Admission Control-Based Joint Bandwidth and Power Allocation in Multi-User DF Relay Networks

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***Abstract*—Joint bandwidth and power allocation in wireless multi-user decode and forward relay networks is proposed in this paper for maximizing the number of admissible users. A suboptimal greedy search algorithm is developed to solve the admission control problem efficiently at low complexity. The condition under which the greedy search is optimal is found. The way we derive such optimality condition for the greedy search can serve as a benchmark for other applications of greedy search. The advantages of the suboptimal greedy search algorithm compared to exhaustive search algorithm are shown.**

I. Introduction

In the case of severe channel conditions in direct links, relays can be deployed to forward the data from a source to a destination [1]. Efficient allocation of available radio resources is one of the critical issues for improving wireless multi-user relay network performance. Power allocation in wireless multi-user networks has been extensively researched. However, the joint allocation of bandwidth and power resources has not attracted the attention it deserves. In practical wireless networks, the available transmission power of the nodes and the total available bandwidth of the network are limited and, therefore, joint bandwidth and power allocation instead of only power allocation should be considered. Moreover, numerous papers on the resource allocation in wireless relay networks (see, for example, [2]– [4]) consider the case of a single user, i.e., a single source-destination pair. The case of multi-user relay networks has been considered only in few works. Power allocation aiming at maximizing the sum capacity of multiple users for four different relay transmission strategies has been studied in [5], while an amplify and forward (AF) based strategy in which multiple sources share multiple relays using power control has been developed in [6]. However, all the works mentioned above (except [4]) have assumed equal and fixed bandwidth allocation for the one-hop links from a source to a destination. Thus, motivated by the fact that it may be inefficient to allocate the bandwidth equally when the total available bandwidth is limited, we consider the problem of joint bandwidth and power allocation in wireless multi-user relay networks in this paper.

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In addition, certain minimum transmission rates often must be guaranteed in a number of applications, which can lead to the situation when the available bandwidth and/or power will not be sufficient to satisfy quality-of-service (QoS) requirements of all users simultaneously. Examples of such applications are the real-time voice and video applications where a minimum rate should be guaranteed for each user to satisfy the delay constraints of the services. When the rate/capacity requirements can not be supported for all users, admission control is necessary to decide which users to be admitted into the network. The admission control in wireless networks typically aims at maximizing the number of admitted users and has been recently considered in several works. A single-stage reformulation approach for a two-stage joint resource allocation and admission control problem is proposed in [7], while another approach, which is based on the idea of user removal, has been developed in [8] and [9].

In this paper, the problem of admission control-base joint bandwidth and power allocation in wireless multi-user decode and forward (DF) networks is considered. The joint bandwidth and power allocation is proposed to maximize the number of admissible users. A greedy search algorithm is developed in order to reduce the computational complexity of solving the admission control problem. The proposed greedy search removes one user at each iteration until the remaining users can be admitted. The optimality conditions of the greedy search algorithm is derived. Our simulation examples show the effectiveness of the proposed method.

II. System Model

Consider a wireless network that consists of M source nodes S_i, $i \in \mathcal{M} = \{1, 2, \cdots, M\}$ and K destination nodes D_i, $i \in \mathcal{K} = \{1, 2, \cdots, K\}$. The network serves N users U_i, $i \in \mathcal{N} = \{1, 2, \cdots, N\}$ where each user represents a two-hop link from a source to a destination. Thus, we assume that there are also L relay nodes R_i, $i \in \mathcal{L} = \{1, 2, \cdots, L\}$ deployed for forwarding the data from the sources to the destinations. To reduce the implementation complexity at the destinations, single relay assignment is adopted so that each user has one designated relay. The set of users served by R_i is denoted by $\mathcal{N}_{R_i}$. Similarly, the set of users which are served by S_i is denoted by $\mathcal{N}_{S_i}$. A spectrum of total bandwidth W is available for the transmission from the sources or the relays. This spectrum can be divided into distinct and nonoverlapping channels of unequal bandwidths, so that the sources or the relays share the available spectrum through frequency division and, therefore, do not interfere with each other.

