Multiuser Amplify-and-Forward Relaying with Delayed Feedback in Nakagami-m Fading

Madushanka Soysa*, Himal A. Suraweera†, Chintha Tellambura* and Hari K. Garg†
*Department of Electrical and Computer Engineering, University of Alberta, Canada
†Department of Electrical and Computer Engineering, National University of Singapore, Singapore
E-mail: {soysa,chintha}@ece.ualberta.ca, {elesaha, eleghk}@nus.edu.sg

***Abstract*—This paper evaluates the impact of using outdated channel estimates in a multiuser Amplify-and-Forward (AF) relay network, under Nakagami-m fading. Both variable gain AF and fixed gain AF schemes are considered. Expressions for the system's outage probability and the average bit error rate (BER) are derived. Since the expressions are barely tractable, we also present approximations for the high signal-to-noise ratio (SNR) regime. By doing so we characterize the impact of network parameters such as the number of relays, correlation between the delayed and current channel state information, chosen user rank and SNR imbalance on the performance degradation.**

I. Introduction

Wireless systems using low complexity relays are capable of efficiently combating fading impairment and provide for coverage extension [1]. Amplify-and-forward (AF) is a well studied protocol to implement relay based communications. Since the relay nodes simply retransmit an amplified version of the received signal, AF relaying is considered to have complexity benefits [2].

Increased demand for high data rates and quality-of-service (QoS) even at the cell boundary is important for future wireless network designing. However, the traditional point-to-multipoint architectures may be unable to support high data rates for users in the far-off distance from the base station or in locations with severe shadowing. Since a relay can cover multiple destinations/users, therefore, new relay based systems are in high demand. The relayed downlink can achieve the best performance, if the system selects the user with the best end-to-end channel quality at a time, which is achieving the so-called *multiuser diversity*.

The performance of point-to-point wireless systems assisted by a single/multiple relays is now well understood, see for eg. [3]–[5] and references therein. Although recent literature has also presented different performance aspects of point-to-multipoint dual-hop links, i.e., multiuser relay networks (MRNs), in many cases of practical interest results are unknown. In MRNs, a single source communicates with multiple destination (user) nodes with the aid of a relay. In [6], the asymptotic outage probability and the error performance of a dual-hop MRN has been investigated. In [7], to reduce the amount of feedback that is needed to select the best user, two methods of SNR-threshold based channel quality information reporting has been proposed. A framework to analyze the multiuser diversity performance in dual-hop relay networks has been presented in [8]. In [9], the capacity of MRNs over Nakagami-m fading has been investigated. In [10], the outage probability and the diversity order of AF based MRNs over Nakagami-m fading has been studied. In [11], the use of multiuser diversity in multi-source multi-relay networks has been investigated. However, to the best of authors' knowledge, all of these works have assumed perfect channel knowledge in the user selection process. So far, despite the practical importance, the case of outdated channel state information (CSI) due to delayed feedback and the performance degradation in MRNs has not been addressed.

In this paper we have considered a MRN in Nakagami-m fading. More specially our new contributions can be summarized as follows:

- We analytically investigate the performance of a MRN when user selection performed based on the outdated CSI. Moreover, instead of only considering the best user selection criterion, our analysis considers the most general case of kth worst user selection [12]. Hence, the presented results can be directly applied to a large set of situations and fading scenarios.
- We derive exact closed-form expressions for the outage probability and the average BER of MRNs equipped with either fixed gain or variable gain relays. This case has previously been studied in [10] but only when user selection is performed using perfect CSI.
- The impact of outdated CSI on the performance is investigated, for both variable gain and fixed gain relaying using high SNR approximations. We obtain the achievable diversity order of MRNs. This result proves that both cases relaying yield the same diversity order and is equal to either the first hop or the second hop Nakagami-m fading parameter, irrespective of the user selection rank.

II. System Model

We consider a relay network where a single source (S) communicates with N_u multiple users through a dedicated relay station (R) as shown in Fig. 1. All terminals are equipped with single antenna while S and each of the N_u users does not have a direct link due to heavy shadowing and path loss. We assume that $\mathsf{S} \rightarrow \mathsf{R}$ and each of the $\mathsf{R} \rightarrow \mathsf{U}_\ell$, $\ell = 1, \ldots, N_u$ channels experience Nakagami-m fading with

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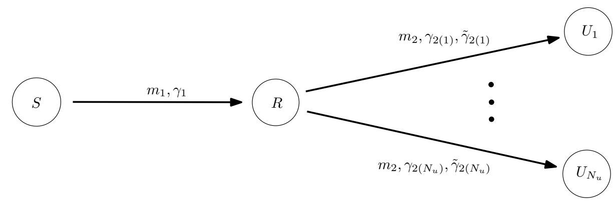

Fig. 1. In the MRN, S selects the user with the k-th worst $\mathsf{R} \rightarrow \mathsf{U}_k$ channel based on outdated feedback obtained at R. The experienced SNR of the link $\mathsf{R} \rightarrow \mathsf{U}_k$, $\widetilde{\gamma}_{2(k)}$, is therefore different from $\gamma_{2(k)}$.

parameter $m_1 \in \mathbb{Z}^+$ and $m_2 \in \mathbb{Z}^+$ respectively. Moreover, S and R are fixed in location while the users are mobile.

