Superimposed Pilot Based Joint CFO and Channel Estimation for CP-OFDM Modulated Two-Way Relay Networks

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***Abstract*—This paper proposes a superimposed training strategy to estimate the individual frequency and channel parameters in an *amplify-and-forward* (AF) two-way relay network (TWRN). Two efficient suboptimal estimation algorithms and an iterative process to further improve the performance are proposed. The estimation Cramér-Rao Bound (CRB) on the proposed estimation strategy is also derived. The simulations confirm that the iterative estimation process converges rapidly and that the resultant estimation mean square error (MSE) approaches the CRB, especially for the case when the carrier frequency offset between the two source terminals is small.**

I. Introduction

Relay networks have attracted much attention since the pioneer work [1] that studied the one way relay network (OWRN), where the data flow is unidirectional from the source to the relay and to the destination. If two source nodes exchange information via a "network coding" manner at the relay node [2], then the result is a two-way relay network (TWRN) [3]. It was reported in [4] that the overall communication rate between the two source terminals in a TWRN is approximately twice as that achieved in OWRN, making TWRN particularly attractive for bidirectional systems.

Most works about TWRN assume perfect synchronization and channel state information (CSI) at the relay node and/or the source terminals, which motivates the crucial efforts in the recent works [5], [6], where the authors design the joint carrier frequency offset (CFO) and channel estimation algorithms for the cyclic prefix (CP) and the zero-padding (ZP) based OFDM modulation in TWRN. However, both these works [5], [6] can only estimate the cascaded channels and cascaded CFO of the uplink and the downlink phases, but not the individual frequency and channel parameters. However, these individual parameters are also crucial to improve the performance of the TWRN in different applications such as precoding and detection [7].

In this paper, we design a new two-phase training strategy by introducing the superimposed pilots from the relay such that the individual channel and CFO parameters can be obtained at both source terminals. Since the optimal minimum mean square error (MMSE) estimator or the maximum a posteriori (MAP) estimator does not have a closed-form expression, we propose to use two suboptimal algorithms for initial estimates and then an iterative process to further improve the estimation accuracy. For completeness, we also derive the estimation Cramér-Rao Bound (CRB) and numerically compare with the estimation mean square error (MSE) of the propose algorithms. Finally, simulation results show promising results of the propose strategies.

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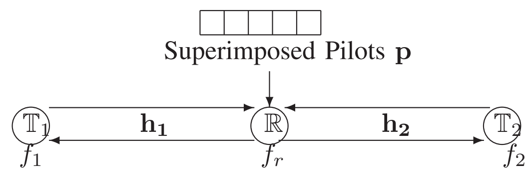

Fig. 1. System configuration for two-way relay network.

II. Problem Formulation

A. System Model

Consider a classical TWRN with two terminal nodes $\mathbb{T}_1$, $\mathbb{T}_2$ and one relay node $\mathbb{R}$, shown in Fig. 1. The channel from $\mathbb{T}_i$ to $\mathbb{R}$ is denoted as $\mathbf{h}_i = [h_{i,0}, \ldots, h_{i,L}]^T$, whose elements are independent and have variances $\sigma_{i,l}^2$. From reciprocity, the channel from $\mathbb{R}$ to $\mathbb{T}_i$ is also $\mathbf{h}_i$.[1] The OFDM block length is set as N. Furthermore, we denote the carrier frequency of $\mathbb{T}_i$ as f_i and that of $\mathbb{R}$ as f_r. The average power of $\mathbb{T}_i$ and $\mathbb{R}$ are denoted as P_i and P_r, respectively. The perfect time synchronization is assumed before our joint CFO and channel estimation.

B. CP-based OFDM Modulation at Terminals

Denote one OFDM block from $\mathbb{T}_i$ as $\tilde{\mathbf{s}}_i = [\tilde{s}_{i,0}, \ldots, \tilde{s}_{i,N-1}]^T$. The corresponding time-domain signal block is obtained from the normalized inverse discrete Fourier transformation (IDFT) as

$$\mathbf{s}_i = \mathbf{F}^H \tilde{\mathbf{s}}_i = [s_{i,0}, s_{i,1}, \ldots, s_{i,N-1}]^T, \qquad (1)$$

[1]There may be a phase difference but is nevertheless ignored in this paper.

$$\begin{aligned}\mathbf{y}_{cp} =& e^{j2\pi(f_r-f_1)LT_s}\mathbf{\Gamma}^{(N)}[f_r-f_1]\mathbf{H}_{cv}^{(N)}[\mathbf{h}_1](\alpha_{cp}\mathbf{r}_{cp}+\mathbf{p})+\mathbf{n}_1\\ =&\alpha_{cp}\mathbf{\Gamma}^{(N)}[f_r-f_1]\mathbf{H}_{cv}^{(N)}[\mathbf{h}_1]\mathbf{\Gamma}^{(N+L)}[f_1-f_r]\mathbf{T}_{cp}^{(L)}\mathbf{H}_{cp}^{(N)}[\mathbf{h}_1]\mathbf{s}_1\\ &+\alpha_{cp}e^{j2\pi(f_2-f_1)LT_s}\mathbf{\Gamma}^{(N)}[f_r-f_1]\mathbf{H}_{cv}^{(N)}[\mathbf{h}_1]\mathbf{\Gamma}^{(N+L)}[f_2-f_r]\mathbf{T}_{cp}^{(L)}\mathbf{H}_{cp}^{(N)}[\mathbf{h}_2]\mathbf{s}_2\\ &+e^{j2\pi(f_r-f_1)LT_s}\mathbf{\Gamma}^{(N)}[f_r-f_1]\mathbf{H}_{cp}^{(N)}[\mathbf{h}_1]\mathbf{p}_0+\underbrace{\alpha_{cp}e^{j2\pi(f_r-f_1)LT_s}\mathbf{\Gamma}^{(N)}[f_r-f_1]\mathbf{H}_{cv}^{(N)}[\mathbf{h}_1]\mathbf{n}_r+\mathbf{n}_1}_{\mathbf{n}_e}.\end{aligned} \tag{7}$$

