# Joint Bandwidth and Power Allocation With Admission Control in Wireless Multi-User Networks With and Without Relaying

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***Abstract*—Equal allocation of bandwidth and/or power may not be efficient for wireless multi-user networks with limited bandwidth and power resources. Optimal joint bandwidth and power allocation strategies for wireless multi-user networks with and without relaying are proposed in this paper for 1) the maximization of the sum capacity of all users; 2) the maximization of the worst user capacity; and 3) the minimization of the total power consumption of all users. It is shown that the proposed allocation problems are convex and, therefore, can be solved efficiently. Moreover, joint bandwidth and power allocation for admission control is considered. A suboptimal greedy search algorithm is developed to solve the admission control problem efficiently. Instructive analysis of the greedy search shows that it can achieve good performance, and the condition under which the greedy search is optimal is derived. The formal and in-depth analysis of the greedy search algorithm presented in this paper can serve as a benchmark for analyzing similar algorithms in other applications. The performance improvements offered by the proposed optimal joint bandwidth and power allocation are demonstrated by simulations. The advantages of the suboptimal greedy search algorithm for admission control are also shown in numerical results.**

***Index Terms*—Admission control, greedy search algorithm, joint bandwidth and power allocation, wireless multi-user networks.**

## I. Introduction

ONE of the critical issues in wireless multi-user networks is the efficient allocation of the available radio resources in order to improve the network performance. Therefore, resource allocation, e.g., power allocation, in wireless multi-user networks has been extensively researched. However, the joint allocation of bandwidth and power resources has not attracted much attention. Indeed, in practical wireless networks, the available transmission power of the nodes and the total available bandwidth of the network are limited and, therefore, joint bandwidth and power allocation must be considered.

It has been shown earlier that communication efficiency can be improved by using relays [1]–[3]. Indeed, in the case of severe channel conditions in direct links, relays can be deployed to forward the data from a source to a destination. There exist numerous papers on the resource allocation in wireless relay networks. For example, power allocation with the decode-and-forward (DF) relaying has been studied in [4] under the assumption that transmitters only know mean channel gains. In [5], a power allocation scheme that aims at maximizing the smallest of two transceiver signal-to-noise ratios (SNRs) has been studied for two-way relay networks. In [6], time/bandwidth allocation strategies with constant transmit power have been developed based on the time division multiple access/frequency division multiple access (TDMA/FDMA) to optimize effective capacity in relay networks. However, [4]–[6] as well as most of the works on the resource allocation in wireless relay networks consider the case of a single user, i.e., a single source-destination pair.

Resource allocation for wireless multi-user relay networks has been investigated only in a few works. Power allocation aiming at optimizing the sum capacity of multiple users for four different relay transmission strategies has been studied in [7], while an amplify-and-forward-based strategy in which multiple sources share multiple relays using power control has been developed in [8].

It is worth noting that the works mentioned above (except [6]) have assumed equal and fixed bandwidth allocation for the source-relay and relay-destination. In fact, it is inefficient to allocate the bandwidth equally when the total available bandwidth is limited. However, the problem of joint allocation of bandwidth and power has never been considered for wireless multi-user relay networks.

Various performance metrics for the resource allocation in multi-user networks have been considered. Sum capacity maximization is taken as an objective for power allocation in [7], while max-min SNR, power minimization, and throughput maximization are used as power allocation criteria in [8]. System throughput maximization and worst user throughput maximization are studied using convex optimization in [9].

In some applications, certain minimum transmission rates must be guaranteed for the users in order to satisfy their quality-of-service (QoS) requirements. For instance, in real-time voice and video applications, a minimum rate should be guaranteed for each user to satisfy the delay constraints of

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the services. However, when the rate/capacity requirements can not be supported for all the users, admission control is adopted to decide which users to be admitted into the network. The admission control in wireless networks typically aims at maximizing the number of admitted users and has been recently considered in several works. A single-stage reformulation approach for a two-stage joint resource allocation and admission control problem is proposed in [10], while another approach based on user removal is developed in [11] and [12].

In this paper, joint bandwidth and power allocation for wireless multi-user networks with and without relaying is considered, which is especially efficient for the networks with both limited bandwidth and limited power. The joint bandwidth and power allocation is proposed to 1) maximize the sum capacity of all users; 2) maximize the capacity of the worst user; 3) minimize the total power consumption of all users. The corresponding joint bandwidth and power allocation problems can be formulated as optimization problems that are shown to be convex. Therefore, these problems can be solved efficiently by using convex optimization techniques. The joint bandwidth and power allocation for admission control is further considered, and a greedy search algorithm is developed in order to reduce the computational complexity of solving the admission control problem. The proposed greedy search removes one user at each iteration until the remaining users can be admitted. Instructive analysis of the greedy search is provided, which shows that it can achieve good performance, and the optimality condition of the greedy search is derived. The formal and in-depth analysis of the greedy search algorithm presented in this paper can serve as a benchmark for analyzing similar algorithms in other applications.

The rest of this paper is organized as follows. The system model is given in Section II. In Section III, joint bandwidth and power allocation problems for the three aforementioned objectives are formulated and studied. Admission control based joint bandwidth and power allocation problem is proposed in Section IV, where a greedy search algorithm is developed and investigated. Numerical results are demonstrated in Section V, followed by concluding remarks in Section VI. This paper is reproducible research and the software needed to generate the simulation results can be obtained from IEEE Xplore together with this paper.

## II. System Model

### A. Without Relaying

Consider a wireless network, which consists of $M$ source nodes $S_i$, $i \in \mathcal{M} \triangleq \{1, 2, \cdots, M\}$, and $K$ destination nodes $D_i$, $i \in \mathcal{K} \triangleq \{1, 2, \cdots, K\}$. The network serves $N$ users $U_i$, $i \in \mathcal{N} \triangleq \{1, 2, \cdots, N\}$, where each user represents a one-hop link from a source to a destination. The set of users which are served by $S_i$ is denoted by $\mathcal{N}_{S_i}$.

A spectrum of total bandwidth $W$ is available for the transmissions from the sources. This spectrum is assumed to be flat fading[1] and can be divided into distinct and nonoverlapping channels of unequal bandwidths, so that the sources share the available spectrum through frequency division and, therefore, do not interfere with each other.

Let $P_i^{\mathrm{S}}$ and $W_i^{\mathrm{S}}$ denote the allocated transmit power and channel bandwidth of the source to serve $U_i$. Then the received SNR at the destination of $U_i$ is

$$\gamma_i^{\mathrm{D}} = \frac{P_i^{\mathrm{S}} h_i^{\mathrm{SD}}}{W_i^{\mathrm{S}} N_0} \tag{1}$$

where $h_i^{\mathrm{SD}}$ denotes the channel gain of the source-destination link of $U_i$ and $W_i^{\mathrm{S}} N_0$ stands for the power of additive white Gaussian noise (AWGN) over the bandwidth $W_i^{\mathrm{S}}$. The channel gain $h_i^{\mathrm{SD}}$ results from such effects as path loss, shadowing, and fading. Due to the fact that the power spectral density (PSD) of AWGN is constant over all frequencies with a constant value denoted by $N_0$, the noise power in the channel is linearly increasing with the channel bandwidth. It can be seen from (1) that a channel with a larger bandwidth introduces higher noise power and, thus, reduces the SNR.

Channel capacity gives the maximum achievable rate of a link. Given $\gamma_i^{\mathrm{D}}$, the source-destination link capacity of $U_i$ is

$$C_i^{\mathrm{SD}} = W_i^{\mathrm{S}} \log\left(1 + \gamma_i^{\mathrm{D}}\right) = W_i^{\mathrm{S}} \log\left(1 + \frac{P_i^{\mathrm{S}} h_i^{\mathrm{SD}}}{W_i^{\mathrm{S}} N_0}\right). \tag{2}$$

It can be seen that $W_i^{\mathrm{S}}$ characterizes the channel's dimension, while $\log(1+\gamma_i^{\mathrm{D}})$ characterizes the data success rate per dimension and, thus, $C_i^{\mathrm{SD}}$ characterizes the amount of data transmitted without error over the source-destination link per unit time.

It can be seen from (2) that, for a fixed $W_i^{\mathrm{S}}$, $C_i^{\mathrm{SD}}$ is a concave increasing function of $P_i^{\mathrm{S}}$. Moreover, it can be shown that $C_i^{\mathrm{SD}}$ is a concave increasing function of $W_i^{\mathrm{S}}$ for a fixed $P_i^{\mathrm{S}}$, although $\gamma_i^{\mathrm{D}}$ is a decreasing function of $W_i^{\mathrm{S}}$. Indeed, it can be proved that $C_i^{\mathrm{SD}}$ is a concave function of $P_i^{\mathrm{S}}$ and $W_i^{\mathrm{S}}$ jointly.

### B. With Relaying

Consider $L$ relay nodes $R_i$, $i \in \mathcal{L} \triangleq \{1, 2, \cdots, L\}$ deployed on basis of the network described in the previous subsection, to forward the data from the sources to the destinations. Then each user represents a two-hop link from a source to a destination via relaying. To reduce the implementation complexity at the destinations, single relay assignment is adopted so that each user has one designated relay. Then the set of users served by $R_i$ is denoted by $\mathcal{N}_{R_i}$. We assume that the relays are preassigned to the users and are fixed, and it is also assumed that perfect channel state information (CSI) is available at the sources and relays such that a resource allocation scheme for the network can be carried out through a central coordination node.[2]

The relays work in a half-duplex manner due to the practical limitation that they can not transmit and receive at the same time. A two-phase decode-and-forward (DF) protocol is assumed, i.e., the relays receive and decode the transmitted data from the sources in the first phase, and re-encode and forward the data to the destinations in the second phase. The sources and relays share the total available spectrum in the first and second phase, respectively. It is assumed that the direct links between

[1]The results of this paper are applicable to narrowband systems where channels are flat fading. However, even if channels are not flat fading as in wideband systems, the results of this paper still give information-theoretic limits by providing upper/lower bounds for achievable performances.

[2]The assumptions of perfect CSI, central coordination, and fixed relay assignments are typical in the context of resource allocation such as [7] and [8]. Note that joint bandwidth and power allocation with admission control is studied in this work based on a relatively simplified system model, but this work is instrumental for generalizing the study to more complicated systems, which is out of the scope of this paper.

the sources and the destinations are blocked and, thus, are not available. Note that although the two-hop relay model is considered in the paper, the results are applicable for the multi-hop relay model.

