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Adaptive Multiple Relay Selection Scheme for Cooperative Wireless Networks

Gayan Amarasuriya, Masoud Ardakani and Chintha Tellambura
Department of Electrical and Computer Engineering,
University of Alberta, Edmonton, AB, Canada T6G 2V4
Email: {amarasur,ardakani,chintha}@ece.ualberta.ca

***Abstract*—In this paper, we propose an output-threshold multiple relay selection scheme for dual-hop multi-branch cooperative wireless networks. The proposed scheme selects the first L_c arbitrary ordered relays out of L relays such that the maximal ratio combined signal-to-noise-ratio (SNR) of the L_c relayed paths and the direct path barely exceeds a preset threshold. Closed-form expressions are derived for the cumulative distribution function, the probability density function, and the moment generating function of an output SNR upper bound for independent and identically distributed Rayleigh fading. Lower bounds for the outage probability, the average symbol error rate, and the average number of selected relays are also derived. Moreover, upper bounds for the average output SNR and the ergodic capacity are also derived. The analytical results are verified via the Monte-Carlo simulation. The performance of our proposed scheme is compared to that of the existing relay selection schemes. The proposed schemes provide more flexibility in utilizing bandwidth and spatial diversity in cooperative wireless networks.**

I. Introduction

Cooperative (relay) wireless networks achieve distributed spatial diversity, wider coverage, low transmit power and reduced interference [1], [2]. Selecting a subset of available relays according to a performance metric can further enhance the performance of cooperative networks. Relay selection (RS) schemes can be derived from classical adaptive diversity combining techniques. In particular, the combiner output is compared against a threshold, and only the diversity branches whose signal-to-noise ratio (SNR) exceed a predefined threshold are used for further processing [3]–[7]. We utilize such an adaptive combining idea to propose a new output-threshold multiple relay selection (OT-MRS) scheme.

A. *Prior related research*

The dual-hop multi-branch cooperative network of Laneman and Wornell [1] uses all available multiple relays, and, hereafter, we call this type of cooperation "all-participate relaying" (APR). APR has a low spectral efficiency due to the use of multiple orthogonal channels. RS schemes [8]–[13], which overcome the low spectral efficiency problem, can be broadly divided into two categories: single relay selection (SRS) schemes and multiple relay selection (MRS) schemes.

Several SRS schemes have been proposed in the literature. The selection of the relay whose path has the maximum end-to-end signal-to-noise ratio (SNR) is the optimal scheme [11], [13]. This scheme achieves the full diversity while maintaining a higher throughput [13] than the other schemes. Reference [9] proposes the nearest-neighbor RS scheme. The best-neighbor RS scheme [13] is a modification of [9], with the selection of the relay with the strongest channel to the source or the destination. It is shown in [13] that this scheme does not achieve any diversity gain if the direct path is not present. Otherwise the nearest-neighbor scheme achieves a diversity order of two. The best-worst channel RS scheme is proposed for dual-hop multi-branch cooperative networks in [10]. This scheme selects the relay whose worst channel is the best and also achieves the full diversity order [13].

In [13], [14] and [15], several MRS schemes are proposed by generalizing the idea of SRS in order to allow for more than one relay to cooperate. The selection method of [14] involves minimizing the error probability under total energy constraints. Several selection methods of [13] involve the maximization of the received SNR subjected to per relay power constraints. However, since the optimal selection rule of [13] has exponential complexity in the number of relays, several suboptimal schemes are proposed, which have linear complexity in the number of relays at the expense of a performance loss. Recently, in [15], a generalized selection combining (GSC) based MRS scheme was proposed and analyzed.

Apart from the RS schemes mentioned above, incremental relaying [2] achieves higher spectral efficiencies than that of APR as the former utilizes the degree of freedom of the channel effectively with the aid of limited feedback from the destination. Recently, in [16], incremental relaying with the best relay selection scheme for AF relaying over fading channels was proposed and analyzed.