$$\begin{aligned}\mathbf{y}_{cp}=&\alpha_{cp}\mathbf{H}_{cp}^{(N)}[(\mathbf{\Omega}^{(L+1)}[f_1-f_r]\mathbf{h}_1)\circledast\mathbf{h}_1]\mathbf{s}_1+e^{j2\pi(f_r-f_1)LT_s}\mathbf{\Gamma}^{(N)}[f_r-f_1]\mathbf{H}_{cp}^{(N)}[\mathbf{h}_1]\mathbf{p}_0+\mathbf{n}_e.\\ &+\alpha_{cp}e^{j2\pi(f_2-f_1)LT_s}\mathbf{\Gamma}^{(N)}[f_2-f_1]\mathbf{H}_{cp}^{(N)}[\ (\mathbf{\Omega}^{(L+1)}[f_2-f_r]\mathbf{h}_1)\circledast\mathbf{h}_2\ ]\mathbf{s}_2\end{aligned} \tag{9}$$

$$\begin{aligned}\mathbf{y}_{cp}&=\alpha_{cp}\mathbf{H}_{cp}^{(N)}[\mathbf{a}]\mathbf{s}_1+\alpha_{cp}\mathbf{\Gamma}_L^{(N)}[v]\mathbf{H}_{cp}^{(N)}[\mathbf{b}]\mathbf{s}_2+\mathbf{\Gamma}_L^{(N)}[w]\mathbf{H}_{cp}^{(N)}[\mathbf{h}_1]\mathbf{p}_0+\mathbf{n}_e\\ &=\alpha_{cp}\mathbf{S}_1\mathbf{a}+\alpha_{cp}\mathbf{\Gamma}_L^{(N)}[v]\mathbf{S}_2\mathbf{b}+\mathbf{\Gamma}_L^{(N)}[w]\mathbf{P}\mathbf{h}_1+\mathbf{n}_e.\end{aligned} \tag{10}$$

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where $\mathbf{F}$ is the $N \times N$ normalized DFT matrix with the (p,q)-th entry given by $\frac{1}{\sqrt{N}}e^{-j2\pi(p-1)(q-1)/N}$. To maintain the subcarrier originality during the overall transmission, we propose to add the cyclic prefix of length $2L$ as did in [5].

Define

$$\mathbf{T}_{cp}^{(P)}=\left[\begin{array}{cc}\mathbf{0} & \mathbf{I}_{P\times P}\\ \hline \multicolumn{2}{c}{\mathbf{I}_{N\times N}}\end{array}\right], \tag{2}$$

for any $P \leq N$. The baseband signal from $\mathbb{T}_i$ after CP insertion can be mathematically expressed as $\mathbf{T}_{cp}^{(2L)}\mathbf{s}_i$ that has length $N+2L$. It will then be up-converted to passband signal by the carrier $e^{j2\pi f_i t}$.

C. Relay Processing

The signals from $\mathbb{T}_1$ and $\mathbb{T}_2$ arrive at $\mathbb{R}$ simultaneously. Relay then down-converts the received signal by the carrier $e^{-j2\pi f_r t}$ and removes only the first L symbols in one block. Define $\mathbf{\Gamma}^{(K)}[f]=\text{diag}\{1,e^{j2\pi fT_s},\ldots,e^{j2\pi f(K-1)T_s}\}$ and

$$\mathbf{H}_{cv}^{(K)}[\mathbf{x}]=\left.\begin{bmatrix}x_P & \ldots & x_0 & \ldots & 0\\ \vdots & \ddots & \ddots & \ddots & \vdots\\ 0 & \ldots & x_P & \ldots & x_0\end{bmatrix}\right\} K \text{ rows}, \tag{3}$$

for $\mathbf{x}=[x_0,x_1,\ldots,x_P]^T$. The resultant baseband signal block at $\mathbb{R}$ is of length $N+L$ and can be expressed as:

$$\mathbf{r}_{cp}=\sum_{i=1}^{2}e^{j2\pi(f_i-f_r)LT_s}\mathbf{\Gamma}^{(N+L)}[f_i-f_r]\mathbf{T}_{cp}^{(L)}\mathbf{H}_{cp}^{(N)}[\mathbf{h}_i]\mathbf{s}_i+\mathbf{n}_r, \tag{4}$$

where $\mathbf{H}_{cp}^{(N)}[\mathbf{h}_i]$ is the $N\times N$ circulant matrix with the first column $[\mathbf{h}_i^T,\mathbf{0}_{1\times(N-L-1)}^T]^T$, and we use the property that

$$\mathbf{H}_{cv}^{(N+L)}[\mathbf{h}_i]\mathbf{T}_{cp}^{(2L)}=\mathbf{T}_{cp}^{(L)}\mathbf{H}_{cp}^{(N)}[\mathbf{h}_i]. \tag{5}$$

Moreover, each element in the noise vector $\mathbf{n}_r$ is assumed to be independent and identically distributed (i.i.d) zero-mean complex white Gaussian, with variance σ_n^2.

