Representation of Composite Fading and Shadowing Distributions by using Mixtures of Gamma Distributions

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***Abstract*—The Nakagami-lognormal distribution is the commonly used composite distribution for modeling multipath fading and shadowing. In this paper[1], simple and new form of distribution which can accurately represent both the mutlipath fading and shadowing effects is introduced. The signal-to-noise ratio (SNR) of the Nakagami-lognormal distribution follows the gamma-lognormal distribution, which is accurately approximated by a weighted mixture of gamma distributions. We show how the weights and other parameters of the summands are obtained. Further, accuracy of the mixture distribution is compared with the K_G distribution – a popular approximation of the Nakagami-lognormal distribution.**

***Index Terms*—Gaussian-Hermite integral, K_G fading model, Nakagami-lognormal distribution.**

I. Introduction

Fading (small-scale signal power fluctuations) is a fundamental characteristic of wireless channels. Several fading models such as Rayleigh, Rice and Nakagami distributions are widely used in wireless research. The large-scale signal power variation is shadowing, which is statistically modeled by the lognormal distribution [1]. Further, the composite fading and shadowing channel is modeled as Rayleigh-lognormal, Rice-lognormal, or Nakagami-lognormal distribution. However, such composite distributions do not lend themselves to performance analysis readily. For example, their cumulative distribution function (CDF) and moment generating function (MGF) may not expressible in simple closed-form analytical formulas. Consequently, in the recent literature, there has been an attempt to develop simpler approximations.

Two more popular channel models as approximation for the Rayleigh-lognormal and Nakagami-lognormal distributions are the K and K_G distributions, respectively. The K distribution is a mixture of the Rayleigh and gamma distributions. In [2], the Rayleigh-lognormal and K distributions are compared with respect to their probability density functions (PDF), and it is shown that both PDFs match well at the tail. However, there is a considerable deviation between two PDFs for small values of shaping factor of gamma distribution (β in [2]). It can be seen that moments are matched only up to the second moment. In [3], error performance is compared for the Rayleigh-lognormal and K distributions with differential phase-shift keying (DPSK) and minimum shift keying (MSK). It is shown that the two distributions do not match well in high signal-to-noise ratio (SNR) range with low shaping factor β. On the other hand, the K_G distribution, which is a mixture of the Nakagami and gamma distributions, has been introduced in [4] to approximate the Nakagami-lognormal distribution. However, no comparison is given for the two distributions.

Although the K and K_G distributions were introduced as simpler alternatives to the composite models, their amplitude and SNR distributions include the modified Bessel function of the second kind ($K_\alpha(\cdot)$) [2] [4] [5]. Further, their CDFs and MGFs include the generalized hypergeometric function and the Whittaker function, respectively. Therefore, average channel capacity and average symbol error probability over the K and K_G fading channels end up with Meijer's G-functions [5].

In [6], different diversity receptions under K_G fading channels have be analyzed in terms of moments, average SNR, and the amount of fading. The PDFs of end-to-end SNR of each combing method are with complicated forms, or the corresponding MGFs do not have closed-form expressions. For example, the MGFs are approximated with the Padé approximation for maximal ratio combining (MRC) and dual branch selection combining (SC), and the outage probabilities are calculated with numerical integration [6]. Performance of generalized SC over K fading channels is analyzed with numerically integrated marginal MGF of K channel when the fading parameter is equal to an integer plus one-half [7].

Although K_G distribution is considered as an approximation and a simple form for the Nakagami-lognormal distribution, we note that most of the performance results are derived using further approximations to avoid mathematical difficulties. As a result, we introduce a better and simple form of distribution in this research in order to approximate the Nakagami-lognormal (amplitude) and the gamma-lognormal (SNR) distributions. The new distribution is derived by replacing the integral of the Nakagami-lognormal distribution with the Gaussian-Hermite quadrature sum [8] [9] [10] .

The rest of the paper is organized as follows. The approximation for the PDFs of the signal amplitude and SNR with the Nakagami-lognormal channel is given in Section II and III, respectively. Statistical properties and performance evaluation can be found in Section IV and Section V, respectively. The

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concluding remarks are made in Section VI.

