

# Performance of Energy Detection: A Complementary AUC Approach

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**Abstract**—This paper<sup>1</sup> investigates detection capability of energy detectors. With the help of receiver operating characteristics (ROC) curve and area under the ROC curve (AUC), a new measure, *Complementary AUC (CAUC)*, is introduced as a proxy for the overall detection capability. When relays are available to help forward the target signal, the upper bound of the CAUC under Rayleigh fading channels is derived without and with a direct path. In addition, the average CAUC is discussed for Nakagami- $m$  fading channels without and with diversity combining. The analytical results are validated by numerical examples.

**Index Terms**—Area under the curve, energy detection, receiver operating characteristics (ROC), relay.

## I. INTRODUCTION

An *energy detector* is a device that can detect the presence of a signal under noisy environments. Energy detection of the presence/absence of a signal is feasible even when little knowledge of the parameters of the signal is available. A conventional energy detector measures the energy of the received signal over a specified time period and a bandwidth. The energy is then compared with an appropriately selected threshold to determine the presence or absence of the unknown signal. With low complexity and no requirement for knowledge of the signal, energy detection has gained renewed interest recently for cognitive radio networks [1]-[3], ultra-wideband communications, and sensor networks [4].

The first comprehensive analysis for the analog energy detector has been introduced in [5] by assuming deterministic signal transmission over a flat band-limited Gaussian noise channel. The receiver operating characteristics (ROC) curve, i.e., the detection probability versus false alarm probability curve, is often used to illustrate and to quantify the detection capability of the energy detector. Based on [5], the ROC analysis over Rayleigh, Rician, Nakagami- $m$ , and  $\eta$ - $\mu$  channel models is discussed in [6]-[9]. Furthermore, diversity reception techniques such as equal gain combining (EGC), selection combining (SC), maximal ratio combining (MRC), square law combining (SLC), and square law selection (SLS), which are used to boost the performance of the energy detector, are discussed in [7]-[11]. The ROC analysis reveals that MRC improves the performance of the energy detector the most among all the diversity reception techniques.

Although the ROC curves fully characterize the performance of an energy detector, it is difficult to compare performance of two energy detectors based on visual perception of their ROC curves, because the curves may cross each other. Therefore, as a desirable measurement to see the overall detection capability, *area under the ROC curve (AUC)* is introduced recently to the wireless communication field as one possible technique to address this problem [12]. AUC clearly shows how the overall detection capability varies with system parameters such as the number of samples, multipath fading parameter, the number of diversity branches, channel estimation error, and channel correlation. However, neither the ROC curve nor the AUC curve is able to show the order of improvement in detection capability when the average signal-to-noise ratio (SNR) increases. On the other hand, using the asymptotic analysis, the impact of channel fading and spatial diversity is quantified by *sensing gain* [13], [14]. The sensing gain is a measurement for detection capability, but not for *overall* detection capability, because it does not characterize the false alarm probability. Therefore, in this paper, *Complementary AUC (CAUC)* is introduced as a measurement for the overall detection capability, which can demonstrate the order of improvement based on a log-log scale when average SNR increases.

This rest of the paper is organized as follows. The system model and CAUC are described in Section II. The average CAUCs for a relay network and for some combining schemes are analyzed with numerical results in Section III and Section IV, respectively. The concluding remarks are made in Section V.

## II. SYSTEM MODEL

### A. Energy Detection

When the primary signal is  $\mathbf{x}(t)$ , the received signal at the receiver,  $\mathbf{y}(t)$ , can be written under two hypotheses:  $\mathcal{H}_0$  (primary signal absent) and  $\mathcal{H}_1$  (primary signal present), as

$$\mathbf{y}(t) = \begin{cases} \mathbf{w}(t) & : \mathcal{H}_0 \\ \mathbf{h}\mathbf{x}(t) + \mathbf{w}(t) & : \mathcal{H}_1 \end{cases} \quad (1)$$

where  $\mathbf{w}(t)$  is the additive white Gaussian noise (AWGN), which is assumed to be a circularly symmetric complex Gaussian random variable with mean zero and one-sided power spectral density  $N_0$  (i.e.  $w(t) \sim \mathcal{CN}(0, N_0)$ ), and  $\mathbf{h}$  is the

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wireless channel gain. The channel gains are modeled as independent and not necessarily identically distributed complex Gaussian fading channels.

