Superimposed Pilots Aided Joint CFO and Channel Estimation for ZP-OFDM Modulated Two-Way Relay Networks

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***Abstract*—Existing works on joint carrier frequency offset (CFO) and channel estimation in two-way relay networks (TWRN) only deal with the composite channel parameters and the mixed CFO values. In this paper, we design a superimposed pilot based training strategy such that the individual frequency and channel parameters can be obtained at the source terminals. We consider the amplify-and-forward (AF) relaying scheme and discuss the zero-padding (ZP) based orthogonal frequency division multiplexing (OFDM) modulation in order to cope with the frequency selective fading channels. We build up the system model and propose the joint estimation method. An iterative process is also proposed to further improve the estimation accuracy. To make the study complete, we also derive the Cramér-Rao Bound (CRB) and compare with the mean square error of our algorithms. Finally, simulation results are provided to corroborate our studies.**

I. Introduction

Wireless relay networks have become a hot research topic ever since the pioneer work [1], [2]. All these works have assumed a unidirectional data transmission from the source to the relay and then to the destination, which is usually referred to as one-way relay network (OWRN). In bidirectional transmission, one may need the two source nodes to exchange information via a "network coding" manner [3] at the relay node, and this new scheme is named as two-way relay network (TWRN). Due to its improved spectral efficiency over OWRN, TWRN has drawn much interest recently [4]–[6]. It was reported in [4] that the overall communication rate between two source terminals in TWRN is approximately twice that achieved in OWRN, making TWRN particularly attractive to bidirectional systems.

The channel estimation problems for TWRN was firstly studied in [7], [8] for frequency-flat and frequency-selective environments, respectively, and these studies show that the channel estimation technique for amplify-and-forward (AF) based TWRN contains essential difference from that in the conventional point-to-point systems. Furthermore, the problem of carrier frequency offset (CFO) estimation in TWRN becomes more challenging because we needs to cope with the mismatch between two source terminals as well as that between the source terminals and relays.

Our first study of joint CFO and channel estimation in TWRN [9] discussed the simplest situation where relay acts only as a repeater. Then in [10] and [11], we considered the general situation and proposed modified models for cyclic prefix (CP) and zero-padding (ZP) based orthogonal frequency division multiplexing (OFDM) modulation, respectively. With certain modification, the OFDM carrier orthogonality in TWRN is kept, and the the joint estimation problem can be solved by efficient linear signal processing.

However, these joint CFO and channel estimation works only provide the results for the convoluted channels between sources and relay, and only the mixed CFO value can be obtained. As seen from [6], individual channel knowledge is also important for optimizing TWRN performance. In this paper, we study the individual channel and CFO estimation in ZP-OFDM based TWRN and propose a superimposed pilot strategy at the relay node.With certain modification, the joint estimation can be solved by the efficient least square (LS) followed by an iterative process to improve the estimation accuracy. To examine the performance of the proposed method, we also derived the analytical expression of the Cramér-Rao Bound (CRB). Finally simulation results are provided to corroborate our studies.

II. System Model

Consider a two-way relay network (TWRN) with two terminal nodes $\mathbb{T}_1$, $\mathbb{T}_2$ and one relay node $\mathbb{R}$, as shown in Fig. 1. Each node has one antenna that cannot transmit and receive simultaneously. The channel from $\mathbb{T}_j$, $j = 1, 2$, to $\mathbb{R}$ is denoted as $\mathbf{h}_j = [h_{j,0}, \ldots, h_{j,L}]^T$, whose elements are independent and have variances $\sigma^2_{j,l}$. From the reciprocity, the channel from $\mathbb{R}$ to $\mathbb{T}_j$ can be assumed as $\mathbf{h}_j$ too. The training block length is set as N, which may or may not be the same as the data block length. Furthermore, we denote the carrier frequency of $\mathbb{T}_j$ as f_j and that of $\mathbb{R}$ as f_r. The average power of $\mathbb{T}_j$ and $\mathbb{R}$ are denoted as P_j and P_r, respectively.

