# Beamforming for Physical Layer Multicasting

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***Abstract*—A systematic scheme is proposed to facilitate arbitrary virtual channel (VC)-to-user mappings in space dimension, through multiple-input multiple-output physical layer multicasting. It is a divide-and-conquer strategy, which breaks down the VC-to-user mapping to manageable orthogonal sub-mappings, each represented in terms of a multicast antenna group (MAG). A generalized form of block diagonalization is proposed to make transmissions pertaining to distinct MAGs orthogonal. Known non-iterative coordinated beamforming techniques are investigated for intra-MAG beamforming. The approach enables physical layer multicasting with non-iterative beamforming techniques alone.**

***Index Terms*—physical layer multicasting, multiuser MIMO, coordinated beamforming, block diagonalization**

## I. Introduction

Multicasting - the ability to send the same information to multiple recipients, is a need many multimedia applications have. It typically happens at higher layers (e.g. at network layer, in IP multicasting [1]). Multicasting at the physical layer, also known as physical layer multicasting (PLM) [2], appears attractive nevertheless, whenever the recipients are in the same wireless network (e.g. a multimedia streaming server delivering content to two notebook computers wireless). The reason is the ability PLM has to conduct a multicast communication using a single spatial degree of freedom (DoF). Multicasting at higher layers, on the other hand, would require repetition of the same content over more than one spatial DoFs.

Since wireless channels are of broadcast nature, multicasting a single stream of data over the air is straightforward. It only requires user selection, i.e. getting the recipients in corresponding multicast group (MG) to listen, while the others remain idle. But neither supporting spatially multiplexed data streams within a MG, nor supporting concurrent MGs via PLM is that straightforward.

Beamforming techniques decompose a multiple-input multiple-output (MIMO) channel into multiple spatial-channels (virtual channels) inflicting zero or negligible interference on one another. Techniques such as zero forcing (at the transmitter) and, more importantly, block diagonalization [3], facilitate the same for MIMO multiuser downlink. However, they produce only multiple point-to-point virtual channels (VCs) at the physical layer; where as PLM requires point-to-multipoint channels. Consequently, none of the known non-iterative beamforming techniques are readily applicable for PLM.

Iterative techniques for PLM have been investigated though, considering both the single- and multiple- antenna user configurations. The references [2], [4]–[9] focus on joint optimization of beamforming matrices to minimize the signal-to-noise plus interference ratio or the mean square error, subjected to constraints on the total transmit power and others. Performance analysis of certain multicast configurations too have been examined [10]–[12]. But attempts of generalizing the non-iterative beamforming techniques on MIMO multiuser downlink for PLM are lacking.

In this paper, we propose a systematic mean for facilitating arbitrary VC-to-user mapping through PLM. It is a divide-and-conquer scheme in a sense that the VC-to-user mapping is reduced to a set of orthogonal sub-mappings, each represented by a multicast antenna group (MAG). The key steps are:

1) selection of MGs and associated users,
2) determination of the MAGs,
3) inter-MAG beamforming for orthogonality of MAGs,
4) intra-MAG beamforming to realize VCs within MAGs.

A choice of MAGs, which ensures the availability of non-iterative inter-MAG and intra-MAG beamforming techniques, is guaranteed, provided the source has sufficient spatial DoFs.

The above procedure is detailed and examined in the subsequent discussion, highlighting the pros and cons of the approach. Our treatment of the proposed approach is introductory. A performance comparison against iterative techniques would be required to justify its use.

The paper is organized as follows: Section II covers background materials pertaining to PLM as proposed in Section III. Section IV provides a numerical example highlighting the feasibility of the approach. Section V concludes the discussion.

**Notations:** Given a matrix $\mathbf{A}$, the transpose, conjugate transpose, and pseudo inverse operations on $\mathbf{A}$ are denoted $\mathbf{A}^T, \mathbf{A}^H$, and $\mathbf{A}^\dagger$. $\mathrm{rank}(\mathbf{A})$ gives its rank, and $\mathrm{Null}(\mathbf{A})$ its null space. $\mathbf{A} \in \mathbb{C}^{m\times n}$ indicates that $\mathbf{A}$ is $m \times n$; and $\mathbf{A} = \mathrm{diag}(a_1, \ldots, a_n)$ implies that $\mathbf{A}$ is rectangular diagonal with elements $a_1, \ldots, a_n$ forming its main diagonal. $\{\mathbf{A}\}_{\mathcal{C}(n)}$ gives the sub-matrix of $\mathbf{A}$ formed with its first $n$ columns. Set operators have their standard semantics.

## II. Background

This section outlines some of the concepts and techniques related to the proposed approach for PLM.