II. Approximation for Signal Amplitude PDF

For the Nakagami-lognormal channel, let X be the fading amplitude, which is a random variable. The composite Nakagami-lognormal distribution $f_{nl}(x)$ is a mixture of Nakagami fading and lognormal shadowing. Its PDF can be expressed as an integral form [1]

$$f_{nl}(x) = \int_0^\infty \frac{2m^m x^{2m-1} e^{-\frac{mx^2}{y}}}{\Gamma(m) y^m} \frac{1}{\sqrt{2\pi}\lambda y}\, e^{-\frac{(\ln y-\mu)^2}{2\lambda^2}} dy \quad (1)$$

where $x \geq 0$, m is the fading parameter in Nakagami-m fading, μ and λ are the mean and the standard deviation of lognormal shadowing, respectively, and $\Gamma(\cdot)$ is the standard gamma function. When $m = 1$, distribution in (1) is Rayleigh-lognormal distribution which has severe fading. The fading effect and shadowing effect diminish for larger m ($m \to \infty$) and smaller λ ($\lambda \to 0$), respectively. A closed-form expression of the composite Nakagami-lognormal PDF is not available in the literature. In the following, we introduce an accurately approximated PDF which can represent both fading and shadowing effects.

Using the substitution $t = \frac{\ln y-\mu}{\sqrt{2}\lambda}$, $f_{nl}(x)$ in (1) can be written as

$$f_{nl}(x) = \frac{2m^m x^{2m-1}}{\sqrt{\pi}\ \Gamma(m)} \cdot \int_{-\infty}^{\infty} e^{-t^2} h(t) dt \quad (2)$$

where $h(t) = e^{-m\left(\sqrt{2}\lambda t+\mu+x^2 e^{-(\sqrt{2}\lambda t+\mu)}\right)}$. The term $\int_{-\infty}^{\infty} e^{-t^2} h(t)dt$ has the form of the Gaussian-Hermite integration which can be approximated as $\int_{-\infty}^{\infty} e^{-t^2} h(t)\ dt \approx \sum_{i=1}^{N} w_i h(t_i)$ where t_i and w_i are abscissas and weight factors for the Gaussian-Hermite integration [11]. t_i and w_i for different N values are available in [11, Table (25.10)] or can be calculated by a simple MATLAB program. Therefore, expression (2) can be re-written as

$$f_{nl}(x) \approx \frac{2m^m x^{2m-1}}{\sqrt{\pi}\ \Gamma(m)} \sum_{i=1}^{N} w_i h(t_i). \quad (3)$$

Using this approximation, we give the expressions of the new PDF and statistical properties of the signal amplitude in the following. The new PDF is denoted as $f_{N_X}(x)$ in the sequel.

A. PDF $f_{N_X}(x)$

The PDF, $f_{N_X}(x)$, is defined as

$$f_{N_X}(x) \triangleq C \sum_{i=1}^{N} a_i x^{2m-1} e^{-b_i x^2} \qquad x \geq 0, \quad (4)$$

where $a_i = 2m^m w_i e^{-m(\sqrt{2}\lambda t_i+\mu)}/\sqrt{\pi}\ \ \Gamma(m)$, $b_i = me^{-(\sqrt{2}\lambda t_i+\mu)}$, and C is the normalization factor to ensure $\int_0^\infty f_{N_X}(x)dx = 1$. It can be shown that $C = \sqrt{\pi}/\sum_{i=1}^{N} w_i$.