For communication, S selects the user (U_k) with the k-th worst channel (i.e., user with the $(N_u - k)$-th highest received SNR) according to the CSI obtained through feedback. Hence, selection of the user who can achieve the largest receive SNR extracting the multiuser diversity is a special case of our analysis. Since in practice, the control link is never perfect, we assume a delay of T_d in the feedback. After user selection, the communication occurs using the half-duplex mode in two signaling intervals: During the first time slot, S communicates with R to send $x(t)$. During the second time slot, R transmits its received signal towards the selected user U_k.

The received signal at R is given by

$$Y_{\mathsf{R}}(t) = \sqrt{P_s} h_{\mathsf{S,R}}(t) x(t) + n_{\mathsf{R}}(t), \tag{1}$$

where P_s is the transmit power at S, the complex fading channel from S to R is denoted by $h_{\mathsf{S,R}}(t)$ and $n_{\mathsf{R}}(t)$ is the additive white Gaussian noise (AWGN) at R satisfying $E\left(|n_{\mathsf{R}}(t)|^2\right) = N_{01}$ with $E(\cdot)$ denoting expectation. A scaling gain, G is applied by R to $Y_{\mathsf{R}}(t)$ and the output is re-transmitted to U_k. The received signal at U_k is given by

$$Y_{\mathsf{U}_k}(t) = h_{\mathsf{R},\mathsf{U}_k}(t) \mathsf{G} Y_{\mathsf{R}_{(k)}}(t) + n_{\mathsf{U}_k}(t), \tag{2}$$

where $h_{\mathsf{R},\mathsf{U}_k}(t)$ is the complex channel between R and U_k, and $n_{\mathsf{U}_k}(t)$ is the AWGN at U_k satisfying $E\left(|n_{\mathsf{U}_k}(t)|^2\right) = N_{02}$.

Let $\gamma_1 = |h_{\mathsf{S,R}}(t)|^2 \eta_1$ and $\widetilde{\gamma}_{2(k)} = |h_{\mathsf{R},\mathsf{U}_k}(t)|^2 \eta_2$, where $\eta_1 = \frac{P_s}{N_{01}}$, $\eta_2 = \frac{P_r}{N_{02}}$ and P_r is the average transmit power of R. We further define $\gamma_{2(k)} = |h_{\mathsf{R},\mathsf{U}_k}(t - T_d)|^2 \eta_2$. With the assumption of Nakagami-m fading, we have $\gamma_1 \sim \mathcal{G}\left(m_1, \frac{\eta_1}{m_1}\right)$ and $\gamma_{2(\ell)} \sim \mathcal{G}\left(m_2, \frac{\eta_2}{m_2}\right)$ where $\mathcal{G}(\lambda, \theta)$ is the gamma distribution with scale parameter θ and shape parameter λ and $\ell = 1 \dots N_u$ represents unordered users. For simplicity, a clustered user setting is assumed, i.e., $\gamma_{2(\ell)}$s are i.i.d distributed. A generalization to non-identical fading (distributed users) is straightforward. Note that the user selection is based on $\gamma_{2(k)}$ while $\widetilde{\gamma}_{2(k)}$, the link SNR experienced by the signal, is a delayed version of $\gamma_{2(k)}$.

Fixed Gain Relaying: Consider the case where R uses a fixed scaling gain such that a constant average transmit power at R is maintained. Assuming that R has the statistics of the $\mathsf{S} \rightarrow \mathsf{R}$ link, we can select a scaling gain of

$$\mathsf{G_F} = \sqrt{\frac{P_r}{P_s E\{|h_{\mathsf{S},\mathsf{R}_{(k)}}(t)|^2\} + N_{01}}}, \tag{3}$$

to apply to $Y_{\mathsf{R}_{(k)}}(t)$. Therefore, at R, fixed gain relaying can avoid the task of continuous monitoring of the $\mathsf{S} \rightarrow \mathsf{R}$ link.

Variable Gain Relaying: At R CSI-based variable gain relaying aims to maintain a constant instantaneous output power for the retransmitted signal. Assuming that R has the $\mathsf{S} \rightarrow \mathsf{R}$ link instantaneous CSI knowledge, we can select the gain factor $\mathsf{G_V}$ as

$$\mathsf{G_V} = \sqrt{\frac{P_r}{P_s |h_{\mathsf{S,R}}(t)|^2 + N_{01}}}. \tag{4}$$

III. Analysis of Fixed Gain Relaying

In this section we analyze the outage probability and the average BER using fixed gain relaying.

It can be shown that the instantaneous end-to-end SNR is given by

$$\gamma_{eq1} = \frac{\gamma_1 \widetilde{\gamma}_{2(k)}}{\widetilde{\gamma}_{2(k)} + C}, \tag{5}$$

where $C = \frac{P_r}{\mathcal{G}_{\mathsf{F}}^2 N_{01}}$. After some manipulations, it is easy to show that, $C = E\{\gamma_1 + 1\} = \eta_1 + 1$.