### A. *Degrees of freedom*

The number of independent data streams a wireless terminal may send and/ or receive is the total *degrees of freedom* it has. The *spatial DoFs* a terminal experiences (in space-dimension) may not exceed the rank of the *effective* MIMO channel it has with other terminals. A particular channelization scheme



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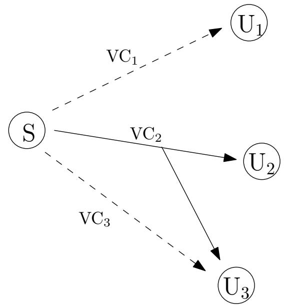


**Virtual Channel to User mapping**

| Virtual Channel \ User | $U_1$ | $U_2$ | $U_3$ |
|---|---|---|---|
| $VC_1$ (unicast) | ✓ | | |
| $VC_2$ (multicast) | | ✓ | ✓ |
| $VC_3$ (unicast) | | | ✓ |

Fig. 1. VC-to-user mapping; and unicast (−−), multicast (—) VCs.

may fail to utilize some of the DoFs. Moreover, in PLM, the end-user requirements may restrict the DoFs of the terminals.

### *B. Beamforming, and virtual channels*

The VCs produced by typical MIMO beamforming techniques (e.g. eigenmode transmission and zero forcing) are point-to-point, to emphasize which be they qualified *unicast*. Coordinated beamforming techniques producing private and common VCs are known [13]. Be such common VCs operating point-to-multipoint denoted *multicast* VCs.

In PLM, the total number of unicast and multicast VCs at a terminal may not exceed the available spatial DoFs. For example, in Fig. 1, which shows a source multicasting to three users, the Source and User 3 should have at least 3 and 2 DoFs respectively. This is a necessary but not sufficient condition.

### *C. Block diagonalization*

Multiuser MIMO decomposition [14], also known as block diagonalization (BD) [3], facilitates multiple unicast VCs from a source to two or more users.

Consider a MIMO capable source terminal communicating with $K > 1$ multiple-antenna users. Let $\mathbf{H}_i, i \in \{1, \ldots, K\}$ represent the channel the source has with the $i^{\text{th}}$ user, and $\tilde{\mathbf{H}}_i = \left[\mathbf{H}_1{}^T \ldots \mathbf{H}_{i-1}{}^T \; \mathbf{H}_{i+1}{}^T \ldots \mathbf{H}_K{}^T\right]^T$. Suppose the columns of a matrix $\tilde{\mathbf{V}}_i^{(0)}$ span the space: $\text{Null}\left(\tilde{\mathbf{H}}_i\right) - \text{Null}(\mathbf{H}_i)$. Then, a channel $\mathbf{G}_i$ defined as $\mathbf{G}_i = \mathbf{H}_i \tilde{\mathbf{V}}_i^{(0)}$ would be orthogonal to $\mathbf{H}_j, j \neq i$ under any linear beamforming scheme. To sum, BD guarantees the orthogonality of communications the source has with different users, by making the effective channel the source has with the users *block diagonal*.

BD requires no more antennas at the source than zero forcing (ZF) [3, Section III.A], which makes BD desirable. However, BD typically provides lesser spatial channels than ZF. This loss is the cost of assuring the orthogonality of users.

Example 1: Consider a '$\{2, 3, 2\} \times 6$' channel [3], representing a 6-antenna source; and 3 users with 2, 3, 2 antennas. Suppose corresponding MIMO channels are not rank-deficient. BD can produce 4 VCs; i.e. 1-, 2- and 1-unicast VC(s) with respective users. ZF would not hold as is, since the transmitter has less antennas than the receivers. But it would hold and yield 6 VCs, if a receiver antenna is disregarded (via antenna selection). Had the configuration been '$\{2, 3, 2\} \times 7$', both schemes could have produced 7-VCs.

Reference [15] suggests partitioning the 'set' of receiver antennas to form correlated antenna groups (CAGs), each comprising antennas from more than one physical user. The notion of CAGs appears to contradict the motive for BD, because it renders receiver-side beamforming impossible. Yet, it inspires the notion of MAGs we propose.

### *D. Multicast groups*

In multicasting, the same information is dispatched to all the receivers in a multicast group (MG). Consider a multiuser MIMO broadcast scenario having the MIMO channel between the source and the $i^{\text{th}}$ user denoted by $\mathbf{H}_i, i \in \{1, \ldots, K\}$, where $K$ is the total number of users. Let $\mathcal{G} \subseteq \{1, \ldots, K\}$ denote a MG of users receiving a multicast made by the common source, and define $m = |\mathcal{G}|$. Assume no interference from outside this MG. Coordinated beamforming can facilitate this MG if a transmit precoding matrix $\mathbf{W}$, and receiver reconstruction matrices $\mathbf{R}_i,\ i \in \mathcal{G}$ can be found such that:

- $\mathbf{D}_i = \mathbf{R}_i \mathbf{H}_i \mathbf{W}$ are diagonal for $i \in \mathcal{G}$, and
- at least $n$ columns of $\mathbf{D} = \left[\mathbf{D}_{i_1}{}^T \ \ldots \ \mathbf{D}_{i_m}{}^T\right]^T, \forall i_j \in \mathcal{G}$ have $m$ non-zero elements, where $n > 0$ is the number of spatially multiplexed multicast VCs desired for this MG.

