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Feedback Delay Effect on Dual-hop MIMO AF Relaying with Antenna Selection

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***Abstract*—In this paper, the effect of feedback delays on the performance of multiple-input multiple-output antenna amplify-and-forward relay networks with the best transmit/receive antenna pair selection over Rayleigh fading is studied. The cumulative distribution function and the moment generating function of the end-to-end signal-to-noise ratio (SNR) are derived. Closed-form expressions for the outage probability, average symbol error rate (SER), and the SNR moments are also derived. To gain further insights, the asymptotic outage probability, average SER, diversity order, and coding gain are presented. Numerical results and Monte-Carlo simulations are provided to illustrate the detrimental effects of feedback delays on the system performance and to verify the accuracy of our analysis.**

I. Introduction

In dual-hop relay networks, the use of multiple-antennas at the source, relay, and the destination can significantly improve the system performance [1], [2]. Transmit/receive diversity techniques can further enhance the benefits of such networks [3]–[5]. For example, transmit and receive (Tx/Rx) antenna subset selection has been widely researched as an efficient approach to achieve better trade-offs among the performance, hardware costs, and computational complexity of multiple-input multiple-output (MIMO) single-hop networks [3], [4]. Among these antenna-selection techniques, the best Tx/Rx antenna pair selection [6] has already been employed as a suboptimal yet simple and efficient diversity transmission technique for dual-hop amplify-and-forward (AF) MIMO relay networks [7].

In [7], Tx/Rx antenna pair selection is proposed for dual-hop MIMO AF relay networks. Here, the end-to-end transmission takes place by selecting the best Tx/Rx antenna pair at both the source-to-relay and relay-to-destination MIMO channels. The probability density function (PDF) of the end-to-end signal-to-noise ratio (SNR) over Rayleigh fading is derived and used to obtain an analytic bit error rate (BER) expression for M-ary phase shift keying. The effect of link unbalance between the first hop and the second hop due to the difference between the average SNR and the number of antennas employed at each terminal is investigated numerically.

In practice, the channel state information (CSI) used for antenna pair selection at the source and relay can be outdated when the CSI feedback rate is not sufficiently high compared to the Doppler fading rate. In this case, the selected Tx/Rx antenna pair may not be the best among the available set of antenna pairs at the instant of data transmission due to the feedback delay. Although this feedback delay may cause severe performance degradation, this problem has not yet been investigated for MIMO relay networks. In this paper, we remedy this gap and study the performance impact of such a networks due to feedback delay.

In [8], the impact of switching constraints on the selection diversity performance is analyzed. In [5], the effect of feedback delays on the performance of systems with transmit antenna selection and maximal ratio combining (TAS/MRC) over Rayleigh fading is investigated. Further, in [9], the symbol error rate (SER) of TAS/MRC over Nakagami-m fading is derived when the transmit antennas are selected based on the outdated CSI. All the above studies consider either single-input multiple-output or MIMO channels for single-hop networks.

The prior related studies which analyze the impact of feedback delays on the dual-hop relay networks are as follows: In [10], the outage probability and the diversity order are analyzed for the opportunistic decode-and-forward relay selection with the outdated CSI. Reference [11] analyzes the impact of the outdated CSI due to feedback delay on the performance of AF k-th best partial relay selection. Moreover, in [12], the effect of feedback delays on the performance of maximal ratio transmission in the down-link of the dual-hop AF relay networks over the Rayleigh fading channels is investigated.

In this paper, the impact of the outdated CSI due to feedback delay on the permanence of the best Tx/Rx antenna pair selection for dual-hop MIMO relay network is studied. Closed-form expressions for the cumulative distribution function (CDF) and the moment generating function (MGF) of the end-to-end SNR are derived. The outage probability, the average SER, and the SNR moments are also derived. To provide valuable insights, the asymptotic outage probability, average SER, diversity order and coding gains are derived. Further, numerical and Monte-Carlo simulation results are presented to analyze the system performance and to verify the accuracy of our analysis.

The rest of this paper is organized as follows. Section II presents the system model. In Section III, the performance analysis is presented. Section IV contains the numerical and simulation results. Section V concludes the paper.

Notations: $\mathcal{J}_\nu(z)$ is the *Bessel function of the first kind* of order ν [13, Eq. (8.404.1)]. $\mathcal{I}_\nu(z)$ is the *Modified Bessel function of the first kind* of order ν [13, Eq. (8.406.1)]. $\mathcal{K}_\nu(z)$ is the *Modified Bessel function of the second kind* of order ν [13, Eq. (8.407.1)]. ${}_2\mathcal{F}_1(\alpha,\beta;\gamma;z)$ is the *Gauss Hypergeometric function* [13, Eq. (9.14.1)]. $\mathcal{Q}(z)$ is the *Q-function* [14, Eq. (26.2.3)]. $\mathcal{E}_\Lambda\{z\}$ denotes the *expected value of z* over Λ.

