Blind Spectrum Sensing in Cognitive Radio

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Abstract— **In this paper, we consider an interesting and practical scenario for spectrum sensing in cognitive radio network, where both the signal power of the primary user and the noise variance are treated as unknowns before the detection. Knowing accurate noise variance and signal power is crucial in most sensing algorithms, e.g., energy detection. By exploiting the received signal structure, we propose blind spectrum sensing methods in the sense that both the signal power of the primary user and the noise variance are estimated, which is a nontrivial task before knowing the status of the primary user. Three different algorithms, direct estimator, approximate maximum likelihood (ML) estimator and pseudo linear minimum mean square error (MMSE) estimator, are proposed based on the moments of received signals. Simulation results confirm that the proposed algorithms can estimate the noise variance and the primary user's signal power with high accuracy.**

I. Introduction

With the rapid development of wireless applications, spectrum resources are facing ever increasing demand. In traditional spectrum management, most of the spectrum bands are exclusively allocated to specific licensed users and no violation from unlicensed users is allowed. Cognitive radio (CR) [1] is a promising technology to remedy the spectrum scarcity by allowing the unlicensed users to opportunistically access the spectrum assigned to the licensed users provided no harmful interference is experienced by incumbent services.

The key technique for successfully applying CR is the spectrum sensing, where the unlicensed user should reliably detect the existence of the primary user on the current bands. Popular spectrum sensing techniques include energy detection [2] and cyclostationary detection [3], of which energy detection is a promising candidate for practical employment due to its very low computational complexity. Specifically, energy detection compares the average received power with a predefined threshold to determine the presence of the primary user. Given a false alarm probability, the detection threshold is a function of the noise variance and the signal power, which are assumed perfectly known in most existing literatures [2]. Although the noise variance can be estimated from the average received power in the absence of primary user, the *a priori* knowledge of the existence of the primary user is never possible before one executes the spectrum sensing. Therefore, this becomes an interesting *chicken and egg* problem.

In this paper, we consider blind spectrum sensing problem in the sense that both the signal power of the primary user and the noise variance are treated as unknown parameters that need to be estimated without knowing the presence of the primary user. Motivated by the signal-to-noise ratio (SNR) estimators in [4]–[6] we consider moment based estimators for estimating both the signal power of the primary user and the noise variance, and propose direct estimator, approximate maximum likelihood (ML) estimator, and pseudo linear minimum mean square error (MMSE) estimator. We find that the signal structure of the primary user is crucial to the estimators' performance. Moreover, we propose a modified energy detector using the estimated SNR, discuss the effect the estimation errors on spectrum sensing, and provide ways to choose the detection threshold under estimation errors. Simulation results corroborate our analysis.

II. System Model

We consider a simple cognitive network with one secondary user and one primary user, denoted by $\mathbb{U}$ and $\mathbb{P}$, respectively. The received signal by $\mathbb{U}$ at time i is

$$y_i=\theta e^{\jmath(\epsilon i+\phi)}x_ih_i+\sigma w_i, \tag{1}$$

where $\theta\in\{0,1\}$ is the primary user indicator, x_i is the transmitted signal from $\mathbb{P}$ belonging to the constellation $\mathcal{C}$ with $E\{x_i\}=0$ and $E\{|x_i|^2\}=1$, h_i is the Rayleigh channel gain between $\mathbb{P}$ and $\mathbb{U}$, w_i is white Gaussian noise with zero mean and unit variance, and ϵ and ϕ are frequency offset and timing offset, respectively. In cognitive radio it is hard to achieve perfect synchronization between the secondary user and the primary user. For a general discussion, we assume that ϵ and ϕ are unknown to the secondary user. The constellation $\mathcal{C}$ contains M elements, $c_1,\ldots,c_M$, e.g., $\mathcal{C}$ is QAM.

Assume N consecutive symbols are observed at $\mathbb{U}$, during which both h_i and θ remain unchanged. For simplicity, we denote h_i as h. The energy detector then compares $\sum_{i=1}^N|y_i|^2$ with a threshold λ. If $\sum_{i=1}^N|y_i|^2>\lambda$, the secondary user decides $\hat{\theta}=1$; otherwise, decision $\hat{\theta}=0$ is made.