Depending on the system design, the other columns of $\mathbf{D}$, if any, would yield unicast VCs; or multicast VCs serving a subset of the users in the MG.

In practice, a MG may not function in isolation; and hence, the orthogonality of the concurrent MGs needs to be assured. Complicating the matters further, the MG-to-user mapping, which depends on the end-user requirements, could be many-to-many. Therefore, multicasting at physical layer poses a challenging design problem, a systematic solution for which is proposed in Section III.

## III. System Design

Consider a MIMO capable base station (BS) having $N$ antennas multicasting to $K$ users ($\text{U}_i$) having $M_i, i \in \{1, \ldots, K\}$ antennas respectively. Define $M = \sum M_i$. Let $\mathbf{H_i} \in \mathbb{C}^{M_i \times N}$ denote the channel the BS has with $\text{U}_i$. Define $\hat{\mathbf{H}} = \left[\mathbf{H_1}^T \ldots \ \mathbf{H_K}^T\right]^T$.

The total number $n$ of possible unicast and multicast VCs is upper bounded by $\text{rank}\left(\hat{\mathbf{H}}\right) \leq \min(M, N)$, the spatial DoFs the BS has. Moreover, any user $\text{U}_i$ may receive no more than $\text{rank}(\mathbf{H}_i) \leq M_i$ VCs. The many-to-many mapping between the users and the VCs is governed by the end-user requirements subjected to the DoF constraints above.

The first hurdle in PLM is deciding if all required VCs can be supported without exhausting the spatial DoFs at each terminal. If not, non-overlapping subsets of the VCs would have to be multiplexed in other orthogonal dimensions (e.g. time). We do not dwell on this issue here, and take for granted that all VCs can be multiplexed in the space-dimension alone.

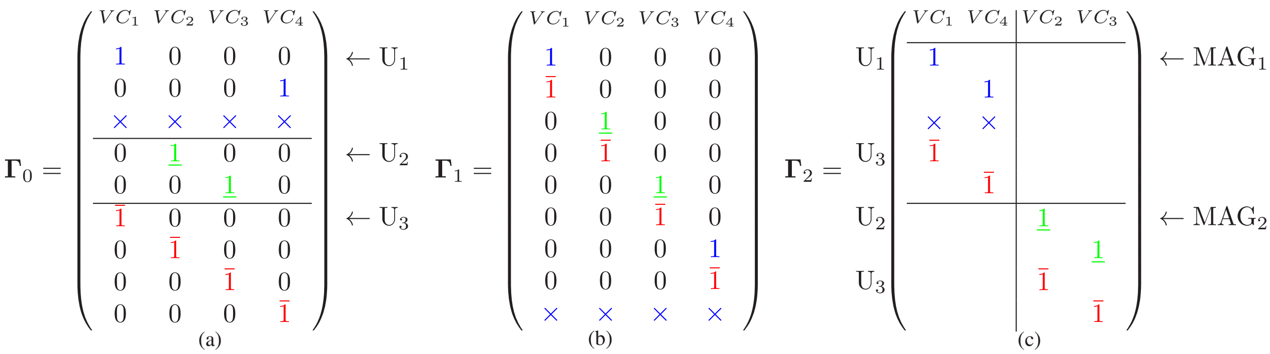


Fig. 2. VC-to-user mapping (3 users and 4 VCs). Non-zero entries corresponding users $\mathrm{U}_1, \mathrm{U}_2$ and $\mathrm{U}_3$ are indicated respectively as $1, \underline{1}$ and $\bar{1}$.

### A. Multicast antenna groups

In one hand, designing beamforming matrices jointly for all $K$ users considering all the VCs is optimal. It might however be prohibitive complexity-wise. Implementing VCs separately, on the other hand, is simpler; but assuring their orthogonality (e.g. via BD) could be costly in terms of spatial DoFs. Hence, a division coarser than the VC-level is desired where possible.

Accommodating the many-to-many mappings between the VCs and the users, in general, requires partitioning the antennas of a given user among several VCs. Therefore, the user level too is coarse for this segmentation.

The solution we propose is an abstract notion: multicast antenna groups (MAGs), a specialization of CAGs in reference [15] having the following characteristics.