The relays work in a half-duplex manner due to the practical limitation that they can not transmit and receive at the same time. A two-phase decode-and-forward (DF) protocol is assumed, i.e., the relays receive and decode the transmitted data from the sources in the first phase, and re-encode and forward the data to the destinations in the second phase. The sources and relays share the total available spectrum in the first and second phase, respectively. It is assumed that the direct links between the sources and the destinations are blocked and, thus, are not available. Note that although the two-hop relay model is considered in the paper, the results are applicable for multi-hop relay models.

Let P_i^{S} and W_i^{S} denote the allocated transmit power and channel bandwidth of the source to serve U_i. Then the one-hop source–relay link capacity of U_i is given by

$$C_i^{\mathrm{SR}} = W_i^{\mathrm{S}} \log\left(1 + \frac{P_i^{\mathrm{S}} h_i^{\mathrm{SR}}}{W_i^{\mathrm{S}} N_0}\right) \tag{1}$$

where h_i^{SR} denotes the channel power gain of the link and N_0 is the noise power spectral density (PSD). Similar, the one-hop relay–destination link capacity of U_i is given by

$$C_i^{\mathrm{RD}} = W_i^{\mathrm{R}} \log\left(1 + \frac{P_i^{\mathrm{R}} h_i^{\mathrm{RD}}}{W_i^{\mathrm{R}} N_0}\right) \tag{2}$$

where P_i^{R} and W_i^{R} denote the allocated transmit power and channel bandwidth of the relay to serve U_i and h_i^{RD} denotes the channel power gain of the link. Therefore, the two-hop source–destination link capacity of U_i is given by

$$C_i^{\mathrm{SD}} = \min\{C_i^{\mathrm{SR}}, C_i^{\mathrm{RD}}\}. \tag{3}$$

It can be seen from (1), (2), and (3) that if equal bandwidth is allocated to W_i^{S} and W_i^{R}, C_i^{SR} and C_i^{RD} can be unequal due to the power limits on P_i^{S} and P_i^{R}. Then the source–destination link capacity C_i^{SD} is constrained by the minimum of C_i^{SR} and C_i^{RD}. Since all users share the total bandwidth of the spectrum, equal bandwidth allocation for all one-hop links can be inefficient. Therefore, the joint allocation of bandwidth and power is necessary.

III. Admission Control for Joint Bandwidth and Power Allocation

We first consider the following admission control-based problem of joint bandwidth and power allocation for one-hop link

$$\max_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}\}, \mathcal{I} \subseteq \mathcal{N}} \quad |\mathcal{I}| \tag{4a}$$

$$\text{s.t.} \sum_{i \in \mathcal{N}_{S_j}} P_i^{\mathrm{S}} \leq P_{S_j}, \ j \in \mathcal{M} \tag{4b}$$

$$\sum_{i \in \mathcal{N}} W_i^{\mathrm{S}} \leq W \tag{4c}$$

$$c_i - C_i^{\mathrm{SD}} \leq 0, \ i \in \mathcal{I} \tag{4d}$$

where $|\mathcal{I}|$ stands for the cardinality of $\mathcal{I}$, and c_i is the minimum acceptable capacity for U_i. The constraint (4b) stands for that the total power at S_j is limited by P_{S_j}. The constraint (4c) indicates that the total bandwidth of the channels allocated to the sources is also limited.

The problem (4a)–(4d) can be solved using exhaustive search among all possible subsets of users. However, the computational complexity of the exhaustive search can be very high since the number of possible subsets of users is exponentially increasing with the number of users, which is not acceptable for practical implementation. Therefore, we develop a suboptimal greedy search algorithm that significantly reduces the complexity of finding the maximum number of admissible users.

Greedy Search Algorithm: Consider the following problem

$$G(\mathcal{I}) \ \triangleq \ \min_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}\}} \quad \sum_{i \in \mathcal{I}} W_i^{\mathrm{S}} \tag{5a}$$

$$\text{s.t. the constraint (4b), (4d).} \tag{5b}$$

The proposition which provides the necessary and sufficient conditions for the admissibility of a set of users is in order.

Proposition 1: *A set of users $\mathcal{I}$ is admissible if and only if $G(\mathcal{I}) \leq W$.*

Proof: See [11].