Let $P_i^{\rm R}$ and $W_i^{\rm R}$ denote the allocated transmit power and channel bandwidth of the relay to serve $U_i$. Similar to (2), the one-hop source-relay link capacity of $U_i$ is given by

$$C_i^{\rm SR} = W_i^{\rm S} \log\left(1 + \frac{P_i^{\rm S} h_i^{\rm SR}}{W_i^{\rm S} N_0}\right) \tag{3}$$

where $h_i^{\rm SR}$ denotes the channel gain of the link, and the one-hop relay-destination link capacity of $U_i$ is given by

$$C_i^{\rm RD} = W_i^{\rm R} \log\left(1 + \frac{P_i^{\rm R} h_i^{\rm RD}}{W_i^{\rm R} N_0}\right) \tag{4}$$

where $h_i^{\rm RD}$ denotes the channel gain of the link. Therefore, the two-hop source-destination link capacity of $U_i$ is given by

$$C_i^{\rm SD} = \min\left\{C_i^{\rm SR}, C_i^{\rm RD}\right\}. \tag{5}$$

It can be seen from (3), (4), and (5) that if equal bandwidth is allocated to $W_i^{\rm S}$ and $W_i^{\rm R}$, $C_i^{\rm SR}$ and $C_i^{\rm RD}$ can be unequal due to the power limits on $P_i^{\rm S}$ and $P_i^{\rm R}$. Then the source-destination link capacity $C_i^{\rm SD}$ is constrained by the minimum of $C_i^{\rm SR}$ and $C_i^{\rm RD}$. Note that since all the users share the total bandwidth of the spectrum, equal bandwidth allocation for all the one-hop links can be inefficient. Therefore, the joint allocation of bandwidth and power is necessary.

## III. Joint Bandwidth and Power Allocation

Different objectives can be considered while jointly allocating bandwidth and power in wireless multi-user networks. The widely used objectives for network optimization are 1) sum capacity maximization; 2) the worst user capacity maximization; and 3) total network power minimization. In this section, the problems of joint bandwidth and power allocation are formulated and solved for the aforementioned objectives.

### A. Sum Capacity Maximization

In the applications without delay constraints, a high data rate from any user in the network is favorable. Thus, it is desirable to allocate the resources to maximize the overall network performance, e.g., the sum capacity of all users.

*1) Without Relaying:* The joint bandwidth and power allocation problem aiming at maximizing the sum capacity for the network without relaying can be mathematically formulated as

$$\max_{\{P_i^{\rm S}, W_i^{\rm S}\}} \sum_{i\in\mathcal{N}} C_i^{\rm SD} \tag{6a}$$

$$\text{s.t.} \sum_{i\in\mathcal{N}_{S_j}} P_i^{\rm S} \le P_{S_j}, \quad j \in \mathcal{M} \tag{6b}$$

$$\sum_{i\in\mathcal{N}} W_i^{\rm S} \le W. \tag{6c}$$

The nonnegativity constraints of the optimization variables are natural and, thus, omitted throughout the paper for brevity. In the problem (6a)–(6c), the constraint (6b) stands for that the total power at $S_j$ is limited by $P_{S_j}$. The constraint (6c) indicates that the total bandwidth of the channels allocated to the sources is also limited.

Note that since $C_i^{\rm SD}$ is a jointly concave function of $P_i^{\rm S}$ and $W_i^{\rm S}$, the objective function (6a) is convex. The constraints (6b) and (6c) are linear and, thus, are convex. Therefore, the problem (6a)–(6c) is convex. Using its convexity, the closed-form solution of the problem (6a)–(6c) is given below.

*Proposition 1:* The optimal solution of the problem (6a)–(6c), denoted by $\{P_i^{\rm S^*} | i \in \mathcal{N}\}$, is $P_i^{\rm S^*} = P_i^{\rm S^\star}$, $W_i^{\rm S^*} = W h_i^{\rm SD} P_i^{\rm S^\star} / \sum_{j\in\mathcal{I}} h_j^{\rm SD} P_j^{\rm S^\star}$, $\forall i \in \mathcal{I}$, and $P_i^{\rm S^*} = W_i^{\rm S^*} = 0, \forall i \notin \mathcal{I}$, where $P_i^{\rm S^\star} = P_{S_k}$ for $i \in \mathcal{N}_{S_k}$ and $\mathcal{I} \triangleq \{i | i = \arg\max_{j\in\mathcal{N}_{S_k}} h_j^{\rm SD}, k \in \mathcal{M}\}$.

*Proof:* See Appendix A.

Proposition 1 shows that for a set of users served by one source, the sum capacity maximization based allocation allocates all the power of the source only to the user with the highest channel gain and, therefore, results in highly unbalanced resource allocation among the users.

*2) With Relaying:* The sum capacity maximization based joint bandwidth and power allocation problem for the network with relaying is given by

$$\max_{\{P_i^{\rm S}, W_i^{\rm S}, P_i^{\rm R}, W_i^{\rm R}\}} \sum_{i\in\mathcal{N}} C_i^{\rm SD} \tag{7a}$$

$$\text{s.t.} \sum_{i\in\mathcal{N}_{S_j}} P_i^{\rm S} \le P_{S_j}, \quad j \in \mathcal{M} \tag{7b}$$

$$\sum_{i\in\mathcal{N}_{R_j}} P_i^{\rm R} \le P_{R_j}, \quad j \in \mathcal{L} \tag{7c}$$

$$\sum_{i\in\mathcal{N}} W_i^{\rm S} \le W \tag{7d}$$

$$\sum_{i\in\mathcal{N}} W_i^{\rm R} \le W. \tag{7e}$$

Introducing new variables $\{T_i | i \in \mathcal{N}\}$, the problem (7a)–(7e) can be equivalently written as

$$\min_{\{P_i^{\rm S}, W_i^{\rm S}, P_i^{\rm R}, W_i^{\rm R}, T_i\}} \quad -\sum_{i\in\mathcal{N}} T_i \tag{8a}$$

$$\text{s.t. } T_i - C_i^{\rm SR} \le 0, \quad i \in \mathcal{N} \tag{8b}$$

$$T_i - C_i^{\rm RD} \le 0, \quad i \in \mathcal{N} \tag{8c}$$

$$\text{the constraints (7b)–(7e).} \tag{8d}$$

Note that the constraints (8b) and (8c) are convex since $C_i^{\rm SR}$ and $C_i^{\rm RD}$ are jointly concave functions of $\{P_i^{\rm S}, W_i^{\rm S}\}$ and $\{P_i^{\rm R}, W_i^{\rm R}\}$, respectively. The constraints (8d) are linear and, thus, convex. Therefore, the problem (8a)–(8d) is convex. It can be seen that the closed-form solution of the problem (8a)–(8d) is intractable due to the coupling of the constraints (8b) and (8c). However, the convexity of the problem (8a)–(8d) allows for standard numerical algorithms for convex optimization to solve the problem efficiently.

Intuitively, the sum capacity maximization based allocation for the network with relaying does not result in as unbalanced resource allocation as that for the network without relaying, since the channel gains in both transmission phases affect the capacity of a user. The proposition below gives a case where the sum capacity maximization based allocation for the network with relaying also starves some users.

*Proposition 2:* If $h_i^{\mathrm{SR}} \geq h_j^{\mathrm{SR}}$ and $h_i^{\mathrm{RD}} \geq h_j^{\mathrm{RD}}$ where $\{i,j\} \subseteq \mathcal{N}_{S_k}$ and $\{i,j\} \subseteq \mathcal{N}_{R_l}$, then $P_j^{\mathrm{S}^*} = W_j^{\mathrm{S}^*} = P_j^{\mathrm{R}^*} = W_j^{\mathrm{R}^*} = 0$.

*Proof:* See Appendix A.

In particular, if two users are served by the same source and relay, and one user has lower channel gains than the other user in both transmission phases, then no resource is allocated to the former user.

### B. Minimum Capacity Maximization

Fairness among users is also an important issue for resource allocation. If the fairness issue is considered, the achievable rate of the worst user is commonly used as the network performance measure. In this case, the joint bandwidth and power allocation problem can be mathematically formulated as

$$\max_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}\}} \quad \min_{i\in\mathcal{N}} C_i^{\mathrm{SD}} \tag{9a}$$
$$\text{s.t. the constraints (6b)–(6c)} \tag{9b}$$

for the network without relaying and

$$\max_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}, P_i^{\mathrm{R}}, W_i^{\mathrm{R}}\}} \quad \min_{i\in\mathcal{N}} C_i^{\mathrm{SD}} \tag{10a}$$
$$\text{s.t. the constraints (7b)–(7e)} \tag{10b}$$

for the network with relaying. Introducing a variable $T$, the problem (10a)–(10b) can be equivalently written as

$$\min_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}, P_i^{\mathrm{R}}, W_i^{\mathrm{R}}, T\}} \quad -T \tag{11a}$$
$$\text{s.t. } T - C_i^{\mathrm{SR}} \leq 0, \quad i \in \mathcal{N} \tag{11b}$$
$$T - C_i^{\mathrm{RD}} \leq 0, \quad i \in \mathcal{N} \tag{11c}$$
$$\text{the constraints (7b)–(7e).} \tag{11d}$$

Similar to the sum capacity maximization based allocation problems, it can be shown that the problems (9a)–(9b) and (11a)–(11d) are convex. Therefore, the optimal solutions can be efficiently obtained.

The next proposition indicates that the minimum capacity maximization based allocation leads to absolute fairness among users, in contrast to the sum capacity maximization based allocation. The proof follows directly from the fact that the total bandwidth is shared by all the users, and is omitted for brevity.

*Proposition 3:* In the problem (9a)–(9b) and (10a)–(10b), the capacities of all users are equal at optimality.

### C. Power Minimization

Another widely considered design objective is the minimization of the total power consumption of all users. This minimization is performed under the constraint that the capacity requirements of all users are satisfied. The corresponding joint bandwidth and power allocation problem can be written as

$$\min_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}\}} \quad \sum_{i\in\mathcal{N}} P_i^{\mathrm{S}} \tag{12a}$$
$$\text{s.t. } c_i - C_i^{\mathrm{SD}} \leq 0, \quad i \in \mathcal{N} \tag{12b}$$
$$\text{the constraints (6b)–(6c)} \tag{12c}$$

for the network without relaying and

$$\min_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}, P_i^{\mathrm{R}}, W_i^{\mathrm{R}}\}} \quad \sum_{i\in\mathcal{N}} \left(P_i^{\mathrm{S}} + P_i^{\mathrm{R}}\right) \tag{13a}$$
$$\text{s.t. } c_i - C_i^{\mathrm{SR}} \leq 0, \quad i \in \mathcal{N} \tag{13b}$$
$$c_i - C_i^{\mathrm{RD}} \leq 0, \quad i \in \mathcal{N} \tag{13c}$$
$$\text{the constraints (7b)–(7e)} \tag{13d}$$

for the network with relaying, where $c_i$ is the minimum acceptable capacity for $U_i$ and the constraints (13b) and (13c) indicate that the one-hop link capacities of $U_i$ should be no less than the given capacity threshold. Similar to the sum capacity maximization- and minimum capacity maximization-based allocation problems, the problems (12a)–(12c) and (13a)–(13d) are convex and, thus, can be solved efficiently.

## IV. Admission Control Based Joint Bandwidth and Power Allocation

In the multi-user networks under consideration, admission control is required if a certain minimum capacity must be guaranteed for each user. The admission control aims at maximizing the number of admitted users.

### A. Without Relaying

The admission control-based joint bandwidth and power allocation problem for the network without relaying can be mathematically expressed as

$$\max_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}\}, \mathcal{I}\subseteq\mathcal{N}} \quad |\mathcal{I}| \tag{14a}$$
$$\text{s.t. } c_i - C_i^{\mathrm{SD}} \leq 0, \quad i \in \mathcal{I} \tag{14b}$$
$$\text{the constraints (6b)–(6c)} \tag{14c}$$

where $|\mathcal{I}|$ stands for the cardinality of $\mathcal{I}$.