The key metric in spectrum sensing is the probability of correct detection and the probability of false alarm, defined as

$$P_d=Pr(\hat{\theta}=1|\theta=1), \quad P_f=Pr(\hat{\theta}=1|\theta=0). \tag{2}$$

Since $\sum_{i=1}^N|y_i|^2$ is a central chi-square random variable when $\theta=0$ and is a non-central chi-square random variable given $\theta=1$ and x_i, the probability of false alarm can be obtained as

$$P_f(\lambda)=\int_{\lambda}^{+\infty}\frac{1}{\sigma^{2N}\Gamma(N)}t^{N-1}\exp\left(-\frac{t}{\sigma^2}\right)dt=\frac{\Gamma\left(N,\frac{\lambda}{\sigma^2}\right)}{\Gamma(N)}, \tag{3}$$

and

$$P_d(\lambda)=\sum_{x_1,\ldots,x_N\in\mathcal{C}}\prod_{i=1}^{N}Pr(x_i)\frac{1}{\Gamma(N)}\sum_{j=0}^{\infty}e^{-\frac{h^2}{\sigma^2}\sum_{i=1}^N x_i^2}\times\frac{\left(\frac{h^2}{\sigma^2}\sum_{i=1}^N x_i^2\right)^j}{j!}\Gamma(N+2j,\lambda), \tag{4}$$

where $\Gamma(\cdot,\cdot)$ is the upper incomplete gamma function, and $\Gamma(\cdot)$ is the gamma function. Since it is hard to gain insight from

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(4), we assume that y_i is Gaussian with zero mean. In this case, $\sum_{i=1}^{N}|y_i|^2$ is a central chi-square random variable when $\theta=1$, which gives

$$P_d(\lambda)=\frac{\Gamma\left(N,\frac{\lambda}{h^2+\sigma^2}\right)}{\Gamma(N)}. \quad (5)$$

In practice, the false alarm probability is constrained by the government regulation. Given the false alarm probability ζ, i.e., $P_f(\lambda)=\zeta$, we can find the threshold λ and compute the corresponding correct detection probability using (4) or (5). Note that the value of threshold ζ depends on σ and h that are typically assumed known by most existing works, see e.g., [2]. However, all parameters in the system should be estimated via certain ways. In CR, the difficulty in estimating σ lies in that we do not know when the period $\theta=0$ and when $\theta=1$.

By defining $\lambda=\tilde{\lambda}\sigma^2$, (3) is only related to $\tilde{\lambda}$, and (5) only depends on $\tilde{\lambda}$ and SNR $\frac{h^2}{\sigma^2}$. The problem is equivalent to estimating σ^2 and SNR (or equivalently σ and h) using only received signals. Hence the energy detection after parameter estimation will be categorized to *blind spectrum sensing*.

III. Noise Variance Estimation and Blind Spectrum Sensing

In this section, we derive several non-data aided noise variance estimators, and then propose the blind spectrum sensing algorithm.

A. ML Noise Variance Estimation

We first consider ML estimation of h and σ to gain insight on the structure of the estimator. Note that if we can estimate θh, θ can be readily obtained with a threshold detector. Note that, this is the key point of our propose method to avoid the chicken-egg problem by considering θh as one unknown parameter, which is defined as h with a slight abuse of notation in the noise variance and signal power estimation algorithms. The joint pdf of $y_1,\ldots,y_N$ given h, σ, ϵ and ϕ is

$$p(y_1,\ldots,y_N|h,\sigma,\epsilon,\phi)$$
$$=\frac{1}{\pi^N\sigma^{2N}}\prod_{i=1}^{N}\sum_{x_i\in\mathcal{C}}Pr(x_i)\exp\left(-\frac{\left|y_i-e^{\jmath(\epsilon i+\phi)}hx_i\right|^2}{\sigma^2}\right). \quad (6)$$

For BPSK with $\mathcal{C}=\{1,-1\}$ and $Pr(1)=Pr(-1)=\frac{1}{2}$, we have

$$p(y_1,\ldots,y_N|h,\sigma,\epsilon,\phi) \quad (7)$$
$$=\frac{1}{\pi^N\sigma^{2N}}\prod_{i=1}^{N}\exp\left(-\frac{|y_i|^2+|h|^2}{\sigma^2}\right)\cosh\frac{2\Re\{e^{\jmath(\epsilon i+\phi)}hy_i^*\}}{\sigma^2}.$$

The ML estimates of h, σ, ϵ and ϕ are obtained from maximizing the joint pdf in (7). As ϵ,ϕ appear in each of the term in the product, the closed-form solution of ϵ,ϕ is complicated. Instead, we define $\psi_i=\epsilon i+\phi$ and maximize each term in (7) over ψ_i individually, which gives

$$p(y_1,\ldots,y_N|h,\sigma)=\frac{1}{\pi^N\sigma^{2N}}\prod_{i=1}^{N}\exp\left(-\frac{|y_i|^2+|h|^2}{\sigma^2}\right)\cosh\frac{2|hy_i^*|}{\sigma^2}. \quad (8)$$