After proper filtering, sampling, squaring and integration, the test statistic of an energy detector is  $T_y = \sum_{i=1}^u |y(i)|^2$ , where  $u$  is the number of complex signal samples. As described in [5], [7], the probability density function (PDF) of  $T_y$  follows a central chi-square distribution with  $2u$  degrees of freedom (DoF) under  $\mathcal{H}_0$ , or a noncentral chi-square distribution with  $2u$  DoF and a noncentrality parameter  $2\gamma$  (where  $\gamma = \frac{E_s |h|^2}{N_0}$  and  $E_s$  is signal power at transmitter side) under  $\mathcal{H}_1$ . The test statistic,  $T_y$ , is compared with a predefined threshold value  $\lambda$ . The probabilities of false alarm  $P_f(\lambda)$ , and detection  $P_d(\gamma, \lambda)$  can be evaluated as  $Pr(T_y > \lambda | \mathcal{H}_0)$  and  $Pr(T_y > \lambda | \mathcal{H}_1)$ , respectively, to yield<sup>2</sup> [7]

$$P_f(\lambda) = \frac{\Gamma(u, \frac{\lambda}{2})}{\Gamma(u)} \quad (2)$$

$$P_d(\gamma, \lambda) = Q_u(\sqrt{2\gamma}, \sqrt{\lambda}), \quad (3)$$

where  $Q_u(\cdot, \cdot)$  is the generalized Marcum- $Q$  function,  $\Gamma(\cdot)$  is the gamma function and  $\Gamma(\cdot, \cdot)$  is the upper incomplete gamma function. Further, missed detection probability,  $P_m(\gamma, \lambda)$ , can be calculated as  $P_m(\gamma, \lambda) = 1 - P_d(\gamma, \lambda)$ .

### B. Complementary Area Under ROC Curve (CAUC)

Generally, for any instantaneous SNR value  $\gamma$ , as the threshold  $\lambda$  in the energy detection varies from  $\infty$  to 0, the false alarm and the detection probabilities vary from value 0 to value 1. The AUC is defined as the area covered by the curve of  $P_d(\gamma, \lambda)$  versus  $P_f(\lambda)$ . Therefore, the AUC is a function of  $\gamma$ , denoted  $A(\gamma)$ , which can be calculated by the threshold averaging method using (2) and (3) [15], as follows. When the value of  $P_f(\lambda)$  varies from  $0 \rightarrow 1$ , it is equivalent to  $\lambda$  ranging from  $\infty \rightarrow 0$ . Therefore,  $A(\gamma)$  can be written as

$$A(\gamma) = - \int_0^\infty P_d(\gamma, \lambda) \frac{\partial P_f(\lambda)}{\partial \lambda} d\lambda \quad (4)$$

where  $\frac{\partial P_f(\lambda)}{\partial \lambda}$  is the partial derivative of  $P_f(\lambda)$  with respect to  $\lambda$ . Since  $P_d(\gamma, \lambda) = 1 - P_m(\gamma, \lambda)$ , (4) can be re-written as

$$A(\gamma) = - \int_0^\infty \frac{\partial P_f(\lambda)}{\partial \lambda} d\lambda + \int_0^\infty P_m(\gamma, \lambda) \frac{\partial P_f(\lambda)}{\partial \lambda} d\lambda. \quad (5)$$

From (2), the first integral in (5) is equal to -1. The second integral in (5) can be written as  $\int_1^0 P_m(\gamma, \lambda) dP_f(\lambda)$ , which is defined as the CAUC, i.e., the area under the complementary ROC curve (the curve of  $P_m(\gamma, \lambda)$  versus  $P_f(\lambda)$ ). Therefore, we have

$$CAUC = 1 - AUC. \quad (6)$$

CAUC for instantaneous SNR value  $\gamma$ ,  $A'(\gamma)$ , can be evaluated in closed-form, with the aid of the AUC expression given in

[12, eq. 9], as

$$A'(\gamma) = \sum_{k=0}^{u-1} \frac{1}{2^k k!} \gamma^k e^{-\frac{\gamma}{2}} - \sum_{k=1-u}^{u-1} \frac{\Gamma(u+k)}{2^{u+k} \Gamma(u)} e^{-\gamma} {}_1\tilde{F}_1\left(u+k; 1+k; \frac{\gamma}{2}\right) \quad (7)$$

where  ${}_1\tilde{F}_1(\cdot; \cdot; \cdot)$  is the regularized confluent hypergeometric function of the confluent hypergeometric function  ${}_1F_1(\cdot; \cdot; \cdot)$  [16]. Note that Eq. (7) gives the CAUC of an energy detector for a specific value of instantaneous SNR  $\gamma$ . Therefore,  $A'(\gamma)$  is defined as *unfaded CAUC*. The average CAUC in closed-form under the AWGN channel can be found from expression (7) after replacing  $\gamma$  by  $\bar{\gamma}$ , where  $\bar{\gamma}$  is the average SNR.