Note that the problem (14a)–(14c) can be solved using exhaustive search among all possible subsets of users. However, the computational complexity of the exhaustive search can be very high since the number of possible subsets of users is exponentially increasing with the number of users, which is not acceptable for practical implementation. Therefore, we develop a suboptimal greedy search algorithm that significantly reduces the complexity of finding the maximum number of admissible users.

*1) Greedy Search Algorithm:* Consider the following problem:

$$G(\mathcal{I}) \triangleq \min_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}\}} \quad \sum_{i\in\mathcal{I}} W_i^{\mathrm{S}} \tag{15a}$$
$$\text{s.t. } c_i - C_i^{\mathrm{SD}} \leq 0, \quad i \in \mathcal{I} \tag{15b}$$
$$\text{the constraints (6b).} \tag{15c}$$

The following proposition which provides the necessary and sufficient conditions for the admissibility of a set of users is in order.

*Proposition 4:* A set of users $\mathcal{I}$ is admissible if and only if $G(\mathcal{I}) \leq W$.

*Proof:* See Appendix B.

Note from Proposition 4 that the optimal value $G(\mathcal{I})$ is the minimum total bandwidth required to support the users in $\mathcal{I}$, given that all the power constraints are satisfied. This is instructive for establishing our greedy search algorithm, which removes users one by one until the remaining users are admissible. The "worst" user, i.e., the user whose removal reduces the total bandwidth requirement to the maximum extent, is removed at each greedy search iteration. In other words, the removal of such "worst" user results in the minimum total bandwidth requirement of the remaining users. Thus, the removal criterion can be stated as

$$n^{(t)} \triangleq \arg\max_{n\in\mathcal{N}^{(t-1)}} \left(G\left(\mathcal{N}^{(t-1)}\right) - G\left(\mathcal{N}^{(t-1)}\setminus\{n\}\right)\right) = \arg\min_{n\in\mathcal{N}^{(t-1)}} G\left(\mathcal{N}^{(t-1)}\setminus\{n\}\right) \tag{16}$$

where $n^{(t)}$ denotes the user removed at the $t$th greedy search iteration, $\mathcal{N}^{(t)} \triangleq \mathcal{N}^{(t-1)}\setminus\{n^{(t)}\}$ denotes the set of remaining users after $t$ greedy search iterations, and $\setminus$ stands for set difference operator.

Note that, intuitively, $\mathcal{N}^{(t)}$ should be the "best" set of $N-t$ users that requires the minimum total bandwidth among all possible sets of $N-t$ users from $\mathcal{N}$, and $G(\mathcal{N}^{(t)})$ is the corresponding minimum total bandwidth requirement. Thus, the stopping rule for the greedy search iterations should be finding such $t^*$ that $G(\mathcal{N}^{(t^*-1)}) > W$ and $G(\mathcal{N}^{(t^*)}) \le W$. In other words, $N-t^*$ is expected to be the maximum number of admissible users, denoted by $d^*$.

*2) Complexity of the Greedy Search Algorithm:* It can be seen from Proposition 4 that using the exhaustive search to find the maximum number of admissible users is equivalent to checking $G(\mathcal{I})$ for all possible $\mathcal{I}\subseteq\mathcal{N}$ and, therefore, the number of times of solving the problem (15a)–(15c) is upper bounded by $\sum_{i=d^*}^{N}\binom{N}{i}$. On the other hand, it can be seen from (16) that using the greedy search, the number of times of solving the problem (15a)–(15c) is upper bounded by $\sum_{i=0}^{t^*-1}(N-i)$. Therefore, the complexity of the proposed greedy search is significantly reduced as compared to that of the exhaustive search, especially if $N$ is large and $d^*$ is small.

The complexity of the greedy search can be further reduced. The lemma given below follows directly from the decomposable structure of the problem (15a)–(15c) that $G(\mathcal{I}) = \sum_{i\in\mathcal{M}} G(\mathcal{I}\cap\mathcal{N}_{S_i})$.

*Lemma 2:* $G(\mathcal{I}) - G(\mathcal{I}\setminus\{n\}) = G(\mathcal{I}\cap\mathcal{N}_{S_i}) - G(\mathcal{I}\cap\mathcal{N}_{S_i}\setminus\{n\})$ for $n\in\mathcal{N}_{S_i}, \forall\mathcal{I}\subseteq\mathcal{N}$.

Lemma 2 shows that the reduction of the total bandwidth requirement is only coupled with the users served by the same source with the user to be removed, and is decoupled with the users served by other sources.

Let $\mathcal{N}_{S_i}^{[t]} \triangleq \mathcal{N}_{S_i}\cap\mathcal{N}^{(t)}$ denote the remaining users served by $S_i$ after $t$ greedy search iterations. Applying Lemma 2 directly to the removal criterion in (16), we have the following proposition with proof omitted.

*Proposition 5:* $n^{(t)} = n_{S_{i^*}}^{[t-1]*}$, where $n_{S_i}^{[t-1]*} \triangleq \arg\max_{n\in\mathcal{N}_{S_i}^{[t-1]}}(G(\mathcal{N}_{S_i}^{[t-1]}) - G(\mathcal{N}_{S_i}^{[t-1]}\setminus\{n\}))$ and $i^* \triangleq \arg\max_{i\in\mathcal{M}}(G(\mathcal{N}_{S_i}^{[t-1]}) - G(\mathcal{N}_{S_i}^{[t-1]}\setminus\{n_{S_i}^{[t-1]*}\}))$.

Proposition 5 provides an equivalent algorithm of searching for the user to be removed at each greedy search iteration with reduced computational complexity. Specifically, we can first find the "worst' user in each set of users served by a source, i.e., $n_{S_i}^{[t-1]*}$, and then determine the user to be removed among these "worst" users. As a result, although the number of times of solving the problem (15a)–(15c) remains the same as that of (16), the number of variables involved in solving each problem is reduced, especially if $N_{S_i}$ is small compared to $N$.

*3) Optimality Conditions of the Greedy Search Algorithm:* In this subsection, we study the conditions under which the proposed greedy search algorithm is optimal. Specifically, the greedy search is optimal if the set of remaining users after each greedy search iteration is the "best" set of users, i.e.,

$$\mathcal{N}^{(t)} = \mathcal{N}_{N-t}^*, \quad \forall 1\le t\le N \tag{17}$$

where $\mathcal{N}_i^* \triangleq \arg\min_{|\mathcal{I}|=i} G(\mathcal{I})$.

Consider applying the greedy search on a set of users served by one source $\mathcal{N}_{S_i}$. Let $n_{S_i}^{(t)} \triangleq \arg\max_{n\in\mathcal{N}_{S_i}^{(t-1)}}(G(\mathcal{N}_{S_i}^{(t-1)}) - G(\mathcal{N}_{S_i}^{(t-1)}\setminus\{n\}))$ denote the user removed at the $t$th greedy search iteration and $\mathcal{N}_{S_i}^{(t)} \triangleq \mathcal{N}_{S_i}^{(t-1)}\setminus\{n_{S_i}^{(t)}\}$ denote the set of remaining users after $t$ greedy search iterations. Also let $\mathcal{N}_{S_i,j}^* \triangleq \arg\min_{\mathcal{I}\subseteq\mathcal{N}_{S_i},|\mathcal{I}|=j} G(\mathcal{I})$ denote the "best" set of $j$ users in $\mathcal{N}_{S_i}$. The next theorem provides the necessary and sufficient conditions for the optimality of the greedy search.

*Theorem 1:* $\mathcal{N}^{(t)} = \mathcal{N}_{N-t}^*, \forall 1\le t\le N$, if and only if the following two conditions hold:

C1) $\mathcal{N}_{S_i}^{(t)} = \mathcal{N}_{S_i,N_{S_i}-t}^*, \forall 1\le t\le N_{S_i}, \forall i\in\mathcal{M}$;

C2) $G(\mathcal{N}_{S_i}^{(t-2)}) - G(\mathcal{N}_{S_i}^{(t-1)}) > G(\mathcal{N}_{S_i}^{(t-1)}) - G(\mathcal{N}_{S_i}^{(t)})$, $\forall 2\le t\le N_{S_i}, \forall i\in\mathcal{M}$.

*Proof:* See Appendix B.

Theorem 1 decouples the optimality conditions of the greedy search into equivalent conditions in the context of applying the greedy search on each set of users served by one source. Specifically, in this context, C1 indicates that the set of remaining users after each greedy search iteration is the "best" set of users; C2 indicates that the reduction of the total bandwidth requirement is decreasing with the greedy search iterations. It will be shown that Theorem 1 is significant in that it allows us to focus on equivalent problems where users are subject to the same power constraint and, therefore, the problems become tractable.

Let $h_i \triangleq h_i^{\mathrm{SD}}/N_0$ denote the channel gain normalized by the noise PSD. Recall that $c_i$ is the minimum acceptable capacity for $U_i$. Define $F_i(p)$ as the unique solution of $w$ in the equation

$$c_i = w\log\left(1+\frac{h_i p}{w}\right) \tag{18}$$

given $h_i$ and $c_i$ for any $p>0$, which represents the minimum bandwidth required by a user for its allocated transmit power. Then the problem (15a)–(15c) for $\mathcal{N}_{S_i}$ can be rewritten as

$$G(\mathcal{N}_{S_i}) \triangleq \min_{\{p_i\}} \sum_{i\in\mathcal{N}_{S_i}} F_i(p_i) \tag{19a}$$

$$\text{s.t.} \sum_{i\in\mathcal{N}_{S_i}} p_i \le P_{S_i}. \tag{19b}$$

The following lemma gives a condition under which C1 holds for a specific $t$.

*Lemma 3:* If there exists $\mathcal{N}_{S_l,k}\subseteq\mathcal{N}_{S_l}$, $|\mathcal{N}_{S_l,k}| = k$, such that $F_i(p) < F_j(p), \forall 0<p<P_{S_l}, \forall i\in\mathcal{N}_{S_l,k}$ and $\forall j\in\mathcal{N}_{S_l}\setminus\mathcal{N}_{S_l,k}$, then $\mathcal{N}_{S_l,k} = \mathcal{N}_{S_l,k}^* = \mathcal{N}_{S_l}^{(N_{S_l}-k)}$.

*Proof:* See Appendix B.

It can be seen from Lemma 3 that since any user in $\mathcal{N}_{S_l,k}$ has a smaller bandwidth requirement than any user in $\mathcal{N}_{S_l} \setminus \mathcal{N}_{S_l,k}$ for the same allocated power over the available power range, the former is more favorable than the latter for reducing the total bandwidth requirement. Therefore, $\mathcal{N}_{S_l,k}$ is the 'best' set of $k$ users and the greedy search removes the users in $\mathcal{N}_{S_l} \setminus \mathcal{N}_{S_l,k}$ before the users in $\mathcal{N}_{S_l,k}$.