From experiments, we find that the performance of the estimator by maximizing (8) is within a constant factor of the modified CRB, which shows that maximizing each term in (7) is a reasonable approximation of the true ML estimator. Taking the derivative of $\log p(y_1,\ldots,y_N|h,\sigma)$ with respect to h and setting the resulting equation to zero gives

$$h=\frac{1}{N}\tanh\frac{|hy_i^*|}{\sigma^2}|y_i|\approx\frac{\sum_{i=1}^{N}|y_i|}{N} \quad (9)$$

in high SNR. Taking the derivative of $\log p(y_1,\ldots,y_N|h,\sigma)$ with respect to σ and using (9) yields

$$\sigma^2\approx\frac{\sum_{i=1}^{N}|y_i|^2}{N}-\left(\frac{\sum_{i=1}^{N}|y_i|}{N}\right)^2. \quad (10)$$

After obtaining h and σ^2, the SNR can be estimated from

$$\rho=\frac{h^2}{\sigma^2}=\frac{\left(\frac{\sum_{i=1}^{N}|y_i|}{N}\right)^2}{\frac{\sum_{i=1}^{N}y_i^2}{N}-\left(\frac{\sum_{i=1}^{N}|y_i|}{N}\right)^2}. \quad (11)$$

Note that the SNR estimator (11) is similar to the squared signal to noise variance (SNV) estimator in [4] for real systems. Our results show that the SNV estimator is an approximate ML estimator for BPSK even in complex systems with synchronization errors. Different from SNR estimation, we, here, are interested in estimating h,σ rather than ρ.

B. Suboptimal Noise Variance Estimation

For higher order constellations, it is hard to derive or approximate the ML estimator in closed-form. The approximate ML estimator for BPSK uses two moments $E\{|y_i|\}$ and $E\{|y_i|^2\}$. Motivated by this, we then propose to use a high moment estimator to approximate the ML estimator.

Note that the pdf of $|y_i|$ is a mixed Ricean distribution. The k-th moment of $|y_i|$ is [6]

$$E\{|y_i|^k\}=h^k\sum_{x_i\in\mathcal{C}}Pr(x_i)\frac{1}{2^{\frac{k}{2}}}\rho^{-\frac{k}{2}}\Gamma\left(\frac{k}{2}+1\right)$$
$$\times\exp\left(-\rho|x_i|^2\right){}_1F_1\left(\frac{k}{2}+1;1;\rho|x_i|^2\right), \quad (12)$$

where ${}_1F_1(\cdot;\cdot;\cdot)$ is the confluent hypergeometric function and $\Gamma(\cdot)$ is the gamma function. After some calculation (12) can be written as

$$m_k=E\{|y_i|^k\}=h^kf_k(\rho), \quad (13)$$

where $f_k(\rho)$ is a function depending only on modulation and ρ. Therefore, we have

$$\frac{\prod_{p=1}^{P}E\{|y_i|^{k_p}\}}{\prod_{q=1}^{Q}E\{|y_i|^{\kappa_q}\}}=\frac{\prod_{p=1}^{P}f_{k_p}(\rho)}{\prod_{q=1}^{Q}f_{\kappa_q}(\rho)}=F(\rho), \quad (14)$$

if $\sum_{p=1}^{P}k_p=\sum_{q=1}^{Q}\kappa_q$. For any SNR, we can optimize $\{k_p\}$ and $\{\kappa_q\}$ by minimizing the average MSE.

1) Direct Estimator: Without considering the distribution of $\sum_{i=1}^{N}|y_i|^k$, the direct SNR estimator can be obtained by replacing $E\{|y_i|^k\}$ with its time average $\frac{1}{N}\sum_{i=1}^{N}|y_i|^k$, i.e.,

$$\hat{\rho}=F^{-1}\left(\frac{\prod_{p=1}^{P}\sum_{i=1}^{N}|y_i|^{k_p}}{\prod_{q=1}^{Q}\sum_{i=1}^{N}|y_i|^{\kappa_q}}\right), \quad (15)$$

where $\sum_{p=1}^{P}k_p=\sum_{q=1}^{Q}\kappa_q$ and $k_p\neq\kappa_q$ for any $1\leq p\leq P$ and $1\leq q\leq Q$. After estimating $\hat{\rho}$, we can obtain

$$\hat{h}=\sqrt{\hat{\rho}}\sum_{l=1}^{L}\alpha_l\sqrt[k_l]{\frac{E\{|y_i|^{k_l}\}}{f_{k_l}(\hat{\rho})}},\quad \hat{\sigma}=\sum_{l=1}^{L}\beta_l\sqrt[k_l]{\frac{E\{|y_i|^{k_l}\}}{f_{k_l}(\hat{\rho})}}, \quad (16)$$

where α_l and β_l are weights to balance different moments and $\sum_{l=1}^{L}\alpha_l=\sum_{l=1}^{L}\beta_l=1$.