B. CDF $F_{N_X}(x)$

The CDF $F_{N_X}(x)$ can be evaluated as $F_{N_X}(x) = \int_0^x f_{N_X}(t)dt$ to yield

$$F_{N_X}(x) = \frac{C}{2} \sum_{i=1}^{N} \frac{a_i}{{b_i}^m} \gamma\left(m, b_i x^2\right) \quad (5)$$

where $\gamma(\cdot;\cdot)$ is the incomplete gamma function defined as $\gamma(a,\rho) \triangleq \int_0^\rho t^{a-1} e^{-\phi} d\phi$ [12, eq. 8.350.1]. Further, $F_{N_X}(x)$ can also be re-written as $F_{N_X}(x) = C/2 \sum_{i=1}^{N} a_i \left(\Gamma(m) - \Gamma(m, b_i x^2)\right)/{b_i}^m$ where $\Gamma(\cdot;\cdot)$ is the upper incomplete gamma function defined as $\Gamma(a,\rho) \triangleq \int_\rho^\infty t^{a-1} e^{-\phi} d\phi$ [12, eq. 8.350.2]. Note that $a_i/{b_i}^m = 2w_i/\sqrt{\pi}\ \Gamma(m)$.

C. MGF $\mathcal{M}_{N_X}(t)$

The MGF $\mathcal{M}_{N_X}(t)$ can be evaluated as $\mathcal{M}_{N_X}(t) = \mathbb{E}(e^{-xt}) = \int_0^\infty e^{-xt} f_{N_X}(x)dx$. Here $\mathbb{E}(\cdot)$ denotes the expectation operator. With the aid of [12, eq. 3.462.1], $\mathcal{M}_{N_X}(t)$ can be evaluated as

$$\mathcal{M}_{N_X}(t) = \frac{C\Gamma(2m)}{2^m} \sum_{i=1}^{N} \frac{a_i}{{b_i}^m} e^{\frac{t^2}{8b_i}} D_{-2m}\left(\frac{t}{\sqrt{2b_i}}\right) \quad (6)$$

where $D_p(\cdot)$ is the parabolic cylinder function [12, eq. 9.240].

D. Amount of Fading

The amount of fading (AoF) is a statistical property which measures the severity of the fading channel. AoF can be expressed as $AoF = \mathbb{E}(X^4)/[\mathbb{E}(X^2)]^2 - 1$ which can be calculated from the second and the forth moments of the amplitude distribution or the first and the second moments of the SNR distribution. AoF of the new distribution given in (4) can be evaluated as

$$AoF_N = \frac{\sqrt{\pi}(m+1)}{m} \frac{\sum_{i=1}^{N} w_i e^{2\sqrt{2}\lambda t_i}}{\left(\sum_{i=1}^{N} w_i e^{\sqrt{2}\lambda t_i}\right)^2} - 1. \quad (7)$$

It can be seen that AoF depends on fading parameter m of the Nakagami fading and standard deviation λ of the lognormal shadowing. However, AoF is independent of the mean (μ) of the lognormal shadowing. When the shadowing effect diminishes, $\lambda \to 0$, $AoF \to 1/m$. It confirms that $AoF = 1$ under Rayleigh fading with no shadowing. Further, $AoF \to \infty$ as $\lambda \to \infty$. Therefore, AoF ranges within $[1/m, \infty)$, which is similar to the AoF of the composite Nakagami-lognormal channel model.

III. Approximation for the SNR PDF

When the fading envelop is X, the transmitted symbol energy is E_s and single sided power spectral density of the complex additive white Gaussian noise (AWGN) is N_0, the instantaneous SNR per symbol is $\gamma = X^2\rho$ where $\rho = E_s/N_0$ is the unfaded SNR. Based on the amplitude distribution given in (4), the SNR distribution $f_{N_\gamma}(x)$ can be expressed as

$$f_{N_\gamma}(x) = \frac{C}{2\rho^m} \sum_{i=1}^{N} a_i \gamma^{m-1} e^{-\frac{b_i}{\rho}\gamma} \quad (8)$$

where $f_{N_\gamma}(x)$ is equal to a *Mixture of Gamma Distributions* [13]. Since the SNR PDF of the Nakagami-lognormal channel is gamma-lognormal distribution, we can accurately approximate the gamma-lognormal distribution through a mixtures of gamma distributions. The CDF of SNR, $F_{N_\gamma}(x)$ can be derived as

$$F_{N_\gamma}(x) = \frac{C}{2}\sum_{i=1}^{N}\frac{a_i}{{b_i}^m}\gamma\left(m, \frac{b_i x}{\rho}\right). \tag{9}$$