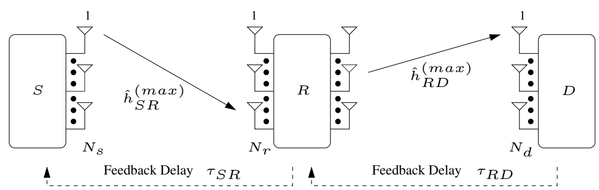

Fig. 1. Dual-hop MIMO AF relaying with best Tx/Rx antenna pair selection

II. System Model

We consider the dual-hop MIMO AF relay network in Fig. 1. The source (S), the relay (R) and the destination (D) are equipped with N_s, N_r and N_d antennas, respectively. The $S \to R$ and $R \to D$ channel matrices are denoted by H_{SR} and H_{RD}, and both have independent and identically distributed Rayleigh fading entries. Further, the (i,j)-th and the (k,l)-th elements of H_{SR} and H_{RD} are denoted by $h_{SR}^{(i,j)}$ and $h_{RD}^{(k,l)}$. All terminals operate in the half-duplex mode, and $S \to D$ communication takes place in two time-slots [15]. In the first time-slot, S selects the best pair of Tx/Rx antennas based on the CSI received by the local feedback channel, which is assumed to experience a time delay τ_{SR}. The corresponding Tx/Rx antenna indices are thus given by $(\hat{I},\hat{J}) = \underset{1\le i\le N_r, 1\le j\le N_s}{\operatorname{argmax}} \left(\left|\hat{h}_{SR}^{(i,j)}\right|\right)$, where $\hat{h}_{SR}^{(i,j)}$ is the (i,j)-th element of the $S \to R$ outdated channel matrix $\hat{H}_{SR}$. Then the received signal at R can be written as

$$y_R = \sqrt{\mathcal{P}_S} h_{SR}^{(\hat{I},\hat{J})} X_s + n_R, \tag{1}$$

where $\mathcal{P}_S$ is the transmit power at S, n_R and X_s are the additive white Gaussian noise (AWGN) at R and the transmit signal, satisfying $\mathcal{E}\left\{|n_R|^2\right\} = \sigma_R^2$ and $\mathcal{E}\left\{|X_s|^2\right\} = 1$, respectively. Then R too selects the best pair of Tx/Rx antennas for its transmission by using the outdated-CSI received via the local feedback channel having a time delay τ_{RD}. Thus, the corresponding Tx/Rx antenna indices are given by $(\hat{K},\hat{L}) = \underset{1\le k\le N_d, 1\le l\le N_r}{\operatorname{argmax}} \left(\left|\hat{h}_{RD}^{(k,l)}\right|\right)$, where $\hat{h}_{RD}^{(k,l)}$ is the (k,l)-th element of $R \to D$ outdated channel matrix $\hat{H}_{RD}$. In the second time-slot, R forwards the received signal to D after amplifying it by a variable gain $G = \sqrt{\frac{\mathcal{P}_R}{\mathcal{P}_S\left|h_{SR}^{(\hat{I},\hat{J})}\right|^2 + c\sigma_R^2}}$, where $\mathcal{P}_R$ is the transmit power at R [15]. Here $c=1$ stands for the channel-assisted amplify-and-forward (CA-AF) relays, whereas $c=0$ stands for the ideal CA-AF relays which invert the channel gain of the previous hop regardless of its fading state. The received signal at D is given by

$$y_D = G Y_R h_{RD}^{(\hat{K},\hat{L})} + n_D, \tag{2}$$

where n_D is the AWGN at D satisfying $\mathcal{E}\left\{|n_D|^2\right\} = \sigma_D^2$. After carrying out some simple mathematical manipulations, the end-to-end SNR γ_{eq} can be written as

$$\gamma_{eq} = \frac{\hat{\gamma}_{SR}^{(max)} \hat{\gamma}_{RD}^{(max)}}{\hat{\gamma}_{SR}^{(max)} + \hat{\gamma}_{RD}^{(max)} + c}, \tag{3}$$

