# Amplify-and-Forward Partial Relay Selection with Feedback Delay

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***Abstract*—This paper evaluates the impact of using outdated channel estimates due to feedback delay for relay selection and signal amplification on the performance of partial relay selection with amplify-and-forward (AF) relays. Both variable and fixed gain AF schemes are considered. Expressions for the system's outage probability and the average bit error rate, and their high signal-to-noise ratio (SNR) approximations are derived. The effect of parameters such as the rank of relay chosen, SNR imbalance and the correlation between the delayed and current channel state information are studied and verified through simulations.**

## I. Introduction

Cooperative communications using low complexity relay terminals are capable of efficiently combating wireless impairments and provide for coverage extension [1]. In such systems, performance can be improved by selecting one of the relay terminals [2]–[8]. Basically, selection schemes can be classified as: (1) opportunistic relay selection; and (2) partial relay selection. In opportunistic relaying, a single relay based on the instantaneous global (two hop) channel state information of the network is selected to assist the source [2]. In partial relay selection (PRS) only local (single-hop) information is used to activate a relay [4].

The PRS approach finds wide applicability especially in low complexity ad-hoc and sensor networks since such networks may not have significant resources to implement complex relay selection protocols. Recent literature includes several works that cover the performance analysis of the PRS scheme. In [4], the probability density function (pdf) of the received signal-to-noise ratio (SNR) and an asymptotic exponential expression have been derived to evaluate the performance of the PRS scheme with variable gain amplify-and-forward (AF) relaying. In [5] and [6], by deriving closed-form expressions for the outage probability, pdf, average bit error rate (BER) and moments of the end-to-end SNR, the performance of PRS with fixed-gain AF relaying has been studied. Very recently in [7], diversity and coding gains of PRS with fixed-gain relaying over Nakagami-$m$ channels have been presented.

So far, only few papers have analytically investigated the impact of outdated channel state information (CSI) on the performance of the relay selection (see eg. [9]–[11]). When PRS is implemented in time-varying channels, there is a possibility that outdated CSI could be used for relay selection due to feedback delay [9], [10]. Moreover, outdated CSI may also be used for signal amplification at the relay. Although outdated CSI corresponds to several realistic scenarios, to the best of our knowledge, the existing literature has not considered these issues.

In this paper, the performance of two reduced complexity PRS system designs in the presence of outdated CSI is studied. Specifically, we investigate fixed gain and CSI-assisted AF systems with $k$-th worst (or choosing $(N_r - k)$-th best) relay selection and outdated CSI due to feedback delay. While it is always desirable to operate the network with the best relay, sometimes it may not be available to assist the source [8]. Systems with fixed gain relays retransmit using a constant amplification factor regardless of the amplitude of the first hop; the advantage is that instantaneous CSI is not required at the relay. The CSI-assisted scheme assumed in this work is different from [10] since the present work considers the impact of using outdated CSI for relay selection and amplification gain. During the data transmission phase, the relay does not estimate the amplitude of the first hop. We derive new outage probability and average BER expressions and confirm their correctness using Monte Carlo simulations. These expressions are general in the sense that they characterize the performance due to $k$-th worst relay selection and arbitrary correlation coefficient between the current and outdated CSI and SNR imbalance.

Fixed gain based PRS systems can also be implemented using pilots transmitted from the relays [5]. At the source, these pilots are used to select a relay. Thus, relays can be relieved channel estimation as required in the case of variable gain relaying. The analysis presented in this paper is also valid for such fixed-gain PRS systems under uplink and downlink imperfect reciprocity conditions [12], eg. scheduling delay.

The rest of the paper is organized as follows. In Section II we present the system model. In Sections III and IV we analyze the outage probability, average BER and their high SNR approximations for fixed gain and variable gain relaying schemes respectively. In Section V simulation results are presented. Finally, concluding remarks appear in Section VI.

## II. System model

We consider a dual-hop AF PRS system, with a single source $S$, a single destination $D$ and $N_r$ relays. In this

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system, we assume that $S$ has no direct link to $D$, which for example may result from high shadowing between $S$ and $D$. $S$ periodically monitors the quality of its connectivity with the relays via transmission of a local feedback, and selects a single relay, $R_{(k)}$, with the $k$-th worst $S-R$ link. We assume that there is a delay ($T_d$) in the feedback. Hence this selection decision is based on outdated channel information. During the first time slot, $S$ communicates with the selected $R_{(k)}$ relay to transmit $x(t)$. In the second time slot the relay transmits its received signal to $D$.