2) Moment ML Estimator: As shown in Section III-A, the exact ML estimator is complicated. Instead, we consider two statistics $\hat{m}_p=\frac{1}{N}\sum_{i=1}^{N}|y_i|^p$ and $\hat{m}_q=\frac{1}{N}\sum_{i=1}^{N}|y_i|^q$. Exact pdfs of $\hat{m}_p$ and $\hat{m}_q$ are hard to derive. Enlightened from the central limited theorem, we can approximate $\hat{m}_p$ and $\hat{m}_q$ as Gaussian random variables with mean

$$m_p=E\{\hat{m}_p\}=\sigma^p f_p(\rho), \quad m_q=E\{\hat{m}_q\}=\sigma^q f_q(\rho), \tag{17}$$

and variance and covariance

$$\begin{aligned}\nu_p&=E\{|\hat{m}_p|^2\}-E\{|\hat{m}_p|\}^2=\sigma^{2p}g_{p,p}(\rho,N)\\ \nu_q&=E\{|\hat{m}_q|^2\}-E\{|\hat{m}_q|\}^2=\sigma^{2q}g_{q,q}(\rho,N)\\ \eta_{p,q}&=E\{\hat{m}_p\hat{m}_q\}-E\{\hat{m}_p\}E\{\hat{m}_q\}=\sigma^{p+q}g_{p,q}(\rho,N),\end{aligned} \tag{18}$$

where $g_{p,q}(\rho,N)=\frac{1}{N}(f_{p+q}(\rho)-f_p(\rho)f_q(\rho))$. Let $\xi(\rho,N)=\sqrt{\frac{\eta_{p,q}^2}{\nu_p\nu_q}}$ denote the correlation coefficient. The joint distribution of $\hat{m}_p,\hat{m}_q$ conditional on σ and ρ is written as

$$\begin{aligned}&\psi(\hat{m}_p,\hat{m}_q|\sigma,\rho)\\ &=\frac{1}{2\pi\sigma^{p+q}\sqrt{g_{p,p}(\rho,N)g_{q,q}(\rho,N)(1-\xi^2(\rho,N))}}\\ &\times\exp\left(-\frac{1}{2(1-\xi^2(\rho,N))}\left(\frac{(\hat{m}_p-m_p)^2}{\sigma^{2p}g_{p,p}(\rho,N)}+\frac{(\hat{m}_q-m_q)^2}{\sigma^{2q}g_{q,q}(\rho,N)}\right.\right.\\ &\left.\left.-\frac{2\xi(\rho,N)(\hat{m}_p-m_p)(\hat{m}_q-m_q)}{\sigma^{p+q}\sqrt{g_{p,p}(\rho,N)g_{q,q}(\rho,N)}}\right)\right).\end{aligned} \tag{19}$$

Then $\hat{m}_p,\hat{m}_q$ can be found by maximizing $\psi(\hat{m}_p,\hat{m}_q|\sigma,\rho)$. For each given ρ, the optimal $\sigma^*(\rho)$ is obtained from the root of the partial derivative of $\psi(\hat{m}_p,\hat{m}_q|\sigma,\rho)$ with respect to σ, i.e.,

$$\begin{aligned}&\frac{p\left(\hat{m}_p^2\sigma^{-2p}-\hat{m}_pf_p(\rho)\sigma^{-p}\right)}{g_{p,p}(\rho,N)}+\frac{q\left(\hat{m}_q^2\sigma^{-2q}-\hat{m}_qf_q(\rho)\sigma^{-q}\right)}{g_{q,q}(\rho,N)}\\ &-\frac{\xi(\rho,N)\left((p+q)\hat{m}_p\hat{m}_q\sigma^{-(p+q)}-q\hat{m}_qf_p(\rho)\sigma^{-q}-p\hat{m}_pf_q(\rho)\sigma^{-p}\right)}{\sqrt{g_{p,p}(\rho,N)g_{q,q}(\rho,N)}}\\ &=-\sqrt{1-\xi^2(\rho,N)}(p+q),\end{aligned} \tag{20}$$

which is polynomial in σ. If there exist several positive roots, we may choose the one that is close to that estimated from (10) or (16). We substitute $\sigma^*(\rho)$ back into (19), and then $\psi(\hat{m}_p,\hat{m}_q|\sigma^*(\rho),\rho)$ is only a function of ρ. Maximizing this function over ρ near that in (9) or (16), we obtain the approximate ML estimate ρ^*, from which we obtain ML estimates $\sigma^*(\rho^*)$ and $h^*=\sigma^*(\rho^*)\sqrt{\rho^*}$. This can be done using a one dimensional grid search.