The relay then superimposes a training vector $\mathbf{p}$ over $\mathbf{r}_{cp}$ and obtain

$$\mathbf{t}_{cp}=\alpha_{cp}\mathbf{r}_{cp}+\mathbf{p}, \tag{6}$$

where α_{cp} is the scaling factor that balances the power between the training from $\mathbb{T}_i$ and the superimposed training from $\mathbb{R}$, and has the range $\left(0,\sqrt{\frac{P_r}{\sum_{j=1}^{2}\sum_{l=0}^{L}\sigma_{j,l}^2P_j+\sigma_n^2}}\right)$. To keep the circular property, $\mathbf{p}$ should has the structure $\mathbf{p}=\mathbf{T}_{cp}^{(L)}\mathbf{p}_0$, where $\mathbf{p}_0$ is an $N\times 1$ vector.

Finally, $\mathbb{R}$ up-converts the resultant signal $\mathbf{t}_{cp}$ to passband by the carrier $e^{j2\pi f_r t}$.

D. Signal Reformulation at Terminals

Due to symmetry, we only illustrate the process at $\mathbb{T}_1$. After down-converting the passband signal by $e^{-j2\pi f_1 t}$, $\mathbb{T}_1$ obtains the baseband block of length $N+L$. It then removes the first L elements and the remaining signal is written as in (7) shown on the top of this paper, where the property

$$\mathbf{H}_{cv}^{(N)}[\mathbf{h}_1]\mathbf{T}_{cp}^{(L)}=\mathbf{H}_{cp}^{(N)}[\mathbf{h}_1]$$

is used. The equivalent Gaussian noise $\mathbf{n}_e$ has the covariance

$$\begin{aligned}\mathbf{R}_n=&\sigma_n^2\Big(\alpha_{cp}^2\mathbf{\Gamma}^{(N)}[f_r-f_1]\mathbf{H}_{cv}^{(N)}[\mathbf{h}_1]\\ &(\mathbf{H}_{cv}^{(N)}[\mathbf{h}_1])^H(\mathbf{\Gamma}^{(N)}[f_r-f_1])^H+\mathbf{I}\Big).\end{aligned} \tag{8}$$

Let $\mathbf{\Omega}^{(K)}[f]=\text{diag}\{e^{j2\pi f(K-1)T_s},\ldots,e^{j2\pi fT_s},1\}$ and $\circledast$ denote the N-point circular convolution. Using the following properties

$$\begin{aligned}\mathbf{H}_{cv}^{(N)}[\mathbf{h}_i]\mathbf{\Gamma}^{(N+L)}[f]&=\mathbf{\Gamma}^{(N)}[f]\mathbf{H}_{cv}^{(N)}[\mathbf{\Omega}^{(L+1)}[f]\mathbf{h}_i],\\ \mathbf{H}_{cp}^{(N)}[\mathbf{x}_1]\mathbf{H}_{cp}^{(N)}[\mathbf{x}_2]&=\mathbf{H}_{cp}^{(N)}[\mathbf{x}_1\circledast\mathbf{x}_2],\end{aligned}$$

we can simplify $\mathbf{y}_{cp}$ as (9) shown on the top of this page.

By defining

$$\begin{aligned}&w=f_r-f_1,\quad \mathbf{a}=(\mathbf{\Omega}^{(L+1)}[-w]\mathbf{h}_1)\circledast\mathbf{h}_1,\\ &v=f_2-f_1,\quad \mathbf{b}=(\mathbf{\Omega}^{(L+1)}[v-w]\mathbf{h}_1)\circledast\mathbf{h}_2,\\ &\mathbf{\Gamma}_L^{(K)}[f]=e^{j2\pi fLT_s}\mathbf{\Gamma}^{(K)}[f],\end{aligned}$$

(9) can be finally expressed as (10), where $\mathbf{S}_j$ is the $N\times(2L+1)$ circulant matrix with first column $\mathbf{s}_i$, and $\mathbf{P}$ is the $N\times(L+1)$ circulant matrix with first column $\mathbf{p}_0$.

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III. Joint estimation algorithm

Based on this new signal model, the task is to estimate the individual channels $\mathbf{h}_1$, $\mathbf{h}_2$, and the CFOs v, w. The overall number of unknown is $2L+4$.