Denote one OFDM block from $\mathbb{T}_j$ as $\tilde{\mathbf{s}}_j = [\tilde{s}_{j,0}, \ldots, \tilde{s}_{j,N-1}]^T$. The corresponding time-domain signal block is obtained from the normalized inverse discrete Fourier

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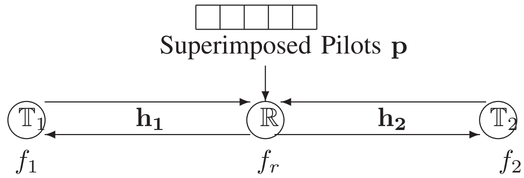

Fig. 1. System configuration for two-way relay network.

transformation (IDFT) as

$$\mathbf{s}_j = \mathbf{F}^H \tilde{\mathbf{s}}_j = [s_{j,0}, s_{j,1}, \ldots, s_{j,N-1}]^T, \tag{1}$$

where $\mathbf{F}$ is the $N \times N$ normalized DFT matrix with the (p, q)-th entry given by $\frac{1}{\sqrt{N}} e^{-\jmath 2\pi(p-1)(q-1)/N}$. To avoid inter-block interference (IBI) in the first transmission phase, L zeros are padded at the end of $\mathbf{s}_i$.

In Phase I, $\mathbb{T}_1$ and $\mathbb{T}_2$ up-convert the baseband signals by the carriers $e^{\jmath 2\pi f_j t}$ and send them to $\mathbb{R}$. As most CFO estimation works [12], the perfect time synchronization is assumed for the time being, i.e., the rely receives the signals from two terminals simultaneously. Then $\mathbb{R}$ down-converts the passband signal by $e^{-\jmath 2\pi f_r t}$ and obtain the $(N+L) \times 1$ baseband signal

$$\mathbf{r} = \sum_{i=1}^{2} \boldsymbol{\Gamma}^{(N+L)}[f_i - f_r]\mathbf{H}_{zp}^{(N)}[\mathbf{h}_i]\mathbf{s}_i + \mathbf{n}_r, \tag{2}$$

where

$$\boldsymbol{\Gamma}^{(K)}[f] = \text{diag}\{1, e^{j2\pi f T_s}, \ldots, e^{j2\pi f(K-1)T_s}\} \tag{3}$$

with T_s representing the sampling period, and

$$\mathbf{H}_{zp}^{(K)}[\mathbf{x}] \triangleq \underbrace{\begin{bmatrix} x_0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ x_P & \ddots & x_0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & x_P \end{bmatrix}}_{K \text{ columns}} \tag{4}$$

for any vector $\mathbf{x} = [x_0, x_1, \ldots, x_P]^T$. Moreover, $\mathbf{n}_r$ is the $(N+L) \times 1$ noise vector, each entry having the variance σ_n^2.

We propose that $\mathbb{R}$ scales the received signal and superimposes a new training vector $\mathbf{p}$ to obtain

$$\mathbf{t} = \alpha\mathbf{r} + \mathbf{p}, \tag{5}$$

where the scaling factor α and the superimposed training should satisfy the relay power constraint

$$\begin{aligned} \alpha^2 \mathrm{E}\{\|\mathbf{r}\|^2\} + \|\mathbf{p}\|^2 =& \alpha^2 N \sum_{j=1}^{2} \sum_{l=0}^{L} \sigma_{j,l}^2 P_j + N\sigma_n^2 + \|\mathbf{p}\|^2 \\ \leq& (N+L)P_r. \end{aligned} \tag{6}$$

Obviously, α belongs to $\left(0, \sqrt{\frac{N+L}{N} \frac{P_r}{\sum_{j=1}^{2} \sum_{l=0}^{L} \sigma_{j,l}^2 P_j + \sigma_n^2}}\right)$ and controls the power allocation between the training from source terminals and the superimposed training from the relay.

Next, $\mathbb{R}$ adds L zeros to the end of $\mathbf{t}$ and up-converts the resultant signal to passband by $e^{\jmath 2\pi f_r t}$. As demonstrated in [11], adding additional L zeros is essential to keep the OFDM carrier orthogonal as well as to reduce the estimation complexity.