The received signal at the selected relay can be written as

$$y_{R_{(k)}}(t) = \sqrt{P_s} h_{S,R_{(k)}}(t)x(t) + n_{R_{(k)}}(t), \tag{1}$$

where $P_s$ is the transmit power at $S$, $h_{S,R_{(k)}}(t)$ is the complex channel between $S$ and $R_{(k)}$ and $n_{R_{(k)}}(t)$ is the additive white Gaussian noise (AWGN) satisfying $E\left(|n_{R_{(k)}}(t)|^2\right) = N_{01}$ with $E(\cdot)$ denoting the expectation. The relay multiplies $y_k(t)$ by a gain, $G$ and the output is transmitted to $D$. The received signal at $D$ is given by

$$y_D(t) = h_{R_{(k)},D}(t) G y_{R_{(k)}}(t) + n_D(t), \tag{2}$$

where $h_{R_{(k)},D}(t)$ is the complex channel between $R_{(k)}$ and $D$, and $n_D(t)$ is the AWGN satisfying $E\left(|n_D(t)|^2\right) = N_{02}$.

Let $\tilde{\gamma}_{1(k)} = |h_{S,R_{(k)}}(t)|^2 \eta_1$ and $\gamma_2 = |h_{R_{(k)},D}(t)|^2 \eta_2$, where $\eta_1 = \frac{P_s}{N_{01}}$, $\eta_2 = \frac{P_r}{N_{02}}$ and $P_r$ is the average transmit power of $R_{(k)}$. We assume Rayleigh fading channels, and $h_{S,R_{(k)}}(t), h_{R_{(k)},D}(t) \sim \mathcal{CN}(0,1)$ are complex Gaussian with zero mean and unit variance. Let $\gamma_{1(1)} \leq \gamma_{1(2)} \leq \cdots \leq \gamma_{1(N_r)}$ be the order statistics obtained by arranging $\gamma_{1(\ell)}$ for $\ell = 1, \ldots, N_r$ in an increasing order of magnitude. Note the relay selection would be based on the $\gamma_{1(k)}$, and $\tilde{\gamma}_{1(k)}$, the link SNR experienced by the signal, is a delayed version of $\gamma_{1(k)}$.

### A. Fixed Gain Relaying

Consider a PRS system in which relays will amplify the received signal using a fixed gain factor [5], [6]. The relay does not require the instantaneous channel state information in the $S-R$ link. Hence, this is a low complexity system, which offers practical relevance.

Assuming that $R_{(k)}$ knows the statistics of the $S-R$ channel, we can choose the fixed gain, $G$ as follows:

$$G = \sqrt{\frac{P_r}{P_s E\{|h_{S,R_{(k)}}(t)|^2\} + N_{01}}}. \tag{3}$$

When the fixed gain of the relay chosen as in (3), it can be shown that the end-to-end SNR is given by

$$\gamma_{eq1} = \frac{\tilde{\gamma}_{1(k)}\gamma_2}{C + \gamma_2}, \tag{4}$$

where $C = \frac{P_r}{G^2 N_{01}}$. From [10, Eq. (9)], we know that the pdf of $\tilde{\gamma}_{1(k)}$ is given by

$$f_{\tilde{\gamma}_{1(k)}}(x) = k\binom{N_r}{k} \sum_{m=0}^{k-1} \frac{(-1)^m}{\eta_1} \binom{k-1}{m} \times \frac{1}{(N_r-k+m)(1-\rho)+1} e^{-\frac{(N_r-k+m+1)x}{((N_r-k+m)(1-\rho)+1)\eta_1}}, \tag{5}$$

where $0 \leq \rho \leq 1$ is the correlation coefficient between $\tilde{\gamma}_{1(k)}$ and $\gamma_{1(k)}$ and $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ denotes the binomial coefficient. Using the Jakes' autocorrelation model, $\rho$ can be expressed as $\rho = J_0\left(2\pi f_{d,AB} T_d\right)$ where $J_0(x)$ is the zeroth order Bessel function of the first kind; $f_{d,AB}$ is the maximum Doppler frequency on the $A-B$ link, and $T_d$ is the time difference between the actual channel value and its estimate.

We obtain an expression for $C = E\left\{\tilde{\gamma}_{1(k)} + 1\right\}$ using (5) given by

$$C = 1 + \sum_{m=0}^{k-1} \frac{k\binom{N_r}{k}(-1)^m \binom{k-1}{m}((N_r-k+m)(1-\rho)+1)\eta_1}{(N_r-k+m+1)^2} \tag{6}$$

### B. Variable Gain Relaying

We now consider a PRS system in which each relay only makes one channel measurement based on which the selection of a relay is made at $S$. The selected relay also uses the same outdated information to amplify the signal, $y_{R_{(k)}}(t)$. Hence the variable gain factor at the relay can be expressed as

$$G = \sqrt{\frac{P_r}{P_s |h_{S,R_{(k)}}(t-T_d)|^2 + N_{01}}}. \tag{7}$$

With the choice of $G$ in (7), this scheme has a reduced implementation complexity compared to the variable gain scheme presented in [10], since that system requires the relay to estimate $h_{S,R_{(k)}}(t-T_d)$ as well as $h_{S,R_{(k)}}(t)$.