A. Outage Probability

The outage probability, P_o, defined as the probability that the end-to-end SNR drops below a predefined threshold γ_T, is an important QoS measure. It is equal to the cumulative distribution function (cdf) value of the end-to-end SNR evaluated at γ_T, i.e., $P_o = F_{\gamma_{eq1}}(\gamma_T)$. Mathematically,

$$F_{\gamma_{eq1}}(\gamma_T) = \Pr\left(\frac{\gamma_1 \widetilde{\gamma}_{2(k)}}{\widetilde{\gamma}_{2(k)} + C} < \gamma_T\right) \tag{6}$$

$$= \int_0^\infty F_{\gamma_1}\left(\frac{\gamma_T (C + x)}{x}\right) f_{\widetilde{\gamma}_{2(k)}}(x) dx, \tag{7}$$

where $\Pr(\cdot)$ denotes probability and $F_{\gamma_1}(x)$ is the cdf of γ_1 and $f_{\widetilde{\gamma}_{2(k)}}(x)$ is the probability density function (pdf) of $\gamma_{2(k)}$. Since $\gamma_1 \sim \mathcal{G}\left(m_1, \frac{\eta_1}{m_1}\right)$ we have

$$F_{\gamma_1}\left(\frac{\gamma_T (C + x)}{x}\right) = 1 - \frac{\Gamma\left(m_1, \frac{m_1 \gamma_T (C+x)}{\eta_1 x}\right)}{\Gamma(m_1)}. \tag{8}$$

Using [16, Sec.(6.5)] for $m_1 \in \mathbb{Z}^+$, (8) can be re-expressed as

$$F_{\gamma_1}\!\left(\frac{\gamma_T (C+x)}{x}\right) = 1 - e^{-\frac{m_1 \gamma_T}{\eta_1}\left(1 + \frac{C}{x}\right)} \sum_{p_1=0}^{m_1-1} \frac{m_1^{p_1} \gamma_T^{p_1} \left(1 + \frac{C}{x}\right)^{p_1}}{\eta_1^{p_1} p_1!}$$

$$= 1 - e^{-\frac{m_1 \gamma_T}{\eta_1}\left(1 + \frac{C}{x}\right)} \sum_{p_1=0}^{m_1-1} \frac{m_1^{p_1} \gamma_T^{p_1}}{\eta_1^{p_1} p_1!} \sum_{q_1=0}^{p_1} \binom{p_1}{q_1} \frac{C^{q_1}}{x^{q_1}}. \tag{9}$$

Using a result from order statistics, and since we have adopted the outdated CSI model used in [13], the pdf of $\widetilde{\gamma}_{2(k)}$ can be written as

$$f_{\widetilde{\gamma}_{2(k)}}(x) = \int_0^\infty f_{\widetilde{\gamma}_{2(k)} | \gamma_{2(k)}}(x|y) f_{\gamma_{2(k)}}(y) dy \tag{10}$$

$$= \int_0^\infty \frac{m_2 \left(\frac{x}{\rho y}\right)^{\frac{m_2 - 1}{2}} e^{-\frac{m_2(\rho y + x)}{(1-\rho)\eta_2}}}{(1-\rho)\eta_2} I_{m-1}\!\left(\frac{2 m_2 \sqrt{\rho x y}}{(1-\rho)\eta_2}\right) f_{\gamma_{2(k)}}(y) dy \tag{11}$$

where $I_\nu(z)$ denotes the νth order modified Bessel function of the first kind [16, Sec. (9.6)]. Since the user with the kth lowest SNR is selected, the pdf of $\gamma_{2(k)}$ is given by

$$f_{\gamma_{2(k)}}(y) \qquad (12)$$
$$= k\binom{N_u}{k}[F_{\gamma_{2(\ell)}}(y)]^{k-1}[1-F_{\gamma_{2(\ell)}}(y)]^{N_u-k} f_{\gamma_{2(\ell)}}(y)$$

Since $\gamma_{2(\ell)} \sim \mathcal{G}\left(m_2, \frac{\eta_2}{m_2}\right)$, by substituting the respective pdf and cdf into (12) we get

$$f_{\gamma_{2(k)}}(y) = k\binom{N_u}{k} \sum_{p_2=N_u-k}^{N_u-1} \frac{(-1)^{p_2+k-N_u}\binom{k-1}{p_2+k-N_u}}{(m_2-1)!} \qquad (13)$$
$$\times e^{-\frac{m_2 y(p_2+1)}{\eta_2}} \sum_{r_2=0}^{p_2(m_2-1)} \phi_{p_2(r_2)}^{m_2-1} \left(\frac{m_2}{\eta_2}\right)^{r_2+m} y^{r_2+m_2-1},$$

where the coefficient $\phi_{b(t)}$ is defined as $\left(\sum_{t=0}^{a} \frac{x^t}{t!}\right)^b = \sum_{t=0}^{ab} \phi_{b(t)}^a x^t$. The value of $\phi_{b(t)}^a$ can be found recursively as [13]

$$\phi_{b(t)}^a = \sum_{\iota=t_1}^{t_2} \frac{\phi_{b-1(\iota)}^a}{(t-\iota)!}, \qquad (14)$$

where $t_1 = \max(0, t-a)$ and $t_2 = \min(t, (b-1)(a-1))$. Using the infinite series expansion of for $I_{m-1}\left(\frac{2m_2\sqrt{\rho x y}}{(1-\rho)\eta_2}\right)$ [16, Eq. (9.6.10)], and following an approach similar to [13], we obtain an expression for $f_{\tilde{\gamma}_{2(k)}}$ given by

$$f_{\tilde{\gamma}_{2(k)}}(x) = \sum_{q_2=0}^{k-1} \frac{k\binom{N_u}{k}(-1)^{q_2}\binom{k-1}{q_2}}{(m_2-1)!} e^{\frac{-m_2(p_2+1)x}{(p_2(1-\rho)+1)\eta_2}} \sum_{r_2=0}^{p_2(m_2-1)} \phi_{p_2(r_2)}^{m_2-1} \qquad (15)$$
$$\times \sum_{s_2=0}^{r_2} \frac{(r_2+m_2-1)!\left(\frac{m_2}{\eta_2}\right)^{s_2+m_2}\binom{r_2}{s_2}\rho^{s_2}(1-\rho)^{r_2-s_2} x^{s_2+m_2-1}}{(p_2(1-\rho)+1)^{r_2+s_2+m_2}(s_2+m_2-1)!}$$

where $p_2 = N_u - k + q_2$.