Due to symmetry, we only look at the signal received at $\mathbb{T}_1$, which is expressed as

$$\mathbf{y} = \boldsymbol{\Gamma}^{(N+2L)}[w]\mathbf{H}^{(N+L)}[\mathbf{h}_1]\mathbf{t} + \mathbf{n}_1. \tag{7}$$

where $w = f_r - f_1$, and $\mathbf{n}_1$ denotes the $(N+2L) \times 1$ noise vector at $\mathbb{T}_1$. Using the following property:

$$\mathbf{H}^{(K)}[\mathbf{x}]\boldsymbol{\Gamma}^{(K)}[f] = \boldsymbol{\Gamma}^{(K+P)}[f]\mathbf{H}^{(K)}\left[\boldsymbol{\Gamma}^{(K)}[-f]\mathbf{x}\right], \tag{8}$$

$\mathbf{y}$ can be rewritten as

$$\begin{aligned} \mathbf{y} =& \alpha\mathbf{H}_{zp}^{(N)}[\mathbf{a}]\mathbf{s}_1 + \alpha\boldsymbol{\Gamma}^{(N+2L)}[v]\mathbf{H}_{zp}^{(N)}[\mathbf{b}]\mathbf{s}_2 \\ &+ \boldsymbol{\Gamma}^{(N+2L)}[w]\mathbf{H}_{zp}^{(N+L)}[\mathbf{h}_1]\mathbf{p} + \mathbf{n}_e, \end{aligned} \tag{9}$$

where

$$\begin{aligned} & v = f_2 - f_1, \quad \mathbf{a} = (\boldsymbol{\Gamma}^{(L+1)}[w]\mathbf{h}_1) \otimes \mathbf{h}_1, \\ & \mathbf{b} = (\boldsymbol{\Gamma}^{(L+1)}[w - v]\mathbf{h}_1) \otimes \mathbf{h}_2, \\ & \mathbf{n}_e = \alpha\boldsymbol{\Gamma}^{(N+2L)}[w]\mathbf{H}_{zp}^{(N)}[\mathbf{h}_1]\mathbf{n}_r + \mathbf{n}_1, \end{aligned}$$

and $\otimes$ denotes the linear convolution. The covariance of $\mathbf{n}_e$ is computed as

$$\begin{aligned} \mathbf{R} = \sigma_n^2 \Big(& \alpha^2 \boldsymbol{\Gamma}^{(N+2L)}[w]\mathbf{H}_{zp}^{(N+L)}[\mathbf{h}_1](\mathbf{H}_{zp}^{(N+L)}[\mathbf{h}_1])^H \\ & \times (\boldsymbol{\Gamma}^{(N+2L)}[w])^H + \mathbf{I}\Big). \end{aligned} \tag{10}$$

In [11], we showed that the orthogonality of the carriers, i.e., the DFT of $\mathbf{y}$, after removing the self-signal part is maintained once channels and CFO are estimated. The equivalent carrier fading factor is a function of both channel and CFO.

Now consider $\mathbf{s}_1$ and $\mathbf{s}_2$ as training sequences from $\mathbb{T}_1$ and $\mathbb{T}_2$, respectively. We rewrite $\mathbf{y}$ as

$$\begin{aligned} \mathbf{y} =& \alpha\mathbf{S}_1^{(N+2L)}\mathbf{a} + \alpha\boldsymbol{\Gamma}^{(N+2L)}[v]\mathbf{S}_2^{(N+2L)}\mathbf{b} \\ &+ \boldsymbol{\Gamma}^{(N+2L)}[w]\mathbf{P}^{(N+2L)}\mathbf{h}_1 + \mathbf{n}_e, \end{aligned} \tag{11}$$

where $\mathbf{S}_j^{(K)}$ is the $K \times (2L+1)$ column-wise circulant matrix with first column $[\mathbf{s}_i^T, \mathbf{0}_{1\times(K-N)}]^T$, and $\mathbf{P}$ is the $K \times (L+1)$ column-wise circulant matrix with first column $[\mathbf{p}^T, \mathbf{0}_{1\times K-(N+L)}]^T$.