The MGF of SNR, $\mathcal{M}_{N_\gamma}(t)$, can be evaluated as

$$\mathcal{M}_{N_\gamma}(t) = \frac{C\Gamma(m)}{2\rho^m}\sum_{i=1}^{N}\frac{a_i}{\left(t+\frac{b_i}{\rho}\right)^m}. \tag{10}$$

The main performance measurement of wireless communication systems is the instantaneous SNR and its statistical properties. Since the mixture of gamma distributions has simple forms for PDF, CDF and MGF expressions, it is a powerful tool in analytical research studies.

IV. Comparison with the K_G Channel Model

The K_G distribution f_{K_G}(x) has been introduced as an approximated and a simple channel model for the Nakagami-lognormal distribution [4]. The K_G channel model has been used in several research studies in the literature to analyze the effect of both fading and shadowing effects in wireless channels [3]–[7]. Therefore, it is reasonable to compare $f_{N_X}(x)$ and $f_{K_G}(x)$ in terms of how they match with $f_{nl}(x)$. In the following, the K_G distribution is introduced first. Then we compare the mean square errors (MSE) of $f_{N_X}(x)$ and $f_{K_G}(x)$ from $f_{nl}(x)$, the moments of the three distributions, and the PDFs of the three distributions.

A. K_G Distribution

For the K_G distribution, the PDF of X is given by [6]:

$$f_{K_G}(x) = \frac{4m^{(\beta+1)/2}x^\beta}{\Gamma(m)\Gamma(k)\Omega^{(\beta+1)/2}}K_\alpha\left[2\left(\frac{m}{\Omega}\right)^{1/2}x\right], \quad x \geq 0 \tag{11}$$

where m is Nakagami fading parameter, $\alpha = k - m$, $\beta = k + m - 1$ and $\Omega = E[X^2]/k$ is the mean signal power.

Using the power series expansion of the logarithm MGFs of Nakagami-lognormal distribution and K_G distribution, the relationships between parameters (λ and μ) of Nakagami-lognormal distribution and parameters (α and β) of K_G distribution can be derived [2] as $\lambda^2 = \Psi^{'}(\beta)$ and $\mu = \Psi(\beta) + \ln(\alpha)$ where $\Psi^{'}(\cdot)$ is the first derivative of the psi function [12, eq. 8.360.1].

B. Mean Square Error (MSE)

The MSE between $f_{N_X}(x)$ and $f_{nl}(x)$ can be calculated numerically for different N values in $f_{N_X}(x)$. If we plot MSE verses N, a lower bound of N can be obtained, which guarantees a given MSE value. Fig. 1 shows the MSE of $f_{K_G}(x)$ and the MSE[2] of $f_{N_X}(x)$ versus N. It can be seen that, if the value of N slightly increases, the MSE decreases significantly. If the MSE requirement is set to be 10^{-4}, the K_G distribution does not meet this requirement when $\beta = 1, 1.5$ or 2, while our new distribution can meet the requirement with $N \geq 11, 7$ or 5 when $\beta = 1, 1.5$ or 2, respectively. This means that we can efficiently get a more accurate approximation for the Nakagami-lognormal distribution than the K_G model. We assume that $m = 2$ and $\alpha = 4$ for the calculations.

[2] MSE between $f_{K_G}(x)$ and $f_{nl}(x)$ does not depend on N.

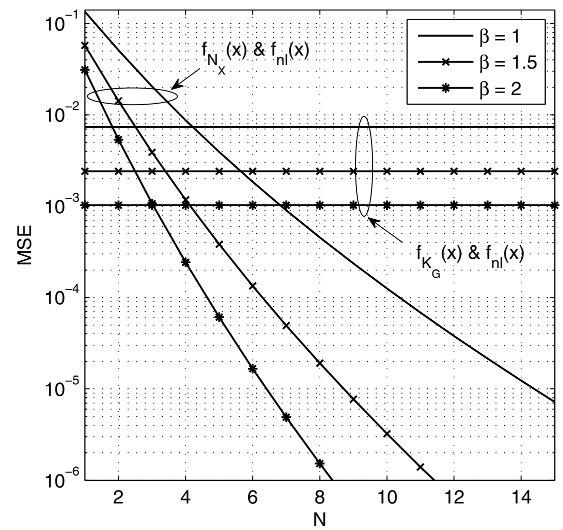

Fig. 1. MSE versus N of $f_{N_X}(x)$ and MSE for $f_{K_G}(x)$ distribution.