where $\hat{\gamma}_{SR}^{(max)} = \frac{\mathcal{P}_S\left|h_{SR}^{(\hat{I},\hat{J})}\right|^2}{\sigma_R^2}$ and $\hat{\gamma}_{RD}^{(max)} = \frac{\mathcal{P}_R\left|h_{RD}^{(\hat{K},\hat{L})}\right|^2}{\sigma_R^2}$. Note that $\hat{\gamma}_{SR}^{(max)}$ and $\hat{\gamma}_{RD}^{(max)}$ are the delayed versions of $\gamma_{SR}^{(max)}$ and $\gamma_{RD}^{(max)}$ by time delays τ_{SD} and τ_{RD}, respectively, where $\gamma_{SR}^{(max)} = \frac{\mathcal{P}_S\left|h_{SR}^{(I,J)}\right|^2}{\sigma_R^2}$ and $\gamma_{RD}^{(max)} = \frac{\mathcal{P}_R\left|h_{RD}^{(K,L)}\right|^2}{\sigma_D^2}$. Here, $(I,J) = \underset{1\le i\le N_r, 1\le j\le N_s}{\operatorname{argmax}} \left(\left|h_{SR}^{(i,j)}\right|\right)$ and $(K,L) = \underset{1\le k\le N_d, 1\le l\le N_r}{\operatorname{argmax}} \left(\left|h_{RD}^{(k,l)}\right|\right)$.

III. Performance Analysis

In this section, the statistics of γ_{eq} are derived in closed-forms and then used to obtain the outage probability, average SER, and the SNR moments. The high SNR analysis of these performance metrics is also derived.

A. Statistical characterization of the end-to-end SNR

The CDF of the end-to-end SNR γ_{eq} when the antennas at S and R are selected based on the outdated-CSI is given by (see the Appendix for the proof)

$$F_{\gamma_{\text{eq}}}(x) = 1 - \sum_{a=0}^{N_sN_r-1}\sum_{b=0}^{N_rN_d-1} \frac{(-1)^{a+b} N_s N_r^2 N_d \nu \binom{N_sN_r-1}{a}\binom{N_rN_d-1}{b}}{(a+1)(b+1)} \times \sqrt{x(x+c)} e^{-\mu x} \mathcal{K}_1\left(\nu\sqrt{x(x+c)}\right), \tag{4}$$

where $\mu = \frac{a+1}{\bar{\gamma}_{SR}(1+a(1-\rho_{SR}))} + \frac{b+1}{\bar{\gamma}_{RD}(1+b(1-\rho_{RD}))}$, $\nu = 2\sqrt{\frac{(a+1)(b+1)}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}(1+a(1-\rho_{SR}))(1+b(1-\rho_{RD}))}}$, and $\bar{\gamma}_{SR}$ and $\bar{\gamma}_{RD}$ are the average SNR of $\gamma_{SR}^{(max)}$ and $\gamma_{RD}^{(max)}$, respectively. Here, $0 \le \rho_{SR} \le 1$ and $0 \le \rho_{RD} \le 1$ are the correlation coefficients between ($h_{SR}^{(i,j)}$, $\hat{h}_{SR}^{(i,j)}$) and ($h_{RD}^{(k,l)}$, $\hat{h}_{RD}^{(k,l)}$), respectively[1]. When $\rho_{SR} = \rho_{RD} = 1$, the pairs ($h_{SR}^{(i,j)}$, $\hat{h}_{SR}^{(i,j)}$) and ($h_{RD}^{(k,l)}$, $\hat{h}_{RD}^{(k,l)}$) are perfectly correlated and the antennas at S and R are selected based on perfect CSI, and maximum achievable diversity gain is achieved. However, when $\rho_{SR} = \rho_{RD} = 0$, the pairs ($h_{SR}^{(i,j)}$, $\hat{h}_{SR}^{(i,j)}$) and ($h_{RD}^{(k,l)}$, $\hat{h}_{RD}^{(k,l)}$) are perfectly uncorrelated, and the antennas at S and R are selected arbitrarily, so no diversity gains are achieved.

The MGF of the end-to-end SNR is a useful statistic which can be used to analyze a wide range of performance metrics. The MGF of γ_{eq} for ideal CA-AF relays can be derived by substituting (4) with $c=0$ into $M_{\gamma_{\text{eq}}}(s) = \int_0^\infty sF_{\gamma_{\text{eq}}}(x)e^{-sx}\mathrm{d}x$, and evaluating the resulting integral by using [13, Eq. (6.621.3)] as follows:

$$M_{\gamma_{\text{eq}}}(s) = 1 - \sum_{a=0}^{N_sN_r-1}\sum_{b=0}^{N_rN_d-1} \frac{16(-1)^{a+b} N_s N_r^2 N_d \nu^2}{3(a+1)(b+1)} \times \binom{N_sN_r-1}{a}\binom{N_rN_d-1}{b} \frac{s\ {}_2\mathcal{F}_1\left(3, \frac{3}{2}; \frac{5}{2}; \frac{s+\mu-\nu}{s+\mu+\nu}\right)}{(s+\mu+\nu)^3}. \tag{5}$$