Plugging (7) into (2) and after some manipulations, the end-to-end SNR can be written as

$$\gamma_{eq2} = \frac{\tilde{\gamma}_{1(k)}\gamma_2}{\gamma_{1(k)} + \gamma_2 + 1}. \tag{8}$$

In order to study the performance metrics of this system, note that a novel analysis is required. This is because the form of the instantaneous end-to-end SNR, $\gamma_{eq2}$, is different from [10, Eq. (3)] and therefore new expressions must be derived.

## III. Fixed Gain Relaying

In this section, we derive important performance metrics, i.e., the outage probability and the average BER for the dual-hop PRS system with fixed gain relaying.

### A. Outage Probability

The outage probability, $P_o$, defined as the probability that the end-to-end SNR drops below a predefined SNR threshold $\gamma_T$, is an important quality of service (QoS) measure. Mathematically, such a metric can be obtained using

$$F_{\gamma_{eq1}}(\gamma_T) = \Pr(\gamma_{eq1} < \gamma_T) = \Pr\left(\frac{\tilde{\gamma}_{1(k)}\gamma_2}{C + \gamma_2} < \gamma_T\right), \tag{9}$$

where $\Pr(\cdot)$ denotes the probability. Eq. (9) can be simplified as

$$F_{\gamma_{eq1}}(\gamma_T) = 1 - \int_{\gamma_T}^{\infty} \Pr\left(\gamma_2 > \frac{C\gamma_T}{x - \gamma_T}\right) f_{\tilde{\gamma}_{1(k)}}(x)dx. \tag{10}$$

Using (5) and the complementary cumulative distribution function (ccdf) of $\gamma_2$, with some algebraic manipulations, we obtain (11).

$$F_{\gamma_{eq1}}(\gamma_T) = 1 - k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m}{\eta_1}\binom{k-1}{m}\frac{e^{-\frac{(N_r-k+m)\gamma_T}{((N_r-k+m)(1-\rho)+1)\eta_1}}}{(N_r-k+m)(1-\rho)+1}\int_0^\infty e^{-\frac{C\gamma_T}{\eta_2 y}-\frac{(N_r-k+m+1)y}{((N_r-k+m)(1-\rho)+1)\eta_1}}dy. \quad (11)$$

$$F_{\gamma_{eq1}}(\gamma_T) = 1 - 2k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m\binom{k-1}{m}}{((N_r-k+m)(1-\rho)+1)}e^{-\frac{(N_r-k+m+1)\gamma_T}{((N_r-k+m)(1-\rho)+1)\eta_1}} \quad (12)$$
$$\times\sqrt{\frac{((N_r-k+m)(1-\rho)+1)C\gamma_T}{(N_r-k+m+1)\eta_1\eta_2}}K_1\left(2\sqrt{\frac{(N_r-k+m+1)C\gamma_T}{((N_r-k+m)(1-\rho)+1)\eta_1\eta_2}}\right)$$

Finally using [13, Eq. (4.5.25)], the outage probability can be expressed as (12). where $K_1(x)$ is the first order modified Bessel function of the second kind [15, Sec. (9.6)]. Outage probability for the special case of $\rho = 1$ and $N_r = k$ is given in [5, Eq. (5)]. Although (12) gives the exact outage probability, an approximation for it in the high SNR region in a much simpler form is also desired in order to gain further insights.

*Corollary 1:* The asymptotic outage probability for large $\eta_1$ and $\eta_2$ with fixed ratio, $\mu = \frac{\eta_2}{\eta_1}$ admits the first order approximation given by

$$F_{\gamma_{eq1}}(\gamma_T) \approx k\frac{\gamma_T}{\eta_1}\sum_{m=0}^{k-1}\frac{(-1)^m\binom{N_r}{k}\binom{k-1}{m}}{((N_r-k+m)(1-\rho)+1)} \quad (13)$$
$$\times\left(\frac{\Lambda}{\mu}\ln\left(\frac{((N_r-k+m)(1-\rho)+1)\eta_1}{(N_r-k+m+1)}\right)+\nu\right),$$

where

$$\nu = e^{-\Lambda} + \Lambda\left(1-\gamma+\mathrm{Ei}(-\Lambda)-\ln(\Lambda)\right), \quad (14)$$

and

$$\Lambda = k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m\binom{k-1}{m}((N_r-k+m)(1-\rho)+1)}{(N_r-k+m+1)^2}. \quad (15)$$

*Proof:* Due to limited space, the proof is omitted. ■

In (14) $\gamma = 0.57721...$ is the Euler–Mascheroni constant and $\mathrm{Ei}(x)$ is the exponential integral function [15, Eq. (5.1.2)]. Note that the simpler form in (13) clearly shows the impact of of parameters such as $k$, $\rho$ and $N_r$ on the outage probability.