It is worth noting that C1 does not hold in general. Indeed, since the reduction of the total bandwidth requirement is maximized only at each single greedy search iteration, the greedy search does not guarantee that the reduction of the total bandwidth requirement is also maximized over multiple greedy search iterations. In other words, it does not guarantee that the set of remaining users is the "best" set of users. In order to demonstrate this point, we present a counterexample as follows.

*Example 1:* Let $\mathcal{N}_{S_1} = \{1,2,3\}$. Also let $h_1 = 4$, $h_2 = 5$, $h_3 = 6$, $c_1 = 1$, $c_2 = 1.1$, $c_3 = 1.2$, and $P_{S_1} = 1.1$. Then, we have $G(\{1,2\}) = 1.3849$, $G(\{1,3\}) = 1.3808$, $G(\{2,3\}) = 1.3573$, $G(\{1\}) = 0.4039$, $G(\{2\}) = 0.4135$, $G(\{3\}) = 0.4292$ and, therefore, $\mathcal{N}_{S_1}^{(1)} = \{2,3\}$, $\mathcal{N}_{S_1}^{(2)} = \{2\}$, $\mathcal{N}_{S_1,1}^{*} = \{1\}$. This shows that $\mathcal{N}_{S_1}^{(2)} \neq \mathcal{N}_{S_1,1}^{*}$.

Example 1 shows that the "worst" user, which is removed first in the greedy search, may be among the users in the 'best' set of users after more users are removed. An intuitive interpretation of this result is that the bandwidth requirement of the "worst" user changes from being larger to smaller than those of the other users for the same allocated power, when the average available power to each user increases after some users are removed in the greedy search.

Applying Lemma 3, the next proposition gives a sufficient condition of C1 and further decouples it into conditions expressed in terms of the bandwidth requirement comparison of two users.

*Proposition 6:* C1 holds if the following condition holds:

C3) for any $j \in \mathcal{N}_{S_i}, \forall i \in \mathcal{M}$, there exists no more than one $k \in \mathcal{N}_{S_i}, k \neq j$, such that the following condition holds:

C4) $F_j(p) = F_k(p), \exists 0 < p < P_{S_i}$.

*Proof:* See Appendix B.

It can be seen that $\Pr\{\text{C3}\}$ increases as $\Pr\{\text{C4}\}$ decreases and $\Pr\{\text{C3}\} \to 1$ as $\Pr\{\text{C4}\} \to 0$, where "Pr" stands for the probability of an event. Moreover, $\Pr\{\text{C3}\}$ increases as $N_{S_i}$ decreases, $i \in \mathcal{M}$, or $M$ decreases.

The next lemma characterizes the bandwidth requirement comparison of two users in terms of the comparison of their capacity requirement ratio and channel gain ratio.

*Lemma 4:* If $i \neq j$ and $h_j/h_i \geq 1$, then

1) there exists such $p'$ that $F_i(p) > F_j(p), \forall p' > p > 0$, and $F_i(p) < F_j(p), \forall p > p'$, if and only if $1 < c_j/c_i < h_j/h_i$; furthermore, $p' \uparrow$ as $h_j/h_i \uparrow$ or $c_j/c_i \downarrow$;
2) $F_i(p) > F_j(p), \forall p > 0$, or $F_i(p) = F_j(p), \forall p > 0$, if and only if $c_j/c_i \leq 1$;
3) $F_i(p) < F_j(p), \forall p > 0$, if and only if $c_j/c_i \geq h_j/h_i$.

*Proof:* See Appendix B.

Lemma 4 gives the conditions under which capacity requirement ratio dominates or is dominated by channel gain ratio in affecting the bandwidth requirement comparison of two users, and the corresponding dominant ranges. Specifically, claim 1 indicates the case where channel gain ratio dominates and is dominated by capacity requirement ratio in the low power range and high power range, respectively. Moreover, the dominant range is increasing and decreasing with channel gain ratio and capacity requirement ratio, respectively. Claims 2 and 3 indicate the cases where capacity requirement ratio dominates and is dominated by channel gain ratio in any power range, respectively.

It can be seen that C4 holds if and only if claim 1 in Lemma 4 holds with $0 < p' < P_{S_i}$. Then it follows from Lemma 4 that $\Pr\{\text{C3}\}$ increases as $h_j/h_i$ increases, $c_j/c_i$ decreases, or $P_{S_i}$ decreases, and $\Pr\{\text{C3}\} \to 1$ as $h_j/h_i \to \infty$, $c_j/c_i \to 1$, or $P_{S_i} \to 0$. This shows that C3 is a mild condition to hold if the diversity of user capacity requirements differs considerably from that of user channel gains, or the available power at a source is small, or the number of users served by a source is small, or the number of sources is small.

The following proposition shows that C2 is true in general, which is a nice property held by the greedy search.

*Proposition 7:* C2 always holds.

*Proof:* The proof is based on two lemmas using the monotonicity and convexity of $F_i(p)$. See Appendix B for details.

Applying Lemma 4 and Propositions 6 and 7 successively, the following corollary follows directly from Theorem 1 and the proof is omitted.

*Corollary 1:* $\mathcal{N}^{(t)} = \mathcal{N}_{N-t}^{*}, \forall 1 \leq t \leq N$, if $c_i = c_j$, $\forall i,j \in \mathcal{N}, i \neq j$.

Note that the optimality condition given in (17) is a sufficient condition under which $\mathcal{N}^{(t^*)} = \mathcal{N}_{N-t^*}^{*} = \mathcal{N}_{d^*}^{*}$. Indeed, the greedy search is optimal if and only if $t^* = N - d^*$. Therefore, even if $\mathcal{N}^{(t^*)} \neq \mathcal{N}_{d^*}^{*}$, the greedy search still gives the maximum number of admissible users if $G(\mathcal{N}_{d^*}^{*}) < G(\mathcal{N}^{(N-d^*)}) \leq W$.

### B. With Relaying

The admission control-based joint bandwidth and power allocation problem for the network with relaying is given by

$$\max_{\{P_i^{\text{S}}, W_i^{\text{S}}, P_i^{\text{R}}, W_i^{\text{R}}\}, \mathcal{I} \subseteq \mathcal{N}} \quad |\mathcal{I}| \tag{20a}$$

$$\text{s.t. } c_i - C_i^{\text{SR}} \leq 0, \ i \in \mathcal{I} \tag{20b}$$

$$c_i - C_i^{\text{RD}} \leq 0, \quad i \in \mathcal{I} \tag{20c}$$

$$\text{the constraints (7b)–(7e).} \tag{20d}$$

The proposed greedy search algorithm can also be used to reduce the complexity of solving the problem (20a)–(20d). Specifically, the problem (20a)–(20d) can be decomposed into

$$\max_{\{P_i^{\text{S}}, W_i^{\text{S}}\}, \mathcal{I} \subseteq \mathcal{N}} \quad |\mathcal{I}| \tag{21a}$$

$$\text{s.t. } c_i - C_i^{\text{SR}} \leq 0, \quad i \in \mathcal{I} \tag{21b}$$

$$\text{the constraints (7b), (7d)} \tag{21c}$$

and

$$\max_{\{P_i^{\text{R}}, W_i^{\text{R}}\}, \mathcal{I} \subseteq \mathcal{N}} \quad |\mathcal{I}| \tag{22a}$$

$$\text{s.t. } c_i - C_i^{\text{RD}} \leq 0, \quad i \in \mathcal{I} \tag{22b}$$

$$\text{the constraints (7c), (7e).} \tag{22c}$$

each of which has the same form as that of the problem (14a)–(14c). Therefore, the greedy search can be applied on each of these two problems separately, and it gives $t_1^*$ and $t_2^*$, respectively, as the number of users removed when the stopping rule is satisfied. Let $d^*$, $d_1^*$, and $d_2^*$ denote the optimal values of the problems (20a)–(20d), (21a)–(21c), and (22a)–(22c), respectively. Since the feasible set of the problem (20a)–(20d) is a subset of those of the problems (21a)–(21c) and (22a)–(22c), we have $d^* \leq \min\{d_1^*, d_2^*\}$. Therefore, $d^*$ should be obtained by solving the problem

$$\max_{\{P_i^{\rm S}, W_i^{\rm S}, P_i^{\rm R}, W_i^{\rm R}\}, \mathcal{I}\subseteq\mathcal{N}, |\mathcal{I}|\leq t'} \quad |\mathcal{I}| \tag{23a}$$

$$\text{s.t. } c_i - C_i^{\rm SR} \leq 0, \quad i \in \mathcal{I} \tag{23b}$$

$$c_i - C_i^{\rm RD} \leq 0, \quad i \in \mathcal{I} \tag{23c}$$

$$\text{the constraints (7b)–(7e)} \tag{23d}$$

where $t' \triangleq \min\{N - t_1^*, N - t_2^*\}$ and the feasible set is reduced as compared to that of the problem (20a)–(20d).

Using the exhaustive search, the number of times of solving the problem (15a)–(15c) is upper bounded by $2\sum_{i=d^*}^{N}\binom{N}{i}$. Using the greedy search, the number of times of solving the problem (15a)–(15c) is upper bounded by $\sum_{i=0}^{t_1^*-1}(N-i) + \sum_{i=0}^{t_2^*-1}(N-i) + 2\sum_{i=d^*}^{t'}\binom{N}{i}$ if $t' \geq d^*$ and $\sum_{i=0}^{t_1^*-1}(N-i) + \sum_{i=0}^{t_2^*-1}(N-i) + 2\binom{N}{t'}$ if $t' < d^*$. Therefore, the greedy search significantly reduces the computational complexity if $N$ is large and $t'$, $d^*$ is small.

It can be seen from comparing the problems (20a)–(20d) and (23a)–(23d) that the greedy search is optimal if and only if $t' \geq d^*$.

## V. Simulation Results

### A. Joint Bandwidth and Power Allocation

Consider a wireless network which consists of four users $\mathcal{N} = \{1, 2, 3, 4\}$, four sources, and two relays. The source and relay assignments to the users are the following: $\mathcal{N}_{S_1} = \{1\}$, $\mathcal{N}_{S_2} = \{2\}$, $\mathcal{N}_{S_3} = \{3\}$, $\mathcal{N}_{S_4} = \{4\}$, $\mathcal{N}_{R_1} = \{1, 2\}$, and $\mathcal{N}_{R_2} = \{3, 4\}$. The sources and destinations are randomly distributed inside a square area bounded by (0,0) and (10,10), and the relays are fixed at (5,3) and (5,7). The path loss and the Rayleigh fading effects are present in all links. The path loss gain is given by $g = (1/d)^2$, where $d$ is the distance between two transmission ends, and the variance of the Rayleigh fading gain is denoted as $\sigma^2$. We set $P_{S_i} = 20$, $\forall i \in \{1, 2, 3, 4\}$, $P_R \triangleq P_{R_1} = P_{R_2} = 40$, $W = 10$, $\sigma^2 = 5$, and $c_i = 1$, $\forall i \in \{1, 2, 3, 4\}$ as default values if no other values are indicated otherwise. The noise PSD $N_0$ equals to 1. All results are averaged over 1000 simulation runs for different instances of random channel realizations.

The following resource allocation schemes are compared to each other: the proposed optimal joint bandwidth and power allocation (OBPA), equal bandwidth with optimal power allocation (EBOPA), and equal bandwidth and power allocation (EBPA). Software package TOMLAB [15] is used to solve the corresponding convex optimization problems.