B. *Motivation and our contribution*

Although SRS schemes have higher bandwidth efficiencies than APR, these schemes suffer a performance loss in terms of the error rate and the outage probability because they do not fully exploit the available degrees of spatial diversity. Moreover, the complexity of the optimal MRS schemes [13] increases exponentially with the available number of relays. Although the GSC-based MRS [15] achieves considerable performance gains, it requires the channel estimation of all the relayed paths. In addition, the combined SNR sometimes may far exceed the requirements of the system. Thus, GSC-based MRS may select more relays unnecessarily. These gaps in the existing RS schemes have motivated us to seek a MRS

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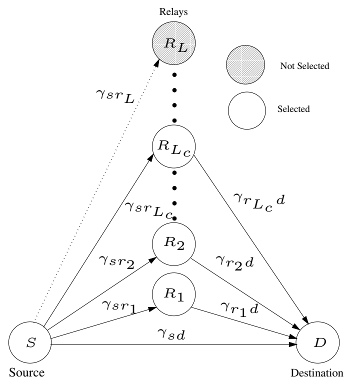

Fig. 1. System model: The available L relays are arbitrarily ordered and labeled from R_1 to R_L.

scheme that offers a better trade-off between error performance and bandwidth efficiency. Thus, in this paper, we propose and analyze the OT-MRS scheme for dual-hop multi-branch cooperative wireless networks. More details of the OT-MRS scheme can be found in [17], which is under review.

The rest of this paper is organized as follows. In Section II, the system and the channel model is presented. Section III describes the mode of operation of the proposed OT-MRS scheme. Section IV presents a performance analysis. Section V contains the numerical results and Section VI concludes the paper.

Notations: $\Gamma(z)$ is the *Gamma function* [18, 8.310.1]. $\gamma(\alpha, z)$ is the *Incomplete gamma function* [18, 8.350.1]. $\mathrm{Q}(z)$ is the *Q-function* [19, 26.2.3]. $\mathcal{E}_\Lambda\{\cdot\}$ denotes the *expected value* over random variable Λ.

II. System and Channel Model

Our analysis considers a cooperative wireless network with $L+2$ terminals that include one source S, one destination D, and L AF relays $R_l|_{l=1}^{L}$ (see Fig. 1). Only single antenna terminals are considered. As usual, source-to-destination communication takes place in two phases. In the first phase (the broadcast phase), S broadcasts its signal to L relays and D. In the second phase (the relaying phase), relay selection is applied; i.e., only L_c $(1 \le L_c \le L)$ relays out of L relays are selected to forward the amplified version of the source signal to D. To facilitate the orthogonal transmission in two phases, a time-division channel allocation scheme with L_c time-slots is used [2]. The channels $S \to R_l$ and $R_l \to D$ are independent and identically distributed (i.i.d.) and undergo flat-Rayleigh fading. Moreover, our model contains an independent flat-Rayleigh fading direct channel from $S \to D$.

The instantaneous output SNR $\Gamma_i|_{i=1}^{L}$ at D with i active channel-assisted AF (CA-AF) relays can be written as [20]

$$\Gamma_i = \gamma_{sd} + \sum_{l=1}^{i} \frac{\gamma_{sr_l}\gamma_{r_ld}}{\gamma_{sr_l} + \gamma_{r_ld} + 1}, \tag{1}$$

where γ_{sd}, γ_{sr_l}, and γ_{r_ld} are instantaneous SNR of the channels $S \to D$, $S \to R_l$, and $R_l \to D$, respectively. For Rayleigh fading channels, γ_{sd}, γ_{sr_l} and γ_{r_ld} are independent exponential random variables with means $\bar{\gamma}_{sd}$, $\bar{\gamma}_{sr}$ and $\bar{\gamma}_{rd}$, respectively.

In order to analyze the system performance, statistics of Γ_i (1) is required. However, the distribution of (1) is not mathematically tractable. To facilitate a comprehensive performance analysis, we express Γ_i in a more mathematically tractable form by approximating Γ_i by a tight upper bound Γ_i^{ub} [15], [21], [22]

$$\Gamma_i \le \Gamma_i^{ub} = \gamma_{sd} + \sum_{l=1}^{i} \gamma_l, \tag{2}$$

where $\gamma_l = \min(\gamma_{sr_l}, \gamma_{r_ld})$. In particular, the performance metrics derived by using (2) serve as benchmarks or lower bounds for systems with practical relays.

III. Proposed Relay Selection Scheme

The proposed scheme selects the first L_c $(1 \le L_c \le L)$ arbitrary ordered relays such that the combined SNR of the first L_c relayed paths and the direct path barely exceeds a preset threshold γ_{th}.