The PDF of γ_{eq} can easily be derived by differentiating the CDF of γ_{eq} with respect to x and by using $x\frac{\partial \mathcal{K}_\nu(x)}{\partial x} +$

[1] For the Clarke's fading model, $\rho_{SR} = \mathcal{J}_0(2\pi B_{f_{SR}}\tau_{SR})$ and $\rho_{RD} = \mathcal{J}_0(2\pi B_{f_{RD}}\tau_{RD})$, where $B_{f_{SR}}$ and $B_{f_{RD}}$ are the Doppler fading bandwidths.

$\nu\mathcal{K}_{\nu}(x) + x\mathcal{K}_{\nu-1}(x) = 0$ [13, Eq. (8.486.12)]. However, the PDF results are omitted for the sake of brevity.

B. Outage probability

The outage probability is the probability that the end-to-end instantaneous SNR falls below a predefined threshold γ_{th}. Hence, it is obtained as: $P_{\text{out}} = \Pr(\gamma_{\text{eq}} \leq \gamma_{th}) = F_{\gamma_{\text{eq}}}(\gamma_{th})$.

C. Average symbol error rate

The conditional error probability (CEP) $P_e|\gamma$, which is valid for a wide range of modulation schemes, can be written as $P_e|\gamma = \eta\mathcal{Q}\left(\sqrt{\zeta\gamma}\right)$, where η and ζ are constants dependent on the modulation scheme. Then the average SER is derived by integrating $P_e|\gamma$ over the PDF of the SNR; $\bar{P}_e = \frac{\eta}{2} - \frac{\eta}{2}\sqrt{\frac{\zeta}{2\pi}}\int_0^{\infty} x^{-\frac{1}{2}} e^{-\frac{\zeta x}{2}}(1-F_{\gamma_{\text{eq}}}(x))\mathrm{d}x$ [16]. By substituting (4) with $c = 0$ into $\bar{P}_e$ and evaluating the resulting integral by using [13, Eq. (6.621.3)], the average SER for the ideal CA-AF relays can be derived as follows:

$$\bar{P}_e = \frac{\eta}{2} - \sqrt{\frac{\zeta}{2}} \sum_{a=0}^{N_sN_r-1} \sum_{b=0}^{N_rN_d-1} \frac{3\pi\eta(-1)^{a+b}N_sN_r^2N_d\nu^2}{4(a+1)(b+1)} \times \binom{N_sN_r-1}{a}\binom{N_rN_d-1}{b} \frac{{}_2\mathcal{F}_1\left(\frac{5}{2},\frac{3}{2};2;\frac{\frac{\zeta}{2}+\mu-\nu}{\frac{\zeta}{2}+\mu+\nu}\right)}{\left(\frac{\zeta}{2}+\mu+\nu\right)^{\frac{5}{2}}}. \quad (6)$$

D. Moments of the SNR

The moments of the end-to-end SNR $\overline{\gamma_{\text{eq}}^n}$ are useful as signal quality measures and can be used as an alternative performance measure to error-rate analysis. The n-th SNR moment of γ_{eq} can be derived by using $\overline{\gamma_{\text{eq}}^n} = \mathcal{E}\{\gamma_{\text{eq}}^n\} = \int_0^{\infty} nx^{n-1}\left(1-F_{\gamma_{\text{eq}}}(x)\right)\mathrm{d}x$. By substituting (4) with $c = 0$ into $\overline{\gamma_{\text{eq}}^n}$ and evaluating the resulting integral by using [13, Eq. (6.621.3)], the n-th moment for the ideal CA-AF relays can be derived as follows:

$$\overline{\gamma_{\text{eq}}^n} = \sum_{a=0}^{N_sN_r-1} \sum_{b=0}^{N_rN_d-1} \frac{2n\sqrt{\pi}(-1)^{a+b}N_sN_r^2N_d\nu^2\Gamma(n)\Gamma(n+2)}{(a+1)(b+1)\Gamma(n+\frac{3}{2})} \times \binom{N_sN_r-1}{a}\binom{N_rN_d-1}{b} \frac{{}_2\mathcal{F}_1\left(n+2,\frac{3}{2};n+\frac{3}{2};\frac{\mu-\nu}{\mu+\nu}\right)}{(\mu+\nu)^{n+2}}. \quad (7)$$

E. High SNR Analysis

In this section, the asymptotic outage probability, the average SER, and the diversity and the coding gains are derived.