III. Joint Estimation Algorithm

A. Estimation Algorithm

For notation simplicity, the superscripts in (11) are all omitted. When $N \geq 3L+5$,[1] there are sufficient degree of freedoms and we can treat v, w, $\mathbf{a}$, $\mathbf{b}$, $\mathbf{h}$ as independent variables. Rewrite $\mathbf{y}$ as

$$\begin{aligned} \mathbf{y} =& [\alpha\mathbf{S}_1 \quad \alpha\boldsymbol{\Gamma}[v]\mathbf{S}_2 \quad \boldsymbol{\Gamma}[w]\mathbf{P}] \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{h}_1 \end{bmatrix} + \mathbf{n}_e \\ =& \mathbf{C}\mathbf{d} + \mathbf{n}_e, \end{aligned} \tag{12}$$

[1]The block length N in most practical system satisfy this condition

$$[v^{(1)}, w^{(1)}] = \arg\min_{v,w} (\mathbf{y} - \alpha\mathbf{S}_1\mathbf{a}^{(0)} - \alpha\mathbf{\Gamma}[v]\mathbf{S}_2\mathbf{b}^{(0)} - \mathbf{\Gamma}[w]\mathbf{P}\mathbf{h}_1^{(0)})^H\mathbf{R}^{-1} \times (\mathbf{y} - \alpha\mathbf{S}_1\mathbf{a}^{(0)} - \alpha\mathbf{\Gamma}[v]\mathbf{S}_2\mathbf{b}^{(0)} - \mathbf{\Gamma}[w]\mathbf{P}\mathbf{h}_1^{(0)}), \tag{19}$$

$$\mathbf{h}_2^{(1)} = \arg\min_{\mathbf{h}_2} (\mathbf{y} - \alpha\mathbf{S}_1\mathbf{a}^{(0)} - \mathbf{\Gamma}[w^{(1)}]\mathbf{P}\mathbf{h}_1^{(0)} - \alpha\mathbf{\Gamma}[v^{(1)}]\mathbf{S}_2\mathbf{H}_{12}^{(1)}\mathbf{h}_2)^H \mathbf{R}^{-1}(\mathbf{y} - \alpha\mathbf{S}_1\mathbf{a}^{(0)} - \mathbf{\Gamma}[w^{(1)}]\mathbf{P}\mathbf{h}_1^{(0)} - \alpha\mathbf{\Gamma}[v^{(1)}]\mathbf{S}_2\mathbf{H}_{12}^{(1)}\mathbf{h}_2), \tag{20}$$

$$\mathbf{h}_1^{(1)} = \arg\min_{\mathbf{h}_1} (\mathbf{y} - \left[\alpha\mathbf{S}_1\mathbf{H}_{11}^{(0)} + \alpha\mathbf{\Gamma}[v^{(1)}]\mathbf{S}_2\mathbf{H}_{zp}^{(L+1)}[\mathbf{h}_2^{(1)}]\mathbf{\Gamma}^{L+1}[w^{(1)} - v^{(1)}] + \mathbf{\Gamma}[w^{(1)}]\mathbf{P}\right]\mathbf{h}_1)^H \mathbf{R}^{-1}(\mathbf{y} - \left[\alpha\mathbf{S}_1\mathbf{H}_{11}^{(0)} + \alpha\mathbf{\Gamma}[v^{(1)}]\mathbf{S}_2\mathbf{H}_{zp}^{(L+1)}[\mathbf{h}_2^{(1)}]\mathbf{\Gamma}^{L+1}[w^{(1)} - v^{(1)}] + \mathbf{\Gamma}[w^{(1)}]\mathbf{P}\right]\mathbf{h}_1). \tag{21}$$

From LS method, we reach

$$\{\hat{v}, \hat{w}\} = \arg\max_{v,w} \mathbf{y}^H\mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H\mathbf{y}, \tag{13}$$

where v and w can be obtained either from the two dimensional search or from the alternating projection that converts the 2-dimensional maximization into a series of 1-D maximization problems. Details on the implementation of alternating projection can be found from [13]. Then

$$\hat{\mathbf{d}} = (\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H\mathbf{y}. \tag{14}$$

Note that

$$\mathbf{a} = \underbrace{\mathbf{H}_{zp}^{(L+1)}[\mathbf{\Gamma}^{L+1}[w]\mathbf{h}_1]}_{\mathbf{H}_{11}}\mathbf{h}_1. \tag{15}$$

Denote $\hat{\mathbf{H}}_{11}$ as the estimated $\mathbf{H}_{11}$ that is constructed from $\hat{w}$ and $\hat{\mathbf{h}}_1$.