In Figs. 1(a)–(c), the performance of the sum capacity maximization based allocation is shown versus $P_R$, $W$, and $\sigma^2$, respectively. These figures show that the OBPA scheme achieves about 30% to 50% performance improvement over the other two schemes for all parameter values. The performance improvement is higher when $P_R$, $W$, or $\sigma^2$ is larger. The observed significant performance improvement for the OBPA can be partly attributed to the fact that the sum capacity maximization based joint bandwidth and power allocation can lead to highly unbalanced resource allocation, while bandwidth is equally allocated in the EBOPA and both bandwidth and power are equally allocated in the EBPA.

Fig. 2(a)–(c) demonstrates the performance of the minimum capacity maximization based allocation versus $P_R$, $W$, and $\sigma^2$, respectively. The performance improvement for the OBPA is about 10% to 30% as compared to the EBOPA for all parameter values. The improvement provided by the OBPA, in this case, is not as significant as that in Fig. 1(a)–(c), which can be attributed to the fact that the minimum capacity maximization based allocation results in relatively balanced resource allocation, while the EBOPA and the EBPA are balanced bandwidth and totally balanced allocation schemes, respectively.

Fig. 3(a)–(c) shows the total power consumption of the sources and relays versus $c$, $W$, and $\sigma^2$ for the power minimization based allocation, where $c \triangleq c_1 = c_2 = c_3 = c_4$ is assumed. Note that the total power of the OBPA is always about 10% to 30% less than that of the EBOPA, and the total power difference between the two tested schemes is larger when $c$ is larger, or when $W$ or $\sigma^2$ is smaller. This shows that more power is saved when the parameters are unfavorable due to the flexible bandwidth allocation in the OBPA.

Fig. 4 depicts admission probability versus $c$, where $c \triangleq c_1 = c_2 = c_3 = c_4$ is assumed. The admission probability is defined as the probability that the capacity requirements can be satisfied for all the users under random channel realizations. The figure shows that the OBPA outperforms the other two schemes for all values of $c$, and the improvement is more significant when $c$ is large. This shows that more users or users with higher capacity requirements can be admitted into the network using the OBPA scheme.

### B. Greedy Search Algorithm

In this example, the performance of the proposed greedy search algorithm is compared to that of the exhaustive search algorithm. We consider eight users $\mathcal{N} = \{1, 2, \cdots, 8\}$ requesting for admission. The sources and the destinations are randomly distributed inside a square area bounded by (0,0) and (10,10). We assume that $c_i$, $i \in \{1, 2, \cdots, 8\}$ are uniformly distributed over the interval $[c, c+4]$ where $c$ is a variable parameter. The channel model is the same as that given in the last example. We set $W = 10$, $\sigma^2 = 10$ as default values. The results are averaged over 20 random channel realizations. We also consider two network setups as follows.

*Setup 1:* In this setup, the optimality condition of the greedy search is satisfied. Specifically, there are four sources and four relays. The source and relay assignments to the users are the following: $\mathcal{N}_{S_1} = \mathcal{N}_{R_1} = \{1, 2\}$, $\mathcal{N}_{S_2} = \mathcal{N}_{R_2} = \{3, 4\}$, $\mathcal{N}_{S_3} = \mathcal{N}_{R_3} = \{5, 6\}$, and $\mathcal{N}_{S_4} = \mathcal{N}_{R_4} = \{7, 8\}$. The relays are fixed at (5,2), (5,4), (5,6), and (5,8), and $P_{S_i} = P_{R_i} = 40$, $\forall i \in \{1, 2, 3, 4\}$. Fig. 5(a) shows the number of admitted users obtained by the greedy search and the corresponding computational complexity in terms of the running time versus $c$. The figure shows that the greedy search gives exactly the same

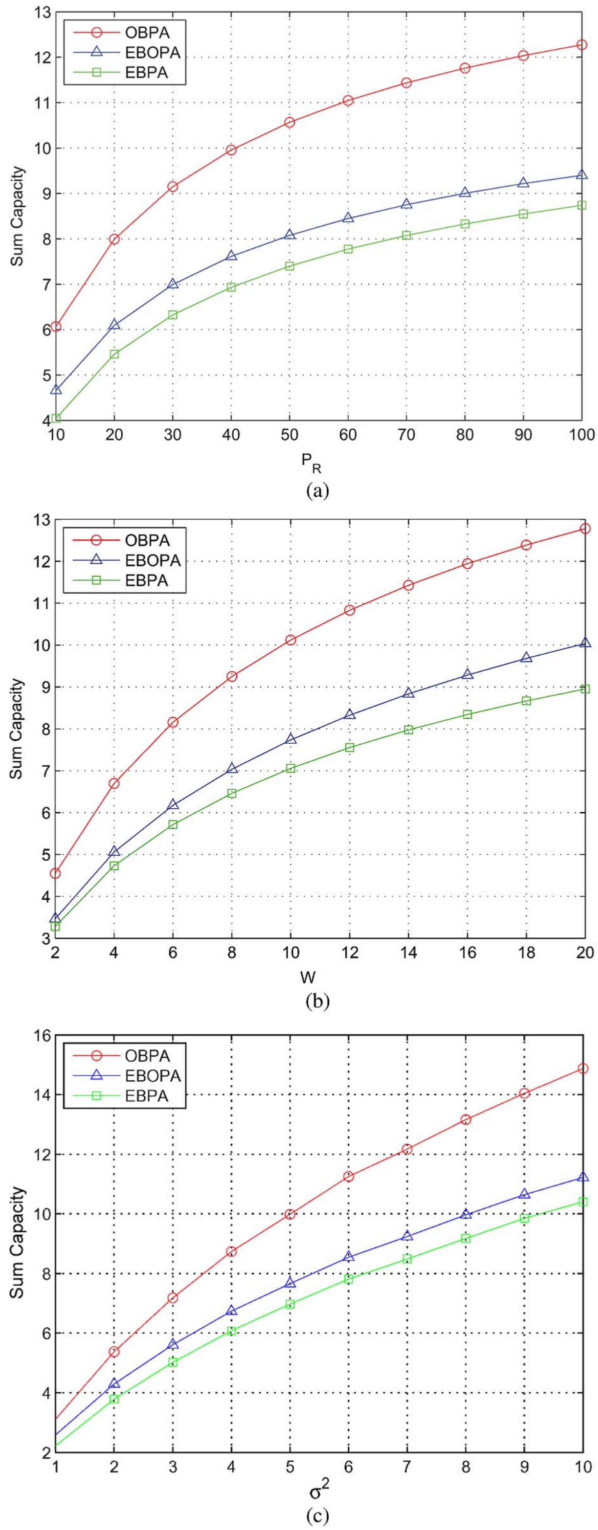


Fig. 1. Sum capacity maximization based allocation. (a) $W = 10, \sigma^2 = 5$; (b) $P_R = 40, \sigma^2 = 5$; and (c) $P_R = 40, W = 10$.

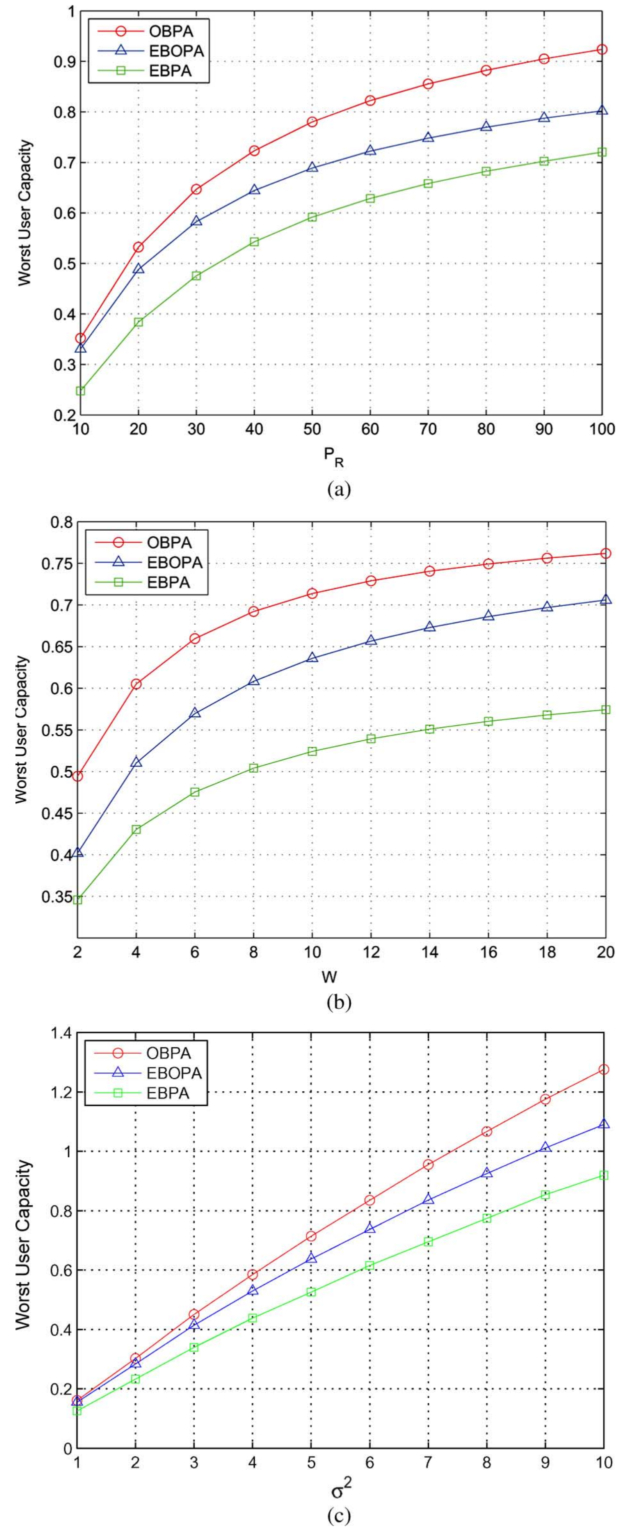


Fig. 2. Minimum capacity maximization based allocation. (a) $W = 10, \sigma^2 = 5$; (b) $P_R = 40, \sigma^2 = 5$; and (c) $P_R = 40, W = 10$.

number of admitted users as that of the exhaustive search for all values of $c$. This confirms that the optimal solution can be obtained when the optimality condition of the greedy search is satisfied. The time consumption of the greedy search is significantly less than that of the exhaustive search, especially when $c$ is large. This shows that the proposed algorithm is especially efficient when the number of users is large and the number of admitted users is small.