1) Asymptotic outage probability : The behavior of the CDF of γ_{eq} for a large $\bar{\gamma}_{SR}$ is equivalent to the behavior of $F_{\gamma_{\text{eq}}}(y)$ around $y = 0$ [17]. By substituting $\bar{\gamma}_{SR} = k\bar{\gamma}_{RD}$ into (4), and by expressing the exponential function and the Bessel function in terms of their Taylor series expansions around $y = 0$ [13, Eq. (1.211) and Eq. (8.446)], the expression with the first-order terms of y can be obtained. Then the asymptotic outage probability can be obtained by substituting $y = \frac{\gamma_{th}}{\bar{\gamma}_{SR}}$ as follows:

$$P_{\text{out}}^{\infty} = \begin{cases} \Omega_1\left(\frac{\gamma_{th}}{\bar{\gamma}_{SR}}\right)^M + o\left(\frac{\gamma_{th}}{\bar{\gamma}_{SR}}\right)^{M+1}, & \rho_{SR} = \rho_{RD} = 1 \\ \Omega_2\frac{\gamma_{th}}{\bar{\gamma}_{SR}} + o\left(\frac{\gamma_{th}}{\bar{\gamma}_{SR}}\right)^2, & 0 \leq \rho_{SR}, \rho_{RD} < 1, \end{cases} \quad (8)$$

where $\Omega_1 = \sum_{a=1}^{N_sN_r}\sum_{b=0}^{N_rN_d-1}\frac{(-1)^{a+b+L}N_rN_d}{(b+1)\,L!}\binom{N_sN_r}{a}\binom{N_rN_d-1}{b}\left(\frac{ak+b+1}{k}\right)^M$, $\Omega_2 = \sum_{a=0}^{N_sN_r-1}\sum_{b=0}^{N_rN_d-1}\frac{(-1)^{a+b}N_sN_r^2N_d}{(a+b)(b+1)}\binom{N_sN_r-1}{a}\binom{N_rN_d-1}{b}\left(\frac{a+1}{1+a(1-\rho_{SR})}+\frac{b+1}{k(1+b(1-\rho_{RD}))}\right)$ and $M = N_r\min(N_s, N_d)$.

2) Asymptotic average SER: The average SER at high SNR can be derived by substituting (8) into the integral representation of SER in Section III-C and then solving it as follows:

$$\bar{P}_e^{\infty} = \begin{cases} \frac{\eta\Omega_1\Gamma\left(M+\frac{1}{2}\right)}{2\sqrt{\pi}\bar{\gamma}_{SR}^M}\left(\frac{2}{\zeta}\right)^{M-\frac{1}{2}} + o\left(\frac{1}{\bar{\gamma}_{SR}}\right)^{M+1}, & \rho_{SR} = \rho_{RD} = 1 \\ \frac{\eta\Omega_2}{4\bar{\gamma}_{SR}}\sqrt{\frac{2}{\zeta}} + o\left(\frac{1}{\bar{\gamma}_{SR}}\right)^2, & 0 \leq \rho_{SR}, \rho_{RD} < 1. \end{cases} \quad (9)$$

In the high SNR regime, the average SER can be represented by $P_e^{\infty} \approx [G_c\bar{\gamma}]^{-G_d}$, where G_d and G_c are referred to as the diversity order and coding gain, respectively [17]. Accordingly, by using $\bar{P}_e^{\infty}$ in (9), G_d and G_c can easily be obtained. When $\rho_{SR} = \rho_{RD} = 1$, the diversity order is given by $G_d = N_r\min(N_s, N_d)$, which is the maximum achievable diversity order for a dual-hop MIMO relay network. However, if either ρ_{SR} or ρ_{RD} is not equal to one, then no diversity gain can be achieved.

IV. Numerical Results

This section presents the numerical and Monte-Carlo simulation results in order to analyze the system performance and to validate our analytical results. The feedback delays τ_{SR} and τ_{RD} at the $R \to S$ and $D \to R$ feedback channels are related to ρ_{SR} and ρ_{RD} by following Clarke's fading model; $\rho_{SR} = \mathcal{J}_0(2\pi B_{f_{SR}}\tau_{SR})$ and $\rho_{RD} = \mathcal{J}_0(2\pi B_{f_{RD}}\tau_{RD})$ [5].

