based on the CSD shows the almost-fixed, reduced complexity at a negligible performance loss. Although detect-and-forward relaying is the main focus, this detection also performs well in amplify-and-forward relaying. The simulation results show that the CSD performs nearly optimal ML performance, while keeping the complexity of MIMO relay detection fixed and reduced, making this algorithm suitable for hardware implementation.**

I. Introduction

Multiple-input multiple-output (MIMO) relay network has been considered recently [1] for detect (or decode)-and-forward (DF) relaying and amplify-and-forward (AF) relaying. The research is mainly concentrated on spatial multiplexing systems. The relay nodes in a cooperative network help the direct source-destination communication to improve the performance. Likewise, MIMO promises significant improvements in terms of the link reliability and data rate. The significant performance improvement by cooperative DF relay networks is shown in [2]. Some studies focus on the performance analysis for the maximum likelihood (ML) detection for DF MIMO relay networks. Reference [3] derives a closed-form expression for the bit error rate (BER) with the assumption of perfect channel state information (CSI).

This paper discusses the receiver design for the relays and destination, and achieves computational complexity reduction with nearly optimal performance. Using the sphere decoder (SD), the optimal performance at a reduced complexity could be achieved compared to the ML detection. In relay networks, because fully detect-and-forward (FDF) at the relays requires a significant amount of resources to achieve the near-optimal performance, a cooperative partial detection (CPD) is proposed in [4]. It performs partial detection at the relay by splitting the detection tree into two groups and detecting only one group. The relay transmits only the detected symbols to the destination. The complexity at the relay is reduced by only detecting a subset of data symbols, while the performance is lowered especially if the detected group is a small subset of the whole transmitted sequence. The detection complexity is a variable over the whole range of the signal noise ratio (SNR)s, and still gets the complexity of the SD for the approximately optimal performance. For the hardware implementation, it is critical to overcome the above challenges. The fixed complexity sphere decoder (FSD) [5] achieves quasi-ML performance at a fixed complexity. The complexity-efficient sphere decoder (CSD) [6] obtains significantly reduced and roughly fixed complexity. We propose the complexity-efficient detection for FDF and AF relaying networks based on the extension of these algorithms.

II. System Model

A basic system model for multi-branch dual-hop relay MIMO network is considered, which contains the source ($\mathbb{S}$), N relays ($\mathbb{R}$) and the destination ($\mathbb{D}$). The number of antennas at the source, the relays and the destination are denoted as N_s, N_r and N_d, respectively. In this paper, we assume half-duplex mode. In the first time slot, the source broadcasts the signals to all the relays and the destination. During the second time slot, the relays retransmit the received and/or processed signals to the destination.

The channels between the source and the ith relay, between the ith relay and the destination, between the source and the destination are denoted by $\mathbf{H}_{sri} \in \mathcal{C}^{N_r \times N_s}$, $\mathbf{H}_{rdi} \in \mathcal{C}^{N_d \times N_r}$ ($i \in \{1, 2, \ldots N\}$) and $\mathbf{H}_{sd} \in \mathcal{C}^{N_d \times N_s}$ respectively. $\mathcal{C}$ is a complex value set. The channels are assumed to be quasi-static flat fading. At the end of the first time slot, the received signal vectors at the ith relay and at the destination are given by

$$\mathbf{y}_{sri} = \mathbf{H}_{sri}\mathbf{s}_s + \mathbf{n}_{sri}, \quad (1)$$

$$\mathbf{y}_{sd} = \mathbf{H}_{sd}\mathbf{s}_s + \mathbf{n}_{sd}, \quad (2)$$

where $\mathbf{s}_s$ is the transmitted signal by the source. $\mathbf{n}_{sri}, \mathbf{n}_{sd} \sim \mathcal{CN}(0,1)$ are additive white Gaussian noise (AWGN) at the ith relay and at the destination, where $\mathcal{CN}(0,1)$ denotes complex Gaussian distribution with zero mean and unit variance. In this paper, the channel matrices with independent elements are considered. It is a circularly symmetric Gaussian random variable with zero mean and variances of $\frac{\mathrm{SNR}_{sri}}{N_s}$, $\frac{\mathrm{SNR}_{rdi}}{N_r}$ and $\frac{\mathrm{SNR}_{sd}}{N_s}$ for $\mathbf{H}_{sri}$, $\mathbf{H}_{rdi}$ and $\mathbf{H}_{sd}$. We define SNR to be [4]

$$\mathrm{SNR}_{sri} = \frac{\mu P}{(d_{sri})^\alpha}, \mathrm{SNR}_{rdi} = \frac{(1-\mu)P}{(d_{rdi})^\alpha}, \mathrm{SNR}_{sd} = \frac{\mu P}{(d_{sd})^\alpha},$$

where $\mu \in (0, 1]$ denotes the proportion factor of transmit power between the source and the relays. The same power and