In the subsequent two sections, we study the average CAUC in Rayleigh fading relay channels and in Nakagami- $m$  fading channels.

## III. AVERAGE CAUC WITH RELAYS

Similar to [17], we consider a dual-hop relay-based spectrum sensing in a cognitive radio network. Note that we use fixed-gain relays and SLC in contrast to variable gain relays and MRC used in [17]. Their use reduces the system complexity. The relay scheme is summarized as follows.

### A. Relay Scheme

A number,  $n$ , of relays are utilized to forward the primary user signal to a fusion center. Instead of making individual hard decision about the presence or absence of the primary user, the relays simply amplify and retransmit the noisy version of the received signals to the fusion center. In this research, we use the fixed-gain relay which has amplification factor  $G$  given as  $G^2 = \frac{E_r}{CN_0}$  [18] where  $E_r$  is the power of transmitted signal at the output of the relay and  $C$  is a constant for a fixed  $G$ . Each communication between a relay and the fusion center occurs in an orthogonal channel to avoid interference. The fusion center is equipped with an energy detector which compares the received signal strength with a pre-defined threshold.

First we consider the  $i$ th relay channel. The received signal at the fusion center's front end can be formulated as two hypotheses given in (1) with  $\mathbf{h} = G\mathbf{h}_{pr_i}\mathbf{h}_{r_i,d}$  and  $\mathbf{w}(t) = G\mathbf{h}_{r_i,d}\mathbf{w}_{r_i}(t) + \mathbf{w}_d(t)$ , where  $\mathbf{h}_{pr_i}$  and  $\mathbf{h}_{r_i,d}$  are channel gains from the primary user to the  $i$ th relay and from the  $i$ th relay to the destination (the fusion center), respectively, and  $\mathbf{w}_{r_i}$  and  $\mathbf{w}_d$  are AWGN at the  $i$ th relay and the fusion center, respectively. Thus, the end-to-end SNR,  $\gamma_i$ , under  $\mathcal{H}_1$  is  $\gamma_i = \frac{\gamma_{pr_i}\gamma_{r_i,d}}{C+\gamma_{r_i,d}}$  where  $\gamma_{pr_i} = \frac{|\mathbf{h}_{pr_i}|^2 E_s}{N_0}$  and  $\gamma_{r_i,d} = \frac{|\mathbf{h}_{r_i,d}|^2 E_r}{N_0}$  are SNRs of the links from the primary user to the  $i$ th relay and from the  $i$ th relay to the fusion center, respectively.

In a multiple relay network, we can consider any combining method at the receiver. Although MRC is one of the popular methods in relay networks, it requires the fusion center should know channel state information (CSI) of all channels in the first and the second hops. On the other hand, CSI may not

<sup>2</sup>Note that the two probabilities are also functions of  $u$ .

be available for energy detection (which is non-coherent). In contrast to MRC, receiver with SLC (which is a non-coherent combiner) does not need instantaneous CSI of the channels in the first hop, and consequently results in a low complexity system. However, CSI of channels in the second hop should be available at the filters for proper noise normalization. It is an unavoidable requirement in non-regenerative (data fusion) receiver structure. The number  $n$  of outputs from all the branches in the SLC, denoted  $\{\mathbf{y}_i\}_{i=1}^n$ , are combined to form the decision statistic  $T_{SLC}$ . Under AWGN channels,  $T_{SLC}$  follows a central chi-square distribution with  $nu$  DoF under  $\mathcal{H}_0$ , and a non-central chi-square distribution with  $nu$  DoF under  $\mathcal{H}_1$ . Further, effective SNR after combiner is  $\gamma_{SLC} = \sum_{i=1}^n \gamma_i$  where  $\gamma_i$  is the equivalent SNR of the  $i$ th relay path. The non-centrality parameter under  $\mathcal{H}_1$  is  $2\gamma_{SLC}$ . The false alarm and detection probabilities can be calculated using (2) and (3), respectively, by replacing  $u$  by  $nu$  and  $\gamma$  by  $\gamma_{SLC}$ .