1) The impact of feedback delays on the outage probability: Fig. 2 shows the impact of feedback delays on the outage probability for the best Tx/Rx antenna pair selection of MIMO AF relay networks with $N_s = 3$, $N_r = 2$ and $N_d = 3$. The outage probability is plotted for several feedback delay scenarios by changing ρ_{SR} and ρ_{RD}. The outage curve with $\rho_{SR} = \rho_{RD} = 1$ corresponds to the best case (i.e., antenna selection with perfect CSI) whereas the curve with $\rho_{SR} = \rho_{RD} = 0$ corresponds to the worst case (i.e., arbitrary antenna selection). As ρ_{SR} and ρ_{RD} decrease from 1 to 0 (i.e., as feedback delay increases), the performance of Tx/Rx antenna selection degrades significantly. Further, the outage curves for the partially outdated CSI ($0 < \rho_{SR}, \rho_{RD} < 1$) are plotted as well. The asymptotic outage curves are also plotted to investigate the diversity gain of the system. Our asymptotic analysis shows that the Tx/Rx antenna selection based on the perfect CSI achieves the full diversity available in the MIMO relay channel; $G_d = N_r\min(N_s, N_d)$. However, when the antennas are selected based on the outdated CSI, this diversity gain decreases to one. The exact match between the Monte-Carlo simulation points and the analytical results verify the accuracy of our analysis.

2) The impact of feedback delays on the average BER: In Fig. 3, the average BER of the binary phase shift skying (BPSK) is plotted for several special cases of time delays on the feedback channels $R \to S$ and $D \to R$. The asymptotic

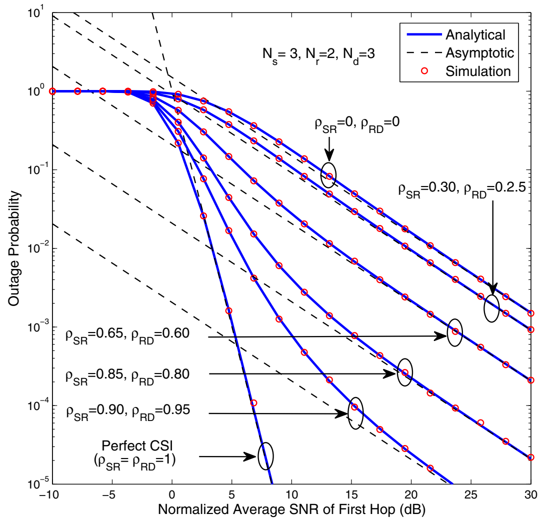

Fig. 2. The impact of feedback delays on the outage probability of dual-hop MIMO AF relaying with the best Tx/Rx antenna pair selection. $\bar{\gamma}_{SR} = 2\bar{\gamma}_{RD}$.

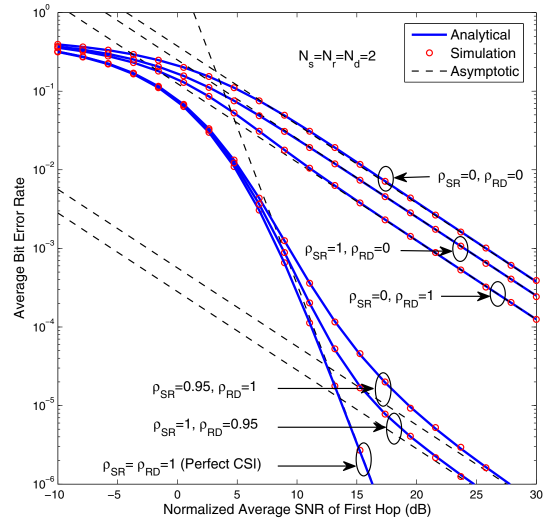

Fig. 3. The impact of feedback delays on the average BER of dual-hop MIMO AF relaying with the best Tx/Rx antenna pair selection. $\bar{\gamma}_{SR} = 2\bar{\gamma}_{RD}$.

average BER curves are also plotted to provide insights on the diversity and coding gains. This figure shows that if a slight feedback delay exists in either hop, then the achievable diversity order diminishes to one. This result is clearly revealed by the curves corresponding to ($\rho_{SR} = 1,\ \rho_{RD} = 0.95$) and ($\rho_{SR} = 0.95,\ \rho_{RD} = 1$), respectively, where one hop has no feedback delays, and the other hop has a slight feedback delay. Fig. 3 thus reveals that Tx/Rx antenna selection based on outdated CSI degrades the system performance considerably. Nevertheless, our asymptotic analysis shows that although diversity gains are non-existent, antenna selection based on partial outdated CSI achieves significant coding gains over that based on the fully outdated CSI. For example, the coding gain achieved by Tx/Rx antenna selection by using feedback channels with (ρ_{SR}=1, $\rho_{SR} = 0.95$) over ($\rho_{SR} = 0, \rho_{SR} = 0$) is about 25 dB.