### B. Average Bit Error Rate

We now proceed to analyze the system's average error performance. For many modulation formats used in wireless applications, the average BER can be expressed as

$$P_b = \alpha\, E[Q(\sqrt{\beta\gamma_{eq1}})], \quad (16)$$

where $\alpha, \beta > 0$ are constants depending on the modulation scheme, and $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{y^2}{2}}dy$ is the Gaussian $Q$-function. Using integration by parts, (16) results in

$$P_b = \frac{\alpha}{\sqrt{2\pi}}\int_0^\infty F_{\gamma_{eq1}}\left(\frac{t^2}{\beta}\right)e^{-\frac{t^2}{2}}dt. \quad (17)$$

Eq. (17) can be solved with the help of [13, Eq. (4.16.33)] and the average BER can be evaluated using (23) where

$$\varsigma_3 = \frac{C(N_r-k+m+1)/\eta_2}{2(N_r-k+m+1)+((N_r-k+m)(1-\rho)+1)\beta\eta_1}.$$

Substituting (13) into (17), and with the help of [14, Eq. (3.381.4)] the BER at high SNR can be approximated as

$$P_b^\infty \approx \frac{\alpha k}{2\beta\eta_1}\sum_{m=0}^{k-1}\frac{(-1)^m\binom{N_r}{k}\binom{k-1}{m}}{((N_r-k+m)(1-\rho)+1)} \quad (19)$$
$$\times\left(\frac{\Lambda}{\mu}\ln\left(\frac{((N_r-k+m)(1-\rho)+1)\eta_1}{(N_r-k+m+1)}\right)+\nu\right).$$

From the above, we can conclude that the system has a diversity order of 1.

## IV. Variable Gain Relaying

In this section, we derive important performance metrics, i.e., the outage probability and the average BER for the dual-hop PRS system with variable gain relaying.

### A. Outage Probability

In order to derive the outage probability of variable gain relaying, it is convenient to obtain a statistical distribution formula for the general form:

$$Y = \frac{\tilde{\gamma}_{1(k)}\gamma_2}{\gamma_{1(k)}+\gamma_2+c}. \quad (20)$$

Note that $c = 1$ gives the exact expression for $\gamma_{eq2}$ in (8), while $c = 0$ can be substituted to obtain an analytically feasible approximation. The cdf of the RV $Y$ is

$$F_Y(\gamma_T) = \Pr\left(\frac{\tilde{\gamma}_{1(k)}\gamma_2}{\gamma_{1(k)}+\gamma_2+c} < \gamma_T\right). \quad (21)$$

After some mathematical manipulations, (21) can be written as follows

$$F_Y(\gamma_T) = 1 - \int_0^\infty\int_0^\infty \Pr\left(\gamma_2 > \frac{\gamma_T(y+c)}{w}\right) \quad (22)$$
$$\times f_{\tilde{\gamma}_{1(k)},\gamma_{1(k)}}(w+\gamma_T, y)dwdy.$$

$$P_b = \frac{\alpha}{2} - \frac{\alpha\sqrt{\beta\eta_1}k}{2\eta_2}\sum_{m=0}^{k-1}\frac{(-1)^m C\binom{N_r}{k}\binom{k-1}{m}\exp(\varsigma_3)}{((N_r-k+m)(1-\rho)+1)}\left(\frac{(N_r-k+m+1)}{((N_r-k+m)(1-\rho)+1)}+\beta\eta_1\right)^{-\frac{3}{2}}(K_1(\varsigma_3)-K_0(\varsigma_3)), \quad (23)$$

$$F_{\gamma_{eq2}}(\gamma_T) \geq 1 - \sum_{p=0}^{\infty}\sum_{m=0}^{k-1}\sum_{n=0}^{p}\frac{(-1)^m\binom{N_r}{k}\binom{k-1}{m}\binom{p}{n}k\rho^p\gamma_T^{p+1}(p+n+1)!}{(1-\rho)^{p-n-1}\eta_1^{p-n}\eta_2^{n+1}p!((N_r-k+m)(1-\rho)+1)^{p+n+2}} \times e^{-\frac{\gamma_T}{\eta_1(1-\rho)}}\Psi\left(p+n+2, n+2; \frac{\gamma_T}{\eta_2((N_r-k+m)(1-\rho)+1)}\right) \quad (24)$$

It is important to note that, $f_{\tilde{\gamma}_{1(k)}|\gamma_{1(k)}}(x|y) = f_{\tilde{\gamma}_{(\ell)}|\gamma_{(\ell)}}(x|y)$, where $\ell$ represents unordered relays. Hence the joint pdf of $\tilde{\gamma}_{1(k)}$ and $\gamma_{1(k)}$ can be established from

$$f_{\tilde{\gamma}_{1(k)},\gamma_{1(k)}}(x,y) = \frac{f_{\tilde{\gamma}_{1(\ell)},\gamma_{1(\ell)}}(x,y)}{f_{\gamma_{1(\ell)}}(y)} \times f_{\gamma_{1(k)}}(y). \quad (25)$$