Now substituting (13) and (15) into (7), and [15, Eq. (4.5.29)], we arrive at the outage probability given by

$$F_{\gamma_{eq1}}(\gamma_T) = 1 - \frac{2k\binom{N_u}{k}e^{-\frac{m_1\gamma_T}{\eta_1}}}{(m_2-1)!} \sum_{p_1=0}^{m_1-1} \frac{\left(\frac{m_1\gamma_T}{\eta_1}\right)^{p_1}}{p_1!} \sum_{q_1=0}^{p_1}\binom{p_1}{q_1}$$
$$\times C^{q_1} \sum_{q_2=0}^{k-1}(-1)^{q_2}\binom{k-1}{q_2} \sum_{r_2=0}^{p_2(m_2-1)} \phi_{p_2(r_2)}^{m_2-1}(r_2+m_2-1)!$$
$$\times \sum_{s_2=0}^{r_2} \frac{\binom{r_2}{s_2}\left(\frac{m_2}{\eta_2}\right)^{\frac{s_2+m_2+q_2}{2}} \rho^{s_2}(1-\rho)^{r_2-s_2}\left(\frac{Cm_1\gamma_T}{(p_2+1)\eta_1}\right)^{\frac{s_2+m_2-q_1}{2}}}{(p_2(1-\rho)+1)^{r_2+\frac{s_2+m_2+q_2}{2}}(s_2+m_2-1)!}$$
$$\times K_{s_2+m_2-q_1}\left(2\sqrt{\frac{C(p_2+1)m_1m_2\gamma_T}{(p_2(1-\rho)+1)\eta_1\eta_2}}\right), \qquad (16)$$

where $K_\nu(z)$ is the νth order modified Bessel function of the second kind [16, Sec. (9.6)].

In the high SNR region with $\eta_1, \eta_2 \to \infty$, for $\rho < 1$, a power series expression for (16) can be obtained substituting the series expansion of $K_\nu(\cdot)$ [14, Eq. (8.446)] and the Mclaurin series expansion of the exponential function. After mathematical manipulations, an asymptotic approximation for $F_{\gamma_{eq1}}(\gamma_T)$ can be obtained as

$$F_{\gamma_{eq1}}(\gamma_T) = \begin{cases} \tau_1\left(\frac{m_1m_2C}{\eta_1\eta_2}\right)^{m_1}\gamma_T^{m_1} & \text{(a)} \\ \left(\frac{k\binom{N_u}{k}\tau_2}{(m_2-1)!} + \ln(\gamma_T)\tau_3\right)\left(\frac{m_1m_2C}{\eta_1\eta_2}\right)^{m_2}\gamma_T^{m_2} & \text{(b)} \end{cases} \qquad (17)$$

where (a) $= m_1 < m_2$ and (b) $= m_1 \geq m_2$. In (17), τ_1, τ_2 and τ_3 are given by

$$\tau_1 = \frac{k\binom{N_u}{k}}{(m_2-1)!}\sum_{p_1=0}^{m_1-1}\sum_{q_1=0}^{p_1}\frac{\binom{p_1}{q_1}}{p_1!}\sum_{q_2=0}^{k-1}(-1)^{q_2}\binom{k-1}{q_2} \qquad (18)$$
$$\times \sum_{r_2=0}^{p_2(m_2-1)}\phi_{p_2(r_2)}^{m_2-1}(r_2+m_2-1)!\sum_{s_2=0}^{r_2}\frac{\binom{r_2}{s_2}\rho^{s_2}(1-\rho)^{r_2-s_2}}{(s_2+m_2-1)!}$$
$$\times \sum_{t_2=0}^{m_1-p_1}\frac{(-1)^{m_1-p_1+1}(r_1-t_2-1)!(p_2+1)^{t_2-r_1}}{(m_1-p_1-t_2)!t_2!(p_2(1-\rho)+1)^{r_2+q_1+t_2}}$$
$$\times \left(\frac{m_1}{\eta_1}\right)^{m_1}\left(\frac{Cm_2}{\eta_2}\right)^{q_1+t_2-m_1},$$