In Fig. 4, the average BER of BPSK is potted against the correlation coefficient ($\rho_{SD} = \rho_{RD} = \rho$) for four different antenna configurations. As ρ decreases from one to zero (i.e., as the feedback delay increases), the BER performance degrades significantly since the antennas are then selected based on the severe outdated CSI. Fig. 4 also reveals that despite the detrimental effect of outdated CSI on the antenna selection, considerable BER improvements can be achieved by increasing the number of antennas at each terminal. Further, the Monte-Carlo simulations validate our analysis.

3) The impact of feedback delays on the SNR moments: Fig. 5 shows the detrimental effect of Tx/Rx antenna selection based on the outdated CSI on the SNR moments. The first and the second moments of the SNR are plotted against the correlation coefficient ($\rho_{SD} = \rho_{RD} = \rho$) for three antenna set-ups. As feedback delay increases (i.e., as ρ decreases), the SNR moments decrease significantly. Fig. 5 also reveals that the impact of the feedback delays is less pronounced for the first moment than that for the second moment.

V. Conclusion

The impact of outdated CSI on the performance of the best Tx/Rx antenna pair selection for dual-hop MIMO AF relay networks was studied. New closed-form expressions for the CDF and the MGF of the end-to-end SNR were derived. The outage probability, the SNR moments and the average SER which are valid for a wide range of modulation schemes were also presented. Moreover, the accurate asymptotic outage probability and the average SER expressions parameterized by the number of antennas at S, R and D, and the correlation coefficients ρ_{SR} and ρ_{RD} were also derived to quantifying the diversity order and coding gain. Our analytical results were verified by comparing them with Monte-Carlo simulation results. Our results reveal that delays in either the $R \rightarrow S$ or the $D \rightarrow R$ feedback channels have a significant detrimental impact on the performance of dual-hop MIMO relay networks with antenna selection. Our analysis can thus be useful for practical MIMO AF relay network designing, and the performance metrics can be used as benchmarks for such networks.

VI. Appendix

The CDF of γ_{eq} in (3) can be derived by using [16] $F_{\gamma_{eq}}(x) = 1 - \int_0^{\infty} \bar{F}_{\hat{\gamma}_{SR}^{(max)}}\left(\frac{(x+z+c)x}{z}\right) f_{\hat{\gamma}_{RD}^{(max)}}(z+x)\mathrm{d}z$, where $\bar{F}_{\hat{\gamma}_{SR}^{(max)}}(x)$ is the complementary cumulative distribution function (CCDF) of $\hat{\gamma}_{SR}^{(max)}$, and $f_{\hat{\gamma}_{RD}^{(max)}}(x)$ is the PDF of $\hat{\gamma}_{RD}^{(max)}$. In order to derive the CDF of γ_{eq}, one needs to obtain the distribution functions of $\hat{\gamma}_{SR}^{(max)}$ and $\hat{\gamma}_{RD}^{(max)}$. They are, in fact, the induced order statistics of the original ordered $\gamma_{SR}^{(max)}$ and $\gamma_{RD}^{(max)}$ [18]. The PDF of $\hat{\gamma}_{SR}^{(max)}$ is given by [8]

$$f_{\hat{\gamma}_{SR}^{(max)}}(x) = \int_0^{\infty} f_{\hat{\gamma}_{SR}^{(max)}\left|\gamma_{SR}^{(max)}\right.}(x|\,y)\, f_{\gamma_{SR}^{(max)}}(y)\,\mathrm{d}y, \quad (10)$$

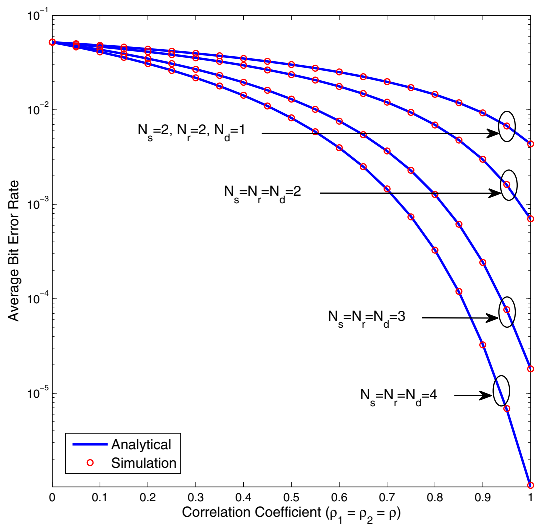

Fig. 4. The average BER vs. the correlation coefficient for dual-hop MIMO AF relaying with the best Tx/Rx antenna pair selection. $\bar{\gamma}_{SR} = \bar{\gamma}_{RD} = 10\,\text{dB}$.