*Setup 2:* In this setup, the optimality condition of the greedy search may not be satisfied. There are two sources and two relays. The source and relay assignments to the users are the following: $\mathcal{N}_{S_1} = \{1, 2, 7, 8\}$, $\mathcal{N}_{S_2} = \{3, 4, 5, 6\}$, $\mathcal{N}_{R_1} =$

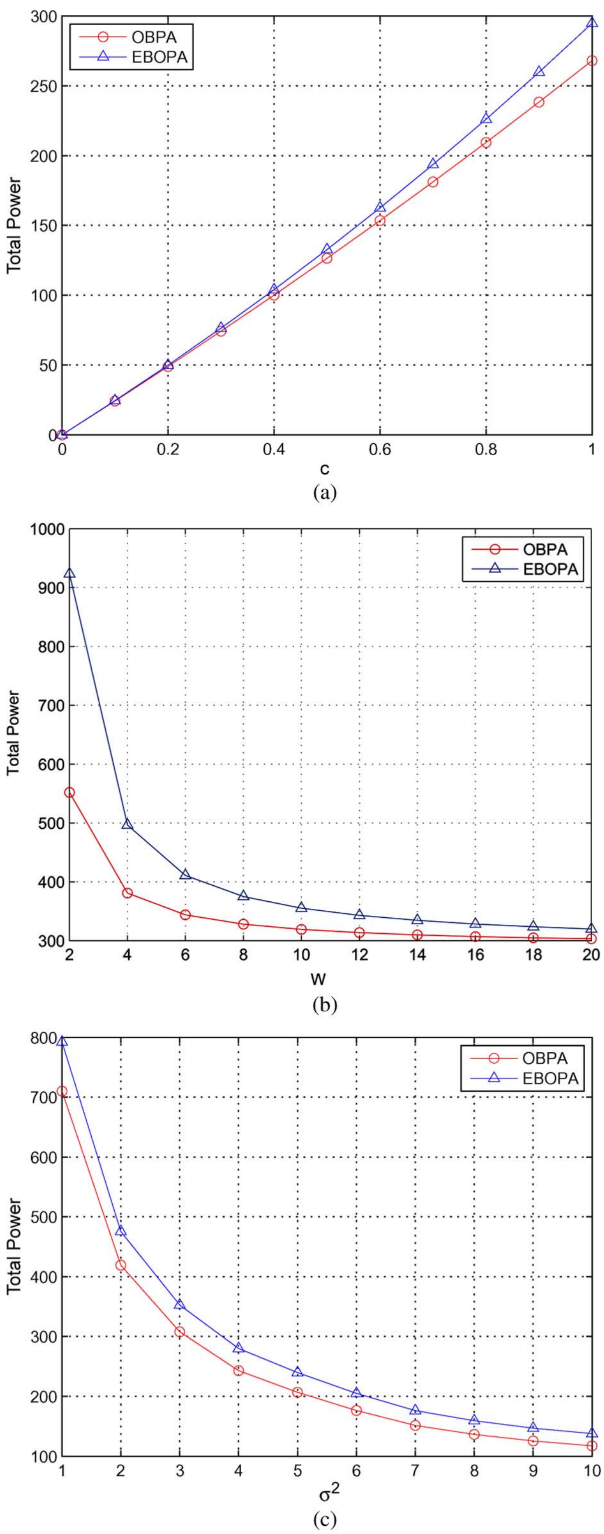


Fig. 3. Power minimization based allocation. (a) $W = 10, \sigma^2 = 5$; (b) $c = 1$, $\sigma^2 = 5$; and (c) $c = 1$, $W = 10$.

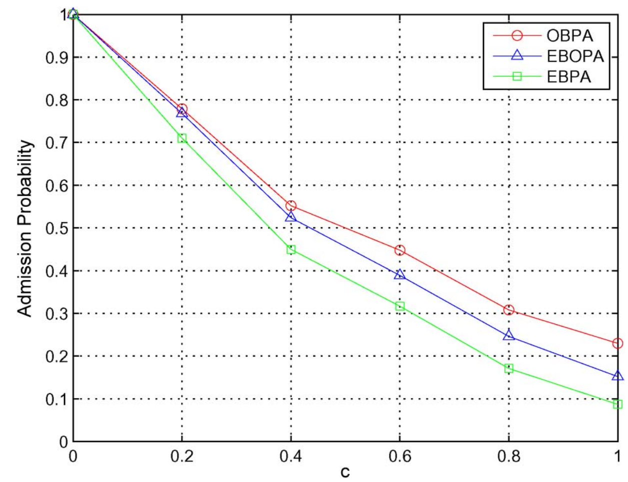


Fig. 4. Admission probability versus capacity threshold.

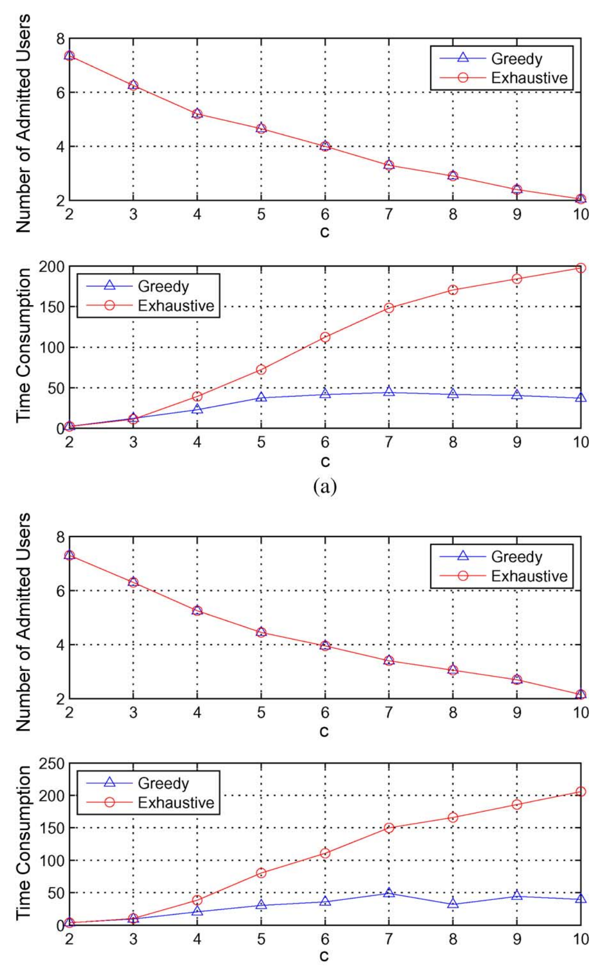


Fig. 5. Greedy search algorithm versus exhaustive search algorithm. (a) Setup 1 and (b) Setup 2.

$\{1, 2, 3, 4\}$, and $\mathcal{N}_{R_2} = \{5, 6, 7, 8\}$. The relays are fixed at (5,3) and (5,7) and $P_{S_i} = P_{R_i} = 80, \forall i \in \{1, 2\}$. Fig. 5(b) demonstrates the performance of the greedy search. Similar conclusions can be obtained as those for Setup 1. This indicates that the greedy search algorithm can still perform well if the optimality condition is not satisfied.

## VI. Conclusion

In this paper, optimal joint bandwidth and power allocation strategies have been proposed for wireless multi-user networks with and without relaying to 1) maximize the sum capacity of all users; 2) maximize the capacity of the worst user; 3) minimize the total power consumption of all users. It is shown that the corresponding resource allocation problems are convex and, thus, can be solved efficiently. Moreover, the admission control-based joint bandwidth and power allocation has been considered. Because of the high complexity of solving the admission control problem, a suboptimal greedy search algorithm with significantly reduced complexity has been developed. Instructive analysis of the greedy search has been provided showing that it can achieve good performance, and the optimality condition of the greedy search has been derived. The formal and in-depth analysis of the greedy search algorithm presented in this paper can serve as a benchmark for analyzing similar algorithms in other applications. Simulation results demonstrate the efficiency of the proposed allocation schemes and the advantages of the greedy search.

## Appendix A
## Proofs of Lemmas, Propositions, and Theorems in Section III

*Proof of Proposition 1*

We first give the following lemma.

*Lemma 1:* The optimal solution of the problem

$$\max_{\{p_i,w_i\}} \quad \sum_{i\in\mathcal{N}} w_i \log\left(1+\frac{h_i p_i}{w_i}\right) \tag{24a}$$

$$\text{s.t.} \sum_{i\in\mathcal{N}} p_i \le p \tag{24b}$$

$$\sum_{i\in\mathcal{N}} w_i \le w \tag{24c}$$

which is denoted by $\{p_i^*|i\in\mathcal{N}\}$, is $p_k^*=p$, $w_k^*=w$, and $p_i^*=w_i^*=0, \forall i\ne k$, where $k=\arg\max_{i\in\mathcal{N}} h_i$.

*Proof of Lemma 1*

Consider the case when $\mathcal{N}=\{1,2\}$. Then the problem (24a)–(24c) is equivalent to

$$\max_{p_1\le p, w_1\le w} \quad g(w,p)=w\log\left(1+\frac{h_1p_1}{w_1}\right) +(w-w_1)\log\left(1+\frac{h_2(p-p_1)}{w-w_1}\right). \tag{25}$$

Assume without loss of generality that $h_1>h_2$. Consider the situation when the constraints $0\le p_1\le p$ and $0\le w_1\le w$ are inactive at optimality. Since the problem (25) is convex, using the Karush–Kuhn–Tucker (KKT) conditions, we have

$$\log\left(1+\frac{h_1p_1^*}{w_1^*}\right)-\frac{h_1p_1^*}{w_1^*+h_1p_1^*}-\log\left(1+\frac{h_2\left(p-p_1^*\right)}{w-w_1^*}\right) +\frac{h_2\left(p-p_1^*\right)}{w-w_1^*+h_2\left(p-p_1^*\right)} = y\left(\frac{h_1p_1^*}{w_1^*}\right)-y\left(\frac{h_2\left(p-p_1^*\right)}{w-w_1^*}\right)=0 \tag{26a}$$

$$\frac{h_1w_1^*}{w_1^*+h_1p_1^*}-\frac{h_2\left(w-w_1^*\right)}{w-w_1^*+h_2\left(p-p_1^*\right)}=0 \tag{26b}$$

where $y(x)\triangleq\log(1+x)-x/(1+x)$. Since $y(x)$ is monotonically increasing, it can be seen from (26a) that

$$\frac{h_1p_1^*}{w_1^*}=\frac{h_2\left(p-p_1^*\right)}{w-w_1^*}. \tag{27}$$

Combining (26b) and (27), we obtain that $h_1=h_2$, which contradicts the condition $h_1>h_2$. Therefore, at least one of the constraints $0\le p_1\le p$ and $0\le w_1\le w$ is active at optimality. Then it can be shown that $p_1^*=p$ and $w_1^*=w$. Note that this is also the optimal solution if $h_1=h_2$ is assumed. Furthermore, this conclusion can be directly extended to the case of $N>2$ by induction. This completes the proof. □