### B. Upper Bound for Average CAUC

The exact average CAUC,  $\bar{A}'$ , can be evaluated as

$$\bar{A}' = \int_0^\infty A'(x) f_{\gamma_{SLC}}(x) dx. \quad (8)$$

But a closed-form solution for  $\bar{A}'$  seems analytically difficult with PDF of  $\gamma_{SLC}$ . Therefore, we resort to the upper bound of the average CAUC instead.

The total SNR  $\gamma_{SLC}$  is upper bounded by  $\gamma_{up}$  as  $\gamma_{SLC} \leq \gamma_{up} = \sum_{i=1}^n \gamma_i^{\min}$ , where  $\gamma_i^{\min} = \min(\gamma_{pr_i}, \gamma_{r_i d})$ . This is popular and a tight upper bound for relay network. The tightness is well studied for error rate and energy detection [19], [17]. For independent channels, moment generating function (MGF) of  $\gamma_{up}$  can be written as  $\mathcal{M}_{\gamma_{up}}(s) = \prod_{i=1}^n \mathcal{M}_{\gamma_i^{\min}}(s)$ . Further, when MGF is defined as  $\mathcal{M}_{\gamma}(s) = \mathbb{E}(e^{-\gamma s})$ , we have  $\mathcal{M}_{\gamma_i^{\min}}(s) = \frac{\Omega_i}{s + \Omega_i}$  where  $\Omega_i = \frac{\gamma_{pr_i} + \gamma_{r_i d}}{\gamma_{pr_i} \gamma_{r_i d}}$  and  $\mathbb{E}(\cdot)$  is the expectation operator. For identically distributed channels (i.e.  $\Omega_i = \Omega, \forall i$ ), with aid of the inverse Laplace transform of  $\mathcal{M}_{\gamma_{up}}(s)$ , PDF of  $\gamma_{up}$  can be found as  $f_{\gamma_{up}}(x) = \mathcal{L}^{-1}[\mathcal{M}_{\gamma_{up}}(s)]$  to yield

$$f_{\gamma_{up}}(x) = \frac{\Omega^n}{\Gamma(n)} x^{n-1} e^{-\Omega x}, \quad x \geq 0. \quad (9)$$

Therefore, the average upper bounded CAUC,  $\bar{A}'_{up}$ , can be evaluated as  $\int_0^\infty A'(x) f_{\gamma_{up}}(x) dx$  to yield

$$\begin{aligned} \bar{A}'_{up} &= \frac{\Omega^n}{\Gamma(n)} \left[ \sum_{k=0}^{u-1} \frac{\Gamma(k+n)}{2^k k! (\Omega + \frac{1}{2})^{k+n}} \right. \\ &\quad \left. - \sum_{k=1-u}^{u-1} \frac{\Gamma(u+k) \Gamma(n) {}_2\tilde{F}_1\left(n, u+k; 1+k; \frac{1}{2(\Omega+1)}\right)}{2^{u+k} \Gamma(u) (\Omega+1)^n} \right] \end{aligned} \quad (10)$$

where  ${}_2\tilde{F}_1(\cdot, \cdot; \cdot; \cdot)$  is the regularized confluent hypergeometric function of the confluent hypergeometric function  ${}_2F_1(\cdot, \cdot; \cdot; \cdot)$ . Note that [20, eq. 3.351.3], [12, eq. 29], and [16, eq. 07.21.21.0004.01] are used to derive (10).

### C. Incorporation with a Direct Link

In preceding subsections, the fusion center receives only signals coming from relays. If the primary user is close to the fusion center, the fusion center can have a strong direct link from the primary user. As the direct link improves the performance of the wireless network in terms of the error rate, we can expect better overall detection performance of an energy detector. The direct signal,  $\mathbf{h}_{pd}\mathbf{x}(t) + \mathbf{w}_d(t)$ , can also be combined at SLC together with relayed signals. Here  $\mathbf{h}_{pd}$  is the channel gain of the primary user to the fusion center. Then the total SNR at the fusion center,  $\gamma^\dagger$ , can be upper bounded as

$$\gamma^\dagger \leq \gamma^\dagger_{up} = \gamma_d + \sum_{i=1}^n \gamma_i^{\min}, \quad (11)$$