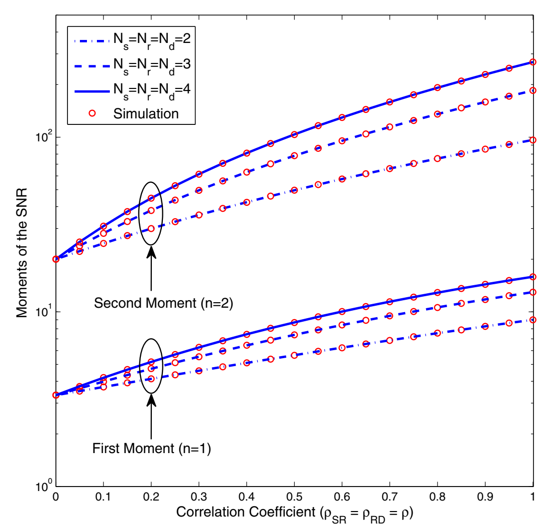

Fig. 5. The SNR moments vs. the correlation coefficient for dual-hop MIMO AF relaying with the best Tx/Rx antenna pair selection. $\bar{\gamma}_{SR} = \bar{\gamma}_{RD} = 10\,\text{dB}$.

where $f_{\hat{\gamma}_{SR}^{(max)} | \gamma_{SR}^{(max)}}(x|\,y) = \frac{f_{\hat{\gamma}_{SR}^{(i,j)},\gamma_{SR}^{(i,j)}}(x,y)}{f_{\gamma_{SR}^{(i,j)}}(y)}$ is the PDF of $\hat{\gamma}_{SR}^{(max)}$ conditioned on $\gamma_{SR}^{(max)}$. Because $\hat{\gamma}_{SR}^{(i,j)}$ and $\gamma_{SR}^{(i,j)}$ are two correlated exponential distributed random variables, their joint PDF can be written as [8]

$$f_{\hat{\gamma}_{SR}^{(i,j)},\gamma_{SR}^{(i,j)}}(x,y) = \frac{e^{-\frac{x+y}{(1-\rho_{SR})\bar{\gamma}_{SR}}}}{(1-\rho_{SR})\,\bar{\gamma}_{SR}^2}\mathcal{I}_0\!\left(\frac{2\sqrt{xy\rho_{SR}}}{(1-\rho_{SR})\,\bar{\gamma}_{SR}}\right). \quad (11)$$

The PDF of $\gamma_{SR}^{(max)}$ is given by $f_{\gamma_{SR}^{(max)}}(y) = N_s N_r \left[F_{\gamma_{SR}^{(i,j)}}(y)\right]^{N_s N_r - 1} f_{\gamma_{SR}^{(i,j)}}(y)$, where $f_{\gamma_{SR}^{(i,j)}}(y) = \frac{1}{\bar{\gamma}_{SR}} e^{-\frac{y}{\bar{\gamma}_{SR}}}$ and $F_{\gamma_{SR}^{(i,j)}}(y) = 1 - e^{-\frac{y}{\bar{\gamma}_{SR}}}$. By substituting (11) into (10) and solving the resulting integral by using [14, Eq. (29.3.81)], the PDF of $\hat{\gamma}_{SR}^{(max)}$ can be obtained. After differentiating it with respect to x, the CCDF of $\hat{\gamma}_{SR}^{(max)}$ can be derived as follows:

$$\bar{F}_{\hat{\gamma}_{SR}^{(max)}}(x) = \sum_{a=0}^{N_s N_r - 1} \frac{(-1)^a N_s N_r \binom{N_s N_R - 1}{a}}{a+1} e^{\frac{-(a+1)x}{\bar{\gamma}_{SR}(1+a(1-\rho_{SR}))}}. \quad (12)$$

By following similar steps, the PDF of $\hat{\gamma}_{RD}^{(max)}$ can be derived as

$$f_{\hat{\gamma}_{RD}^{(max)}}(x) = \sum_{b=0}^{N_r N_d - 1} \frac{(-1)^b N_r N_d \binom{N_r N_d - 1}{b}}{\bar{\gamma}_{RD}(1+b\,(1-\rho_{RD}))} e^{\frac{-(b+1)x}{\bar{\gamma}_{RD}(1+b(1-\rho_{RD}))}}. \quad (13)$$

Now, $F_{\gamma_{eq}}(x)$ given in (4) can be derived by substituting (12) and (13) into the integral representation of $F_{\gamma_{eq}}(x)$ and evaluating the resulting integral by using [13, Eq. (3.471.9)].

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