Now we are ready to prove Proposition 1. It can be seen from Lemma 1 that $P_i^{\mathrm{S}^*}=P_i^{\mathrm{S}\star}, \forall i\in\mathcal{I}$, and $P_i^{\mathrm{S}^*}=0, \forall i\notin\mathcal{I}$. Then the problem (6a)–(6c) is equivalent to

$$\max_{\{W_i^{\mathrm{S}}\}} \sum_{i\in\mathcal{I}} W_i^{\mathrm{S}}\log\left(1+\frac{P_i^{\mathrm{S}\star}h_i^{\mathrm{SD}}}{W_i^{\mathrm{S}}N_0}\right) \tag{28a}$$

$$\text{s.t.} \sum_{i\in\mathcal{I}} W_i^{\mathrm{S}}\le W. \tag{28b}$$

Since the problem (28a)–(28b) is convex, using the KKT conditions, we have

$$\log\left(1+\frac{P_i^{\mathrm{S}\star}h_i^{\mathrm{SD}}}{W_i^{\mathrm{S}^*}N_0}\right)-\frac{P_i^{\mathrm{S}\star}h_i^{\mathrm{SD}}}{W_i^{\mathrm{S}^*}N_0+P_i^{\mathrm{S}\star}h_i^{\mathrm{SD}}}-\lambda^* = y\left(\frac{P_i^{\mathrm{S}\star}h_i^{\mathrm{SD}}}{W_i^{\mathrm{S}^*}N_0}\right)-\lambda^*=0, \quad \forall i\in\mathcal{I} \tag{29a}$$

$$W-\sum_{i\in\mathcal{I}} W_i^{\mathrm{S}^*}=0 \tag{29b}$$

where $\lambda^*$ denotes the optimal Lagrange multiplier and $y(x)$ has been introduced above. Since $y(x)$ is monotonically increasing, it follows from (29a) that

$$\frac{P_i^{\mathrm{S}\star}h_i^{\mathrm{SD}}}{W_i^{\mathrm{S}^*}N_0}=\frac{P_j^{\mathrm{S}\star}h_j^{\mathrm{SD}}}{W_j^{\mathrm{S}^*}N_0}, \quad \forall i,j\in\mathcal{I}, i\ne j. \tag{30}$$

Solving the system of equations (29b) and (30), we obtain $W_i^{\mathrm{S}^*}=Wh_i^{\mathrm{SD}}P_i^{\mathrm{S}\star}/\sum_{j\in\mathcal{I}}h_j^{\mathrm{SD}}P_j^{\mathrm{S}\star}, \forall i\in\mathcal{I}$. This completes the proof. □

*Proof of Proposition 2*

It can be seen that

$$C_i^{\mathrm{SD}}+C_j^{\mathrm{SD}}=\min\left\{C_i^{\mathrm{SR}},C_i^{\mathrm{RD}}\right\}+\min\left\{C_j^{\mathrm{SR}},C_j^{\mathrm{RD}}\right\} \le\min\left\{C_i^{\mathrm{SR}}+C_j^{\mathrm{SR}},C_i^{\mathrm{RD}}+C_j^{\mathrm{RD}}\right\}. \tag{31}$$

When $P_j^{\mathrm{S}}=W_j^{\mathrm{S}}=P_j^{\mathrm{R}}=W_j^{\mathrm{R}}=0$, it follows from Lemma 1 that the maximum value of the right-hand side of (31) is achieved and equals to $C_i^{\mathrm{SD}}$ and, on the other hand, the left-hand side of (31) also equals to $C_i^{\mathrm{SD}}$. Therefore, the maximum value of $C_i^{\mathrm{SD}}+C_j^{\mathrm{SD}}$ is achieved when $P_j^{\mathrm{S}}=W_j^{\mathrm{S}}=P_j^{\mathrm{R}}=W_j^{\mathrm{R}}=0$. This completes the proof. □

## Appendix B
## Proofs of Lemmas, Propositions, and Theorems in Section IV

*Proof of Proposition 4*

It is equivalent to show that there exists a feasible point $\{P_i^{\mathrm{S}},W_i^{\mathrm{S}}|i\in\mathcal{I}\}$ of the problem (14a)–(14c) if and only if $G(\mathcal{I})\le W$. If $\{P_i^{\mathrm{S}},W_i^{\mathrm{S}}|i\in\mathcal{I}\}$ is a feasible point of the problem (14a)–(14c), then since it is also a feasible point of the problem (15a)–(15c), we have $G(\mathcal{I})\le\sum_{i\in\mathcal{I}}W_i^{\mathrm{S}}\le W$.

If we have $G(\mathcal{I}) \leq W$, then the optimal solution of the problem (15a)–(15c) for $\mathcal{I}$, denoted by $\{P_i^{\mathrm{S}*}, W_i^{\mathrm{S}*} | i \in \mathcal{I}\}$, is a feasible point of the problem (14a)–(14c) since $\sum_{i\in\mathcal{I}} W_i^{\mathrm{S}*} = G(\mathcal{I}) \leq W$. This completes the proof. □

*Proof of Theorem 1*

We first show that C1 and C2 are sufficient conditions.

Define $V(n) \triangleq G(\mathcal{N}^{(t-1)}) - G(\mathcal{N}^{(t)})$ for $n = n^{(t)}$, $1 \leq t \leq N$. It follows from C2 that $V(n_{S_i}^{(1)}) > V(n_{S_i}^{(2)}) > \cdots > V(n_{S_i}^{(N_{S_i})}), \forall i \in \mathcal{M}$. Then using Proposition 5, we have $n^{(t)} = \arg\max_{n\in\mathcal{N}^{(t-1)}} V(n)$, $1 \leq t \leq N$. Therefore, we obtain

$$V\left(n^{(1)}\right) > V\left(n^{(2)}\right) > \cdots > V\left(n^{(N)}\right). \tag{32}$$

It can be seen from C1 that $\mathcal{N} \setminus \mathcal{N}_{N-t}^* \cap \mathcal{N}_{S_i} = \arg\min_{\mathcal{I}\subseteq\mathcal{N}_{S_i}, |\mathcal{I}|=t_i} G(\mathcal{N}_{S_i}\setminus\mathcal{I}) = \{n_{S_i}^{(j)} | 1 \leq j \leq t_i\}, \forall i \in \mathcal{M}$, where $t_i \triangleq |\mathcal{N} \setminus \mathcal{N}_{N-t}^* \cap \mathcal{N}_{S_i}|$. Then we have $\mathcal{N} \setminus \mathcal{N}_{N-t}^* = \{n_{S_i}^{(j)} | 1 \leq j \leq t_i, i \in \mathcal{M}\}$ and $G(\mathcal{N}) - G(\mathcal{N}_{N-t}^*) = \sum_{i\in\mathcal{M}} \sum_{j=1}^{t_i} V(n_{S_i}^{(j)})$. Therefore, we obtain $\{t_i | i \in \mathcal{M}\} = \arg\max_{\{k_i : \sum_{i\in\mathcal{M}} k_i = t\}} \sum_{i\in\mathcal{M}} \sum_{j=1}^{k_i} V(n_{S_i}^{(j)})$. Since it follows from C2 that $V(n_{S_i}^{(1)}) > V(n_{S_i}^{(2)}) > \cdots > V(n_{S_i}^{(N_{S_i})}), \forall i \in \mathcal{M}$, we have $\mathcal{N} \setminus \mathcal{N}_{N-t}^* = \arg\max_{\mathcal{I}\in\mathcal{N}, |\mathcal{I}|=t} \sum_{n\in\mathcal{I}} V(n) = \{n^{(i)} | 1 \leq i \leq t\} = \mathcal{N} \setminus \mathcal{N}^{(t)}$, where the second equality is from (32). This completes the proof for sufficiency of C1 and C2.

We next show that C1 and C2 are necessary conditions by giving two instructive counter examples.

Consider the case when C1 does not hold. Assume without loss of generality that $\mathcal{M} = \{1\}$. Then it can be seen that C1 is equivalent to the condition (17) and, therefore, the condition (17) does not hold, either.

Consider the case when C2 does not hold. Assume without loss of generality that $\mathcal{M} = \{1, 2\}$, $N_{S_2} = 1$ and $G(\mathcal{N}_{S_1}^{(1)}) - G(\mathcal{N}_{S_1}^{(2)}) > G(\mathcal{N}_{S_2}) - G(\mathcal{N}_{S_2}^{(1)}) > G(\mathcal{N}_{S_1}) - G(\mathcal{N}_{S_1}^{(1)})$. Then we have $\mathcal{N}_{N-2}^* = \mathcal{N} \setminus \{n_{S_1}^{(1)}, n_{S_1}^{(2)}\}$, while it follows from Proposition 5 that $\mathcal{N}^{(2)} = \mathcal{N} \setminus \{n_{S_1}^{(1)}, n_{S_2}^{(1)}\}$. Therefore, $\mathcal{N}_{N-2}^* \neq \mathcal{N}^{(2)}$. This completes the proof for necessity of C1 and C2. □

*Proof of Lemma 3*

Assume $\mathcal{N}_{S_l,k} \neq \mathcal{N}_{S_l,k}^*$. Then there exist $a \in \mathcal{N}_{S_l,k}^*$ and $b \in \mathcal{N}_{S_l} \setminus \mathcal{N}_{S_l,k}^*$ such that $F_a(p) > F_b(p), \forall 0 < p < P_{S_l}$. Let $\{p_i^* | i \in \mathcal{N}_{S_l,k}^*\}$ denote the optimal solution of the problem (19a)–(19b) for $G(\mathcal{N}_{S_l,k}^*)$. Then there always exists $\mathcal{N}'_{S_l,k} \triangleq \mathcal{N}_{S_l,k}^* \cup \{b\} \setminus \{a\}$ such that

$$\begin{aligned} G\left(\mathcal{N}_{S_l,k}^*\right) &= \sum_{i\in\mathcal{N}_{S_l,k}^*, i\neq a} F_i\left(p_i^*\right) + F_a\left(p_a^*\right) \\ &> \sum_{i\in\mathcal{N}_{S_l,k}^*, i\neq a} F_i\left(p_i^*\right) + F_b\left(p_a^*\right) \\ &\geq \min_{\{p_i : \sum_{i\in\mathcal{N}'_{S_l,k}} p_i \leq P_{S_l}\}} \sum_{i\in\mathcal{N}'_{S_l,k}} F_i(p_i) \\ &= G\left(\mathcal{N}'_{S_l,k}\right) \end{aligned} \tag{33}$$

which contradicts the definition of $\mathcal{N}_{S_l,k}^*$. Then it follows that $\mathcal{N}_{S_l,k} = \mathcal{N}_{S_l,k}^*$. Using similar arguments, it can be shown that $\mathcal{N}_{S_l,k} = \mathcal{N}_{S_l}^{(N_{S_l}-k)}$. This completes the proof. □

*Proof of Proposition 6*

It suffices to show that C1 holds for $i = 1$ if C3 holds for $i = 1$. It can be seen that for any $1 \leq k \leq N_{S_1}$, only two cases are under consideration: 1) there exists $\mathcal{N}_{S_1,k}$ that satisfies the condition given in Lemma 3 and, therefore, $\mathcal{N}_{S_1,k}^* = \mathcal{N}_{S_1}^{(N_{S_1}-k)}$; 2) there exist $\mathcal{N}_{S_1,k-1}$ and $\mathcal{N}_{S_1,k+1}$ that satisfy the condition given in Lemma 3, respectively and, therefore, $\mathcal{N}_{S_1,k-1}^* = \mathcal{N}_{S_1}^{(N_{S_1}-k+1)} \subseteq \mathcal{N}_{S_1}^{(N_{S_1}-k-1)} = \mathcal{N}_{S_1,k+1}^*$. Then it follows that $\mathcal{N}_{S_1,k}^* = \mathcal{N}_{S_1}^{(N_{S_1}-k)}$. This completes the proof. □