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distance between the source and N relays are assumed. d_{sri}, d_{rdi} and d_{sd} denote the distance between the source and the ith relay, between the ith relay and the destination, between the source and the destination. P is the total transmitted power, and $\alpha \in [2, 6]$ is the path loss exponent.

A. Fully Detect-and-Forward Relaying

In FDF relaying, it is assumed that the relays detect the signals correctly and the perfect CSI is available at the relays and destination. After the second time slot, the received signal vector at the destination from the ith relay is

$$\mathbf{y}_{rdi} = \mathbf{H}_{rdi}\mathbf{s}_{ri} + \mathbf{n}_{rdi}, \tag{3}$$

where $\mathbf{s}_{ri}$ is the detected signal at the ith relay and $\mathbf{n}_{rdi} \sim \mathcal{CN}(0, 1)$.

B. Amplify-and-Forward Relaying

For AF relaying, during the second time slot, the ith relay scales $\mathbf{y}_{sri}$ by a fixed-gain parameter α_i and then transmits to the destination.

According to [7], the fixed-gain constraint is given by

$$\alpha_i = \sqrt{\frac{1}{1 + 1/\text{SNR}_{sri}}} \tag{4}$$

to satisfy the power constraint. Therefore, the received signal at the destination from the ith relay is

$$\begin{aligned} \mathbf{y}_{rdi} &= \mathbf{H}_{rdi}(\alpha_i \mathbf{y}_{sri}) + \mathbf{n}_{rdi} \\ &= \alpha_i \mathbf{H}_{rdi}(\mathbf{H}_{sri}\mathbf{s}_s + \mathbf{n}_{sri}) + \mathbf{n}_{rdi} \\ &= \alpha_i \mathbf{H}_{rdi}\mathbf{H}_{sri}\mathbf{s}_s + \mathbf{n}'. \end{aligned} \tag{5}$$

Here, $\mathbf{n}' = \alpha_i \mathbf{H}_{rdi}\mathbf{n}_{sri} + \mathbf{n}_{rdi}$.

III. Detection in MIMO relay network

This section introduces the detection rule for both FDF and AF relaying networks, which uses the SD at the destination. The equivalent ML rule for multi-branch dual-hop MIMO relay network is derived. It works for both FDF and AF relaying.

A. Fully Detect-and-Forward Relaying

The ML detection at the relay and destination for a single relay network is proposed in [4] for FDF relaying. In this paper, the general ML detection rule is extended into an uncoded multi-branch dual-hop MIMO relay network.

For the first step at the ith relay, the optimal ML detection rule is given by

$$\mathbf{s}_{ri} = \arg\min_{\mathbf{s}_s \in \mathcal{Q}^{N_s}} ||\mathbf{y}_{sri} - \mathbf{H}_{sri}\mathbf{s}_s||^2, \tag{6}$$

which means that $\mathbf{s}_{ri}$ is the element in $\mathbf{s}_s \in \mathcal{Q}^{N_s}$ obtaining the minimum of $||\mathbf{y}_{sri} - \mathbf{H}_{sri}\mathbf{s}_s||^2$, and $\mathcal{Q}^{N_s}$ is the set of constellation symbols in the N_s dimensional constellation $\mathcal{Q}$. Because the computational complexity of ML detection rule increases exponentially with the number of transmit antennas N_s and with the size of constellation, the SD [8] is proposed to reduce the complexity.

The SD achieves the optimum performance with significantly reduced complexity. The main idea is to limit the points search within a hypersphere at a certain radius d around the received signal rather than to search all the constellation points. The Fincke-Pohst SD (FP-SD) and the Schnorr-Euchner SD (SE-SD) [9] are two efficient methods to realize the SD.