- VC-to-MAG relationship is *Many-to-One*:
  A MAG supports one or more unicast/ multicast virtual channels $\mathrm{VC}_k, \exists k \in \{1, \ldots, n\}$; each VC is associated with just one MAG. VCs catering a certain subset of users may be accommodated in the same MAG, to keep the number of MAGs small.
- Antenna-to-MAG relationship is *Many-to-One*:
  For each supported VC, a MAG has a sufficient number of antennas from corresponding users allocated.
  Let $\Xi_i$ denote the set of antennas the user $\mathrm{U}_i$ has (excluding any antenna disregarded through *antenna selection*[1]); and $\Theta_j$ represent the set of antennas associated with $\mathrm{MAG}_j$, the $j^{th}$ MAG.
  - $\Xi_i \cap \Theta_j \neq \phi$ iff $\exists \, \mathrm{VC}_k$ in $\mathrm{MAG}_j$ that caters $\mathrm{U}_i$.
  - $|\Xi_i \cap \Theta_j|$ should be no less than number of $\mathrm{VC}_k$ in $\mathrm{MAG}_j$ that cater $\mathrm{U}_i$.
  - $\Theta_j$s form a partition of $\bigcup_{\forall i} \Xi_i$.

Beamforming techniques can be used to realize the MAGs.

- Inter-MAG beamforming:
  The effective MIMO channel $\mathbf{G}_j$ pertaining to each $\mathrm{MAG}_j$ is formed concatenating the rows of $\hat{\mathbf{H}}$ that correspond to antennas in $\Theta_j$. Inter-MAG beamforming uses BD to make the communications over $\mathbf{G}_j$s orthogonal.
- Intra-MAG beamforming:
  Since elements in $\Theta_j$ may come from more than one physical user, coordinated beamforming techniques are required for beamforming within the MAGs.

To summarize, MAGs allow a compromise be made between the two extreme cases: beamforming for the whole system, and, considering the VCs one-by-one.

### B. Determination of MAGs

Let $\mathbf{\Gamma}_0$ be an $M \times n$ binary matrix indicating the mapping the receiver antennas have with the VCs, such that

1) the $i^{\text{th}}$ antenna of the $j^{\text{th}}$ user corresponds to row number $i + \sum_{k=1}^{j-1} M_k$ of $\mathbf{\Gamma}_0$,
2) columns of $\mathbf{\Gamma}_0$ correspond to VCs, and
3) each non-zero element associates a VC with a corresponding antenna.

For the sake of simplicity, we assume the antennas of a given user are interchangeable[2].

A MAG should have sufficient number of antennas alloted from respective users to facilitate the VCs they receive. Excess antennas the users may have can be disregarded or assigned to a MAG. Thus, each row of $\mathbf{\Gamma}_0$ would have a single unit element; and each column would have at least one unit element (see Fig. 2-a). Excess antennas may be treated as *Don't Cares* (designated as: '$\times$' on $\mathbf{\Gamma}_0$) to gain more leeway for determination of the MAGs.

Since the VC-to-MAG relationship is Many-to-One, the first phase of determining the MAGs is interchanging the rows of $\mathbf{\Gamma}_0$ to make any non-zero elements on the columns adjacent. Don't Cares can be disregarded in this process (as in $\mathbf{\Gamma}_1$ of Fig. 2-b). Notably, a block diagonal form $\mathbf{\Gamma}_1$ with each block having just one column (i.e. one MAG per VC) is always realizable.

The cost of inter-MAG beamforming makes it preferable to keep the number of MAGs minimal. Therefore, more than one VC may be associated with a MAG considering the similarity of the VCs, while ensuring that the VC mapping within each MAG remains realizable. The rows and columns of $\mathbf{\Gamma}_1$ (of Fig. 2-b) may be interchanged, for instance, to obtain

[1] Antenna selection is not desirable in a beamforming point-of-view. However, leaving out user antennas could ease assuring the orthogonality of MAGs.

[2] Distinguishing them apart requires (i) the non-zero elements of $\mathbf{\Gamma}_i$-mappings to represent the quality of the instantaneous multiple-input single-output channel each receiver antenna sees; and (ii) considering all permutations when allotting antennas of a given physical user among the MAGs.

a block diagonal form as in $\mathbf{\Gamma}_2$ (of Fig. 2-c) having the rows corresponding to each MAG grouped by users.

Following example focuses on the VC-to-user mapping in Fig. 2, and illustrates the determination of MAGs as well as the implications of the divide-and-conquer approach.

Example 2:

- The system considered in Fig. 2 can be broken down into 2 MAGs:
  - $\mathrm{MAG}_1$ using 3 antennas from $\mathrm{U}_1$ and 2 antennas from $\mathrm{U}_3$ to support 2 VCs, and
  - $\mathrm{MAG}_2$ using 2 antennas each from $\mathrm{U}_2$ and $\mathrm{U}_3$ to support 2 VCs.

  $\mathbf{\Gamma}_0$ being a simple mapping, only the antennas of $\mathrm{U}_3$ have to be split between the MAGs; and similar VCs (e.g. $\mathrm{VC}_1$ and $\mathrm{VC}_4$) are easily grouped together.

  Effective channel matrix for $\mathrm{MAG}_1$ is $\mathbf{G}_1 \in \mathbb{C}^{5\times N}$, where $N$ is the number of antennas at the BS. $\mathbf{G}_1$ has, say, the first 3 rows from $\mathbf{H}_1$, and the first 2 rows of $\mathbf{H}_3$ concatenated. The effective channel matrix $\mathbf{G}_2 \in \mathbb{C}^{4\times N}$ for $\mathrm{MAG}_2$ may be defined similarly.