Let's omit all redundant superscripts and subscripts for notation simplicity and rewrite (10) as

$$\mathbf{y} = \alpha\mathbf{S}_1\mathbf{a} + \alpha\mathbf{\Gamma}[v]\mathbf{S}_2\mathbf{b} + \mathbf{\Gamma}[w]\mathbf{P}\mathbf{h}_1 + \mathbf{n}_e. \quad (11)$$

Clearly, the optimal MMSE or MAP estimator is hard to be expressed in a closed form. Instead, we propose several efficient suboptimal estimators.

A. A General Training Scheme

Define $\mathcal{K}_1$, $\mathcal{K}_2$, and $\mathcal{K}_r$ as the frequency domain pilot index sets from $\mathbb{T}_1$, $\mathbb{T}_2$, and $\mathbb{R}$ with cardinality K_1, K_2, and K_r respectively. We require $K_1 \geq L+1$, $K_2 \geq L+1$, $K_r \geq L+1$ and $\mathcal{K}_1 \bigcup \mathcal{K}_2 \bigcup \mathcal{K}_r = \{1,\ldots,N\}$. From (11), we know that the frequency domain pilots are $\tilde{\mathbf{s}}_j = \mathbf{F}\mathbf{s}_j$, and $\tilde{\mathbf{p}}_0 = \mathbf{F}\mathbf{p}_0$.

Let us put non-zero pilots from $\mathbb{T}_j$ into a $K_j \times 1$ vector $\breve{\mathbf{s}}_j$ and non-zero pilots from $\mathbb{R}$ into a $K_r \times 1$ vector $\breve{\mathbf{p}}_0$. Since $\mathbf{S}_j$ and $\mathbf{P}$ are columnwise circulant matrices, they can be represented as

$$\begin{aligned}\mathbf{S}_j =& \mathbf{F}^H \mathrm{diag}\{\tilde{\mathbf{s}}_j\}\mathbf{F}_{[:,1:2L+1]} \\ =& \mathbf{F}^H_{[:,\mathcal{K}_j]}\mathrm{diag}\{\breve{\mathbf{s}}_j\}\mathbf{F}_{[\mathcal{K}_j,1:2L+1]} \quad (12)\\ \mathbf{P} =& \mathbf{F}^H \mathrm{diag}\{\tilde{\mathbf{p}}_0\}\mathbf{F}_{[:,1:L+1]} \\ =& \mathbf{F}^H_{[:,\mathcal{K}_r]}\mathrm{diag}\{\breve{\mathbf{p}}_0\}\mathbf{F}_{[\mathcal{K}_r,1:L+1]}. \quad (13)\end{aligned}$$

Note that MATLAB notations for rows and columns of a matrix are adopted here.

Define $\bar{\mathcal{K}}_1$ as the complement set of $\mathcal{K}_1$. Multiplying both sides of (11) with $\mathbf{F}_{[\bar{\mathcal{K}}_1,:]}$ yields

$$\mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{y} = \underbrace{\left[\alpha\mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{\Gamma}[v]\mathbf{F}^H_{[:,\mathcal{K}_2]}\mathrm{diag}\{\breve{\mathbf{s}}_2\}, \mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{\Gamma}[w]\mathbf{P}\right]}_{\mathbf{C}_1}\underbrace{\begin{bmatrix}\breve{\mathbf{b}}\\ \mathbf{h}_1\end{bmatrix}}_{\mathbf{d}_1} + \mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{n}_e, \quad (14)$$

where $\breve{\mathbf{b}} = \mathbf{F}_{[\mathcal{K}_2,1:2L+1]}\mathbf{b}$ is the DFT response of $\mathbf{b}$ on the subcarrier set $\mathcal{K}_2$, while $\mathbf{C}_1$ and $\mathbf{d}_1$ are the corresponding matrices.

As long as $N-K_1-K_2-K_r \geq 2$, namely when there is sufficient degree of freedom to estimate the two known CFO, the least square (LS) estimation of $\mathbf{d}_1$ can be obtained as

$$\hat{\mathbf{d}}_1 \;=\; (\mathbf{C}_1^H\mathbf{C}_1)^{-1}\mathbf{C}_1^H\mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{y}. \quad (15)$$

Substituting (15) back to (14), the CFOs can be found from

$$\{\hat{v},\hat{w}\} = \arg\max_{v,w}\; \mathbf{y}^H\mathbf{F}^H_{[\bar{\mathcal{K}}_1,:]}\mathbf{C}_1(\mathbf{C}_1^H\mathbf{C}_1)^{-1}\mathbf{C}_1^H\mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{y}. \quad (16)$$

Considering the range of K_j and K_r, the minimum training length is $N \geq 3L+5$, which is greater than the number of unknown variables.

By definition

$$\breve{\mathbf{b}} = \mathbf{F}_{[\mathcal{K}_2,1:2L+1]}\mathbf{H}_{zp}^{(L+1)}[\mathbf{\Omega}^{(L+1)}[v-w]\mathbf{h}_1]\mathbf{h}_2. \quad (17)$$

Then, $\mathbf{h}_2$ can be estimated from

$$\hat{\mathbf{h}}_2 = (\mathbf{F}_{[\mathcal{K}_2,1:2L+1]}\mathbf{H}_{zp}^{(L+1)}[\mathbf{\Omega}^{(L+1)}[v-w]\mathbf{h}_1])^{\dagger}\breve{\mathbf{b}}, \quad (18)$$

where $(\cdot)^\dagger$ denotes the pseudo-inverse of a matrix.