Then we can update the estimation of $\mathbf{h}_1$ as

$$\hat{\hat{\mathbf{h}}}_1 = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H(\mathbf{y} - \alpha\mathbf{\Gamma}[v]\mathbf{S}_2\hat{\mathbf{b}}), \tag{16}$$

where $\mathbf{A} = \alpha\mathbf{S}_1\hat{\mathbf{H}}_{11} + \mathbf{\Gamma}[w]\mathbf{P}$. This process can be continued until certain stopping criterion is satisfied.

Similarly, from

$$\mathbf{b} = \underbrace{\mathbf{H}_{zp}^{(L+1)}[\mathbf{\Gamma}^{L+1}[w - v]\mathbf{h}_1]}_{\mathbf{H}_{12}}\mathbf{h}_2, \tag{17}$$

the estimation of $\mathbf{h}_2$ can be found from

$$\hat{\mathbf{h}}_2 = (\hat{\mathbf{H}}_{12}^H\hat{\mathbf{H}}_{12})^{-1}\hat{\mathbf{H}}_{12}^H\hat{\mathbf{b}}. \tag{18}$$

B. Iterative Algorithm to Improve the Performance

Now that we have the initial estimation of every parameters, we can apply an iterative approach to improve the estimation accuracy. Assume the initial estimate as $v^{(0)}, w^{(0)}, \mathbf{h}_1^{(0)}, \mathbf{h}_2^{(0)}$, respectively. We will estimate $v^{(1)}, w^{(1)}$ simultaneously from the ML estimation process as in (19) that is shown on the top of this page. The complexity here is not significant even if the 2-dimensional search is applied because the fine search region is small. Then $\mathbf{h}_2^{(1)}$ and $\mathbf{h}_1^{(1)}$ can be updated based on (20) and (21).

The interactive processing could gain the improvement from the fact that the initial estimation does not exploit the correlation between $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{h}_1$.

Remark 1: The super-imposed pilots used in our paper are different from traditional ones in two aspects. On one hand, traditional super-imposed pilots [14] are transmitted together with data while our super-imposed pilots are sent together with pilots. On the other hand, traditional super-imposed pilots [15] select the first order statistics for channel estimation while here we use the nulling-based LS method for joint CFO and channel estimation.

IV. Cramér-Rao Bound

In this section, we derive the analytical expression of the CRB of our proposed joint estimation strategy. Let

$$\boldsymbol{\mu} = \alpha\mathbf{S}_1\mathbf{a} + \alpha\mathbf{\Gamma}[v]\mathbf{S}_2\mathbf{b} + \mathbf{\Gamma}[w]\mathbf{P}\mathbf{h}_1, \tag{22}$$

and define

$$\boldsymbol{\eta} \triangleq [v, w, \Re\{\mathbf{h}_1\}^T, \Im\{\mathbf{h}_1\}^T, \Re\{\mathbf{h}_2\}^T, \Im\{\mathbf{h}_2\}^T]^T \tag{23}$$

as a real vector. According to [16], the $(i,j)^{th}$ entry of the Fisher Information Matrix (FIM) can be calculated as

$$[\mathbf{F}]_{i,j} = 2\Re\left[\frac{\partial\boldsymbol{\mu}^H}{\partial\eta_i}\mathbf{R}^{-1}\frac{\partial\boldsymbol{\mu}}{\partial\eta_j}\right] + \mathrm{tr}\left[\mathbf{R}^{-1}\frac{\partial\mathbf{R}}{\partial\eta_i}\mathbf{R}^{-1}\frac{\partial\mathbf{R}}{\partial\eta_j}\right]. \tag{24}$$