Here the optimal value $G(\mathcal{I})$ is the minimum total bandwidth required to support the users in $\mathcal{I}$, given that the power constraints are satisfied. Our greedy search algorithm removes users one by one until the remaining users are admissible. Then the 'worst' user, i.e., the user whose removal reduces the total bandwidth requirement to the maximum extent, should be removed at each greedy search iteration. Thus, the removal criterion can be stated as

$$\begin{aligned} n^{(t)} &\triangleq \arg \max_{n \in \mathcal{N}^{(t-1)}} \left(G(\mathcal{N}^{(t-1)}) - G(\mathcal{N}^{(t-1)} \setminus \{n\}) \right) \\ &= \arg \min_{n \in \mathcal{N}^{(t-1)}} G(\mathcal{N}^{(t-1)} \setminus \{n\}) \end{aligned} \tag{6}$$

where $n^{(t)}$ denotes the user removed at the t-th greedy search iteration, $\mathcal{N}^{(t)} \triangleq \mathcal{N}^{(t-1)} \setminus \{n^{(t)}\}$ denotes the remaining users after t greedy search iterations, and the symbol '$\setminus$' stands for set difference operator.

Intuitively, $\mathcal{N}^{(t)}$ should be the 'best' set of $N - t$ users that requires the minimum total bandwidth among all possible sets of $N - t$ users from $\mathcal{N}$, and $G(\mathcal{N}^{(t)})$ is the corresponding minimum total bandwidth requirement. Thus, the stopping rule for the greedy search iterations should be finding such t^* that $G(\mathcal{N}^{(t^*-1)}) > W$ and $G(\mathcal{N}^{(t^*)}) \leq W$. In other words, $N-t^*$ is expected to be the maximum number of admissible users, denoted by d^*.

Complexity of the Greedy Search Algorithm: Using the exhaustive search to find the maximum number of admissible users is equivalent to checking $G(\mathcal{I})$ for all possible $\mathcal{I} \subseteq \mathcal{N}$ and, therefore, the number of times of solving the problem (5a)–(5b) is $\sum_{i=t^*}^{N} \binom{N}{i}$. On the other hand, it can be seen from (6) that using the greedy search, the number of times of solving the problem (5a)–(5b) is $\sum_{i=0}^{t^*-1} N-i$. Therefore, the complexity of the proposed greedy search is significantly reduced as compared to that of the exhaustive search.

The complexity of the greedy search can be further reduced. The lemma given below follows directly from the decomposable structure of the problem (5a)–(5b), that is, $G(\mathcal{I}) = \sum_{i\in\mathcal{M}} G(\mathcal{I} \cap \mathcal{N}_{S_i})$.

Lemma 1: $G(\mathcal{I}) - G(\mathcal{I} \setminus \{n\}) = G(\mathcal{I} \cap \mathcal{N}_{S_i}) - G(\mathcal{I} \cap \mathcal{N}_{S_i} \setminus \{n\})$ *for* $n \in \mathcal{N}_{S_i}$, $\forall \mathcal{I} \subseteq \mathcal{N}$.

Lemma 1 shows that the reduction of the total bandwidth requirement is only coupled with the users served by the same source which serves the user to be removed, and is decoupled with the users served by other sources. Let $\mathcal{N}_{S_i}^{[t]} \triangleq \mathcal{N}_{S_i} \cap \mathcal{N}^{(t)}$ denote the remaining users served by S_i after t greedy search iterations. Applying Lemma 1 directly to the removal criterion in (6), we have the following proposition.

Proposition 2: $n^{(t)} = n_{S_{i^*}}^{[t-1]^*}$, *where* $n_{S_i}^{[t-1]^*} \triangleq \arg\max_{n\in n_{S_i}^{[t-1]}} \left(G(\mathcal{N}_{S_i}^{[t-1]}) - G(\mathcal{N}_{S_i}^{[t-1]} \setminus \{n\})\right)$ *and* $i^* \triangleq \arg\max_{i\in\mathcal{M}} \left(G(\mathcal{N}_{S_i}^{[t-1]}) - G(\mathcal{N}_{S_i}^{[t-1]} \setminus \{n_{S_i}^{[t-1]^*}\})\right)$.

Proposition 2 provides an equivalent algorithm for searching for the user to be removed at each greedy search iteration with reduced computational complexity. Specifically, we can first find the 'worst' user in each set of users served by a source, i.e., $n_{S_i}^{[t-1]^*}$, and then determine the user to be removed among these 'worst' users. As a result, the number of variables involved in solving each problem (5a)–(5b) is reduced, especially if N_{S_i} is small compared to N.