B. Minimum Training Length: A Special Case

In practical applications, the relay terminal is a simple device while the two source terminals may have complicated functionality that can afford high precision algorithms. Therefore, we can assume that $v \approx 0$ or $v \ll 1/N$, i.e., one subcarrier spacing. This is also true at the CFO tracking stage when the frequency difference between two terminals are quite small. In this case, we a training strategy can be designed with the minimum training length $N = 2L+3$, i.e., the same number of the unknowns variables.

Let us choose the same frequency pilot sets for $\mathbb{T}_1$ and $\mathbb{T}_2$, i.e., $\mathcal{K}_1 = \mathcal{K}_2$. Left multiplying the received signal $\mathbf{y}$ by $\mathbf{F}_{[\bar{\mathcal{K}}_1,:]}$ gives

$$\begin{aligned}\mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{y} =& \underbrace{\alpha\mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{\Gamma}[v]\mathbf{F}^H_{[:,\mathcal{K}_2]}}_{\approx\mathbf{0}}\mathrm{diag}\{\breve{\mathbf{s}}_2\}\mathbf{F}_{[\mathcal{K}_i,1:2L+1]}\mathbf{b} \\ &+ \underbrace{\mathbf{G}_{[\bar{\mathcal{K}}_1,:]}\mathbf{\Gamma}[w]\mathbf{P}}_{\mathbf{C}_2}\mathbf{h}_1, \quad (19)\end{aligned}$$

where $\mathbf{C}_2$ is an $(N-K_1)\times(L+1)$ matrix, and the first term can be approximated by 0 because $v \approx 0$. As long as $(N-K_1-L-1) \geq 1$, we can obtain

$$\hat{\mathbf{h}}_1 = \mathbf{C}_2^{\dagger}\mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{y}, \quad (20)$$

and

$$\hat{w} = \arg\max_{w}\; \mathbf{y}^H\mathbf{F}^H_{[\bar{\mathcal{K}}_1,:]}\mathbf{C}_2(\mathbf{C}_2^H\mathbf{C}_2)^{-1}\mathbf{C}_2^H\mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{y}. \quad (21)$$

Since the estimation of $\mathbf{a}$ can be found from $\hat{w}$ and $\hat{\mathbf{h}}_1$, then $\mathbf{h}_2$ can be estimated as

$$\hat{\mathbf{h}}_2 = \mathbf{C}_3^{\dagger}(\mathbf{y} - \alpha\mathbf{S}_1\hat{\mathbf{a}} - \mathbf{\Gamma}[\hat{w}]\mathbf{P}\hat{\mathbf{h}}_1), \quad (22)$$

where $\mathbf{C}_3 = \alpha\mathbf{S}_2\mathbf{H}_{zp}^{(L+1)}[\mathbf{\Omega}^{(L+1)}[\hat{v}-\hat{w}]\hat{\mathbf{h}}_1]$.

C. Iterative Algorithm to Improve the Performance

Now that we have the initial estimation of every parameters, we can apply an iterative approach to improve the estimation accuracy. Re-denote the initial estimate as $v^{(0)}, w^{(0)}, \mathbf{h}_1^{(0)}, \mathbf{h}_2^{(0)}$, respectively, with the superscript representing the number of the iterations. We will estimate $v^{(1)}, w^{(1)}$ simultaneously from the weighted LS estimation process as in (23) shown on the top of the next page, where $\mathbf{R}_n^{-1}$ is always obtained using the newest estimates of w and $\mathbf{h}_1$. The complexity here is not significant even if the 2-dimensional search is applied, since the search region for fine estimation is around the initial estimation and is thus very small. Moreover, we can obtain $\mathbf{h}_2^{(1)}$ and $\mathbf{h}_1^{(1)}$ from (24) and (25) respectively.

The iterative processing could gain the improvement from the fact that the initial estimation does not fully exploit the correlation between $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{h}_1$.

$$[v^{(1)}, w^{(1)}] = \arg\min_{v,w} (\mathbf{y} - \alpha\mathbf{S}_1\mathbf{a}^{(0)} - \alpha\mathbf{\Gamma}[v]\mathbf{S}_2\mathbf{b}^{(0)} - \mathbf{\Gamma}[w]\mathbf{P}\mathbf{h}_1^{(0)})^H\mathbf{R}_n^{-1}(\mathbf{y} - \alpha\mathbf{S}_1\mathbf{a}^{(0)} - \alpha\mathbf{\Gamma}[v]\mathbf{S}_2\mathbf{b}^{(0)} - \mathbf{\Gamma}[w]\mathbf{P}\mathbf{h}_1^{(0)}), \quad (23)$$