Since $\tilde{\gamma}_{1(\ell)}$ and $\gamma_{1(\ell)}$ are two correlated exponentially distributed RVs, their joint pdf is given by

$$f_{\tilde{\gamma}_{1(\ell)},\gamma_{1(\ell)}}(x,y) = \frac{e^{-\frac{x+y}{(1-\rho)\eta_1}}}{(1-\rho)\eta_1^2}I_0\left(\frac{2\sqrt{\rho xy}}{(1-\rho)\eta_1}\right), \quad (26)$$

where $I_0(x)$ is the zeroth order modified Bessel function of the first kind [15, Eq. (9.6.16)]. Following the approach in [10], we know that the pdf $f_{\gamma_{1(k)}}(y)$ is given by

$$f_{\gamma_{1(k)}}(y) = k\binom{N_r}{k}[F_{\gamma_{1(\ell)}}(y)]^{k-1}[1-F_{\gamma_{1(\ell)}}(y)]^{N_r-k}f_{\gamma_{1(\ell)}}(y) \quad (27)$$

where $f_{\gamma_{1(\ell)}}(y) = \frac{1}{\eta_1}e^{-\frac{y}{\eta_1}}$ and $F_{\gamma_{1(\ell)}}(y) = 1 - e^{-\frac{y}{\eta_1}}$. Using the above results in (25) and after some simplifications, the joint pdf of $\tilde{\gamma}_{1(k)}$ and $\gamma_{1(k)}$, $f_{\tilde{\gamma}_{1(k)},\gamma_{1(k)}}(x,y)$, can be written as

$$f_{\tilde{\gamma}_{1(k)},\gamma_{1(k)}}(x,y) = \frac{k\binom{N_r}{k}e^{-\frac{x}{(1-\rho)\eta_1}}}{(1-\rho)\eta_1^2}I_0\left(\frac{2\sqrt{\rho xy}}{(1-\rho)\eta_1}\right) \times \sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}e^{-\frac{((N_r-k+m)(1-\rho)+1)y}{(1-\rho)\eta_1}}. \quad (28)$$

Substituting (28) and the ccdf of $\gamma_2$ into (22), we obtain (29).

$$F_Y(\gamma_T) = 1 - \frac{k\binom{N_r}{k}}{(1-\rho)\eta_1^2}\int_0^\infty\int_0^\infty e^{-\frac{\gamma_T(y+c)}{\eta_2 w}-\frac{w+\gamma_T}{(1-\rho)\eta_1}} \times I_0\left(\frac{2\sqrt{\rho(w+\gamma_T)y}}{(1-\rho)\eta_1}\right) \times \sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}e^{-\frac{((N_r-k+m)(1-\rho)+1)y}{(1-\rho)\eta_1}}dwdy. \quad (29)$$

Using the infinite series expansion $I_0(x) = \sum_{p=0}^{\infty}\frac{x^{2p}}{2^{2p}(p!)^2}$ from [14, Eq. (8.447.1)] in (29) and [13, Eq. (4.5.29)], (29) can be expressed as (30).

$$F_Y(\gamma_T) = 1 - \sum_{p=0}^{\infty}\sum_{m=0}^{k-1}\sum_{n=0}^{p}\frac{(-1)^m\binom{k-1}{m}k\binom{N_r}{k}\rho^p e^{-\frac{\gamma_T}{(1-\rho)\eta_1}}}{(1-\rho)^{2p+1}(p!)^{2p}\eta_1^{2p+2}} \times \binom{p}{n}\gamma_T^{p-n}\int_0^\infty 2y^p e^{-\frac{((N_r-k+m)(1-\rho)+1)y}{(1-\rho)\eta_1}} \times \left(\frac{\gamma_T(y+c)(1-\rho)\eta_1}{\eta_2}\right)^{\frac{n+1}{2}}K_{n+1}\left(\sqrt{\frac{4\gamma_T(y+c)}{\eta_1\eta_2(1-\rho)}}\right)dy. \quad (30)$$

To the best of authors' knowledge, the integral in (30) does not have a closed-form solution. Hence, we substitute $c = 0$ and using [13, Eq. (4.16.37)], a tight lower bound for the cdf as given in (24) is obtained, where $\Psi(a, b; z)$ is the confluent hypergeometric function of the second kind [15, Eq. (13.1.3)].

In Section IV, extensive simulation results to complement (24) are presented. We stress out that the outage probability predicted from (24) and simulations match perfectly even at low SNRs as $\eta_1 = \eta_2 = 5$ dB. For the special case of $\rho = 1$ and $N_r = k$, the outage probability is given in [4, Eq. (2)].

After lengthy manipulations, a high SNR approximation for the outage probability presented as *Corollary 2* below can be obtained.