Optimality Conditions of the Greedy Search Algorithm: The greedy search is optimal if the remaining users after each greedy search iteration form the 'best' set of users, i.e.,

$$\mathcal{N}^{(t)} = \mathcal{N}_{N-t}^*, \forall 1 \leq t \leq N$$

where $\mathcal{N}_i^* \triangleq \arg\min_{|\mathcal{I}|=i} G(\mathcal{I})$.

Let the greedy search be applied on a set of users served by one source $\mathcal{N}_{S_i}$. Let $n_{S_i}^{(t)} \triangleq \arg\max_{n\in\mathcal{N}_{S_i}^{(t-1)}} \{G(\mathcal{N}_{S_i}^{(t-1)}) - G(\mathcal{N}_{S_i}^{(t-1)} \setminus \{n\})\}$ denote the user removed at the t-th greedy search iteration, and $\mathcal{N}_{S_i}^{(t)} \triangleq \mathcal{N}_{S_i}^{(t-1)} \setminus \{n_{S_i}^{(t)}\}$ denote the remaining users after t greedy search iterations. Also let $\mathcal{N}_{S_i,j}^* \triangleq \arg\min_{\mathcal{I}\subseteq\mathcal{N}_{S_i}, |\mathcal{I}|=j} G(\mathcal{I})$ denote the 'best' set of j users in $\mathcal{N}_{S_i}$. The next theorem provides the necessary and sufficient conditions for the optimality of the greedy search.

Theorem 1: $\mathcal{N}^{(t)} = \mathcal{N}_{N-t}^*$, $\forall 1 \leq t \leq N$, *if and only if the following two conditions hold*

C1: $\mathcal{N}_{S_i}^{(t)} = \mathcal{N}_{S_i,N_{S_i}-t}^*$, $\forall 1 \leq t \leq N_{S_i}$, $\forall i \in \mathcal{M}$*;*

C2: $G(\mathcal{N}_{S_i}^{(t-2)}) - G(\mathcal{N}_{S_i}^{(t-1)}) > G(\mathcal{N}_{S_i}^{(t-1)}) - G(\mathcal{N}_{S_i}^{(t)})$, $\forall 2 \leq t \leq N_{S_i}$, $\forall i \in \mathcal{M}$.

Proof: See [11].

Theorem 1 decouples the optimality conditions of the greedy search into equivalent conditions. Specifically, C1 indicates that the set of remaining users after each greedy search iteration is the 'best' set of users, while C2 indicates that the reduction of the total bandwidth requirement is decreasing with the greedy search iterations.

Let $h_i \triangleq h_i^{\mathrm{SD}}/N_0$ denote the channel gain normalized by the noise PSD. Recall that c_i is the minimum acceptable capacity for U_i. Define $F_i(p)$ as the unique solution for w of the equation

$$c_i = w \log\left(1 + \frac{h_i p}{w}\right). \tag{7}$$

given h_i and c_i for any $p > 0$, which represents the minimum bandwidth required by a user for its allocated transmit power. Then the problem (5a)-(5b) for $\mathcal{N}_{S_i}$ can be rewritten as

$$G(\mathcal{N}_{S_i}) \triangleq \min_{p_i} \sum_{i\in\mathcal{N}_{S_i}} F_i(p_i) \tag{8a}$$

$$\text{s.t.} \sum_{i\in\mathcal{N}_{S_i}} F_i(p_i) \leq P_{S_i}. \tag{8b}$$

The following lemma gives a condition under which C1 holds for a specific t.

Lemma 2: *If there exists* $\mathcal{N}_{S_l,k} \subseteq \mathcal{N}_{S_l}$, $|\mathcal{N}_{S_l,k}| = k$, *such that* $F_i(p) < F_j(p)$, $\forall 0 < p < P_{S_i}$, $\forall i \in \mathcal{N}_{S_l,k}$ *and* $\forall j \in \mathcal{N} \setminus \mathcal{N}_{S_l,k}$, *then* $\mathcal{N}_{S_l,k} = \mathcal{N}_{S_l,k}^* = \mathcal{N}_{N_{S_l}}^{(N_{S_l}-k)}$.