After some tedious simplifications, we obtain

$$\begin{aligned}
\frac{\partial\boldsymbol{\mu}}{\partial v} &= j\alpha\big(\mathbf{D}_1\mathbf{\Gamma}^{(N+2L)}[v]\mathbf{S}_2\mathbf{b} \\
&\quad - \mathbf{\Gamma}^{(N+2L)}[v]\mathbf{S}_2\mathbf{H}_{zp}^{(L+1)}[\mathbf{h}_2]\mathbf{D}_2\mathbf{\Gamma}^{(L+1)}[w-v]\mathbf{h}_1\big), \\
\frac{\partial\boldsymbol{\mu}}{\partial w} &= j\alpha\big(\mathbf{S}_1\mathbf{H}_{zp}^{(L+1)}[\mathbf{h}_1]\mathbf{D}_2\mathbf{\Gamma}^{(L+1)}[w]\mathbf{h}_1 \\
&\quad + \mathbf{\Gamma}^{(N+2L)}[v]\mathbf{S}_2\mathbf{H}_{zp}^{(L+1)}[\mathbf{h}_2]\mathbf{D}_2\mathbf{\Gamma}^{(L+1)}[w-v]\mathbf{h}_1\big) \\
&\quad + j\mathbf{D}_1\mathbf{\Gamma}^{(N+2L)}[w]\mathbf{P}\mathbf{h}_1, \\
\frac{\partial\boldsymbol{\mu}}{\partial\Re\{\mathbf{h}_1\}^T} &= \alpha\mathbf{S}_1\mathbf{H}_{zp}^{(L+1)}[\mathbf{\Gamma}^{(L+1)}[w]\mathbf{h}_1] + \mathbf{\Gamma}(w)\mathbf{P} \\
&\quad + \alpha\mathbf{\Gamma}^{(N+2L)}[v]\mathbf{S}_2\mathbf{H}_{zp}^{(L+1)}[\mathbf{h}_2]\mathbf{\Gamma}^{(L+1)}[w-v] \\
&\quad + \alpha\mathbf{S}_1\mathbf{H}_{zp}^{(L+1)}[\mathbf{h}_1]\mathbf{\Gamma}^{(L+1)}[w], \\
\frac{\partial\boldsymbol{\mu}}{\partial\Re\{\mathbf{h}_2\}^T} &= \alpha\mathbf{\Gamma}^{(N+2L)}[v]\mathbf{S}_2\mathbf{H}_{zp}^{(L+1)}[\mathbf{\Gamma}^{(L+1)}[w-v]\mathbf{h}_1],
\end{aligned}$$

$$\frac{\partial \mathbf{R}}{\partial w} = \sigma_n^2\alpha^2\Big(j\mathbf{D}_1\boldsymbol{\Gamma}^{(N+2L)}[w]\mathbf{H}_{zp}^{(N+L)}[\mathbf{h}_1](\mathbf{H}_{zp}^{(N+L)}[\mathbf{h}_1])^H(\boldsymbol{\Gamma}^{(N+2L)}[w])^H - j\boldsymbol{\Gamma}^{(N+2L)}[w]\mathbf{H}_{zp}^{(N+L)}[\mathbf{h}_1](\mathbf{H}_{zp}^{(N+L)}[\mathbf{h}_1])^H\mathbf{D}_1(\boldsymbol{\Gamma}^{(N+2L)}[w])^H\Big), \quad (25)$$

$$\frac{\partial \mathbf{R}}{\partial \Re\{h_{1i}\}} = \sigma_n^2\alpha^2\Big(\boldsymbol{\Gamma}^{(N+2L)}[w]\mathbf{H}_{zp}^{(N+L)}[\mathbf{e}_i](\mathbf{H}_{zp}^{(N+L)}[\mathbf{h}_1])^H(\boldsymbol{\Gamma}^{(N+2L)}[w])^H + \boldsymbol{\Gamma}^{(N+2L)}[w]\mathbf{H}_{zp}^{(N+L)}[\mathbf{h}_1](\mathbf{H}_{zp}^{(N+L)}[\mathbf{e}_i])^H(\boldsymbol{\Gamma}^{(N+2L)}[w])^H\Big). \quad (26)$$

where

$$\mathbf{D}_1 = 2\pi T_s \text{diag}\{0, 1, \ldots, N+2L-1\},$$
$$\mathbf{D}_2 = 2\pi T_s \text{diag}\{0, 1, \ldots, L\}.$$

The derivatives of $\mathbf{R}$ with respect to w and $\mathbf{h}_1$ can also be found as (25) and (26) where $\mathbf{e}_i$ is an $(L+1)\times 1$ vector with its i^{th} element 1 and others 0. And it can be easily known that

$$\frac{\partial \mathbf{R}}{\partial \Re\{h_{2i}\}} = \frac{\partial \mathbf{R}}{\partial v} = \mathbf{0}_{(N+2L)\times(N+2L)}.$$

Substituting the above results back to (24), we can obtain the CRB by inverting the FIM $\mathbf{F}$.