The basic principle of the SD algorithm is introduced briefly here. Based on Eq. (6) and the QR factorization of $\mathbf{H}_{sri}$ ($\mathbf{H}_{sri} = \mathbf{Q}_{sri}\mathbf{R}_{sri}$, where $\mathbf{R}_{sri}$ is an upper-triangular matrix and $\mathbf{Q}_{sri}$ is an unitary matrix), then let $\mathbf{z}_{sri} = \mathbf{Q}_{sri}^H \mathbf{y}_{sri}$ ($(.)^H$ denotes the Hermitian of the matrix), so Eq. (6) is equivalent to

$$\mathbf{s}_{ri} = \arg\min_{\mathbf{s}_s \in \Phi} ||\mathbf{z}_{sri} - \mathbf{R}_{sri}\mathbf{s}_s||^2. \tag{7}$$

Φ should be the set of all points in the hypersphere which satisfies $||\mathbf{z}_{sri} - \mathbf{R}_{sri}\mathbf{s}_{sri}||^2 \leq d^2$.

In the second step, the detected signals by the relays are transmitted to the destination by applying the same constellation. Hence, $N + 1$ vectors are received at the destination, which are the received signals from N relays in the second time slot and the directly received signal from the source in the first time slot. The ML detection rule at the destination is given as

$$\begin{aligned} \hat{\mathbf{s}}_d &= \arg\min_{\mathbf{s}_s \in \mathcal{Q}^{N_s}} \left(\sum_{i=1}^{N} ||\mathbf{y}_{rdi} - \mathbf{H}_{rdi}\mathbf{s}_s||^2 + ||\mathbf{y}_d - \mathbf{H}_{sd}\mathbf{s}_s||^2 \right) \\ &= \arg\min_{\mathbf{s}_s \in \mathcal{Q}^{N_s}} ||\mathbf{y}'_N - \mathbf{H}'_N \mathbf{s}_s||^2. \end{aligned} \tag{8}$$

By expanding each of the norms in Eq. (8) and regrouping some terms, we can get

$$\mathbf{H}'_N = \left(\sum_{i=1}^{N} \mathbf{H}_{rdi}^H \mathbf{H}_{rdi} + \mathbf{H}_{sd}^H \mathbf{H}_{sd} \right)^{1/2}, \tag{9}$$

$$\mathbf{y}'_N = (\mathbf{H}'_N)^{-1} \left(\sum_{i=1}^{N} \mathbf{H}_{rdi}^H \mathbf{y}_{rdi} + \mathbf{H}_{sd}^H \mathbf{y}_{sd} \right). \tag{10}$$

Therefore, at the destination, we can also use the SD for the optimal symbol error rate (SER) performance. The difference with Eq. (6) is that the SD at the destination is performed by the newly combined matrix of channel matrix and received signal vector from Eq. (9) and (10).

B. Amplify-and-Forward Relaying

Similar to FDF relaying, the ML detection rule at the destination for AF relaying is given as Eq. (8). Here,

$$\mathbf{H}'_N = \left(\sum_{i=1}^{N} (\alpha_i \mathbf{H}_{rdi}\mathbf{H}_{sri})^H (\alpha_i \mathbf{H}_{rdi}\mathbf{H}_{sri}) + \mathbf{H}_{sd}^H \mathbf{H}_{sd} \right)^{1/2}, \tag{11}$$

$$\mathbf{y}'_N = (\mathbf{H}'_N)^{-1} \left(\sum_{i=1}^{N} (\alpha_i \mathbf{H}_{rdi}\mathbf{H}_{sri})^H \mathbf{y}_{rdi} + \mathbf{H}_{sd}^H \mathbf{y}_{sd} \right). \tag{12}$$

To sum up, the SD is appropriate for the receiver in both FDF relaying and AF relaying network to reduce the complexity, while achieving nearly-optimal performance.

IV. Complexity-Efficient Detection

In this section, complexity-efficient detection for MIMO relay network by extending the FSD and CSD is introduced. We propose the CSD for MIMO systems in [6], which significantly reduces the complexity of the SD. In this paper, this algorithm is extended to multi-branch dual-hop MIMO relay networks to efficiently reduce the computational complexity at both the relays and the destination. Further, the FSD is applied [5] to obtain the fixed complexity in the MIMO relay networks.