*Corollary 2:* The asymptotic outage probability for large $\eta_1$ and $\eta_2$ with fixed ratio, $\mu = \frac{\eta_2}{\eta_1}$ admits the first order approximation given by

$$F_{\gamma_{eq2}}(\gamma_T) \approx \frac{k\binom{N_r}{k}\gamma_T}{\eta_1}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m} \times \left(\frac{p_1+p_2}{N_r-k+m+1}+\frac{\ln(\eta_1/p_1)}{\mu(1-\rho)\omega^2}\right) \quad (31)$$

where $p_1 = \frac{N_r-k+m+1}{(N_r-k+m)(1-\rho)+1}$, $p_2 = \frac{\rho}{\mu((N_r-k+m)(1-\rho)+1)^2}$ and $\omega = \frac{((N_r-k+m)(1-\rho)+1)}{(1-\rho)}$.

*Proof:* Due to limited space, the proof is omitted. ■

### *B. Average Bit Error Rate*

Using (17), and [13, Eq (4.22.16)], we derive a tight lower bound for the average BER of the PRS system with variable gain relaying as (32). In (32)

$$\varsigma_1 = \frac{1}{(1-\rho)\beta\eta_1} - \frac{1}{2((N_r-k+m)(1-\rho)+1)\beta\eta_2} + \frac{1}{2},$$

$$P_b \geq \frac{\alpha}{2} - \frac{\alpha k}{\sqrt{8\pi}} \sum_{p=0}^{\infty}\sum_{m=0}^{k-1}\sum_{n=0}^{p} \frac{(-1)^m\binom{N_r}{k}\binom{k-1}{m}\binom{p}{n}(p+n+1)!\rho^p\beta^{\frac{n}{2}-p}}{(1-\rho)^{p-n-1}\eta_1^{p-n}\eta_2^{\frac{n}{2}}p!((N_r-k+m)(1-\rho)+1)^{p+1+\frac{n}{2}}} \times \frac{\Gamma(p+\frac{3}{2})\Gamma(p-n+\frac{1}{2})\varsigma_2^{\frac{n}{2}+1}}{\Gamma(2p+\frac{5}{2})(\varsigma_1+\frac{\varsigma_2}{2})^{p+\frac{3}{2}}}\, {}_2F_1\left(p+\frac{3}{2}, p+n+2, 2p+\frac{5}{2}; \frac{\varsigma_1-\frac{\varsigma_2}{2}}{\varsigma_1+\frac{\varsigma_2}{2}}\right), \tag{32}$$

$$\varsigma_2 = \frac{1}{((N_r-k+m)(1-\rho)+1)\beta\eta_2},$$

and ${}_2F_1(a,b;c;x)$ is the Gauss hypergeometric function [15, Eq. (15.1.1)].

Consider the average BER at high SNR. Following a similar approach as in the case of fixed gain relaying, the average BER for the variable gain relaying, in the high SNR regime can be written as

$$P_b^{\infty} \approx \frac{\alpha k\binom{N_r}{k}}{2(1-\rho)\beta\eta_1} \sum_{m=0}^{k-1} (-1)^m \binom{k-1}{m} \times \left(\frac{p_1+p_2}{N_r-k+m+1} + \frac{\ln(\eta_1/p_1)}{\mu(1-\rho)\omega^2}\right). \tag{33}$$

The average output power at the relay in the case of variable gain relaying would be different from $P_r$ due to selecting the amplification gain factor using outdated CSI. Hence, for a fair comparison of the fixed and variable gain schemes, we make an average power normalization, so that the average output power at the relay is equal to unity. In order to do so, we introduce a modified amplification gain factor, $G = \sqrt{\frac{P_r/\xi}{P_s|h_{S,R_{(k)}}(t-T_d)|^2+N_{01}}}$, where

$$\begin{aligned} \xi &= E\left\{\frac{\tilde{\gamma}_{1(k)}+1}{\gamma_{1(k)}+1}\right\} \\ &= \frac{k\binom{N_r}{k}}{(1-\rho)\eta_1^2} \sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}\int_0^{\infty}\int_0^{\infty}\left(\frac{x+1}{y+1}\right) \\ &\times I_0\left(\frac{2\sqrt{\rho x y}}{(1-\rho)\eta_1}\right) e^{-\left(\frac{x}{(1-\rho)\eta_1}+\frac{((N_r-k+m)(1-\rho)+1)y}{(1-\rho)\eta_1}\right)} dx dy. \end{aligned} \tag{34}$$

Using [16, Eq. (9)] and [13, Eq. (4.2.6)], we can evaluate (34) to arrive at the expression for the scaling factor given by

$$\xi = k\binom{N_r}{k}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m} \times \left(\frac{\rho}{\eta_1\Upsilon} - (1-\rho)\left(1+\frac{1}{\eta_1}\right)e^{\Upsilon}\,\mathrm{Ei}\left(-\Upsilon\right)\right), \tag{35}$$

where $\Upsilon = (N_r-k+m+1)/\eta_1$.