3) Approximate Moment ML Estimator: The ML estimator (19) is complicated to solve because estimation of ρ and σ are coupled. We further consider a decoupled ML estimator by using a special statistic

$$\hat{F}=\frac{\hat{m}_p^q}{\hat{m}_q^p}=\frac{(m_p+\epsilon_p)^q}{(m_q+\epsilon_q)^p}\approx\frac{m_p^q}{m_q^p}+\frac{qm_p^{q-1}}{m_q^p}\epsilon_p-\frac{pm_p^q}{m_q^{p+1}}\epsilon_q, \tag{21}$$

where ϵ_p and ϵ_q are errors between $\hat{m}_p$, $\hat{m}_q$ and m_p, and the approximation follows from the first order Taylor expansion. The variance of $\hat{F}$ can be computed as

$$\begin{aligned}&\nu_F(\rho,N)=E\left\{\left|\frac{qm_p^{q-1}}{m_q^p}\epsilon_p-\frac{pm_p^q}{m_q^{p+1}}\epsilon_q\right|^2\right\}\\ &=F^2(\rho)\left(\frac{q^2}{f_p^2(\rho)}g_{p,p}(\rho,N)+\frac{p^2}{f_q^2(\rho)}g_{q,q}(\rho,N)-\frac{2pq}{f_p(\rho)f_q(\rho)}g_{p,q}(\rho,N)\right),\end{aligned} \tag{22}$$

where $F(\rho)=\frac{f_p^q(\rho)}{f_q^p(\rho)}$. Therefore, $\hat{F}$ is approximately a Gaussian random variable with pdf

$$\psi(\hat{F}|\rho)=\frac{1}{\sqrt{2\pi\nu_F(\rho,N)}}\exp\left(-\frac{\left(\hat{F}-F(\rho)\right)^2}{2\nu_F(\rho,N)}\right). \tag{23}$$

The approximate ML estimator for ρ can be obtained by maximizing $\psi(\hat{F}|\rho)$ over ρ. A remarkable property of $\psi(\hat{F}|\rho)$ is that it only depends on ρ but not on σ. Therefore, the estimation of ρ and σ are decoupled.

After getting ρ^* from maximizing $\psi(\hat{F}|\rho)$, we can substitute it into (20) and estimate σ. We may only use either $\hat{m}_p$ or $\hat{m}_q$ to estimate σ so that the complexity involved is not high. For example, we consider $\hat{m}_p$ as Gaussian with mean m_p and variance ν_p. Maximizing the pdf of $\hat{m}_p$ conditioned on ρ,σ over σ gives the ML estimate of σ as

$$\sigma^*=\sqrt[p]{\frac{2\hat{m}_p}{f_p(\rho^*)+\sqrt{f_p^2(\rho^*)+4g_{p,p}(\rho^*,N)}}}. \tag{24}$$

Comparing (24) with the direct estimator (16), we find that the former reduces to the latter if we choose $g_{p,p}(\rho^*,N)=0$ in (24), i.e., $N\to\infty$. Therefore, the direct estimator is asymptotically ML. From (20) we can see that the true ML estimate σ depends on both $\hat{m}_p$ and $\hat{m}_q$. Let σ_p^* and σ_q^* denote the solution of (24) using $\hat{m}_p$ and $\hat{m}_q$, respectively. We can use a linear combination of σ_p^* and σ_q^*, i.e., $(1-\gamma)\sigma_p^*+\gamma\sigma_q^*$ as the final estimate.

4) Pseudo LMMSE Estimator: We next consider another popular estimator, i.e., linear MMSE. From the direct estimator, we obtain

$$\begin{aligned}\hat{\rho}&=F^{-1}\left(\frac{(m_p+\epsilon_p)^q}{(m_q+\epsilon_q)^p}\right)\approx\rho+\frac{1}{F'(\rho)}\left(\frac{qm_p^{q-1}}{m_q^p}\epsilon_p-\frac{pm_p^q}{m_q^{p+1}}\epsilon_q\right)\\ \hat{\sigma}&=\sqrt[p]{\frac{m_p+\epsilon_p}{f_p(\rho+\epsilon_\rho)}}\approx\sigma+\frac{1}{p}\left(\frac{1}{\sigma^{p-1}f_p(\rho)}\epsilon_p-\frac{\sigma f_p'(\rho)}{f_p(\rho)}\epsilon_\rho\right),\end{aligned} \tag{25}$$

where ϵ_ρ is the error between $\hat{\rho}$ and ρ. The variances of ρ and σ can be computed as