Define $\boldsymbol{\eta}_0 = [v, w, \mathbf{h}_1^T, \mathbf{h}_2^T]^T$ as the original set of complex-valued parameters. It can be readily checked that

$$\boldsymbol{\eta}_0 = \boldsymbol{\Xi}\boldsymbol{\eta}, \quad (27)$$

where

$$\boldsymbol{\Xi} = \begin{bmatrix} 1, & 0, & \mathbf{0}^T, & \mathbf{0}^T, & \mathbf{0}^T, & \mathbf{0}^T \\ 0, & 1, & \mathbf{0}^T, & \mathbf{0}^T, & \mathbf{0}^T, & \mathbf{0}^T \\ \mathbf{0}, & \mathbf{0}, & \mathbf{I}_{L+1}, & j\mathbf{I}_{L+1}, & 0\cdot\mathbf{I}_{L+1}, & 0\cdot\mathbf{I}_{L+1} \\ \mathbf{0}, & \mathbf{0}, & 0\cdot\mathbf{I}_{L+1}, & 0\cdot\mathbf{I}_{L+1}, & \mathbf{I}_{L+1}, & j\mathbf{I}_{L+1} \end{bmatrix}.$$

Hence, the CRB matrix of $\boldsymbol{\eta}_0$ is

$$\mathbf{CRB} = \boldsymbol{\Xi}\cdot\mathbf{F}^{-1}\cdot\boldsymbol{\Xi}^H. \quad (28)$$

V. NUMERICAL RESULTS

In this section we numerically study the performance of our proposed estimation algorithm. Three-tap channel model is assumed for both $\mathbf{h}_i$, while each tap is Gaussian with unit variance. The variance of the noise is set as $\sigma_n^2 = 1$. We take $N = 16$ and $\alpha = 0.5$. The normalized frequencies f_1, f_r, and f_2 are set as 0.94, 1 and 1.06, respectively. The MSE is chosen as the figure of merit, defined by

$$\text{MSE}(v) = \frac{1}{1000}\sum_{i=1}^{1000}(\hat{v}_i - v)^2,$$
$$\text{MSE}(\mathbf{x}) = \frac{1}{1000}\sum_{i=1}^{1000}\frac{1}{K}(\hat{\mathbf{x}}_i - \mathbf{x})^2,$$

where $\mathbf{x}$ represents $\mathbf{h}_1$ or $\mathbf{h}_2$, K is taken as 3, and 1000 is the number of the Monte-Carlo trials used for average.

First we examine the performance of the individual CFO estimation for v and w. The corresponding MSEs versus SNR

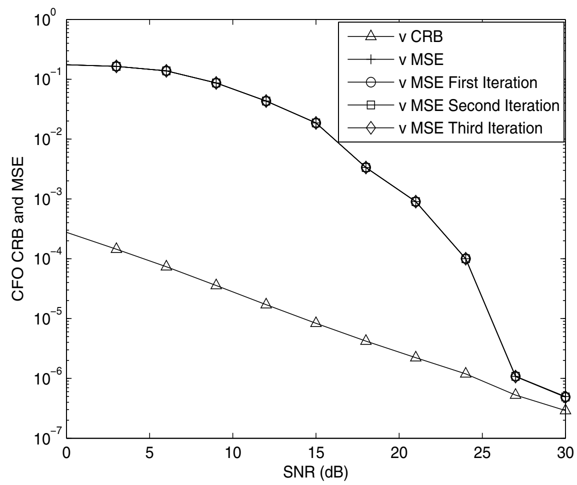

Fig. 2. v CFO estimation when N=16, $\alpha = 0.5$

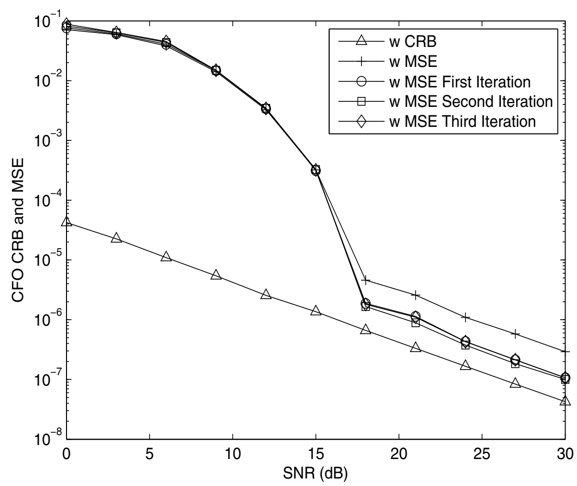

Fig. 3. w CFO estimation when N=16, $\alpha = 0.5$

curves are shown in Fig. 2 and Fig. 3, respectively. The analytical CRBs are also displayed for comparison. It is seen that iteration process does not improve the performance for v much, while it does improve the performance for w at high SNR region. The distance between the numerical results and