C. Moments

The n^{th} moment of a PDF $f_X(x)$ can be defined as $M_X^n \triangleq \mathbb{E}(X^n)$. Therefore the n^{th} moment of $f_{N_X}(x)$ distribution, $M_{N_X}^n$, can be evaluated as

$$M_{N_X}^n = \frac{C\Gamma(\frac{2m+n}{2})}{2}\sum_{i=1}^{N}\frac{a_i}{{b_i}^m}\frac{1}{{b_i}^{\frac{n}{2}}}. \tag{12}$$

By interchanging the order of integrations, The n^{th} moment of $f_{nl}(x)$ distribution can be evaluated as

$$M_{nl}^n = \frac{\Gamma(\frac{n}{2}+m)e^{-(\frac{n\mu}{2}+\frac{n^2\lambda^2}{8})}}{m^{\frac{n}{2}}\Gamma(m)}.$$

The expression for the n^{th} moment of K_G channel model, $M_{K_G}^n$, is available in [4] as

$$M_{K_G}^n = \frac{\Gamma(\frac{n}{2}+\beta)\Gamma(\frac{n}{2}+m)}{\Gamma(\beta)\Gamma(m)}\left(\frac{\alpha}{m}\right)^{\frac{n}{2}}.$$

Table I shows some numerical examples to compare the moments (n) of the three distributions. For the $f_{N_X}(x)$, the value of N is selected such that an MSE requirement (say, $< 10^{-4}$) is met. We assume that $m = 2$ and $\alpha = 4$ for the calculations. It can be seen that the moments (n) of $f_{N_X}(x)$ and $f_{nl}(x)$ distributions are almost the same. On the other hand, the differences between moments of $f_{nl}(x)$ and $f_{K_G}(x)$ distributions increase for the third moment and beyond.

TABLE I
THE FIRST FIVE MOMENTS OF THREE DISTRIBUTIONS.

	$\beta = 1$, $N = 11$			$\beta = 2$, $N = 5$		
n	$f_{N_X}(x)$	$f_{nl}(x)$	$f_{K_G}(x)$	$f_{N_X}(x)$	$f_{nl}(x)$	$f_{K_G}(x)$
1	1.7	1.7	1.7	2.5	2.5	2.5
2	5.1	5.1	4.0	8.4	8.4	8.0
3	25.1	25.1	12.5	36.6	36.6	31.2
4	202	203	48	203	203	144
5	2610	2654	219	1421	1421	765

D. PDFs

Fig. 2 illustrates the three distributions for two cases $\beta = 1$ and $\beta = 2$, with $m = 2$ and $\alpha = 4$. $f_{N_X}(x)$ is almost the same as $f_{nl}(x)$. The mixture sizes of $f_{N_X}(x)$ for the two cases are $N = 11$ and $N = 5$, respectively. On the other hand, $f_{K_G}(x)$ has more deviation from $f_{nl}(x)$, particularly for small values of x and β.

V. PERFORMANCE ANALYSIS WITH NUMERICAL EXAMPLES

Since the mixture of gamma distributions has a simple form and accurately approximate the gamma-lognormal distribution, it is motivated to derive closed-form expressions for some important performance measures such as the channel capacity, outage probability, and symbol error rate as follows.