  Suppose the channels are not rank deficient. $N \geq 7$ would make the MAGs orthogonal. If the excess antenna of $\mathrm{U}_1$ (i.e. the one corresponding to Don't Cares in $\mathbf{\Gamma}_2$) is disregarded, $N = 6$ would be sufficient to achieve the same VCs.
- Alternatively the system can be implemented with 4 MAGs, each supporting a distinct multicast VC. Requirement for assuring the orthogonality is $N \geq 7$, even when the excess antenna is disregarded. Each MAG is effectively a signal-input multiple-output channel, intra-MAG beamforming for which is straightforward.
- Multicasting at higher layers, i.e. the use of multiple unicast VCs in place of each multicast VC, achieves the same for $N \geq 8$ (again with the excess antenna disregarded).

To summarize, PLM achieves the 4 multicast VCs using $\min(N, 9)$ spatial DoFs. The divide-and-conquer approach exhausts some DoFs, increasing the lower bound of $N$; nevertheless it makes beamforming within the MAGs simpler.

### C. Inter-MAG beamforming

The focus of Subsection III-B was reducing the PLM problem to manageable subproblems, of course, risking the loss of spatial DoFs. Inter-MAG beamforming is the mean for assuring the orthogonality of these 'subproblems'.

Let $\mathbf{G}_j \in \mathbb{C}^{|\Theta_j|\times N}$ be the effective channel corresponding to each $\mathrm{MAG}_j$. Consider $\mathbf{V}_j^{(0)} \in \mathbb{C}^{N\times n_j}$ whose columns span the space: $\bigcap_{k\neq j} \mathrm{Null}(\mathbf{G}_k) - \mathrm{Null}(\mathbf{G}_j)$. $n_j = \mathrm{rank}\left(\hat{\mathbf{H}}\right) - \mathrm{rank}\left(\tilde{\mathbf{G}}_j\right)$ gives the maximal number of VCs $\mathrm{MAG}_j$ may support, where $\tilde{\mathbf{G}}_j = \left[{\mathbf{G}_1}^T \ldots {\mathbf{G}_{j-1}}^T \, {\mathbf{G}_{j+1}}^T \ldots {\mathbf{G}_K}^T\right]^T$. It is noteworthy that $\sum n_j$ cannot exceed the total spatial DoFs at the BS.

Let $r_j \leq n_j$ be the number of unicast and multicast VCs end-users require $\mathrm{MAG}_j$ to support.

1) Consider an effective channel matrix $\hat{\mathbf{G}}_\mathbf{j} = \mathbf{G}_j \mathbf{V}_j^{(0)}$, and compute a precoding matrix $\mathbf{W}_j$ along with the receiver reconstruction matrices using any desired intra-MAG beamforming technique. Then, the matrices $\hat{\mathbf{W}}_\mathbf{j}$, each formed with selected $r_j$ columns of $\mathbf{V}_j^{(0)}\mathbf{W}_j$, would jointly ensure that all VC-to-user mappings are realized.
2) Alternatively, $\hat{\mathbf{G}}_\mathbf{j}$ may be defined as $\mathbf{G}_j \left\{\mathbf{V}_j^{(0)}\right\}_{\mathcal{C}(r_j)}$, and the precoding matrix computed for intra-MAG beamforming be modified as $\hat{\mathbf{W}}_\mathbf{j} = \left\{\mathbf{V}_j^{(0)}\right\}_{\mathcal{C}(r_j)} \mathbf{W}_j$.

The approach 2) above apparently yields lower diversity gains than approach 1). However, approach 2) simplifies intra-MAG beamforming some of the times (e.g. when $r_j = 1$). Receiver reconstruction matrices produced by intra-MAG beamforming require no modification with either approach.

### D. Intra-MAG beamforming

This section examines the intra-MAG beamforming techniques usable to facilitate the VCs within each MAG $j$, through joint computation of the precoding matrix $\mathbf{W}_j$ and the reconstruction matrices $\mathbf{R}_{j,k}, k \in \{1, \ldots, K\}$.

Rows of $\hat{\mathbf{G}}_j$ correspond to distinct antennas, each of which belongs to one of the physical users associated with the VCs served by $\mathrm{MAG}_j$. Without a loss of generality, assume that the rows corresponding to each user are group together as blocks within $\hat{\mathbf{G}}_j$. That is, $\hat{\mathbf{G}}_j = \left[\hat{\mathbf{G}}_{j,1}^T \ldots \hat{\mathbf{G}}_{j,K}^T\right]^T$ where the submatrix $\hat{\mathbf{G}}_{j,i} \in \mathbb{C}^{|\Xi_i \cap \Theta_j|\times n_j}$, possibly having zero rows, represents the $\mathrm{U}_i$'s contribution to $\mathrm{MAG}_j$.