and

$$\tau_2 = \sum_{p_1=0}^{m_1-1}\sum_{q_1=0}^{p_1}\frac{\binom{p_1}{q_1}}{p_1!}\sum_{q_2=0}^{k-1}(-1)^{q_2+1}\binom{k-1}{q_2} \qquad (19)$$
$$\times \sum_{r_2=0}^{p_2(m_2-1)}\phi_{p_2(r_2)}^{m_2-1}(r_2+m_2-1)!\sum_{s_2=\max(0,q_1-m_2)}^{r_2}\frac{\binom{r_2}{s_2}\rho^{s_2}(1-\rho)^{r_2-s_2}}{(s_2+m_2-1)!}$$
$$\times \sum_{t_2=0}^{\min(m_2-p_1,r_1-1)}\frac{(p_2+1)^{t_2-r_1}(-1)^{m_1-p_1}(r_1-t_2-1)!}{t_2!(m_2-p_1-t_2)!(p_2(1-\rho)+1)^{r_2+q_1+t_2}}$$
$$\times \left(\frac{Cm_2}{\eta_2}\right)^{q_1+t_2-m_2} + \sum_{q_2=0}^{k-1}\frac{(-1)^{q_2+1}\binom{k-1}{q_2}}{p_1!(m_2-1)!}$$
$$\times \sum_{r_2=0}^{p_2(m_2-1)}\frac{\phi_{p_2(r_2)}^{m_2-1}(r_2+m_2-1)!(1-\rho)^{r_2}}{(p_2(1-\rho)+1)^{r_2+s_2+m_2}}\left(\sum_{p_1=m_2+1}^{m_1-1}\frac{(-r_1-1)!}{p_1!}\right.$$
$$\left. + \sum_{p_1=0}^{\min(m_2,m_1-1)}\frac{1}{p!}\left(\ln\left(\frac{C(p_2+1)m_1m_2}{(p_2(1-\rho)+1)\eta_1\eta_2}\right) - \widehat{\tau}\right)\right),$$

and

$$\tau_3 = \frac{k\binom{N_u}{k}}{(m_2-1)!}\sum_{p_1=0}^{\min(m_2,m_1-1)}\frac{1}{p_1!}\left(\sum_{q_2=0}^{k-1}(-1)^{q_2+1}\right. \qquad (20)$$
$$\left.\times \binom{k-1}{q_2}\sum_{r_2=0}^{p_2(m_2-1)}\frac{\phi_{p_2(r_2)}^{m_2-1}(r_2+m_2-1)!(1-\rho)^{r_2}}{m_2-1}\right).$$

where $r_1 = s_2 + m_2 - q_1$, $\widehat{\tau} = \psi(1) - \psi(r_1+1)$ and $\psi(x)$ is the digamma function [16, Eq. (6.3.1)].

Remark 1: We note that when $\rho < 1$ (outdated CSI), the diversity order of the MRN is given by $\min(m_1, m_2)$. The diversity order, if S had perfect CSI is $\min(m_1, m_2k)$. If $m_1 < m_2$, the impact of outdated CSI is not very significant, as the S $\rightarrow$ R link acts as the performance bottleneck. However, if $m_1 > m_2$, and for large N_u and k, the impact of outdated CSI is large, particularly in the high SNR region. This is due to the potential loss of the system's diversity order, compared to the perfect CSI case.

In the special case of Rayleigh fading, by substituting $m_1 = m_2 = 1$ in (17) yields

$$F_{\gamma_{eq1}}(\gamma_T)=k\binom{Nu}{k}\sum_{p=0}^{k-1}\frac{(-1)^{p+1}\binom{k-1}{p}C\gamma_T}{\eta_1\eta_2((N_u-k+p)(1-\rho)+1)} \times\left(\ln\left(\frac{C(p_2+1)m_1m_2\gamma_T}{(p_2(1-\rho)+1)\eta_1\eta_2}\right)+\gamma-\psi(2)\right)+\frac{\gamma_T}{\eta_1}, \quad (21)$$

where $\gamma = 0.57721...$ is the Euler-Gamma constant [16, Sec. (6.1.3)].

B. Average Bit Error Rate

We now proceed to analyze the system's average BER. For many modulation formats used in wireless applications, the average BER can be expressed as

$$P_b = \alpha\, E[Q(\sqrt{\beta\gamma_{eq1}})] = \frac{\alpha}{\sqrt{2\pi}}\int_0^\infty F_{\gamma_{eq1}}\left(\frac{t^2}{\beta}\right)e^{-\frac{t^2}{2}}dt, \quad (22)$$

where $\alpha, \beta > 0$ are constants depending on the modulation scheme, and $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{y^2}{2}}dy$ is the Gaussian Q-function. Using the result of (16) and [14, Eq. (6.631.3)] we can arrive at the following expression for the BER given by (23), where $\Psi(a, b; z)$ is the confluent hypergeometric function of the second kind [16, Eq. (13.1.3)].

$$P_b = \frac{\alpha}{2} - \frac{\alpha\sqrt{\beta}k\binom{N_u}{k}}{\sqrt{\pi}(m_2-1)!}\sum_{p_1=0}^{m_1-1}\frac{1}{p_1!}\sum_{q_1=0}^{p_1}\binom{p_1}{q_1} \quad (23)$$
$$\times\sum_{q_2=0}^{k-1}(-1)^{q_2}\binom{k-1}{q_2}\sum_{r_2=0}^{p_2(m_2-1)}\phi_{p_2(r_2)}^{m_2-1}(r_2+m_2-1)!$$
$$\times\sum_{s_2=0}^{r_2}\frac{2^{p_1+r_1-1}\binom{r_2}{s_2}C^{r_1+q_1}\rho^{s_2}(1-\rho)^{r_2-s_2}}{(p_2(1-\rho)+1)^{r_2+s_2+m_2}(s_2+m_2-1)!}\left(\frac{m_2}{\eta_2}\right)^{s_2+m_2}$$
$$\times\left(\frac{m_1^{p_1+r_1}\sqrt{\eta_1}}{(2m_1+\eta_1\beta)^{p_1+r_1+\frac{1}{2}}}\right)\Gamma\left(p_1+r_1+\frac{1}{2}\right)\Gamma\left(p_1+\frac{1}{2}\right)$$
$$\times\Psi\left(p_1+r_1+\frac{1}{2}, r_1+1; \frac{2C(p_2+1)m_1m_2}{(p_2(1-\rho)+1)(2m_1+\eta_1\beta)\eta_2}\right)$$