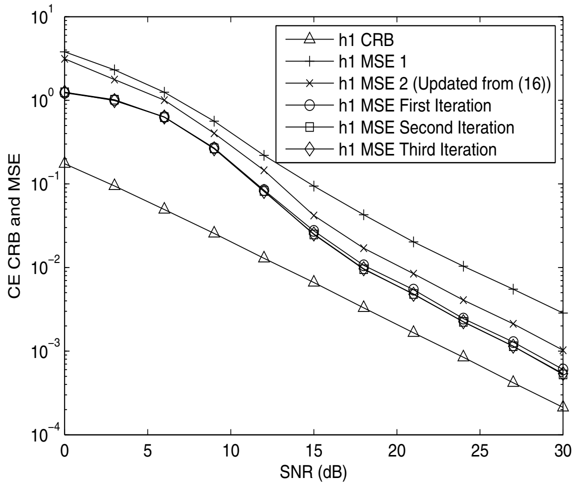

Fig. 4. $\mathbf{h}_1$ channel estimation when N=16, $\alpha = 0.5$

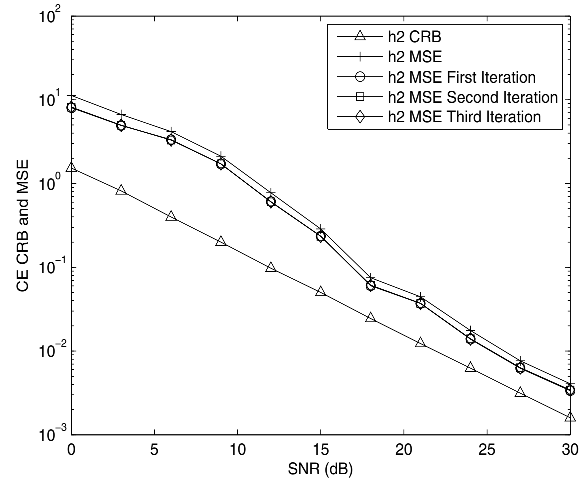

Fig. 5. $\mathbf{h}_2$ channel estimation when N=16, $\alpha = 0.5$

the CRB is about 3dB.

We then demonstrate the individual channel estimation results in for $\mathbf{h}_1$ and $\mathbf{h}_2$ in Fig. 4 and Fig. 5, as well as their corresponding CRBs. For $\mathbf{h}_1$, it can be seen that the iterative estimation from (16) is much better than the original one from (14). Next, our iterative algorithms further improve the estimation accuracy. Similar observation as in CFO estimation is that, the channel estimation of $\mathbf{h}_1$ is improved much but the channel estimation of $\mathbf{h}_2$ improves little. The distance between the numerical results and the CRB is about 2 dB.

Since the iteration mainly improves by combing $\mathbf{a}$ and $\mathbf{h}_1$, so the directly related estimation of $\mathbf{h}_1$ and w can be immediately improved, while the improvement over the indirect parameters $\mathbf{h}_2$ and v is not that obvious.

We also find that the proposed iteration converges only in 2 or 3 steps.

Though the proposed method and the iteration demonstrate effectiveness, we see that there still exist 3 dB space to further improve the estimation accuracy. How to reduce such a gap is an open question that remains to be solved.

VI. Conclusions

In this paper, we designed a new training strategy to obtain the individual frequency and channel parameters in a ZP-OFDM modulated TWRN systems. By introducing the transmission of superimposed pilots from the relay node, the CFO estimation problem can be solved as a 2-dimensional search and the channel estimates can be obtained from the low complexity LS method. We also provide an iterative process to improve the estimation accuracy. The analytical CRB is derived and compared with the numerical results via various examples. From the simulations, we also find that there is room to improve the estimation accuracy. This serves as a potential future research topic.

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