$$\begin{aligned}&\nu_\rho(\rho,N)=E\left\{\frac{1}{F'^2(\rho)}\left(\frac{qm_p^{q-1}}{m_q^p}\epsilon_p-\frac{pm_p^q}{m_q^{p+1}}\epsilon_q\right)^2\right\}\\ &=\frac{\frac{q^2}{f_p^2(\rho)}g_{p,p}(\rho,N)+\frac{p^2}{f_q^2(\rho)}g_{q,q}(\rho,N)-\frac{2pq}{f_p(\rho)f_q(\rho)}g_{p,q}(\rho,N)}{\left(q\frac{f_p'(\rho)}{f_p(\rho)}-p\frac{f_q'(\rho)}{f_q(\rho)}\right)^2}.\end{aligned} \tag{26}$$

and

$$\begin{aligned}&\nu_\sigma(\rho,N)=E\left\{\frac{1}{p^2}\left(\frac{1}{\sigma^pf_p(\rho)}\epsilon_p-\frac{f_p'(\rho)}{f_p(\rho)}\epsilon_\rho\right)^2\right\}\\ &=\frac{f_q'^2(\rho)g_{p,p}(\rho,N)+f_p'^2(\rho)g_{q,q}(\rho,N)-2f_p'(\rho)f_q'(\rho)g_{p,q}(\rho,N)}{\left(qf_q(\rho)f_p'(\rho)-pf_p(\rho)f_q'(\rho)\right)^2}.\end{aligned} \tag{27}$$

From experiments (not shown in this paper), we find that both variances are good approximation to the true variances. The MMSE estimator takes the form

$$\hat{\rho}=\alpha F^{-1}\left(\frac{(\hat{m}_p)^q}{(\hat{m}_q)^p}\right),\quad \hat{\sigma}=\beta\sqrt[p]{\frac{\hat{m}_p}{f_p(\hat{\rho})}}, \tag{28}$$

where α and β are two scalers to be determined. To find α, we minimize the MSE between ρ and $\hat{\rho}$, i.e., $E\{|\rho-\hat{\rho}|^2\}$, which gives

$$\alpha(\rho,N)=\frac{\rho^2}{\rho^2+\nu_\rho(\rho,N)}, \tag{29}$$

where $\nu_\rho(\rho,N)$ is defined in (26). Substituting (29) into (28), ρ is found from the root of

$$\hat{\rho}=\alpha(\hat{\rho},N)F^{-1}\left(\frac{(\hat{m}_p)^q}{(\hat{m}_q)^p}\right). \tag{30}$$

From the expression of $\hat{\sigma}$ in (25) and by minimizing the MSE $E\{(\hat{\sigma}-\sigma)^2\}$, we obtain

$$\beta(\rho,N)=\frac{1}{1+\nu_\sigma(\rho,N)}. \tag{31}$$

Finally, substituting (31) into (28), we get

$$\hat{\sigma}=\beta(\rho,N)\sqrt[p]{\frac{\hat{m}_p}{f_p(\hat{\rho})}}\approx\beta(\hat{\rho},N)\sqrt[p]{\frac{\hat{m}_p}{f_p(\hat{\rho})}}, \tag{32}$$

where $\hat{\rho}$ is from (30). Interestingly, the LMMSE estimator can be seen as an approximation of the ML estimator (27) when $\nu_\sigma(\rho,N)$ is small. As the true LMMSE estimator requires substituting the true ρ into (29) and (31), the derived MMSE estimators are named as pseudo LMMSE estimators.

Remarks:

- The SNV estimator in [4] or (11) is a special case of (15) if we choose $P=2$, $k_1=k_2=1$, and $Q=1$, $\kappa_1=2$. The second- and fourth-order moments M_2M_4 estimator in [4] is obtained by choosing $P=2$, $k_1=k_2=2$, and $Q=1$, $\kappa_1=4$ in (15). However, different from the SNV [4] which estimates SNR from (11), we compute the SNR from an inverse function F^{-1}. Furthermore, different from the M_2M_4 estimator in [4] which estimates h and σ first and then compute ρ, we estimate ρ directly from (15).
- Even though the proposed estimators involve complicated function evaluation, we can build a look-up table for practical usage.

C. Blind Spectrum Sensing

There are two ways to conduct blind spectrum sensing. First, we can use the estimated h and σ without taking into account the history of noise variance estimation. Second, we can estimate noise variance using previous estimates assuming noise variances at different time are correlated and then apply the energy detector in Section II.