First, D receives the signal transmitted by S during the broadcast phase. Thus, the combiner output γ_c is set to γ_{sd}. Then, the first relay (labeled as R_1) forwards the amplified version of the source message to D in the first time-slot of the relaying phase. The combiner at D combines this signal with the signal received via the direct path, and the output SNR is given by $\gamma_c = \gamma_{sd} + \gamma_1$. Next, γ_c is compared with the preset threshold γ_{th}. If γ_c exceeds γ_{th}, no more relays are selected, and γ_c is set as the output SNR. Otherwise, the remaining relays $R_2, ..., R_{L-1}$ are selected in subsequent time-slots until the output SNR exceeds the threshold. If the combined SNR of the first $L-1$ relays and the direct path does not exceed γ_{th}, then all L relays are selected, and this event corresponds to the worst case. However, in the best case, only one relay is selected arbitrarily. A desired feature of OT-MRS scheme is that D only needs to estimate L_c relayed paths. Further, the destination does not need to perform any ordering of relays according to their channel conditions. In contrast, the best relay and GSC-based MRS schemes require the channel knowledge of all L relayed paths and relay ordering.

IV. Performance Analysis

In this section, the statistics of the output SNR of the OT-MRS scheme is derived and it is used to derive various performance metrics.

A. Statistical characterization of the output SNR

The instantaneous output SNR γ_c of the OT-MRS scheme can be written as follows:

$$\gamma_c = \begin{cases} \gamma_{sd} + \gamma_{R_1}, & \gamma_{sd} + \gamma_{R_1} \ge \gamma_{th} \\ \gamma_{sd} + \sum_{l=1}^{L_c} \gamma_{R_l}, & \gamma_{sd} + \sum_{l=1}^{L_c} \gamma_{R_l} \ge \gamma_{th} \\ & \text{and } 0 \le \gamma_{sd} + \sum_{l=1}^{L_c-1} \gamma_{R_l} \le \gamma_{th} \\ \gamma_{sd} + \sum_{l=1}^{L} \gamma_{R_l}, & \text{otherwise,} \end{cases} \tag{3}$$

where $\gamma_{R_l} = \frac{\gamma_{sr_l}\gamma_{r_ld}}{\gamma_{sr_l} + \gamma_{r_ld} + 1}$ is the arbitrarily ordered SNR of the relayed path via the l-th relay R_l. The cumulative distribution function (CDF) of the output SNR γ_c can be written by using (3) as follows:

$$
\begin{aligned}
F_{\gamma_c}(x) &= \sum_{i=1}^{L} \Pr\left(\left[\gamma_c = \gamma_{sd} + \sum_{l=1}^{i} \gamma_{R_l}\right] \cap [\gamma_c \le x]\right) \\
&= \Pr\left(\gamma_{th} \le \Gamma_1 \le x\right) \\
&\quad + \sum_{i=2}^{L} \Pr\left([\gamma_{th} \le \Gamma_i \le x] \cap [0 \le \Gamma_{i-1} < \gamma_{th}]\right) \\
&\quad + \Pr\left([0 \le \Gamma_L \le x] \cap [0 \le \Gamma_{L-1} < \gamma_{th}]\right), \qquad (4)
\end{aligned}
$$

where Pr $(\cdot)$ is the probability assignment, and Γ_i is the combined SNR of the first i relayed paths and the direct path, defined in (1).

After some manipulations, we simplify (4) into a more mathematically tractable form

$$
F_{\gamma_c}(x) = \begin{cases} F_{\Gamma_L}(x), & x < \gamma_{th} \\ F_{\Gamma_1}(x) - F_{\Gamma_1}(\gamma_{th}) + \\ \sum_{i=2}^{L} \int_0^{\gamma_{th}} \int_{\gamma_{th}-\Gamma_{i-1}}^{x-\Gamma_{i-1}} f_{\Gamma_{i-1},\gamma_{R_i}}(\Gamma_{i-1},\gamma_{R_i}) d\gamma_{R_i} d\Gamma_{i-1} \\ + \int_0^{\gamma_{th}} \int_0^{\gamma_{th}-\Gamma_{L-1}} f_{\Gamma_{L-1},\gamma_{R_L}}(\Gamma_{L-1},\gamma_{R_L}) d\gamma_{R_L} d\Gamma_{L-1}, & x \ge \gamma_{th}, \end{cases} \qquad (5)
$$