$$\mathbf{h}_2^{(1)} = \arg\min_{\mathbf{h}_2} (\mathbf{y} - \alpha\mathbf{S}_1\mathbf{a}^{(0)} - \mathbf{\Gamma}[w^{(1)}]\mathbf{P}\mathbf{h}_1^{(0)} - \alpha\mathbf{\Gamma}[v^{(1)}]\mathbf{S}_2\mathbf{H}_{12}^{(1)}\mathbf{h}_2)^H\mathbf{R}_n^{-1} \times(\mathbf{y} - \alpha\mathbf{S}_1\mathbf{a}^{(0)} - \mathbf{\Gamma}[w^{(1)}]\mathbf{P}\mathbf{h}_1^{(0)} - \alpha\mathbf{\Gamma}[v^{(1)}]\mathbf{S}_2\mathbf{H}_{12}^{(1)}\mathbf{h}_2), \quad (24)$$

$$\mathbf{h}_1^{(1)} = \arg\min_{\mathbf{h}_1} \left(\mathbf{y} - \left[\alpha\mathbf{S}_1\mathbf{H}_{11}^{(0)} + \alpha\mathbf{\Gamma}[v^{(1)}]\mathbf{S}_2\mathbf{H}_{zp}^{(L+1)}[\mathbf{h}_2^{(1)}]\mathbf{\Omega}^{L+1}[v^{(1)} - w^{(1)}] + \mathbf{\Gamma}[w^{(1)}]\mathbf{P}\right]\mathbf{h}_1\right)^H \times\mathbf{R}_n^{-1}\left(\mathbf{y} - \left[\alpha\mathbf{S}_1\mathbf{H}_{11}^{(0)} + \alpha\mathbf{\Gamma}[v^{(1)}]\mathbf{S}_2\mathbf{H}_{zp}^{(L+1)}[\mathbf{h}_2^{(1)}]\mathbf{\Omega}^{L+1}[v^{(1)} - w^{(1)}] + \mathbf{\Gamma}[w^{(1)}]\mathbf{P}\right]\mathbf{h}_1\right). \quad (25)$$

IV. Cramér-Rao Bound

In this section, we derive the CRB that defines the theoretical bound of the estimation accuracy. Let

$$\boldsymbol{\mu} \triangleq \alpha\mathbf{S}_1\mathbf{a} + \alpha\mathbf{\Gamma}[v]\mathbf{S}_2\mathbf{b} + \mathbf{\Gamma}[w]\mathbf{P}\mathbf{h}_1, \quad (26)$$

$$\boldsymbol{\eta} \triangleq [v, w, \Re\{\mathbf{h}_1\}^T, \Im\{\mathbf{h}_1\}^T, \Re\{\mathbf{h}_2\}^T, \Im\{\mathbf{h}_2\}^T]^T. \quad (27)$$

According to [8], the (i,j)th entry of the Fisher Information Matrix (FIM) can be calculated as

$$[\mathbf{F}]_{i,j} = 2\Re\left[\frac{\partial\boldsymbol{\mu}^H}{\partial\eta_i}\mathbf{R}_n^{-1}\frac{\partial\boldsymbol{\mu}}{\partial\eta_j}\right] + \mathrm{tr}\left[\mathbf{R}_n^{-1}\frac{\partial\mathbf{R}_n}{\partial\eta_i}\mathbf{R}_n^{-1}\frac{\partial\mathbf{R}_n}{\partial\eta_j}\right]. \quad (28)$$

After some tedious simplifications, we can derive

$$\frac{\partial\boldsymbol{\mu}}{\partial v} = j\mathbf{\Gamma}[v]\mathbf{S}_2\mathbf{H}_{zp}^{(L+1)}[\mathbf{h}_2]\mathbf{D}_0\mathbf{\Omega}^{(L+1)}[v-w]\mathbf{h}_1 + j\alpha\mathbf{D}_1\mathbf{\Gamma}[v]\mathbf{S}_2\mathbf{b},$$

$$\frac{\partial\boldsymbol{\mu}}{\partial w} = \alpha\mathbf{S}_1\mathbf{H}_{zp}^{(L+1)}[\mathbf{h}_1](-j\mathbf{D}_0)\mathbf{\Omega}^{(L+1)}[-w]\mathbf{h}_1 + j\mathbf{D}_1\mathbf{\Gamma}[w]\mathbf{P}\mathbf{h}_1 + \alpha\mathbf{\Gamma}[v]\mathbf{S}_2\mathbf{H}_{zp}^{(L+1)}[\mathbf{h}_2](-j\mathbf{D}_0)\mathbf{\Omega}^{(L+1)}[v-w]\mathbf{h}_1,$$

$$\frac{\partial\boldsymbol{\mu}}{\partial\Re\{\mathbf{h}_1\}^T} = \alpha\mathbf{S}_1\mathbf{H}_{zp}^{(L+1)}[\mathbf{\Omega}^{(L+1)}[-w]\mathbf{h}_1] + \mathbf{\Gamma}[w]\mathbf{P} + \alpha\mathbf{\Gamma}[v]\mathbf{S}_2\mathbf{H}_{zp}^{(L+1)}[\mathbf{h}_2]\mathbf{\Omega}^{(L+1)}[v-w] + \alpha\mathbf{S}_1\mathbf{H}_{zp}^{(L+1)}[\mathbf{h}_1]\mathbf{\Omega}^{(L+1)}[-w],$$

$$\frac{\partial\boldsymbol{\mu}}{\partial\Re\{\mathbf{h}_2\}^T} = \alpha\mathbf{\Gamma}[v]\mathbf{S}_2\mathbf{H}_{zp}^{(L+1)}[\mathbf{\Omega}^{(L+1)}[v-w]\mathbf{h}_1],$$