*Proof of Lemma 4*

Consider the case when $F_i(p)$ intersects $F_j(p)$ at a point $(p', w')$. Then we obtain

$$\frac{c_j}{c_i} = \frac{w' \log\left(1 + \frac{h_j p'}{w'}\right)}{w' \log\left(1 + \frac{h_i p'}{w'}\right)} = q\left(\frac{p'}{w'}\right) \tag{34}$$

where $q(x) = \log(1 + h_j x)/\log(1 + h_i x)$, $0 < x < \infty$. It can be shown that $\lim_{x\to 0} q(x) = h_j/h_i$, $\lim_{x\to\infty} q(x) = 1$, and $q(x)$ is monotonically decreasing with $x$. Therefore, the range of $q(x)$ is $(1, h_j/h_i)$. If $c_j/c_i \in (1, h_j/h_i)$, there exists a unique solution $x'$ such that $q(x') = c_j/c_i$. Hence, $F_i(p)$ and $F_j(p)$ have a unique intersection point given by $w' = c_j/\log(1 + h_j x')$, $p' = w'x'$, and claim 1 follows. If $c_j/c_i \notin (1, h_j/h_i)$, there is a special case that $F_i(p) = F_j(p), \forall p > 0$ if $h_j/h_i = c_j/c_i = 1$. Otherwise, the solution of (34) does not exist, i.e., $F_i(p)$ does not intersect $F_j(p)$ and, therefore, claims 2 and 3 also follow. This completes the proof. □

*Proof of Proposition 7*

The proof of this proposition is built upon the following two lemmas. It suffices to show that C2 holds for $i = 1$ and $t = 2$.

*Lemma 5:* If $p_1 > p_2 > \Delta p > 0$, the following inequality holds:

$$F_i(p_1 - \Delta p) - F_i(p_1) < F_i(p_2 - \Delta p) - F_i(p_2). \tag{35}$$

*Proof of Lemma 5*

It can be shown that $F_i(p)$ is a strictly convex and decreasing function of $p$. Using the first order convexity condition, we have

$$F_i(p_2 - \Delta p) - F_i(p_2) > -F_i'(p_2)\Delta p \tag{36}$$

and

$$F_i(p_1 - \Delta p) - F_i(p_1) < -F_i'(p_1 - \Delta p)\Delta p \tag{37}$$

where $F_i'$ is the first order derivative of $F_i$. Consider two cases: 1) If $p_2 \leq p_1 - \Delta p$, then $F_i'(p_2) \leq F_i'(p_1 - \Delta p)$ due to the convexity of $F_i(p_2)$. Therefore, using $\Delta p > 0$ together with (36) and (37), we obtain (35); 2) If $p_2 \geq p_1 - \Delta p$, using $p_1 > p_2$ and a similar argument as in 1), we can show that $F_i(p_2) -$

$F_i(p_1) < F_i(p_2 - \Delta p) - F_i(p_1 - \Delta p)$, which is equivalent to (35). This completes the proof. □

$G(\mathcal{N}_{S_1})$ can be written as $G(\mathcal{N}_{S_1}, P_{S_1})$ if $P_{S_1}$ is considered as a variable.

*Lemma 6:* $p_i^*$, $\forall i \in \mathcal{N}_{S_1}$ are increasing with $P_{S_1}$, where $\{p_i^* | i \in \mathcal{N}_{S_1}\}$ denotes the optimal solution of the problem (19a)–(19b) for $\mathcal{N}_{S_1}$ and $P_{S_1}$.

*Proof of Lemma 6*

The inverse function of $w = F_i(p)$ is $p = F_i^{-1}(w) = (e^{c_i/w} - 1)w/h_i$. Then we have

$$G(\mathcal{N}_{S_1}, P_{S_1}) = \min_{\{w_i\}} \sum_{i \in \mathcal{N}_{S_1}} w_i \tag{38a}$$

$$\text{s.t.} \sum_{i \in \mathcal{N}_{S_1}} F_i^{-1}(w_i) \le P_{S_1}. \tag{38b}$$

Since the problem (38b)–(38b) is convex, using the KKT conditions, the optimal solution and the optimal Lagrange multiplier of this problem, denoted by $\{w_i^* | i \in \mathcal{N}_{S_1}\}$ and $\lambda^*$, respectively, satisfy the following equations:

$$1 + \frac{\lambda^*}{h_i}\left(e^{\frac{c_i}{w_i^*}}\left(\frac{c_i}{w_i^*} - 1\right) + 1\right) = 0, \quad \forall i \in \mathcal{N}_{S_1}. \tag{39}$$

It can be shown that $(e^{c_i/w_i^*}(c_i/w_i^* - 1) + 1)/h_i$ is monotonically decreasing with $w_i^*$. Therefore, $w_i^*$, $\forall i \in \mathcal{N}_{S_1}$, and, correspondingly, $p_i^* = F_i^{-1}(w_i^*)$, $\forall i \in \mathcal{N}_{S_1}$ are decreasing and increasing, respectively, with $\lambda^*$. Then it follows from (38b) that $p_i^*$, $\forall i \in \mathcal{N}_{S_1}$ are increasing with $P_{S_1}$. This completes the proof. □

We are now ready to prove the proposition. Let $P_1 > P_2 > 0$ and $\mathcal{N}_{S_1}^{-k} \triangleq \mathcal{N}_{S_1} \setminus \{k\}$ for some $k \in \mathcal{N}_{S_1}$. Let $\{p_i^\star | i \in \mathcal{N}_{S_1}^{-k}\}$ denote the optimal solution of the problem (19a)–(19b) for $\mathcal{N}_{S_1}^{-k}$ and $P_2$. Using Lemma 6, the optimal solution of the problem (19a)–(19b) for $G(\mathcal{N}_{S_1}, P_2)$ can be expressed as $p_i^\star - \Delta p_i$, $i \in \mathcal{N}_{S_1}^{-k}$, and $p_k^\star$, respectively, where $\Delta p_i > 0$ and $\sum_{i \in \mathcal{N}_{S_1}^{-k}} \Delta p_i = p_k^\star$. Then we have

$$G(\mathcal{N}_{S_1}, P_2) - G\left(\mathcal{N}_{S_1}^{-k}, P_2\right) = \sum_{i \in \mathcal{N}_{S_1}^{-k}} \left(F_i\left(p_i^\star - \Delta p_i\right) - F_i\left(p_i^\star\right)\right) + F_k\left(p_k^\star\right). \tag{40}$$

Let $\{p_i^+ | i \in \mathcal{N}_{S_1}^{-k}\}$ denote the optimal solution of the problem (19a)–(19b) for $\mathcal{N}_{S_1}^{-k}$ and $P_1$. Then we have

$$\begin{aligned} G(\mathcal{N}_{S_1}, P_1) &- G\left(\mathcal{N}_{S_1}^{-k}, P_1\right) \\ &= \min_{\{p_i : \sum_{i \in \mathcal{N}_{S_1}} p_i \le P_1\}} \sum_{i \in \mathcal{N}_{S_1}} F_i(p_i) - \sum_{i \in \mathcal{N}_{S_1}^{-k}} F_i\left(p_i^+\right) \\ &\le \sum_{i \in \mathcal{N}_{S_1}^{-k}} \left(F_i\left(p_i^+ - \Delta p_i\right) - F_i\left(p_i^+\right)\right) + F_k\left(p_k^\star\right). \end{aligned} \tag{41}$$

Since $P_1 > P_2$, it follows from Lemma 6 that $p_i^+ > p_i^\star > \Delta p_i > 0$, $\forall i \in \mathcal{N}_{S_1}^{-k}$. Using Lemma 5, we obtain $F_i(p_i^+ - \Delta p_i) - F_i(p_i^+) < F_i(p_i^\star - \Delta p_i) - F_i(p_i^\star), \forall i \in \mathcal{N}_{S_1}^{-k}$. Therefore, comparing (40) with (41), we have

$$G(\mathcal{N}_{S_1}, P_1) - G\left(\mathcal{N}_{S_1}^{-k}, P_1\right) < G(\mathcal{N}_{S_1}, P_2) - G\left(\mathcal{N}_{S_1}^{-k}, P_2\right) \tag{42}$$

which can be rewritten as

$$G\left(\mathcal{N}_{S_1}^{-k}, P_2\right) - G\left(\mathcal{N}_{S_1}^{-k}, P_1\right) < G(\mathcal{N}_{S_1}, P_2) - G(\mathcal{N}_{S_1}, P_1). \tag{43}$$

Let $\{p_i^* | i \in \mathcal{N}_{S_1}\}$ denote the optimal solution of the problem (19a)–(19b) for $\mathcal{N}_{S_1}$ and $P_{S_1}$. Then we have

$$\begin{aligned} G\left(\mathcal{N}_{S_1} \setminus \left\{n_{S_1}^{(2)}\right\}, P_{S_1}\right) &- G\left(\mathcal{N}_{S_1}^{(2)}, P_{S_1}\right) \\ &\le F_{n_{S_1}^{(1)}}\left(p^*_{n_{S_1}^{(1)}}\right) + G\left(\mathcal{N}_{S_1}^{(2)}, P_{S_1} - p^*_{n_{S_1}^{(1)}}\right) \\ &\quad - G\left(\mathcal{N}_{S_1}^{(2)}, P_{S_1}\right) \\ &< F_{n_{S_1}^{(1)}}\left(p^*_{n_{S_1}^{(1)}}\right) + G\left(\mathcal{N}_{S_1}^{(1)}, P_{S_1} - p^*_{n_{S_1}^{(1)}}\right) \\ &\quad - G\left(\mathcal{N}_{S_1}^{(1)}, P_{S_1}\right) \\ &= G(\mathcal{N}_{S_1}, P_{S_1}) - G\left(\mathcal{N}_{S_1}^{(1)}, P_{S_1}\right) \end{aligned} \tag{44}$$

where the second inequality follows from (43). On the other hand, we have

$$\begin{aligned} G\left(\mathcal{N}_{S_1} \setminus \left\{n_{S_1}^{(2)}\right\}, P_{S_1}\right) &- G\left(\mathcal{N}_{S_1}^{(2)}, P_{S_1}\right) \\ &\ge G\left(\mathcal{N}_{S_1} \setminus \left\{n_{S_1}^{(1)}\right\}, P_{S_1}\right) - G\left(\mathcal{N}_{S_1}^{(2)}, P_{S_1}\right) \\ &= G\left(\mathcal{N}_{S_1}^{(1)}, P_{S_1}\right) - G\left(\mathcal{N}_{S_1}^{(2)}, P_{S_1}\right). \end{aligned} \tag{45}$$

Therefore, comparing (44) with (45), we complete the proof. □


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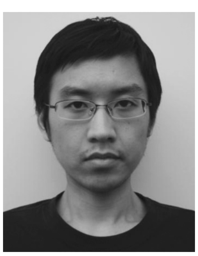

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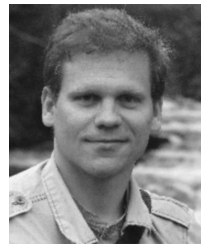

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