Only a few non-iterative beamforming techniques are currently known. They are summarized below.

*1) SIMO diversity combining:* This technique may be applied whenever the effective channel $\hat{\mathbf{G}}_j$ is a column vector. It is similar to diversity combining in a multiuser signal-input multiple-output (SIMO) channels; and, arbitrary number of users can be catered. A scalar $\mathbf{W}_j = 1$ would be used for precoding; and vectors $\mathbf{R}_{j,i} = \hat{\mathbf{G}}_{j,i}^H$ be used for maximal-ratio combining. The simplicity and the straightforwardness of this approach might even justify setting aside a separate MAG for each VC whenever DoFs at the common source is adequate.

*2) SVD-based beamforming:* In the trivial case of having a MAG catering a single physical user, intra-MAG beamforming reduces to that of a point-to-point MIMO channel. Eigenmode transmission based on the singular value decomposition (SVD) [16, (5)] is a suitable candidate for this case.

*3) GSVD-based beamforming:* Generalized singular value decomposition (GSVD) facilitates joint decomposition of two matrices having same number of columns. It can be exploited [13] for coordinated beamforming in two-user MIMO downlink channels, to obtain both unicast and multicast VCs. Hence, *GSVD-based beamforming* is usable for implementing MAGs associated with two physical users.

Let $\hat{\mathbf{G}}_{j,k} \in \mathbb{C}^{m_k \times n_j}, k \in \{p, q\}$, where $m_k = |\Xi_k \cap \Theta_j|$ correspond to the two physical users $\mathrm{U}_p, \mathrm{U}_q$ of $\mathrm{MAG}_j$; $\Theta_j \subseteq \Xi_p \cup \Xi_q$. Define $t = \mathrm{rank}\left(\hat{\mathbf{G}}_j\right) > 0$, $r = t -$

$\mathrm{rank}\left(\hat{\mathbf{G}}_{j,q}\right) \geq 0$, $s = \mathrm{rank}\left(\hat{\mathbf{G}}_{j,p}\right) + \mathrm{rank}\left(\hat{\mathbf{G}}_{j,q}\right) - t \geq 0$. GSVD yields a decomposition

$$\hat{\mathbf{G}}_{j,p} = \mathbf{V}_p \mathbf{\Sigma}_{j,p} \mathbf{Q}, \qquad \hat{\mathbf{G}}_{j,q} = \mathbf{V}_q \mathbf{\Sigma}_{j,q} \mathbf{Q}, \quad (1)$$

where (i) $\mathbf{V}_p \in \mathbb{C}^{m_p \times m_p}, \mathbf{V}_q \in \mathbb{C}^{m_q \times m_q}$ are unitary, (ii) $\mathbf{Q} \in \mathbb{C}^{t \times n_j}$ non-singular, and (iii) $\mathbf{\Sigma}_{j,p} \in \mathbb{C}^{m_p \times t}, \mathbf{\Sigma}_{j,q} \in \mathbb{C}^{m_q \times t}$ have structures

$$\mathbf{\Sigma}_{j,p} \triangleq \begin{pmatrix} \mathbf{I}_p & & \\ & \tilde{\mathbf{\Sigma}}_{j,p} & \\ & & \mathbf{0}_p \end{pmatrix}, \ \mathbf{\Sigma}_{j,q} \triangleq \begin{pmatrix} \mathbf{0}_q & & \\ & \tilde{\mathbf{\Sigma}}_{j,q} & \\ & & \mathbf{I}_q \end{pmatrix}.$$

$\mathbf{I}_p \in \mathbb{C}^{r \times r}$ and $\mathbf{I}_q \in \mathbb{C}^{(t-r-s)\times(t-r-s)}$ are identity matrices. $\mathbf{0}_p \in \mathbb{C}^{(m_p-r-s)\times(t-r-s)}$, and $\mathbf{0}_q \in \mathbb{C}^{(m_q-t+r)\times r}$ are zero matrices possibly having no rows or no columns. $\tilde{\mathbf{\Sigma}}_{j,p} = \mathrm{diag}(\sigma_1, \ldots, \sigma_s), \tilde{\mathbf{\Sigma}}_{j,q} = \mathrm{diag}(\lambda_1, \ldots, \lambda_s) \in \mathbb{C}^{s \times s}$ such that $1 > \sigma_1 \geq \ldots \geq \sigma_s > 0$, and $\sigma_i^2 + \lambda_i^2 = 1$ for $i \in \{1, \ldots, s\}$.