$$\frac{\partial\mathbf{R}_n}{\partial w} = \sigma_n^2\mathbf{\Gamma}^{(N)}[w]\Big(j\alpha^2\mathbf{D}_N\mathbf{H}_{cv}^{(N)}[\mathbf{h}_1](\mathbf{H}_{cv}^{(N)}[\mathbf{h}_1])^H - j\alpha^2\mathbf{H}_{cv}^{(N)}[\mathbf{h}_1](\mathbf{H}_{cv}^{(N)}[\mathbf{h}_1])^H\mathbf{D}_N\Big)\mathbf{\Gamma}^{(N)}[-w],$$

$$\frac{\partial\mathbf{R}_n}{\partial\Re\{\mathbf{h}_{1i}\}^T} = \sigma_n^2\alpha^2\mathbf{\Gamma}^{(N)}[w]\mathbf{H}_{cv}^{(N)}[\mathbf{e}_{1i}](\mathbf{H}_{cv}^{(N)}[\mathbf{h}_1])^H\mathbf{\Gamma}^{(N)}[-w],$$

$$\frac{\partial\mathbf{R}_n}{\partial\Re\{\mathbf{h}_{2i}\}^T} = \frac{\partial\mathbf{R}_n}{\partial v} = \mathbf{0}_{N\times N},$$

where

$$\mathbf{D}_0 = 2\pi T_s\mathrm{diag}\{L, (L-1), \ldots, 1, 0\},$$
$$\mathbf{D}_1 = 2\pi T_s\mathrm{diag}\{L, \ldots, (L+N-1)\},$$
$$\mathbf{D}_N = 2\pi T_s\mathrm{diag}\{0, 1, \ldots, (N-1)\},$$

and $\mathbf{e}_i$ is a $(L+1)\times 1$ vector whose i^{th} element equals 1 and others 0.

The CRB of $\boldsymbol{\eta}$ is then obtained by inverting the FIM $\mathbf{F}$. Define $\boldsymbol{\eta}_0 = [v, w, \mathbf{h}_1^T, \mathbf{h}_2^T]^T$ as the original set of complex-valued parameters and let $\mathbf{I}_0 = \mathbf{I}_{(L+1)\times(L+1)}$. It can be readily checked that

$$\boldsymbol{\eta}_0 = \mathbf{\Xi}\boldsymbol{\eta}, \quad (29)$$

where

$$\mathbf{\Xi} = \begin{bmatrix} 1, & 0, & \mathbf{0}^T, & \mathbf{0}^T, & \mathbf{0}^T, & \mathbf{0}^T \\ 0, & 1, & \mathbf{0}^T, & \mathbf{0}^T, & \mathbf{0}^T, & \mathbf{0}^T \\ \mathbf{0}, & \mathbf{0}, & \mathbf{I}_0, & j\mathbf{I}_0, & 0\cdot\mathbf{I}_0, & 0\cdot\mathbf{I}_0 \\ \mathbf{0}, & \mathbf{0}, & 0\cdot\mathbf{I}_0, & 0\cdot\mathbf{I}_0, & \mathbf{I}_0, & j\mathbf{I}_0 \end{bmatrix}.$$

The CRB of $v, w, \mathbf{h}_1$, and $\mathbf{h}_2$ can be expressed as

$$\mathbf{CRB} = \mathbf{\Xi}\cdot\mathbf{F}^{-1}\cdot\mathbf{\Xi}^H. \quad (30)$$

V. Simulation Results

In this section, the performance of our proposed three estimation algorithms and the iterative method is studied. A four-tap model for both $\mathbf{h}_i$ is assumed, where each tap is complex Gaussian with unit variance. The variance of the noise is taken as $\sigma_n^2 = 1$. The normalized frequencies f_1, f_r, and f_2 are set as 0.94, 1 and 1.06, respectively and the variance of the noise $\sigma_n^2 = 1$. Let $\alpha_{cp} = 0.5\sqrt{\frac{P_r}{\sum_{j=1}^{2}\sum_{l=0}^{L}\sigma_{j,l}^2P_j+\sigma_n^2}}$. The MSE is chosen as the figure of merit and 10000 Monte-Carlo trials are used for averaging.

A. General Case

First we choose $N = 3L+5 = 14$. The initial CFO and channel estimation can be obtained from (15), (16), and (18). The iteration (23) - (25) are found to convergence in ten iterations. The MSEs and CRBs versus the signal-to-noise ratio (SNR) for both CFO and channel estimation are displayed in Fig. 2 and Fig. 3, respectively. It is observed that at the high SNR region, the MSEs approach the CRBs after ten iterations. The iterative algorithm can improve the estimation of $\mathbf{h}_1$ in the whole SNR region while for $\mathbf{h}_2$, the iterative algorithm only works well at the high SNR region. A possible reason is as follows. Since only the second item of $\mathbf{y}_{cp}$ in (10) contains the information of $\mathbf{h}_2$ and iterations require reconstruction of $\mathbf{a}$ from the initial estimate $\hat{w}$ and $\hat{\mathbf{h}}_1$, the ambiguity of $\hat{\mathbf{h}}_2$ somehow is enlarged at the low SNR region from errors of all factors.