where  $\gamma_d = \frac{|\mathbf{h}_{pd}|^2 E_s}{N_0}$  is the instantaneous SNR of the direct path. Therefore, the corresponding MGF of  $\gamma^\dagger_{up}$ , assuming independent fading channels, can be written as  $\mathcal{M}_{\gamma^\dagger_{up}}(s) = \mathcal{M}_{\gamma_d}(s) \prod_{i=1}^n \mathcal{M}_{\gamma_i^{\min}}(s)$  where  $\mathcal{M}_{\gamma_d}(s)$  is given by  $\frac{\Omega_d}{(\Omega_d + s)}$ , and  $\Omega_d = \frac{1}{\mathbb{E}(\gamma_d)}$ . For identically distributed relay channels (i.e.  $\Omega_i = \Omega, \forall i$ ), the PDF of  $\gamma^\dagger_{up}$  can be derived with the inverse Laplace transform of  $\mathcal{M}_{\gamma^\dagger_{up}}(s)$  as

$$f_{\gamma^\dagger_{up}}(x) = \frac{\Omega^n \Omega_d e^{-\Omega_d x}}{(\Omega - \Omega_d)^n} \left[ 1 - \frac{\Gamma(n, (\Omega - \Omega_d)x)}{\Gamma(n)} \right] \quad (12)$$

where  $x \geq 0$  and  $\Omega \neq \Omega_d$ . Further, (12) can be re-written as

$$f_{\gamma^\dagger_{up}}(x) = \frac{\Omega^n \Omega_d}{(\Omega - \Omega_d)^n} \left[ e^{-\Omega_d x} - \sum_{j=1}^n \frac{e^{-\Omega_d x} x^{j-1}}{(j-1)! (\Omega - \Omega_d)^{1-j}} \right]. \quad (13)$$

With the aid of [20, eq. 3.351.3], [12, eq. 29], and [16, eq. 07.21.21.0004.01], the average upper bounded CAUC,  $\bar{A}'_{up^\dagger}$ , can be evaluated as  $\int_0^\infty A'(x) f_{\gamma^\dagger_{up}}(x) dx$  to yield expression in (14) on the next page.

### D. Numerical Results

In the numerical results, channels (i.e., from the primary user to a relay, from a relay to the fusion center, and from the primary user to the fusion center) are independent and identically distributed (i.i.d.) with the average SNR  $\bar{\gamma} = 5$  dB. And  $u = 2$ .

Fig. 1 shows the analytical results for average CAUC for a relay network with  $n$  relays and the direct path under Rayleigh fading channels. The analytical results are based on (10) and (14). When the number of relays increases, the average CAUC decreases, which means the overall detection capability increases. Two curves with legend ‘‘without direct path ( $n = 1$ )’’ and ‘‘only direct path’’ also show that the direct path gives a major impact on the detection performance. Due to the noise amplification at the relay, single relay network, i.e., ‘‘without direct path ( $n = 1$ )’’, has lower end-to-end SNR than the direct communication, i.e., ‘‘only direct path’’.

However, Fig. 1 does not illustrate the diversity effect clearly. From this figure, one may (mistakenly) think that the different detection capabilities are due to end-to-end SNR

$$\begin{aligned} \overline{A'_{up\dagger}} = & \frac{\Omega^n \Omega_d}{(\Omega - \Omega_d)^n} \left[ \sum_{i=0}^{u-1} \frac{1}{2^i i!} \left[ \frac{\Gamma(i+1)}{(\Omega_d + \frac{1}{2})^{i+1}} - \sum_{j=1}^n \frac{\Gamma(i+j)}{(j-1)! (\Omega - \Omega_d)^{j-1} (\Omega + \frac{1}{2})^{i+j}} \right] \right. \\ & \left. - \sum_{i=1-u}^{u-1} \frac{\Gamma(u+i)}{2^{u+i} \Gamma(u)} \left[ \frac{{}_2\tilde{F}_1\left(1, u+i; 1+i; \frac{1}{2(\Omega_d+1)}\right)}{(\Omega_d+1)} - \sum_{j=1}^n \frac{\Gamma(j) {}_2\tilde{F}_1\left(j, u+i; 1+i; \frac{1}{2(\Omega_d+1)}\right)}{(j-1)! (\Omega - \Omega_d)^{1-j} (\Omega+1)^j} \right] \right]. \end{aligned} \quad (14)$$