Proof: See [11].

Since any user in $\mathcal{N}_{S_l,k}$ has a smaller bandwidth requirement than any user in $\mathcal{N} \setminus \mathcal{N}_{S_l,k}$ for the same allocated power over the available power range, the former is more favorable than the latter in the perspective of reducing the total bandwidth requirement. Therefore, $\mathcal{N}_{S_l,k}$ is the 'best' set of k users and the greedy search removes users in $\mathcal{N} \setminus \mathcal{N}_{S_l,k}$ before $\mathcal{N}_{S_l,k}$. Note, however, that C1 does not hold in general. Indeed, since the reduction of the total bandwidth requirement is maximized only at each single greedy search iteration, the greedy search does not guarantee that the reduction of the total bandwidth requirement is also maximized over multiple greedy search iterations.

Applying Lemma 2, the next proposition gives a sufficient condition for C1 and further decouples it into two other conditions.

Proposition 3: *C1 holds if the following condition holds*

C3: for any $j \in \mathcal{N}_{S_i}$, $\forall i \in \mathcal{M}$, *there exists no more than one* $k \in \mathcal{N}_{S_i}$, $k \neq j$, *such that the following condition holds*

C4: $F_j(p) < F_k(p)$, $\forall 0 < p < P_{S_i}$, *or* $F_j(p) > F_k(p)$, $\forall 0 < p < P_{S_i}$ *does not hold.*

Proof: See [11].

It can be seen that Pr{C3} ↑ as Pr{C4} ↓ and Pr{C3} → 1 as Pr{C4} → 0, where Pr{·} stands for the probability of an event. Moreover, Pr{C3} ↑ as $N_{S_i} \downarrow$, $\forall i \in \mathcal{M}$, or $M \downarrow$.

The next lemma characterizes the bandwidth requirement comparison of two users in terms of the comparison of their capacity requirement ratio and channel gain ratio.

Lemma 3: *If $i \neq j$ and $h_j/h_i \geq 1$, then*

1) there exists such p' that $F_i(p) > F_j(p)$, $\forall p' > p > 0$, and $F_i(p) < F_j(p)$, $\forall p > p'$, if and only if $1 < c_j/c_i < h_j/h_i$; furthermore, $p' \uparrow$ as $h_j/h_i \uparrow$ or $c_j/c_i \downarrow$;

2) $F_i(p) > F_j(p)$, $\forall p > 0$, or $F_i(p) = F_j(p)$, $\forall p > 0$, if and only if $c_j/c_i \leq 1$;

3) $F_i(p) < F_j(p)$, $\forall p > 0$, if and only if $c_j/c_i \geq h_j/h_i$.

Proof: See [11].

Here claim (1) indicates the case when channel gain ratio dominates and is dominated by capacity requirement ratio in the low power range and high power range, respectively; claims (2) and (3) indicate the cases where capacity requirement ratio dominates and is dominated by channel gain ratio in any power range, respectively. Then C4 holds if and only if the claim (1) of Lemma 3 holds with $0 < p' < P_{S_i}$. Moreover, it follows from Lemma 3 that Pr{C3} ↑ as Pr{C4} ↓ as $h_j/h_i \uparrow$, $c_j/c_i \downarrow$, or $P_{S_i} \downarrow$, and Pr{C3} → 1 as Pr{C4} → 0 as $h_j/h_i \to \infty$, $c_j/c_i \to 1$, or $P_{S_i} \to 0$. This shows that C3 is a mild condition.

The following proposition shows that C2 is true in general.

Proposition 4: *C2 always holds.*

Proof: See [11].

Applying Lemma 3, Proposition 3, and Proposition 4 successively, the following corollary can be drawn from Theorem 1.

Corollary 1: $\mathcal{N}^{(t)} = \mathcal{N}^*_{N-t}$, $\forall 1 \leq t \leq N$, *if* $c_i = c_j$, $\forall i, j \in \mathcal{N}$, $i \neq j$.