The choice of (i) precoding matrix $\mathbf{Q}^\dagger$, and (ii) receiver reconstruction matrices $\mathbf{V}_k^H, k \in \{p, q\}$ ensures intra-MAG beamforming, with the non-zero elements of $\mathbf{\Sigma}_{j,k}$ governing the VC mapping for user $\mathrm{U}_k$. As elaborated in [13],

- columns 1 through $r$, if any, provide unicast VCs in $\mathrm{MAG}_j$ for $\mathrm{U}_p$,
- columns $r+1$ through $r+s$, if any, provide multicast VCs in $\mathrm{MAG}_j$ for $\mathrm{U}_p$ and $\mathrm{U}_q$,
- columns $r+s+1$ through $t$, if any, provide unicast VCs in $\mathrm{MAG}_j$ for $\mathrm{U}_q$.

The choice of the number of antennas for $\mathrm{MAG}_j$ and other MAGs governs the values of $r, s, t$; therefore the number of available unicast and multicast VCs. This fact appears to restrict[3] the use of GSVD-based beamforming for PLM. Finer control on the unicast vs. multicast breakdown of the VCs can nevertheless be achieved through careful selection of columns of $\mathbf{V}_k, k \in \{p, q\}$ and $\mathbf{Q}^\dagger$ for beamforming.

Each receiver reconstruction matrix $\mathbf{V}_k^H, k \in \{p, q\}$ ensures that all antennas in $\Xi_k \cap \Theta_j$ are utilized to receive the VCs. Hence, an explicit antenna-to-VC mapping is not realizable, just as in eigenmode transmission over MIMO channels (but unlike in ZF).

Intra-MAG beamforming for more complicated mappings is desirable in terms of the spatial DoFs savings. Only iterative algorithms are known presently for the purpose (e.g. [2]).

## IV. Numerical Results

Monte-Carlo simulation based performance analysis results are presented here for the multiuser MIMO configuration considered in Example 2. Quasi-static independent and identically distributed (i.i.d.) Rayleigh fading is assumed between the transmit- and receive- antenna pairs.

Fig. 3 shows average symbol error rate (SER) experienced by each recipient of Example 2. $N = 7$ antennas are assumed at the BS so as to implement PLM based on both the VC-to-user mappings: $\mathbf{\Gamma}_1$ and $\mathbf{\Gamma}_2$. SIMO diversity combining is used within the MAGs based on $\mathbf{\Gamma}_1$; GSVD-based beamforming is used for those based on $\mathbf{\Gamma}_2$. Monte-Carlo simulation uses $10^6$ channel instantiations, with a block of 10 quadrature phase shift keying (QPSK) modulated symbols per VC transmitted for each channel instantiation. Average transmit power per block is normalized to be $N$.

[3]This can be overcome through antenna selection, or more generally, using an additional set of suitable recombination matrices.

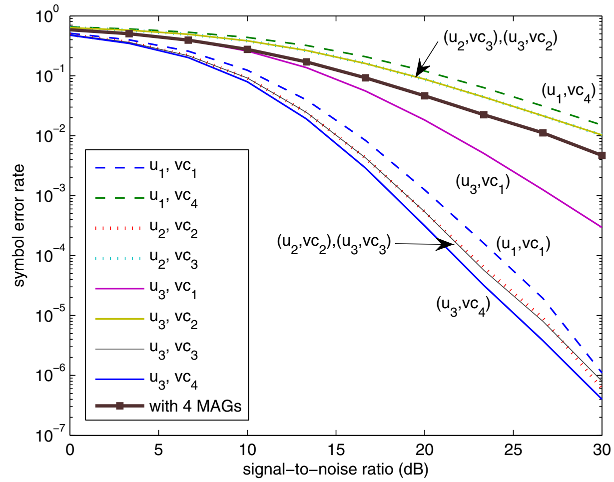


Fig. 3. Average SER at each recipient of Example 2 with $N = 7$. The notation '$(\mathrm{u}_k, \mathrm{VC}_j)$' refers to $j^{\mathrm{th}}$ VC of user $k$, assuming $\Gamma_2$ mapping. Curves corresponding to $\Gamma_1$ overlap, and indicated as 'with 4 MAGs'.

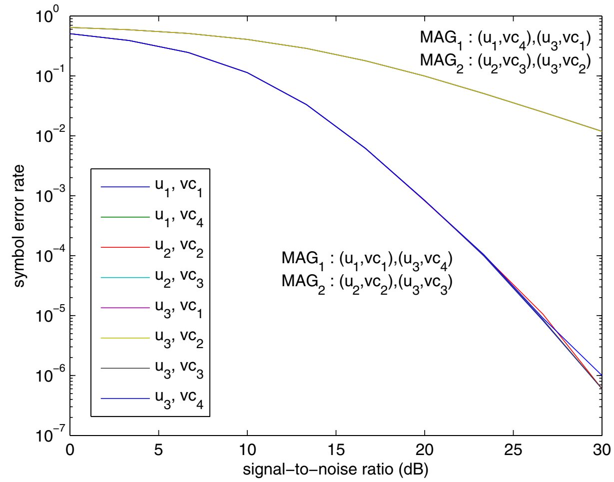


Fig. 4. Average SER at each recipient of Example 2 with $N = 6$. The notation '$(\mathrm{u}_k, \mathrm{VC}_j)$' refers to $j^{\mathrm{th}}$ VC of user $k$, assuming $\Gamma_2$ mapping with the excess antenna disregarded.