IV. Analysis of Variable Gain Relaying

In the case of variable gain relaying, the instantaneous end-to-end SNR of the selected user (γ_{eq2}) is given by

$$\gamma_{eq2} = \frac{\gamma_1\widetilde{\gamma}_{2(k)}}{\gamma_1+\widetilde{\gamma}_{2(k)}+1}. \quad (24)$$

Therefore, for further analysis, it is desirable to define a new RV of the form: $Y = \frac{\gamma_1\widetilde{\gamma}_{2(k)}}{\gamma_1+\widetilde{\gamma}_{2(k)}+c}$, where $c \geq 0$ is a constant. Note that $c = 0$ gives the exact form of γ_{eq2} in (24), while $c = 0$ gives a mathematically more tractable tight approximation for γ_{eq2} in the medium-to-high SNR region.

A. Outage Probability

The cdf of Y given by $\Pr\left(\frac{\gamma_1\tilde{\gamma}_{2(k)}}{\gamma_{1(k)}+\tilde{\gamma}_{2(k)}+c} < \gamma_T\right)$ can be evaluated as

$$F_Y(\gamma_T) = \int_0^{\gamma_T} f_{\tilde{\gamma}_{2(k)}}(x)dx \quad (25)$$
$$+\int_{\gamma_T}^\infty \Pr\left(\gamma_1 < \frac{x(\gamma_T+c)}{x-\gamma_T}\right)f_{\tilde{\gamma}_{2(k)}}(x)dx$$
$$= 1 - \int_0^\infty\frac{\Gamma\left(m_1, \frac{m_1\gamma_T(x+\gamma_T+c)}{\eta_1 x}\right)}{\Gamma(m_1)}f_{\tilde{\gamma}_{2(k)}}(x+\gamma_T)dx.$$

Using the series expansion for $\Gamma\left(m_1, \frac{m_1\gamma_T(x+\gamma_T+c)}{\eta_1 x}\right)$ [16, Sec. (6.5)], subsequent binomial expansions and substituting (15) and $c = 1$, we arrive at (31) for $F_{\gamma_{eq2}}(\gamma_T)$.

Although (31) gives the exact outage probability, asymptotic results are also of interest due to the insights they offer on high SNR behavior of the systems.

It has been shown that in high SNR, the system performance is governed by the weakest link [17]. Since $\gamma_1 \sim \mathcal{G}\left(m_1, \frac{\eta_1}{m_1}\right)$, we have

$$f_{\gamma_1}(x) = \frac{m_1^{m_1}x^{m_1-1}}{\eta_1^{m_1}\Gamma(m_1)}e^{-\frac{m_1x}{\eta_1}} \quad (26)$$
$$= \frac{m_1^{m_1}}{\eta_1^{m_1}\Gamma(m_1)}\,x^{m_1-1}+o(x^{m_1}).$$

From (15) substituting the Mclaurin series expansion for the exponential function and selecting only the lowest power of γ_T with a non-zero coefficient, yields

$$f_{\tilde{\gamma}_{2(k)}}(x) = \frac{\kappa}{\eta_2^{m_2}}\,x^{m_2-1}+o(x^{m_2}), \quad (27)$$

where

$$\kappa = \frac{k\binom{N_u}{k}m_2{}^{m_2}}{((m_2-1)!)^2}\sum_{q_2=0}^{k-1}(-1)^{q_2}\binom{k-1}{q_2} \times\sum_{r_2=0}^{p_2(m_2-1)}\frac{\phi_{p_2(r_2)}^{m_2-1}(r_2+m_2-1)!(1-\rho)^{r_2}}{(p_2(1-\rho)+1)^{r_2+m_2}}. \quad (28)$$

Using the results of [17], we arrive at the following asymptotic result for the outage probability. As $\eta_1, \eta_2 \rightarrow \infty$, for $\rho < 1$

$$F_{\gamma_{eq2}}(\gamma_T) = \begin{cases}\frac{m_1^{m_1}}{\eta_1^{m_1}m_1!}\gamma_T^{m_1} & m_1 < m_2,\\ \left(\frac{m_1^{m_1}}{\eta_1^{m_1}m_1!}+\frac{\kappa}{m_1\eta_2^{m_1}}\right)\gamma_T^{m_1} & m_1 = m_2,\\ \frac{\kappa}{m_2\eta_2^{m_2}}\gamma_T^{m_2} & m_1 > m_2.\end{cases} \quad (29)$$

Remark 2: Similar to the fixed gain case, we observe that the diversity order of the system is $\min(m_1, m_2)$. As discussed in Section III, the performance loss due to outdated CSI will be most significant in the case $m_1 > m_2$, as imperfect CSI

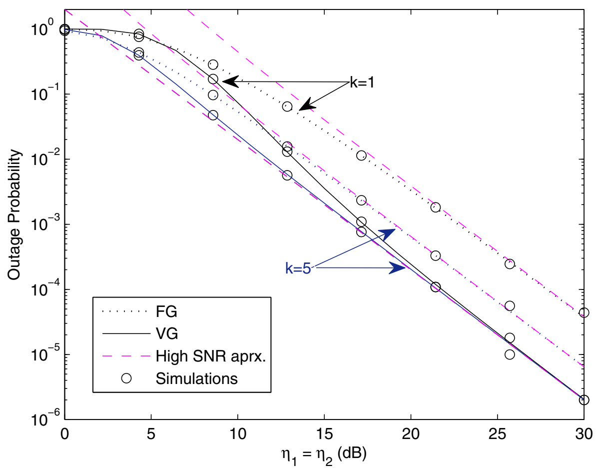

Fig. 2. The outage probability, for best ($k = 5$) and worst ($k = 1$) user selection. ($N_u = 5, m_1 = 2, m_2 = 3, \rho = 0.8, \gamma_T = 1$).