## V. Simulation results and Comparisons

Figures 1-3 show the performance of the two systems investigated in this paper, and also the system analyzed previously in [10], where the relay benefits from the perfect instantaneous CSI, as a reference (labeled "VG II"). The variable gain relaying system and the fixed gain relaying system analyzed in this paper are labeled as "VG II" and "FG" respectively.

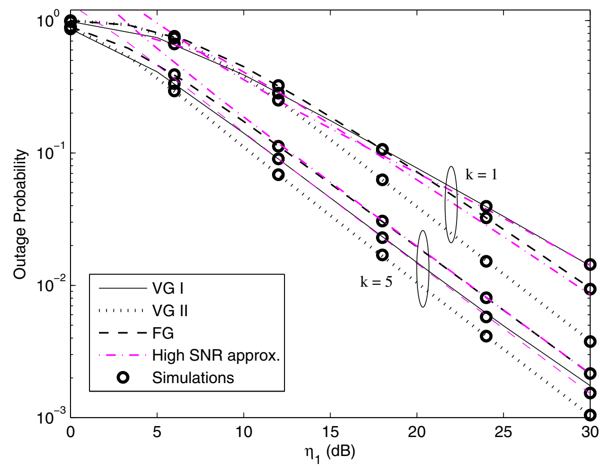


Fig. 1. The outage probability with best and worst relay selection.

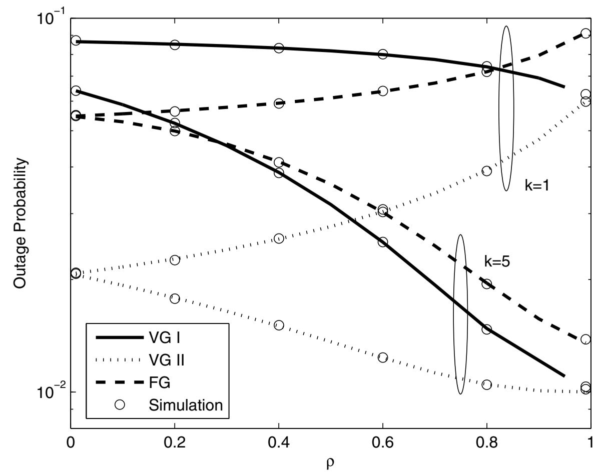


Fig. 2. Outage probability versus $\rho$, for best ($k=5$) and worst ($k=1$) relay selection ($N_r=5, \gamma_T=5, \eta_1=\eta_2=20$ dB).

All investigated cases revealed an excellent agreement between analytical and Monte Carlo simulation results. Figure 1 shows the outage probability against $\eta_1$ in dB. The number of relays $N_r$ considered was five, $\mu=1$ and the correlation $\rho$ was taken to be 0.8. Two cases where the best ($k=5$) and worst ($k=1$) relay is chosen is presented. We can see that, when the best relay is chosen, the variable gain relaying performs better than fixed relaying. In the case of worst relay selection, at low SNR variable gain relaying outperforms fixed gain relaying,

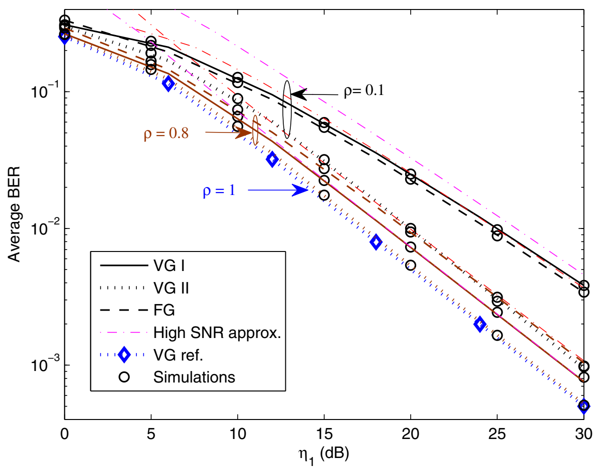


Fig. 3. The average BER under different values of correlation $\rho$.

while at high SNR it becomes worse. The simple high SNR approximations obtained show good proximity to the exact results. Simulation results not included here showed for higher values of $k$ (e.g. best relay) and $\rho$, the overlap of the high SNR approximation with the exact result happens as early as 15 dB, while for low $k$ and $\rho$ values it happens around $40$ dB.

Figure 2 shows the influence of the correlation $\rho$ on the outage probability. When the best relay is chosen, i.e. $k = 5$, the performance improves with increasing $\rho$. The performance of the variable gain systems is better than the fixed gain system at high $\rho$ ($> 0.3$). In the case of worst relay selection, the fixed gain relaying performs better than the "VG I" system in $\rho < 0.8$ region and the performance of the FG and VG II systems improve with decreasing $\rho$ as expected. The performance gaps between best and worst selection curves in both those systems vanishes as $\rho \to 0$, because when the decision and actual link SNR values aren't correlated, the ranking of relays would have no effect on the performance. The performance of 'VG I' under worst relay selection, in contrast, degrades with decreasing $\rho$, as the influence of incorrectly selecting the amplification gain factor at the relay becomes more significant.