The overall admission control problem of joint bandwidth and power allocation for multi-user DF relay networks is given by

$$\max_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}, P_i^{\mathrm{R}}, W_i^{\mathrm{R}}\}, \mathcal{I} \subseteq \mathcal{N}} \quad |\mathcal{I}| \tag{9a}$$

$$\text{s.t.} \sum_{i \in \mathcal{N}_{S_j}} P_i^{\mathrm{S}} \leq P_{S_j}, \; j \in \mathcal{M} \tag{9b}$$

$$\sum_{i \in \mathcal{N}_{R_j}} P_i^{\mathrm{R}} \leq P_{R_j}, \; j \in \mathcal{L} \tag{9c}$$

$$\sum_{i \in \mathcal{N}} W_i^{\mathrm{S}} \leq W \tag{9d}$$

$$\sum_{i \in \mathcal{N}} W_i^{\mathrm{R}} \leq W \tag{9e}$$

$$c_i - C_i^{\mathrm{SR}} \leq 0, \; i \in \mathcal{I} \tag{9f}$$

$$c_i - C_i^{\mathrm{RD}} \leq 0, \; i \in \mathcal{I}. \tag{9g}$$

The proposed greedy search algorithm for one stage of the transmission can be used to reduce the complexity of solving the problem (9a)–(9g). Specifically, this problem can be decomposed into

$$\max_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}\}, \mathcal{I} \subseteq \mathcal{N}} \quad |\mathcal{I}| \tag{10a}$$

$$\text{s.t. the constraint (9b), (9d), (9f)} \tag{10b}$$

and

$$\max_{\{P_i^{\mathrm{R}}, W_i^{\mathrm{R}}\}, \mathcal{I} \subseteq \mathcal{N}} \quad |\mathcal{I}| \tag{11a}$$

$$\text{s.t. the constraint (9c), (9e), (9g),} \tag{11b}$$

each of which has the same form as that of the problem (4a)–(4d). Therefore, the greedy search can be applied on each of these two problems separately, and it gives t_1^* and t_2^*, respectively, as the number of users removed when the stopping rule is satisfied. Let d^*, d_1^*, and d_2^* denote the optimal values of the problems (9a)–(9g), (10a)–(10b), and (11a)–(11b), respectively. Since the feasible set of the problem (9a)–(9g) is a subset of those of the problems (10a)–(10b) and (11a)–(11b), we have $d^* \leq \min\{d_1^*, d_2^*\}$. Therefore, d^* can be obtained by solving the problem

$$\max_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}, P_i^{\mathrm{R}}, W_i^{\mathrm{R}}\}, \mathcal{I} \subseteq \mathcal{N}, |\mathcal{I}| \leq t'} \quad |\mathcal{I}| \tag{12a}$$

$$\text{s.t. the constraints (9b)–(9g)} \tag{12b}$$

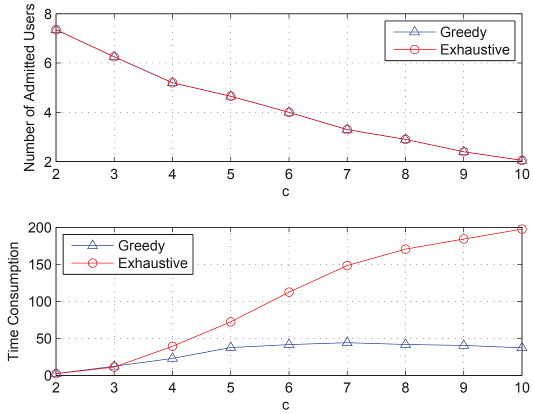
where $t' \triangleq \min\{N - t_1^*, N - t_2^*\}$ and the feasible set is reduced as compared to that of the problem (9a)–(9g).

Using the exhaustive search, the number of times of solving the problem (5a)–(5b) is upper bounded by $2\sum_{i=d^*}^{N} \binom{N}{i}$. Using the greedy search, the number of times of solving the problem (5a)–(5b) is upper bounded by $\sum_{i=0}^{t_1^*-1}(N-i) + \sum_{i=0}^{t_2^*-1}(N-i) + 2\sum_{i=d^*}^{t'} \binom{N}{i}$ if $t' \geq d^*$ and $\sum_{i=0}^{t_1^*-1}(N-i) + \sum_{i=0}^{t_2^*-1}(N-i) + 2\binom{N}{t'}$ if $t' < d^*$. Therefore, the greedy search significantly reduces the computational complexity. Moreover, comparing the problems (9a)–(9g) and (12a)–(12b) we see that the greedy search is optimal if and only if $t' \geq d^*$.