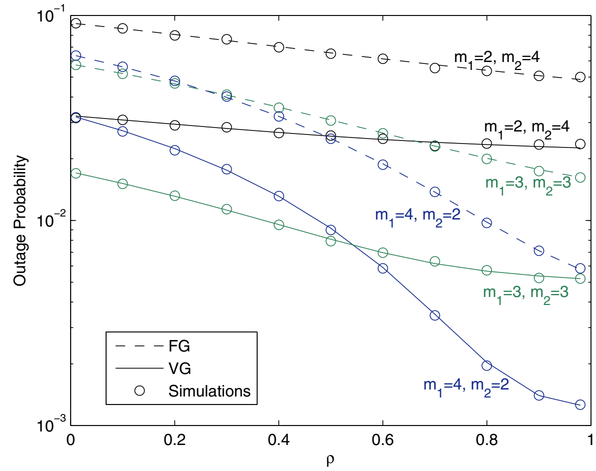

Fig. 3. The effect of correlation on the outage probability, for different fading parameters m_1, m_2. ($N_u = 5, k = 5, \gamma_T = 1, \eta_1 = \eta_2 = 10$ dB).

would cause a reduction in the achievable diversity order for the considered multiuser relay network.

An asymptotic approximation for outage probability in Rayleigh fading can be found by substituting $m_1 = m_2 = 1$ in (29). Under Rayleigh fading, as $\eta_1, \eta_2 \to \infty$, $F_{\gamma_{eq2}}(\gamma_T)$ can be approximated as

$$F_{\gamma_{eq2}}(\gamma_T) = \frac{\gamma_T}{\eta_1} + \sum_{p=0}^{k-1} \frac{k\binom{N_u}{k}(-1)^p\binom{k-1}{p}\gamma_T}{((N_u - k + p)(1-\rho) + 1)\eta_2}. \quad (30)$$

B. Average Bit Error Rate

We substitute (31) into (12) to derive the average BER. However, since there is no closed-form solution, we substitute $c = 0$ in (25) and proceed. Hence, with the help of [14, Eq.(6.621.3)], a tight lower bound for the average BER can be obtained as (32), where $r_1 = p_1 + s_2 + m_2$ and ${}_2F_1(a, b; c; x)$ is the Gauss hypergeometric function [16, Eq. (15.1.1)].

V. Numerical and Simulation Results

Figures 2-4 show the performance of the fixed gain (FG) and variable gain(VG) relaying schemes discussed earlier.

Figure 2 shows the outage probability of the systems for the best ($k = 5$) and worst ($k = 1$) user selection, with $N_u = 5, \rho = 0.8, \gamma_T = 1, m_1 = 3$ and $m_2 = 2$. The VG scheme outperforms the FG scheme in terms of the outage probability. We notice that at high SNR, for VG, the performance is the same for both best and worst user selection. This is so since we have considered $m_1 < m_2$ and the high SNR performance is dependent only on the $S - R$ link which is common to all users. The simulations and asymptotic results shows excellent agreement with the analytical results. We notice that the outage diversity order is equal to m_1 in all cases. Figure 3 shows the outage probability of FG, VG systems with varying correlation ρ, with $N_u = k = 5$ and $\eta_1 = \eta_2 = 10$ dB. Under all conditions, the VG relaying outperforms FG relaying. A more important observation to notice is that, the highest variation with ρ is shown for

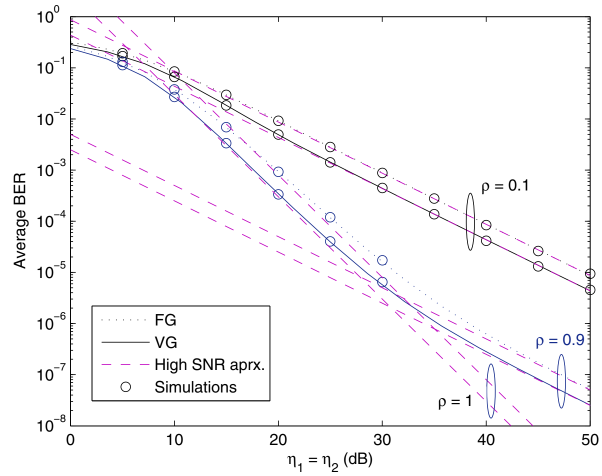

Fig. 4. The average BER for QPSK ($\alpha = \beta = 1$), under high ($\rho = 0.9$) and low($\rho = 0.1$) correlation.