Figure 3 presents the average BER performance with quadrature phase shift keying (QPSK) modulation ($\alpha = \beta = 1$), for two cases where the correlation between the outdated channel estimate and the actual channel is high ($\rho = 0.8$) and low ($\rho = 0.1$). The best relay ($k = 5$) out of all the ($N_r = 5$) relays was chosen and $\eta_1 = \eta_2$. The infinite series in (32) was truncated at 45 terms for calculations. In low SNR regions, the variable gain systems outperform the fixed gain counterparts. When the correlation is high ($\rho = 0.8$), variable gain relaying outperforms fixed gain relaying. If the correlation is low ($\rho = 0.1$), at high SNR, the fixed gain system outperforms variable gain system. In all cases, the reference "VG II" system demonstrates better performance. A reference curve for variable gain relaying with $\rho = 1$ is plotted to observe the performance loss due to not having perfect information at the relay.

## VI. Conclusion

In this paper we analyzed the effect of using outdated channel information at the source and the relay on the performance of dual-hop variable and fixed gain systems using partial relay selection. The two schemes have a low implementation complexity compared to schemes analyzed in the existing literature. We presented new analytical expressions for the outage probability and the average BER when the $k$-th worse relay is chosen, hence the derived results are general. The derived high SNR approximations are simple and can be conveniently used to gain quick insights on the influence of system parameters on the performance. We found that for low correlation values and with best relay selection, fixed gain relaying gives better performance than variable gain relaying. However as correlation increases, variable gain relaying outperforms fixed gain relaying.

## References

[1] M. Uysal (Ed.), *Cooperative Communications for Improved Wireless Network Transmission: Frameworks for Virtual Antenna Array Applications*. IGI-Global, 2009.

[2] A. Bletsas, A. Khisti, D. P. Reed and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE J. Select. Areas Commun.*, vol. 24, pp. 659-672, Mar. 2006.

[3] Y. Zhao, R. Adve and T. J. Lim, "Symbol error rate of selection amplify-and-forward relay systems," *IEEE Commun. Lett.*, vol. 10, pp. 757-759, Nov. 2006.

[4] I. Krikidis, J. S. Thompson, S. McLaughlin and N. Goertz, "Amplify and forward with partial relay selection," *IEEE Commun. Lett.*, vol. 12, pp. 235-237, Apr. 2008.

[5] H. A. Suraweera, D. S. Michalopoulos and G. K. Karagiannidis, "Semi-blind amplify-and-forward with partial relay selection," *Electron. Lett.*, vol. 45, pp. 317-318, Mar. 2009.

[6] D. B. da Costa and S. Aïssa, "End-to-end performance of dual-hop semi-blind relaying systems with partial relay selection," *IEEE Trans. Wireless Commun.* vol. 8, pp. 4306-4315, Aug. 2009.

[7] H. Ding, J. Ge, D. B. da Costa and Z. Jiang, Diversity and coding gains of fixed-gain amplify-and-forward with partial relay selection in Nakagami-$m$ fading," *IEEE Commun. Lett.*, vol. 14, pp. 734-736, Aug. 2010.

[8] S. S. Ikki and M. H. Ahmed, "On the analysis of the $N$th best-relay cooperative-diversity networks," *IEEE Trans. Commun.*, Accepted.

[9] J. L. Vicario, A. Bel, J. A. Lopez-Salcedo and G. Seco, "Opportunistic relay selection with outdated CSI: Outage probability and diversity analysis," *IEEE Trans. Wirless Commun.*, vol. 8, pp. 2872-2876, June 2009.

[10] H. A. Suraweera, M. Soysa, C. Tellambura and H. K. Garg, "Performance analysis of partial relay selection with feedback delay," *IEEE Signal Processing Lett.*, vol. 17, pp. 531-534, June 2010.

[11] D. S. Michalapoulos, H. A. Suraweera, G. K. Karagiannidis and R. Schober, "Relay selection with outdated channel estimates," in *Proc. IEEE GLOBECOM 2010*, Miami, FL, Dec. 2010.

[12] G. Andrieux, J.-F. Diouris and Y. Wang, "Channel time variation effects on transmit beamforming in the TDD mode of UMTS," in *Proc. IEEE PIMRC 2002*, Sept. 2002, Lisbon, Portugal, pp. 1987-1991.

[13] A. Erdelyi, W. Magnus, F. Oberhettinger and F. G. Tricomi, *Tables of Integral Transforms*, vol. 1, ser. Bateman Manuscript Project, California Institute of Technology. McGraw-Hill Book Company, Inc, 1954.

[14] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. Academic Press, Inc., 2000.

[15] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions*. Dover Publications, Inc., New York, 1970.

[16] A. H. Nuttall, "Some integrals involving the $Q$-function," Naval Underwater Systems Center (NUSC) Technical Report 4297, April 1972.