A. Average Channel Capacity

Based on Shannon's theorem, the average channel capacity, $\mathcal{C}$ can be calculated by averaging the instantaneous channel capacity over SNR as $\mathcal{C} = \int_0^\infty B\log_2(1+x) f_{N_\gamma}(x)dx$, where B is the signal transmission bandwidth. Using a similar approach to that in [14], $\mathcal{C}$ can be calculated for integer m as

$$\mathcal{C} = \frac{CB(m-1)!}{2\rho^m \ln 2} \sum_{i=1}^{N} a_i e^{\frac{b_i}{\rho}} \sum_{j=1}^{m} \left(\frac{\rho}{b_i}\right)^j \Gamma(j-m, \frac{b_i}{\rho}). \quad (13)$$

B. Outage Probability

The outage probability is the probability that the receiver SNR is below a given threshold γ_{th}. It can be straightforwardly calculated as $P_{out}(\gamma_{th}) = F_{N_\gamma}(\gamma_{th})$, where $F_{N_\gamma}(x)$ is in (9).

C. Average Symbol Error Rate

We present average symbol error rate (SER) for M-PSK and M-QAM in the following.

1) M-PSK: The average SER for M-PSK, P_e^{PSK}, is given in [15, eq. (9.15)]. With the MGF given in (10), P_e^{PSK} can be evaluated with the aid of [15, eq. (5A.17)], the average SER of M-PSK modulation can be evaluated in closed-form for integer m.

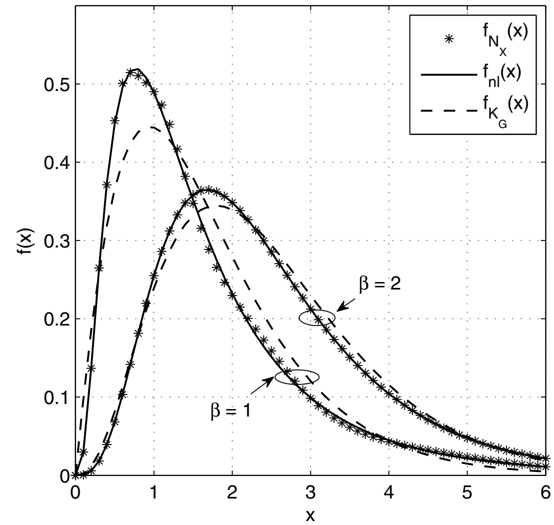

Fig. 2. Plots of three distributions.

2) M-QAM: Square M-QAM signals that have constellation size $M = 2^k$ with an even integer k are considered. The average SER for M-QAM, P_e^{QAM}, over the generalized fading channels is given in [15, eq. (9.22)]. With the MGF given in (10), P_e^{QAM} can be evaluated as

$$\begin{aligned} P_e^{QAM} =& K \sum_{i=1}^{N} \frac{a_i}{b_i^{\,m}} \left[\int_0^{\pi/2} \left(\frac{\sin^2\theta}{\sin^2\theta + \frac{g_{QAM}\rho}{b_i}} \right)^m d\theta \right. \\ & \left. - \left(1 - \frac{1}{\sqrt{M}}\right) \int_0^{\pi/4} \left(\frac{\sin^2\theta}{\sin^2\theta + \frac{g_{QAM}\rho}{b_i}} \right)^m d\theta \right]. \end{aligned} \quad (14)$$

where $K = 2C\Gamma(m)(1 - 1/\sqrt{M})/\pi$ and $g_{QAM} = 3/2(M-1)$. The first and the second integrals in expression (14) can be evaluated with the aid of [15, eq. (5A.3)] and [15, eq. (5A.13)], respectively. Therefore, average SER of M-QAM modulation can be evaluated in closed-form for any m.