A. Complexity-Efficient Sphere Decoder

Although the original SD achieves optimal performance with much lower complexity compared to the naive ML detector [10], it still has two main drawbacks: (i) the high complexity in the low SNR region and (ii) the variability of complexity with the SNRs. The main idea of the CSD is to scale the searching radius based on the SNR, which overcomes these disadvantages of the naive SD. The radius of the hypersphere in the CSD [6] is defined to be

$$d^2_{CSD} = \frac{\rho}{\rho + C_0} \times d^2, \tag{13}$$

where d^2 is the initial radius for the original SD, ρ is the SNR of the MIMO system, and C_0 is a certain constant to guarantee that more points are pruned in the low SNR region and fewer points are pruned in the high SNR region. Because of

$$\lim_{\rho \to \infty} \frac{\rho}{\rho + C_0} = 1, \tag{14}$$

the performance of the new CSD reverts to that of the naive SD when the SNR is sufficiently high.

The SE-SD is more efficient by contrast with the FP-SD [9]. In this paper, we apply the complexity-efficient SE-SD in MIMO relay systems.

B. Complexity measurement for MIMO relays

Because the updating of the searching radius and the cost ordering at each level in the SE-SD, it is difficult to analyze the complexity theoretically. Therefore, we resorts to simulations to evaluate the complexity of the MIMO relay systems. Considering the number of nodes visited by the searching levels in one symbol detection, the expected complexity is given by

$$C = \sum_{\text{all levels}} (\text{The number of nodes visited at each level within the hypersphere of radius } d). \tag{15}$$

For MIMO relay network, we focus on FDF and AF relaying by using the SD at the receivers. For AF relaying, the complexity is same as Eq. (15) at the destination. While the complexity of FDF relaying is evaluated by summing up the complexity at the relays and destination. So it is

$$C_{all} = \sum_{i=1}^{N} C_i + C_d, \tag{16}$$

where C_i is the complexity evaluated at the ith relay, and C_d is the complexity of detection at the destination.

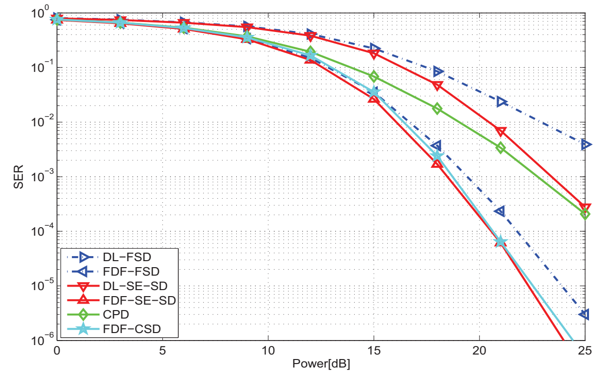

Fig. 1. Error probability of sphere decoders in a single relay MIMO network.

On the other hand, the measure method about the variability of complexity C in [6] is exploited here, shown as

$$\eta = \frac{E(C - \bar{C})^2}{\bar{C}^2}, \tag{17}$$

where $E(C)$ and $\bar{C}$ denote the expectation and the mean of the variable C, respectively. $\eta = 0$ means that the complexity is a constant and fixed. The variability of C reduces with the decreasing η.

V. Simulation Results

Both the performance and the complexity are simulated in MIMO relay network (FDF and AF relaying) with $N_s = N_r = N_d = 4$ and 16-QAM constellation. Only the results for single relay and two relays network are shown in this paper. The FSD and the CSD are compared with the SE-SD and the CPD [4]. In all the following results, it is assumed that $d_{sd} = d_{sri} + d_{rdi}$ with $\frac{d_{sri}}{d_{sd}} = 0.2,\ i \in \{1, 2, \ldots N\}$, the path loss exponent $\alpha = 3$, and $\mu = 0.5$.

Fig. 1 shows the SER performance of the FSD, the CSD, the SE-SD and the direct link (DL) in single relay network for different transmitted power. $p = 1$ (p is the number of levels where the maximum number of nodes are considered) for the FSD [5] and $C_0 = 5$ for the CSD [6] are set. For the CPD, we let $ef = 3$ (ef is the expansion factor [4]). Our proposed detection methods (the FDF-FSD and the FDF-CSD) greatly improve the SER performance for a single-relay network compared to the CPD. The relay causes the SER performance improvement than the DL. Further, the CSD achieves nearly-optimal performance and outperforms the FSD.

The complexity comparison is given in Fig. 2 corresponding to the performance in Fig. 1. The complexity of the DL is lower than the case with one relay by using FSD in higher power region. The CSD with one relay reduces the complexity by contrast to the CPD and the SE-SD in the DL for the lower power region. This is because of the path loss due to the long distance. Therefore, the CSD is most suitable for MIMO relay networks due to the reduced complexity and the almost optimal SER performance.