where $F_{\Gamma_i}(x)$ is the CDF of the combined SNR of the first i relayed paths and the direct path, and $f_{\Gamma_i,\gamma_{R_i}}(\Gamma_i, \gamma_{R_i})$ is the joint probability density function (PDF) of Γ_i and γ_{R_i}. To evaluate the CDF of γ_c, one needs to find $f_{\Gamma_i,\gamma_{R_i}}(\Gamma_i, \gamma_{R_i})$. This can easily be obtained by using (1) and identifying the statistically independence of Γ_i and γ_{R_i} as follows:

$$
f_{\Gamma_{i-1},\gamma_{R_i}}(\Gamma_{i-1}, \gamma_{R_i}) = f_{\Gamma_{i-1}}(\Gamma_{i-1}) f_{\gamma_{R_i}}(\gamma_{R_i}). \qquad (6)
$$

The probability density function (PDF) of Γ_{i-1}^{ub} in (2) can be written in closed-form [22]

$$
f_{\Gamma_{i-1}^{ub}}(x) = \beta_{sd,i-1} e^{-\frac{x}{\bar{\gamma}_{sd}}} + \sum_{l=1}^{i-1} \frac{\beta_{l,i-1}}{(l-1)!} x^{l-1} e^{-\frac{x}{\bar{\gamma}}}, \qquad (7)
$$

where $\beta_{sd,k} = \frac{\bar{\gamma}_{sd}^{k-1}}{(\bar{\gamma}_{sd}-\bar{\gamma})^k}$, $\beta_{l,k} = \frac{(-\bar{\gamma}_{sd})^{k-l}}{\bar{\gamma}^{l-1}(\bar{\gamma}-\bar{\gamma}_{sd})^{k-l}}$ and $\bar{\gamma} = \frac{\bar{\gamma}_{sr}\bar{\gamma}_{rd}}{\bar{\gamma}_{sr}+\bar{\gamma}_{rd}}$. The CDF of Γ_{i-1}^{ub} can be easily evaluated as

$$
F_{\Gamma_{i-1}^{ub}}(x) = \beta_{sd,i-1}\bar{\gamma}_{sd}\left(1 - e^{-\frac{x}{\bar{\gamma}_{sd}}}\right) + \sum_{l=1}^{i-1} \frac{\beta_{l,i-1}}{(l-1)!} \bar{\gamma}^l \gamma\left(l, \frac{x}{\bar{\gamma}}\right). \qquad (8)
$$

By using (5), (7), and (8), the CDF of an upper bound of the output SNR γ_c^{ub}, which is obtained by replacing γ_c with γ_c^{ub} in (3) is derived in closed-form

$$
F_{\gamma_c^{ub}}(x) = \begin{cases} \beta_{sd,L}\bar{\gamma}_{sd}\left(1 - e^{-\frac{x}{\bar{\gamma}_{sd}}}\right) + \sum_{l=1}^{L} \frac{\beta_{l,L}}{(l-1)!} \bar{\gamma}^l \gamma\left(l, \frac{x}{\bar{\gamma}}\right), & 0 \le x < \gamma_{th} \\ \beta_{sd,1}\bar{\gamma}_{sd} e^{-\frac{\gamma_{th}}{\bar{\gamma}_{sd}}} \left(1 - e^{-\frac{(x-\gamma_{th})}{\bar{\gamma}_{sd}}}\right) \\ + \lambda e^{-\frac{\gamma_{th}}{\bar{\gamma}}} \left(1 - e^{-\frac{(x-\gamma_{th})}{\bar{\gamma}}}\right) + \kappa, & x \ge \gamma_{th}, \end{cases}
$$

where

$$
\begin{aligned}
\lambda &= \beta_{1,1}\bar{\gamma} + \sum_{i=2}^{L} \left(\beta_{sd,i-1}\left(\frac{\bar{\gamma}\bar{\gamma}_{sd}}{\bar{\gamma}-\bar{\gamma}_{sd}}\right)\left(1 - e^{-\frac{\gamma_{th}(\bar{\gamma}-\bar{\gamma}_{sd})}{\bar{\gamma}\bar{\gamma}_{sd}}}\right)\right. \\
&\quad \left. + \sum_{l=1}^{i-1} \frac{\beta_{l,i-1}}{l!} \gamma_{th}^l\right) \text{ and}
\end{aligned}
$$