D. Numerical Results

Figs. 3-5 show normalized average capacity, outage probability and SER, respectively. Continuous and dashed lines represent the performance over the gamma-lognormal distribution, while discrete symbols represent the performance over the mixture of gamma distributions with $N = 15$. With $N = 15$, we can achieve MSE less than 10^{-6} for the following examples. In Fig. 4 and Fig. 5, σ denotes the standard deviation in dB given as $\sigma = (10/\ln 10)\lambda$ dB. Further, $\sigma = 4.5, 8, 13$ dB represents urban, typical macrocells and microcells, respectively in cellular mobile systems. We assume $\alpha = 4$ for all the three figures and $m = 2$ for outage and SER calculations. The performance curves for gamma-lognormal are plotted by Monte-Carlo simulations. All figures show that both the gamma-lognormal distribution

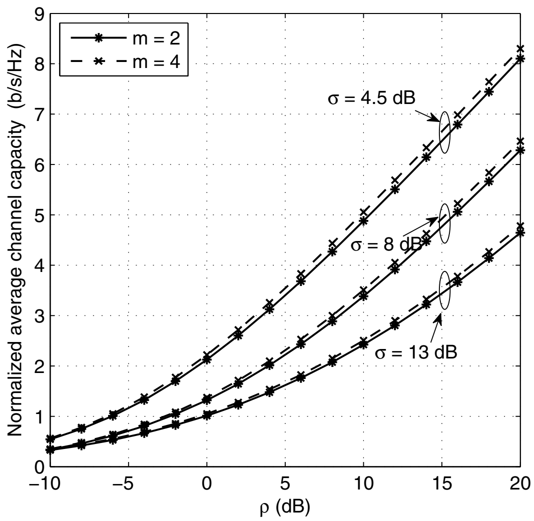

Fig. 3. Normalized average channel capacity versus unfaded SNR ρ.

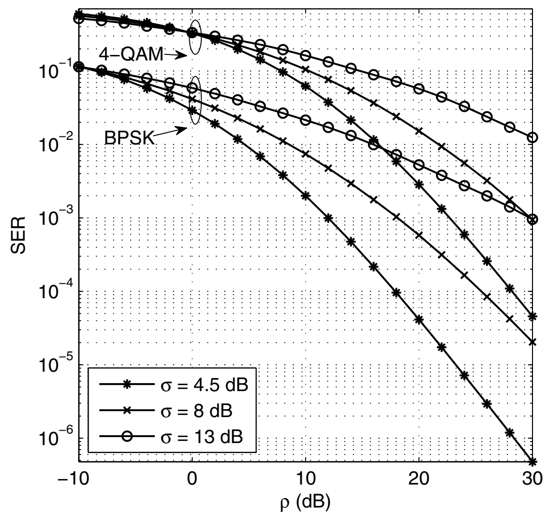

Fig. 5. SER versus unfaded SNR ρ.

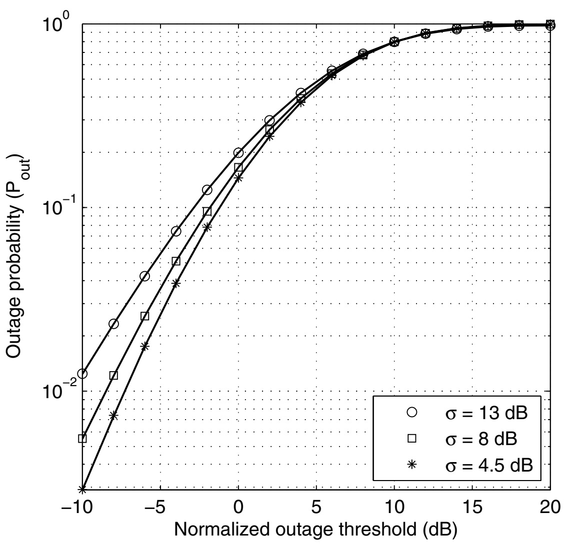

Fig. 4. Outage probability versus normalized outage threshold.

and the mixture of gamma distributions have well-matched performance curves.

VI. Conclusions

This paper proposes a mixture distribution to model both multipath and shadowing effects in wireless channels. In this new model, the SNR distribution follows a mixture of gamma distributions, resulting in simple forms of PDF, CDF and MGF. This approach leads to excellent accuracy in representing the composite distributions. For examples, in terms moment matching, the new approach can be highly accurate match a large number of moments (Table I), whereas the K_G distribution can only match the first two moments. We believe that the proposed distribution provides a highly flexible and simpler alternative to several currently popular channel distributions.

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