$m_1 > m_2$. With $m_1 < m_2$, the performance loss with reducing ρ is small, particularly for the VG scheme. The performance bottleneck in the case of $m_1 < m_2$ is the S $\to$ R link. As such the impact of outdated CSI is not so significant. However, in the case of $m_1 > m_2$, there is a reduction in the diversity order due to outdated CSI, and hence has a highly negative impact on the system's performance. Figure 4 shows the average BER of the two schemes under QPSK modulation with $N_u = k = 5$, $m_1 = 2$ and $m_2 = 1$. We notice that the VG scheme outperforms the FG scheme, exhibiting a performance gain of approximately 3 dB in the high SNR region. We further observe that there is an increase in the coding gain at high correlation ($\rho = 0.9$) over low correlation ($\rho = 0.1$). As expected, the diversity gain with imperfect CSI is equal to m_2. We have plotted high SNR approximation for the perfect CSI case ($\rho = 1$) as a reference and there we observe a higher diversity order equal to m_1

$$F_{\gamma_{eq2}}(\gamma_T) = 1 - \frac{2k\binom{N_u}{k}}{(m-1)!} \sum_{p_1=0}^{m_1-1} \frac{m_1^{p_1}}{p_1!\eta_1^{p_1}} \sum_{q_1=0}^{p_1} \binom{p_1}{q_1} \sum_{q_2=0}^{k-1} (-1)^{q_2} \binom{k-1}{q_2} \sum_{r_2=0}^{p_2(m_2-1)} \phi_{p_2(r_2)}^{m_2-1} (r_2+m_2-1)!$$
$$\times \sum_{s_2=0}^{r_2} \frac{\binom{r_2}{s_2}\left(\frac{m_2}{\eta_2}\right)^{s_2+m_2} \rho^{s_2}(1-\rho)^{r_2-s_2} e^{-\gamma_T\left(\frac{m_1}{\eta_1}+\frac{m_2(p_2+1)}{(p_2(1-\rho)+1)\eta_2}\right)}}{(p_2(1-\rho)+1)^{r_2+s_2+m_2}(s_2+m_2-1)!} \sum_{q_2=0}^{s_2+m_2-1} \binom{s_2+m_2-1}{q_2} \left(\frac{(p_2(1-\rho)+1)m_1\eta_2}{(p_2+1)m_2\eta_1}\right)^{\frac{q_2-q_1+1}{2}}$$
$$\times \gamma_T^{m_2+p_1+s_2-\frac{(q_2+q_1+1)}{2}} (1+\gamma_T)^{\frac{q_2+q_1+1}{2}} K_{q_2-q_1+1}\left(2\sqrt{\frac{(p_2+1)m_1m_2(\gamma_T+1)\gamma_T}{(p_2(1-\rho)+1)\eta_1\eta_2}}\right). \quad (31)$$

$$P_b \geq \frac{\alpha}{2} - \frac{\alpha\sqrt{\beta}k\binom{N_u}{k}}{(m_2-1)!} \sum_{p_1=0}^{m_1-1} \frac{m_1^{p_1}}{p_1!\eta_1^{p_1}} \sum_{q_1=0}^{p_1} \binom{p_1}{q_1} \sum_{q_2=0}^{k-1} (-1)^{q_2} \binom{k-1}{q_2} \sum_{r_2=0}^{p_2(m_2-1)} \phi_{p_2(r_2)}^{m_2-1} (r_2+m_2-1)! \quad (32)$$
$$\times \sum_{s_2=0}^{r_2} \frac{\binom{r_2}{s_2}\left(\frac{m_2}{\eta_2}\right)^{s_2+m_2} \rho^{s_2}(1-\rho)^{r_2-s_2}}{(p_2(1-\rho)+1)^{r_2+s_2+m_2}(s_2+m_2-1)!} \sum_{q_2=0}^{s_2+m_2-1} \binom{s_2+m_2-1}{q_2} \left(\frac{(p_2(1-\rho)+1)m_1^3\eta_2}{(p_2+1)m_2\eta_1^3}\right)^{\frac{q_2-q_1+1}{2}}$$
$$\times \frac{2^{2q_2-2q_1+\frac{3}{2}}\Gamma\left(r_1+q_2-q_1+\frac{3}{2}\right)\Gamma\left(r_1-q_2+q_1+\frac{1}{2}\right)}{\left(\frac{m_1}{\eta_1}+\frac{m_2(p_2+1)}{(p_2(1-\rho)+1)\eta_2}+2\sqrt{\frac{(p_2+1)m_1m_2}{(p_2(1-\rho)+1)\eta_1\eta_2}}+\frac{1}{2}\right)(r_1)!}$$
$$\times\ {}_2F_1\left(r_1+q_2-q_1+\frac{3}{2}, q_2-q_1+\frac{3}{2}; r_1+1; \frac{\frac{m_1}{\beta\eta_1}+\frac{m_2(p_2+1)}{(p_2(1-\rho)+1)\beta\eta_2}-\frac{2}{\beta}\sqrt{\frac{(p_2+1)m_1m_2}{(p_2(1-\rho)+1)\eta_1\eta_2}}+\frac{1}{2}}{\frac{m_1}{\beta\eta_1}+\frac{m_2(p_2+1)}{(p_2(1-\rho)+1)\beta\eta_2}+\frac{2}{\beta}\sqrt{\frac{(p_2+1)m_1m_2}{(p_2(1-\rho)+1)\eta_1\eta_2}}+\frac{1}{2}}\right),$$

VI. Conclusion

In this paper, we presented new expressions for the outage probability and the average BER of an AF based MRN over Nakagami-m fading channels, when user selection is performed using outdated CSI. The derived expressions quantify the performance degradation in the presence of outdated CSI for both fixed gain and variable gain AF relaying protocols. Additionally, high SNR approximations for the outage probability and the average BER were presented. By doing so, we were able to quantify the diversity order and the array gain of the considered MRN. We have also verified all our derivations using extensive Monte-Carlo simulations.

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