$$
\begin{aligned}
\kappa &= \beta_{sd,L-1}\bar{\gamma}_{sd}\left(1 + \frac{1}{\bar{\gamma}-\bar{\gamma}_{sd}}\left(\bar{\gamma}_{sd} e^{-\frac{\gamma_{th}}{\bar{\gamma}_{sd}}} - \bar{\gamma} e^{-\frac{\gamma_{th}}{\bar{\gamma}}}\right)\right) \\
&\quad + \sum_{l=1}^{L-1} \beta_{l,L-1}\left(\frac{\bar{\gamma}^l}{(l-1)!}\gamma\left(l, \frac{\gamma_{th}}{\bar{\gamma}}\right) - \frac{\gamma_{th}^l}{l!} e^{-\frac{\gamma_{th}}{\bar{\gamma}}}\right). \qquad (9)
\end{aligned}
$$

The PDF of γ_c^{ub} can easily be obtained by differentiating (9) as follows:

$$
f_{\gamma_c^{ub}}(x) = \begin{cases} \beta_{sd,L} e^{-\frac{x}{\bar{\gamma}_{sd}}} + \sum_{l=1}^{L} \frac{\beta_{l,L}}{(l-1)!} x^{l-1} e^{-\frac{x}{\bar{\gamma}}}, & 0 \le x < \gamma_{th} \\ \beta_{sd,1} e^{-\frac{x}{\bar{\gamma}_{sd}}} + \frac{\lambda}{\bar{\gamma}} e^{-\frac{x}{\bar{\gamma}}}, & x \ge \gamma_{th} \end{cases}. \qquad (10)
$$

Moreover, the moment generating function (MGF) of γ_c^{ub} is derived by using (10) as

$$
\begin{aligned}
M_{\gamma_c^{ub}}(s) &= \mathcal{E}\{e^{-sx}\} = \frac{\beta_{sd,L} + (\beta_{sd,1} - \beta_{sd,L}) e^{-\gamma_{th}\left(s + \frac{1}{\bar{\gamma}_{sd}}\right)}}{s + \frac{1}{\bar{\gamma}_{sd}}} \\
&\quad + \sum_{l=1}^{L} \frac{\beta_{l,L}}{(i-1)!(s+\frac{1}{\bar{\gamma}})^l} \gamma\left(l, \gamma_{th}\left(s + \frac{1}{\bar{\gamma}}\right)\right) \\
&\quad + \frac{\frac{\lambda}{\bar{\gamma}} e^{-\gamma_{th}\left(s+\frac{1}{\bar{\gamma}}\right)}}{s + \frac{1}{\bar{\gamma}}}. \qquad (11)
\end{aligned}
$$

B. Outage probability

The outage probability is defined as the probability that the instantaneous output SNR γ_c falls below the preset SNR threshold γ_{th}. Thus, a lower bound for the outage probability P_{out} can immediately be obtained by using (9) as follows:

$$
P_{out}^{lb} = \Pr(0 \le \gamma_c^{ub} \le \gamma_{th}) = F_{\gamma_c^{ub}}(\gamma_{th}). \qquad (12)
$$

C. Average symbol error rate

The average symbol error rate (SER) is one of the most widely used performance metrics of wireless systems. A lower bound for the average SER can be derived by integrating the conditional error probability (CEP) $P_e|\gamma$ over the PDF of the output SNR γ_c^{ub}. The CEP of coherent binary frequency shift keying (C-BFSK) and M-ary pulse amplitude modulation (PAM) can be expressed as $P_e|\gamma = \zeta \mathrm{Q}\left(\sqrt{\eta\gamma}\right)$ [23], where ζ and η are modulation dependent constants. For example, $(\zeta = 1, \eta = 2)$ and $(\zeta = 1, \eta = 1)$ represent coherent binary phase shift keying (C-BPSK) and C-BFSK, respectively. Further, the SER of M-ary PAM is obtained by using $(\zeta = 2(M-1)/M)$ and $(\eta = 6\log_2 M/M^2 - 1)$. A lower bound of the average SER is given by the following closed-form expression:

$$
\begin{aligned}
\bar{P}_e^{lb} &= 0.5\zeta\beta_{sd,L}\bar{\gamma}_{sd}(1-\mu_{sd}) + \zeta\left(\beta_{sd,1} - \beta_{sd,L}\right)\bar{\gamma}_{sd} \\
&\quad \times \left(\mathrm{Q}\left(\sqrt{\eta\gamma_{th}}\right) e^{-\frac{\gamma_{th}}{\bar{\gamma}_{sd}}} - \mu_{sd}\mathrm{Q}\left(\sqrt{2\nu_{sd}}\right)\right) \\
&\quad + \zeta\lambda\left(e^{-\frac{\gamma_{th}}{\bar{\gamma}}}\mathrm{Q}\left(\sqrt{\eta\gamma_{th}}\right) - \mu\mathrm{Q}\left(\sqrt{2\nu}\right)\right) \\
&\quad + \sum_{l=1}^{L} \zeta\beta_{l,L}\bar{\gamma}^l \left(\frac{1}{2} - \mathrm{Q}\left(\sqrt{\eta\gamma_{th}}\right)\left(1 - \frac{\gamma\left(l, \frac{\gamma_{th}}{\bar{\gamma}}\right)}{(l-1)!}\right)\right.
\end{aligned}
$$

$$- \sum_{j=0}^{l-1} \frac{2^{j-1}\mu^{(2j+1)}\gamma\big(j+\frac{1}{2},\nu\big)}{\sqrt{\pi}\; j!(\eta\bar{\gamma})^{j}} \Bigg), \tag{13}$$

where $\mu_{sd} = \sqrt{\frac{\eta\bar{\gamma}_{sd}}{2+\eta\bar{\gamma}_{sd}}}$, $\mu = \sqrt{\frac{\eta\bar{\gamma}}{2+\eta\bar{\gamma}}}$, $\nu_{sd} = \frac{\gamma_{th}(2+\eta\bar{\gamma}_{sd})}{2\bar{\gamma}_{sd}}$, and $\nu = \frac{\gamma_{th}(2+\eta\bar{\gamma})}{2\bar{\gamma}}$.

D. Average output SNR

The moments of the output SNR γ_c is useful as signal quality indicators. They can be used as an alternative performance measure to error-rate analysis. The n-th moment of γ_c^{ub} can be obtained as

$$\overline{(\gamma_c^{ub})^n} = \mathcal{E}\Big\{\big(\gamma_c^{ub}\big)^n\Big\} = (-1)^n \left.\frac{\mathrm{d}^n M_{\gamma_c^{ub}}(s)}{\mathrm{d}s^n}\right|_{s=0}. \tag{14}$$

Among these moments, the average output SNR $\bar{\gamma}_c$ is one of the most commonly used performance metrics. Thus, an upper bound for $\bar{\gamma}_c$ is obtained by substituting (10) into (14) and letting $n = 1$:

$$\overline{\gamma_c^{ub}} = \bar{\gamma}_{sd}^2 \left(\beta_{sd,L} + (\beta_{sd,1} - \beta_{sd,L})\left(1 + \frac{\gamma_{th}}{\bar{\gamma}_{sd}}\right) e^{-\frac{\gamma_{th}}{\bar{\gamma}_{sd}}}\right) + \bar{\gamma}\lambda\left(1 + \frac{\gamma_{th}}{\bar{\gamma}}\right) e^{-\frac{\gamma_{th}}{\bar{\gamma}}} + \sum_{l=1}^{L} \frac{\beta_{l,L}}{(l-1)!}\bar{\gamma}^{(l+1)}\gamma\left(l+1, \frac{\gamma_{th}}{\bar{\gamma}}\right). \tag{15}$$