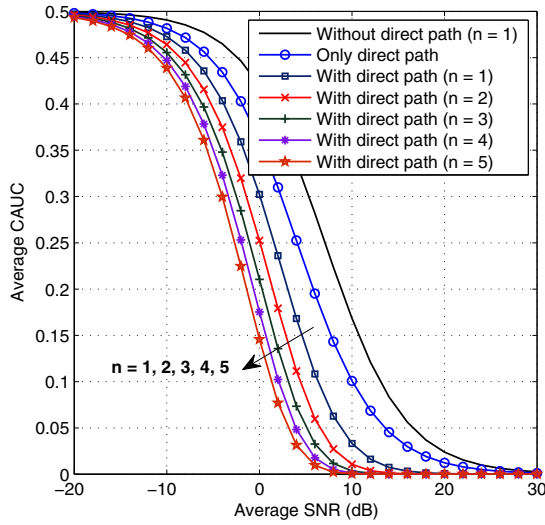


Fig. 1. Average CAUC versus average SNR for different number of relays in semi-log scale.

differences as the only reason. Actually, for high SNRs, (10) and (14) can be approximated as  $\overline{A'_{up}} \approx g_{up}(u, n) \bar{\gamma}^{-(n)}$  and  $\overline{A'_{up\dagger}} \approx g_{up\dagger}(u, n) \bar{\gamma}^{-(1+n)}$  where  $g_{up}(u, n)$  and  $g_{up\dagger}(u, n)$  are constant for given  $u$  and  $n$ . From these approximations, it can be seen that the CAUC decreases according to the  $n$  and  $(n+1)$  orders of the average SNR in the cases without and with direct path, respectively. Therefore, we say the detection diversity gain orders are  $n$  and  $(n+1)$  for the two cases. The detection diversity gain order can be demonstrated clearly if we plot Fig. 1 in log-log scale, as shown in Fig. 2. In Fig. 2, the curves “without direct path ( $n=1$ )” and “only direct path” are parallel to each other when the average SNR is high. This means the detection diversity gain orders are the same (both are equal to one). The gap between the two curves is because of the different end-to-end SNR values. On the other hand, the other five curves (with direct path) have different slopes, which means the detection diversity gain order is different, varying from 2 to 6.

#### IV. AVERAGE CAUC IN NAKAGAMI- $m$ FADING CHANNELS

The average CAUC for Nakagami- $m$  fading channels without diversity reception,  $\overline{A'_{Nak}}$ , can be obtained as (15) on the next page with the aid of (6) and average AUC for Nakagami-

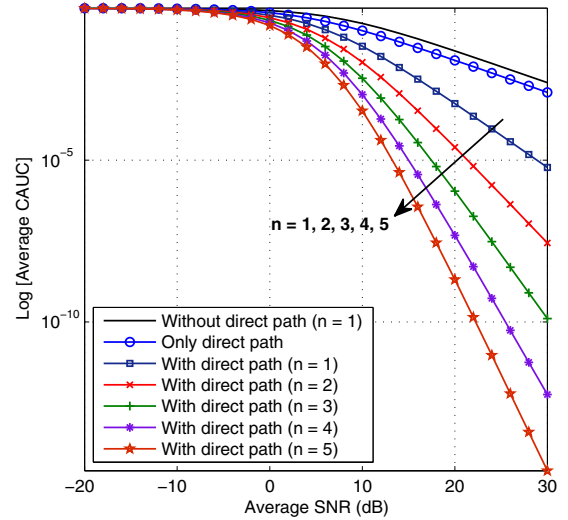


Fig. 2. Average CAUC versus average SNR for different number of relays in log-log scale.

$m$  fading model given in [12, eq. 13]. Here the channels are i.i.d. with average SNR  $\bar{\gamma}$ . Further, the average CAUC for different diversity schemes such as MRC, SC, and SLC over Nakagami- $m$  fading channels, denoted commonly as  $\overline{A'_{Com}}$ , can be derived easily with the aid of (6) and average AUCs given in [12, eqs. 16 and 20].

In the numerical result, the average SNR on each fading channel is  $\bar{\gamma} = 5$  dB. And  $u = 2$ .