By the first approach, we compare the estimated SNR $\hat{\rho}$ with a threshold λ. If $\hat{\rho}>\lambda$, the secondary user decides $\hat{\theta}=1$; otherwise, $\hat{\theta}=0$. For the direct estimator (15), we can approximate $\hat{\rho}$ using (25) as a Gaussian random variable. The false alarm probability can be approximated by

$$P_f=Pr(\hat{\theta}=1|\theta=0)=\int_\lambda^\infty\frac{1}{\sqrt{2\pi\nu_\rho(0,N)}}\exp\left(-\frac{\hat{\rho}^2}{2\nu_\rho(0,N)}\right)d\hat{\rho}$$
$$=Q\left(\frac{\lambda}{\sqrt{\nu_\rho(0,N)}}\right), \tag{33}$$

where $\nu_\rho(0,N)$ is defined in (26). Similarly, the correction detection probability is

$$P_d=Pr(\hat{\theta}=1|\theta=1)=Q\left(\frac{\lambda-\rho}{\sqrt{\nu_\rho(\rho,N)}}\right). \tag{34}$$

For the second approach, let $\hat{\sigma}_n^2$ denote the n-th noise variance estimate. To smooth the estimation of noise variance, we can update noise variance using time average $\bar{\sigma}^2=\frac{1}{K}\sum_{n=1}^K\hat{\sigma}_n^2$, if we assume that the noise variance remains constant during the past K estimations. Or we can smooth noise variance using a first order infinite impulse response (IIR) filter

$$\bar{\sigma}^2=(1-\gamma)\bar{\sigma}^2+\gamma\hat{\sigma}_n^2, \tag{35}$$

where $1>\gamma>0$ is a coefficient.

By using the energy detector in Section II, we compare $\sum_{i=1}^N|y_i|^2$ with a threshold $\lambda\bar{\sigma}^2$. If $|y|^2>\lambda$, the secondary user decides $\hat{\theta}=1$; otherwise, $\hat{\theta}=0$. The parameter λ is chosen such that the false alarm probability is ζ, i.e., $P_f(\lambda)=\zeta$.

We can use (3) directly or take into account the estimation error in $\bar{\sigma}^2$. Assuming that the variance of the estimate $\hat{\sigma}_n^2$ is $\sigma^4\nu_{\sigma,n}$ and the variance of $\bar{\sigma}^2$ is $\sigma^4\nu_{\bar{\sigma}}$, from (35) gives

$$\nu_{\bar{\sigma}}=(1-\gamma)^2\nu_{\bar{\sigma}}+\gamma^2\nu_{\sigma,n}, \tag{36}$$

where $\nu_{\sigma,n}$ can be obtained from Section III-B. Let $\bar{\sigma}^2=\sigma^2+\epsilon_\sigma$, where ϵ_σ is a Gaussian random variable with zero mean and variance $\sigma^4\nu_{\bar{\sigma}}$. Substituting $\bar{\sigma}^2$ into (3), we obtain

$$P_f(\lambda)=\frac{1}{\sqrt{2\pi\sigma^2\nu_{\bar{\sigma}}}}\int_{\lambda(\sigma^2+\epsilon_\sigma)}^{+\infty}\frac{1}{\sigma^{2N}\Gamma(N)}t^{N-1}$$
$$\times\exp\left(-\frac{t}{\sigma^2}\right)dt\exp\left(-\frac{\epsilon_\sigma^2}{2\sigma^4\nu_{\bar{\sigma}}}\right)d\epsilon_\sigma$$
$$=\frac{1}{\sqrt{2\pi\nu_{\bar{\sigma}}}\Gamma(N)}\int_{-\infty}^{\infty}\Gamma(N,\lambda(1+x))\exp\left(-\frac{x^2}{2\nu_{\bar{\sigma}}}\right)dx. \tag{37}$$

This method constitutes two parts: noise variance estimation and tracking of the variance of noise variance estimation.

IV. Simulation Results

In this section, we present simulation results to complement our theoretical analysis. We estimate h and σ with $N=192$. BPSK and 64QAM are simulated. The ML estimator for BPSK by using (7) is denoted as "ML BPSK", which is also applied to 64QAM. The estimator by using (11) is denoted as SNV. The moment ML estimator by maximizing (19) is denoted as "ML Moment". The simulation is done for SNR between -10 dB and 20 dB. We restrict the maximum ρ to be 10^3 in the moment estimators.

We first show simulation results for the estimation of h, σ and ρ. We use normalized MSE (NMSE) as performance

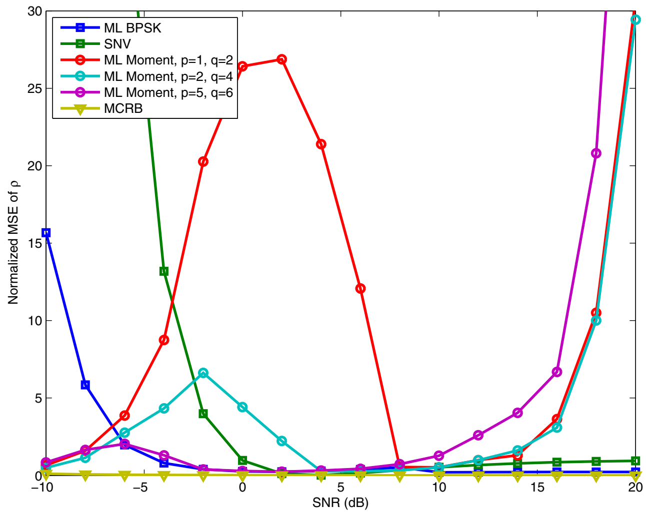

Fig. 1. Comparison of normalized MSE for ρ between different estimators with 64QAM and N=192.