E. Average number of selected relays

The number of selected relays L_c by the OT-MRS scheme fluctuates with the channel fading states. Thus, L_c may change in subsequent time-slots during the relaying phase. Accordingly, L_c is a discrete random variable with the range $1 \le L_c \le L$. When L_c is less than L, higher bandwidth efficiency and lower power consumption can be achieved. In contrast, a low L_c results in a weaker output signal and leads to considerable performance degradation. The average number of selected relays $\bar{L}_c$ can be obtained as follows:

$$\bar{L}_c = \sum_{l=1}^{L} l \,\Pr(L_c = l), \tag{16}$$

where $\Pr(L_c = l)$ is the probability that the selected number of relays is l. In order to obtain $\bar{L}_c$, we first derive $\Pr(L_c = l)$ by using the definition of γ_c given in (3) as follows:

$$\Pr(L_c = l) = \begin{cases} \Pr\left(\gamma_{sd} + \gamma_{R_1} \ge \gamma_{th}\right), & l = 1 \\ \Pr\left(\left[\gamma_{sd} + \sum_{i=1}^{l}\gamma_{R_i} \ge \gamma_{th}\right] \cap \left[\gamma_{sd} + \sum_{i=1}^{l-1}\gamma_{R_i} < \gamma_{th}\right]\right), & l \in \{2, ..., L-1\} \\ \Pr\left(\gamma_{sd} + \sum_{i=1}^{L-1}\gamma_{R_i} < \gamma_{th}\right), & l = L. \end{cases}$$

By substituting (7), (8) into (17) and after some manipulations, we obtain $\Pr(L_c = l)$ in closed-form

$$\Pr(L_c = l) = \begin{cases} \beta_{sd,1}\bar{\gamma}_{sd} e^{-\frac{\gamma_{th}}{\bar{\gamma}_{sd}}} + \beta_{1,1}\bar{\gamma} e^{-\frac{\gamma_{th}}{\bar{\gamma}}}, & l = 1 \\ \beta_{sd,l-1}\left(\frac{\bar{\gamma}\bar{\gamma}_{sd}}{\bar{\gamma}-\bar{\gamma}_{sd}}\right) e^{-\frac{\gamma_{th}}{\bar{\gamma}}} \times \left(1 - e^{-\frac{\gamma_{th}(\bar{\gamma}-\bar{\gamma}_{sd})}{\bar{\gamma}\bar{\gamma}_{sd}}}\right) + \sum_{i=1}^{l-1} \frac{\beta_{i,l-1}}{i!}\gamma_{th}^{i} e^{-\frac{\gamma_{th}}{\bar{\gamma}}}, & l \in \{2, ..., L-1\} \\ \beta_{sd,L-1}\bar{\gamma}_{sd}\left(1 - e^{-\frac{\gamma_{th}}{\bar{\gamma}_{sd}}}\right) + \sum_{i=1}^{L-1} \frac{\beta_{i,L-1}}{(i-1)!}\bar{\gamma}^{i}\gamma\left(i, \frac{\gamma_{th}}{\bar{\gamma}}\right), & l = L. \end{cases} \tag{17}$$

By substituting (17) into (16), a lower bound of the average number of selected relays $\bar{L}_c$ can be obtained as

$$\bar{L}_c^{lb} = L\beta_{sd,L-1}\bar{\gamma}_{sd} + (\beta_{sd,1} - L\beta_{sd,L-1})\,\bar{\gamma}_{sd} e^{-\frac{\gamma_{th}}{\bar{\gamma}_{sd}}} + \beta_{1,1}\bar{\gamma} e^{-\frac{\gamma_{th}}{\bar{\gamma}}} + \sum_{i=1}^{L-1} \frac{L\beta_{i,L-1}}{(i-1)!}\bar{\gamma}^{i}\gamma\left(i, \frac{\gamma_{th}}{\bar{\gamma}}\right) + \sum_{l=2}^{L-1}\left(l\beta_{sd,l-1}\left(\frac{\bar{\gamma}\bar{\gamma}_{sd}}{\bar{\gamma}-\bar{\gamma}_{sd}}\right) \times \left(1 - e^{-\frac{\gamma_{th}(\bar{\gamma}-\bar{\gamma}_{sd})}{\bar{\gamma}\bar{\gamma}_{sd}}}\right) e^{-\frac{\gamma_{th}}{\bar{\gamma}}} + \sum_{i=1}^{l-1} \frac{l\beta_{i,l-1}}{i!}\gamma_{th}^{i} e^{-\frac{\gamma_{th}}{\bar{\gamma}}}\right). \tag{18}$$