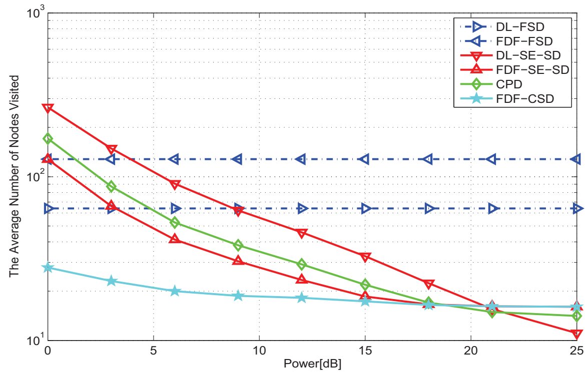

Fig. 2. Complexity comparison of sphere decoders in a single relay MIMO network.

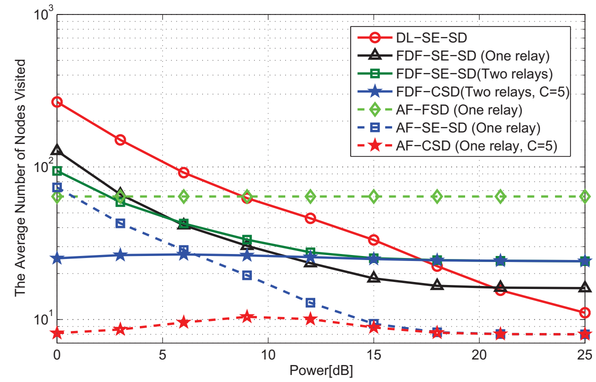

Fig. 4. Complexity comparison of sphere decoders in a multi-branch MIMO relay network.

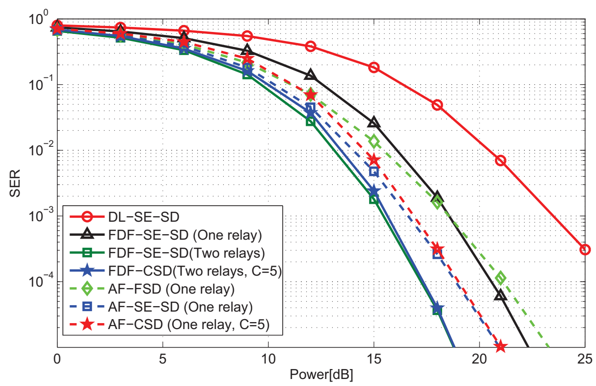

Fig. 3. Error probability of sphere decoders in a multi-branch MIMO relay network system.

The SER performance for DF and AF relaying in multi-branch dual-hop MIMO relay network is shown in Fig. 3. The performance improves when the number of relays increases, and the CSD performs nearly optimal detection in the two relays network just like in the single relay network as shown in Fig. 1. The CSD also obtains similar performance with the SE-SD in AF relaying. It outperforms the FDF relaying about 2dB due to the assumption of perfectly correct detection at the relay. However, the FSD gets worse SER performance than FDF relaying when the power is larger than 18dB.

Fig. 4 shows the complexity comparison between DF and AF relaying. Due to the path loss by long distance, the network with two relays achieves lower complexity for the SE-SD than both one relay network and the DL in the lower power region. The complexity of the CSD is lowest in the lower power region for all the cases and roughly fixed, as $\eta = 0.0014$ according to Eq. (17). The almost fixed complexity is very suitable for hardware implementation. The complexity of AF relaying including SE-SD and CSD is much lower than that of FDF. The CSD also obtains lower complexity than the SE-SD and FSD in AF relaying, just like FDF relaying in Fig. 2. Therefore, the CSD efficiently reduces the detection complexity in MIMO relay networks.

VI. Conclusions

In this paper, one complexity-efficient detection for multi-branch dual-hop relay network is proposed. The SD is exploited at the relays, while it can be also applied at the destination for FDF and AF relaying according to the equivalent ML detection rule we derived. The complexity is reduced and fixed by the FSD and the CSD by contrast to the CPD and the SE-SD. The simulation results show that our CSD is better than the FSD because of the near-optimal SER performance and the approximately fixed, reduced complexity. It is more suitable for the practical implementation.

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