All VCs are seen to exhibit identical SER performance in the 4-MAG implementation based on $\mathbf{\Gamma}_1$. Even though an exact analytical characterization of the diversity order is not available for GSVD-based beamforming, the relative differences of diversity order in Fig. 3 can be interpreted easily. Consider $\mathrm{VC}_2$ and $\mathrm{VC}_3$ that correspond to $\mathrm{MAG}_2$ (in Example 2). At any given instant, signal-to-noise ratio (SNR) of multicast VCs produced by GSVD-based beamforming is in descending order for the first user; and in ascending order for the other. Therefore $\mathrm{U}_2$ experiences higher diversity order for $\mathrm{VC}_2$ and lower for $\mathrm{VC}_3$. It is the other way around for $\mathrm{U}_3$. Somewhat similar observations can be made regarding

Code 1: MATLAB code & output

```
    H1 = (randn(2,6)+1i*randn(2,6))/sqrt(2); % for U_1; MAG_1
    H2 = (randn(2,6)+1i*randn(2,6))/sqrt(2); % for U_2; MAG_2
    H3 = (randn(4,6)+1i*randn(4,6))/sqrt(2); H3a = H3(1:2,:); H3b = H3(3:4,:); % for U_3; MAG_1, MAG_2
    % compute basis for each null space (for inter-MAG beamforming)
    G1 = [H1; H3a]; [~,S,V] = svd(G1); V02 = V(:,rank(S)+1:end);
    G2 = [H2; H3b]; [~,S,V] = svd(G2); V01 = V(:,rank(S)+1:end);
    H1hat = H1 * V01; H3ahat = H3a * V01; H2hat = H2 * V02; H3bhat = H3b * V02; % effective channel matrices
    % perform GSVD (for intra-MAG beaforming) - adjustments made for differences in MATLAB implementation
    [V1,U1,X,~,~] = gsvd(H3ahat,H1hat); Q = X'; Qinv1 = Q'/(Q*Q');
    [V2,U2,X,~,~] = gsvd(H3bhat,H2hat); Q = X'; Qinv2 = Q'/(Q*Q');
    % find joint beamforming matrices (all columns used since this case has only point-to-2 point VCs)
    R1 = U1; R2 = U2; R3 = blkdiag(V1,V2); W = [V01*Qinv1, V02*Qinv2];
    R3 = R3(:,[1,3,4,2]); W = W(:,[1,3,4,2]); % fix order of antennas and VCs
    map = [R1'*H1;R2'*H2;R3'*H3]*W; % compute the mapping (i.e. effective ch. coef. for each VC at recipient)
    inhibit = @(x)x.*(abs(x)>1e-15); inhibit(real(map)) + 1i* inhibit(imag(map)) % inhibit values < 1e-15
>> ans =
    0.8615         0         0         0
         0         0         0    0.5441
         0    0.9473         0         0
         0         0    0.2772         0
    0.5077         0         0         0
         0    0.3203         0         0
         0         0    0.9608         0
         0         0         0    0.8390
```

$\text{VC}_1$ and $\text{VC}_4$ supported by $\text{MAG}_1$. The performance of $\text{U}_1$ and $\text{U}_3$ in this case is not similar, because of the effect the excess antenna of $\text{U}_1$ has on the beamforming matrices. The fact is evident from Fig. 4, which assume $N = 6$ disregarding the excess antenna altogether. All $4$ VCs exhibit similar SER performance in this case. The observation is expected for i.i.d. fading, since each VC is point-to-2 point.

MATLAB code snippet above (see Code 1) illustrates the operations required to realize the mapping $\mathbf{\Gamma}_0$ in Fig. 2. $N = 6$ is assumed disregarding the excess antenna of $\text{U}_1$ corresponding to *Don't Cares* in Fig. 2-a. The coefficients of each VC for a single instantiation of the channels is included as an output. The correspondence of the non-zero entries of $\mathbf{\Gamma}_0$ (in Fig. 2) and the code output '`ans`' confirms that perfect VC-to-user mapping has been achieved!

## V. Conclusion

A divide-and-conquer strategy for realizing physical layer multicasting in multiple-input multiple-output multiuser downlink has been proposed and examined.

It involves forming multicast antenna groups (MAGs) to cater the unicast/ multicast virtual channels which need to be facilitated, apportioning the receiver antennas among these MAGs. A generalized form of BD is used to ensure orthogonality of communications on different MAGs; while non-iterative coordinated beamforming techniques are suggested for beamforming within each MAG. Numerical results are provided verifying the ability to realize a desired VC-to-user mapping. Several possibilities that realize a given VC-to-user mapping may exist (e.g. Example 2). Present work merely outlines the proposed approach and does consider choosing between these possibilities.

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