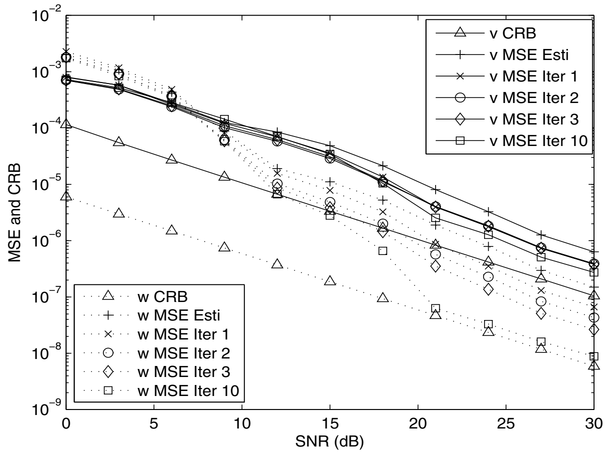

Fig. 2. CFO estimation: the general case

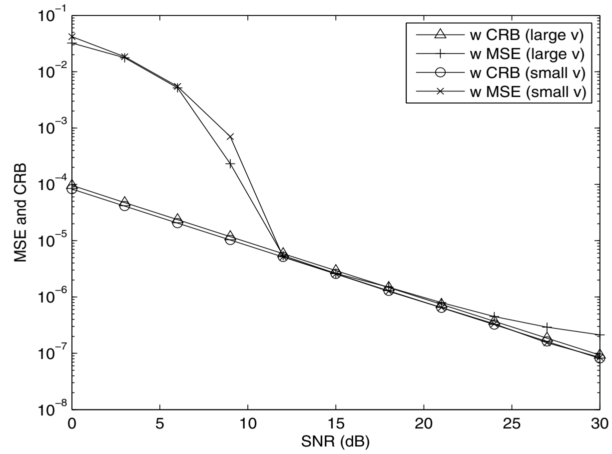

Fig. 4. CFO estimation with minimum N: the special case

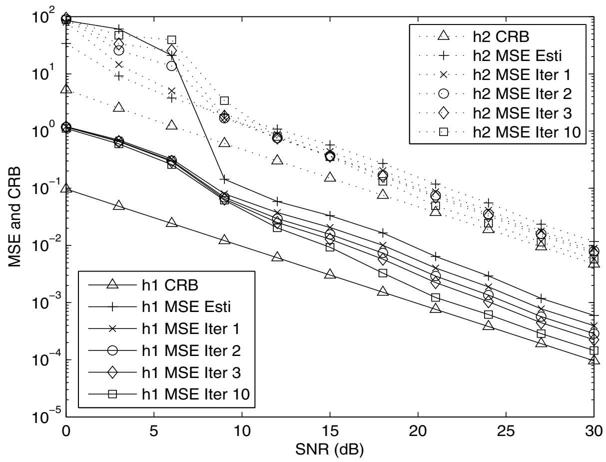

Fig. 3. Channel estimation: the general case

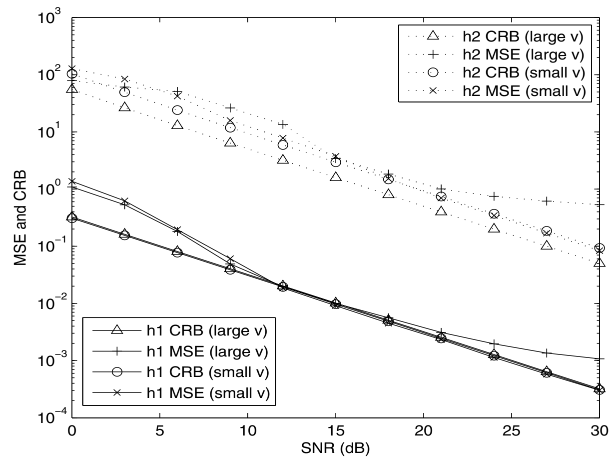

Fig. 5. Channel estimation with minimum N: the special case

B. Minimum N: A Special Case

In the last example, we take two values of f_2 as 0.95 and 0.9401, respectively, such that the CFO between the two terminals are $v = 0.01$ and $v = 0.0001$. We then choose the minimum training length as $N = 2L + 3 = 9$. The CFO and channel estimation results can be obtained from (21), (20), and (22). It should be mentioned that for this case, the iterations are not required since v is assumed to be as small as 0 and does not require the estimation. The estimation MSEs of w and channels, as well as their corresponding CRBs are shown in Fig. 4 and Fig. 5, respectively. These figures show that the estimation accuracy is quite close to the CRB. The reason for this is that the orthogonality can reduce the interference between the pilots.

VI. Conclusions

In this paper, we proposed a superimposed based training strategy for CP-OFDM modulated TWRN for the estimation of the individual channel and CFO parameters. Two suboptimal estimation algorithms and an iterative algorithm to improve the performance were developed. The analytical CRBs were also derived. When the CFO between the two source terminals is small, the estimation MSE approaches the CRB in the high SNR region, which suggests the optimality of the proposed algorithm.

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