Fig. 3 shows the average CAUC, which demonstrates the effect of fading parameter  $m$  of Nakagami- $m$  fading model on overall detection capability. The detection diversity gain in high SNR is equivalent to order  $m$  because  $\overline{A'_{Nak}} \approx g_{Nak}(u, m) \bar{\gamma}^{-m}$ . From Fig. 3, it can be clearly seen that the detection performance increases with order  $m$  as  $m$  increases from 1 to 5.

Similarly, Fig. 4 shows the effect on the overall detection capability due to different combining techniques with different values of diversity branches  $L$ . It can be seen that, by increasing  $L$ , the average CAUC in all combining methods approaches to zero in the same order  $Lm$  because  $\overline{A'_{Com}} \approx g_{Com}(u, m, L) \bar{\gamma}^{-Lm}$  in high SNR. For a particular  $L$ , although all three curves corresponding to MRC, SLC, and SC are parallel to each other, MRC always outperforms SLC and SC due to higher end-to-end SNR associated with MRC.

$$\overline{A'_{Nak}} = \frac{1}{\Gamma(m)} \left( \frac{2m}{2m+\bar{\gamma}} \right)^m \sum_{k=0}^{u-1} \frac{\Gamma(k+m)}{k!} \left( \frac{\bar{\gamma}}{2m+\bar{\gamma}} \right)^k - \left( \frac{m}{m+\bar{\gamma}} \right)^m \sum_{k=1-u}^{u-1} \frac{\Gamma(u+k)}{2^{u+k}\Gamma(u)} {}_2\tilde{F}_1 \left( m, u+k; 1+k; \frac{\bar{\gamma}}{2(m+\bar{\gamma})} \right). \quad (15)$$

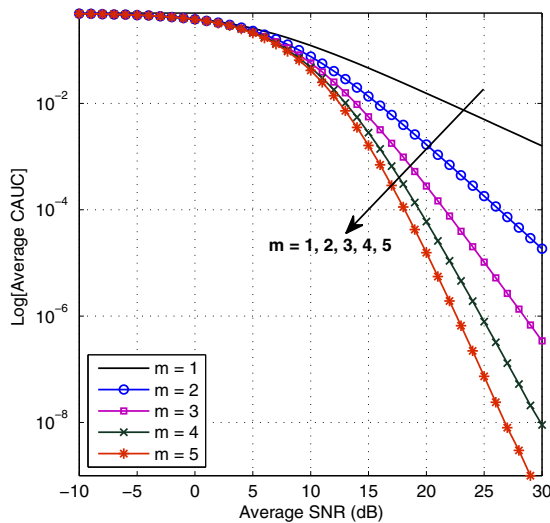


Fig. 3. Average CAUC versus average SNR in a Nakagami- $m$  fading environment.

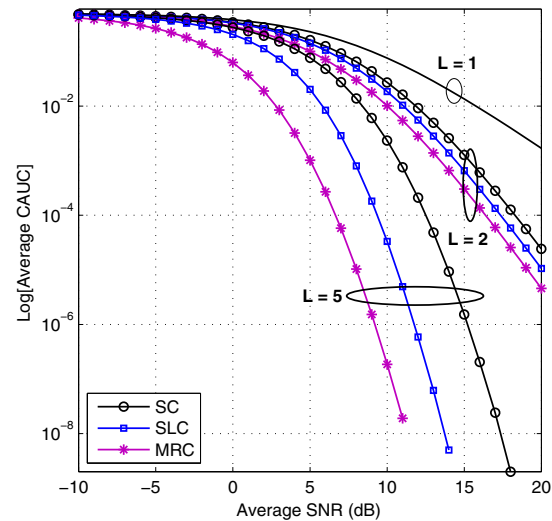


Fig. 4. Average CAUC versus average SNR for different  $L$  in diversity receptions.

## V. CONCLUSION

We have introduced the CAUC as a measure of the overall detection capability for an energy detector. An upper bound of average CAUC is derived for the relay network. It shows that the detection diversity gain order is  $n$  and  $(n+1)$  for an  $n$ -relay network without and with direct path, respectively, in Rayleigh fading channels. Moreover, a multipath fading environment modeled with Nakagami- $m$  fading has a detection diversity gain order of  $m$ , and if there are  $L$  diversity branches, then the detection diversity gain order is shown to be  $Lm$ . The analytical results are validated by numerical examples by plotting average CAUC versus average SNR in a log-log scale. Therefore, the CAUC is a good performance parameter to evaluate the detection diversity gain order of energy detectors.

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