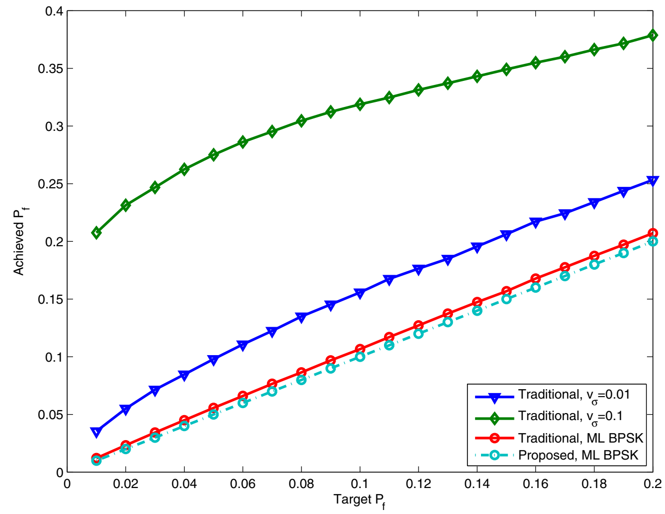

Fig. 2. Comparison of achieved false alarm probability P_f with BPSK, 16QAM and N=48.

metric defined as $E\{|\hat{\rho}-\rho|^2\}/\rho^2$. We also compare with the modified CRB (MCRB). The NMSEs are obtained after 2000 simulation runs for each SNR.

Fig. 1 shows the NMSE of ρ with 64QAM and N= 192, respectively. All estimators' NMSEs diverge (but MSEs converge to zero as SNR goes to infinity). Interestingly, the NMSEs of ρ and σ by using ML BPSK are still good though the NMSE of h is not good. This can be explained as the BPSK approximation of the amplitude of 64QAM is not accurate. We also find that the high moment estimator with p=5 and q=6 achieve better performance than those with smaller moments in high SNR. These confirm that to achieve a better performance in high SNR a high moment pair is required. We have also found that the ML moment estimator generally achieves a better performance than the direct moment estimator. The peak of ML moment estimator is due to the use of the NMSE metric.

Next, we apply the noise variance estimator to spectrum sensing. To evaluate the effect of estimation error on the performance of spectrum sensing, we model the estimated noise variance $\hat{\sigma}^2$ as a Gaussian random variable with mean σ^2 and variance $\sigma^4\nu_{\bar{\sigma}}$, where $\nu_{\bar{\sigma}}$ is equal to the normalized MSE. The energy detector is used along with the estimated noise variance. We compare the performance of the traditional spectrum sensing computing the detection threshold λ by substituting $\hat{\sigma}^2$ into (3) with that of the proposed algorithm by computing λ using (37). In addition, we include the result using the ML BPSK estimator, where estimated noise variance and signal power are obtained after 1000 simulation runs. Simulations are performed at SNR=−5 dB with 16QAM and N=48. Fig. 2 shows the achieved false alarm probability P_f given a target P_f. We find that the traditional method by using the estimated noise variance directly increases P_f especially when $\nu_{\bar{\sigma}}$ is large, while the proposed algorithm can achieve the desired P_f. This shows that computing P_f in the conventional energy detector depends crucially on the accuracy of noise variance estimate. As the proposed noise variance and signal power estimators can achieve a very high accuracy, the impact of the estimation error on the performance of the proposed spectrum sensing algorithm is negligible.

V. Conclusion

In this paper, we considered blind energy detection based spectrum sensing without *a priori* knowledge of the signal power of the primary user and the noise variance. Direct estimator, approximate ML estimator and pseudo linear MMSE estimator were proposed by using the moments of received signal at the secondary user. The proposed estimators exploit the signal structure of the primary user. To remedy the problem of unknown primary user signal constellation, we approximated a finite PAM constellation using a continuous uniform distribution. The way to find the optimal moment pair and to choose the spectrum sensing detection threshold under estimation error was discussed. The proposed estimators can also be used to enhance the performance of SNR estimation and for applications such as turbo decoding.

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