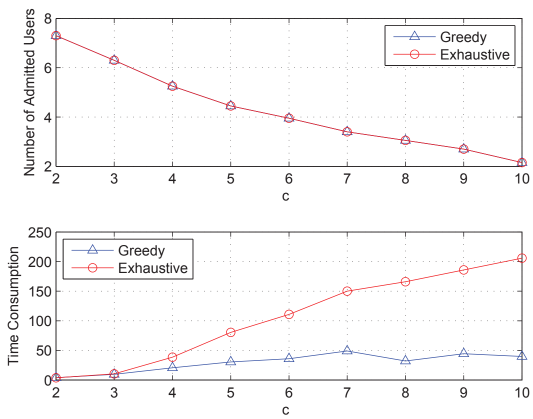
IV. Simulation Results

A wireless network consists of eight users $\mathcal{N} = \{1, 2, \cdots, 8\}$ requesting for admission. The sources and destinations are randomly distributed inside a square area bounded by (0,0) and (10,10). The path loss and the Rayleigh fading effects are present in all links. The path loss gain is given by $g = (1/d)^2$ where d is the distance between two transmission ends, and the variance of the Rayleigh fading gain is denoted as σ^2. We set $W = 10$, and $\sigma^2 = 10$ as default values if no other values are indicated otherwise. The noise PSD N_0 equals to 1. We assume that c_i, $i \in \{1, 2, \cdots, 8\}$ is uniformly distributed over the interval $[c, c+4]$ where c is a variable parameter. The results are averaged over 20 random channel realizations.

We compare the performance of the proposed greedy search algorithm to that of the exhaustive search algorithm for the following two network setups. *Setup 1:* The optimality condition



(a) Setup 1



(b) Setup 2

Fig. 1. Greedy search algorithm vs exhaustive search algorithm.

of the greedy search is satisfied. Specifically, there are four sources and four relays. The source and relay assignments to the users are the following: $\mathcal{N}_{S_1} = \mathcal{N}_{R_1} = \{1,2\}$, $\mathcal{N}_{S_2} = \mathcal{N}_{R_2} = \{3,4\}$, $\mathcal{N}_{S_3} = \mathcal{N}_{R_3} = \{5,6\}$, and $\mathcal{N}_{S_4} = \mathcal{N}_{R_4} = \{7,8\}$. The relays are fixed at (5,2), (5,4), (5,6), and (5,8), and $P_{S_i} = P_{R_i} = 40$, $\forall i \in \{1,2,3,4\}$. *Setup 2:* The optimality condition of the greedy search may not be satisfied. Specifically, there are two sources and two relays. The source and relay assignments to the users are the following: $\mathcal{N}_{S_1} = \{1,2,7,8\}$, $\mathcal{N}_{S_2} = \{3,4,5,6\}$, $\mathcal{N}_{R_1} = \{1,2,3,4\}$, and $\mathcal{N}_{R_2} = \{5,6,7,8\}$. The relays are fixed at (5,3) and (5,7) and $P_{S_i} = P_{R_i} = 80$, $\forall i \in \{1,2\}$.

Fig. 1(a) shows the number of admitted users obtained by the greedy search and the corresponding computational complexity in terms of the running time versus c for Setup 1. It can be seen that the greedy search gives exactly the same number of admitted users as that of the exhaustive search for all values of c. This confirms that the optimal solution is obtained when the optimality condition of the greedy search is satisfied. The time consumption of the greedy search is significantly less than that of the exhaustive search, especially when c is large. This shows that the proposed algorithm is very efficient. Fig. 1(b) demonstrates the performance of the greedy search for Setup 2. Similar conclusions can be obtained as those for Setup 1. This indicates that the greedy search algorithm can still be optimal even if the sufficient optimality condition is not satisfied.

V. Conclusion

Admission control for joint bandwidth and power allocation has been proposed for wireless multi-user DF relay networks. A suboptimal greedy search algorithm with significantly reduced complexity has been developed. A sufficient optimality condition for the proposed greedy search algorithm has been found. The way we derive this condition is novel and can be used as a benchmark for similar optimality analysis of greedy search in other applications. Simulation results demonstrate the advantages of the greedy search.

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