F. Ergodic Capacity

The channel capacity is defined as the maximum rate at which information can be transmitted across a noisy channel with arbitrary reliability. The ergodic capacity C is defined as the expected value of the instantaneous maximum mutual information I between the source and the destination. For a cooperative relay network operating with an orthogonal TDMA channeling, I is given by; $I = \frac{1}{L_c+1}\log_2(1 + \gamma_c)$, where L_c is the number of selected relays, and γ_c is the instantaneous output SNR at the destination [2]. We express C as a joint expectation of I with respect to γ_c and L_c:

$$C = \mathcal{E}_{\gamma_c, L_c}\left\{\frac{1}{L_c + 1}\left(\log_2(1 + \gamma_c)\right)\right\}. \tag{19}$$

Since the analytical evaluation of (19) is tedious, we present an approximation of C as follows. First, the dependency of L_c in the expectation operation given in (19) is removed by replacing L_c by its expected value $\bar{L}_c$ given in (18):

$$C \approx \mathcal{E}_{\gamma_c}\left\{\frac{1}{\bar{L}_c + 1}\left(\log_2(1 + \gamma_c^{ub})\right)\right\}. \tag{20}$$

Then, by knowing that $\log_2(\cdot)$ is a concave function and by using the Jensen's inequality [18], C can be approximated as

$$C \approx \frac{1}{\bar{L}_c + 1}\log_2\left(1 + \mathcal{E}\{\gamma_c^{ub}\}\right), \tag{21}$$

where $\mathcal{E}\{\gamma_c^{ub}\}$ is the average output SNR defined in (15).

V. Numerical Results and Discussion

In this section, the numerical results are presented to show the performance of the proposed OT-MRS scheme. The Monte-Carlo simulation results are presented as an independent verification of our analytical results.

Fig.2 shows the average BER of BPSK against the normalized average SNR per branch ($\bar{\gamma}^* = \bar{\gamma}/\gamma_{th}$) for 4-relay, 8-relay, and 12-relay cooperative networks. In very high SNR regime,

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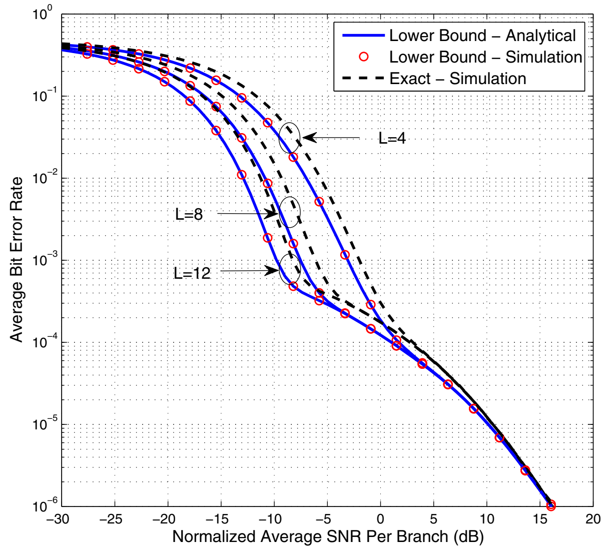

Fig. 2. The average BER of BPSK vs. the normalized average SNR per branch $\bar{\gamma}^* = \bar{\gamma}/\gamma_{th}$.

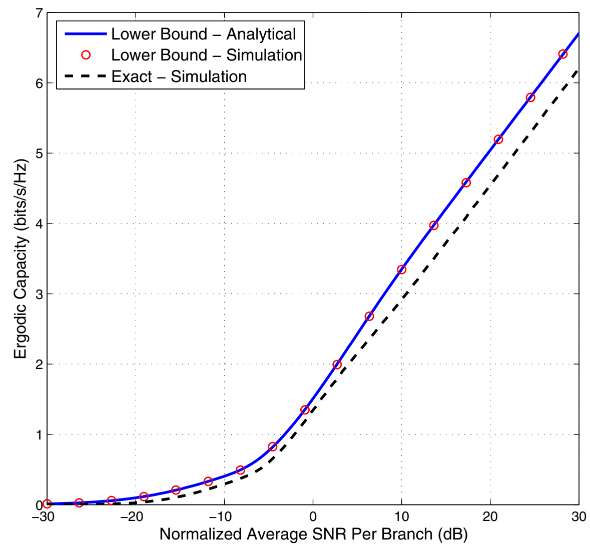

Fig. 4. The Ergodic capacity vs. the normalized average SNR per branch $\bar{\gamma}^* = \bar{\gamma}/\gamma_{th}$. $L = 10$.

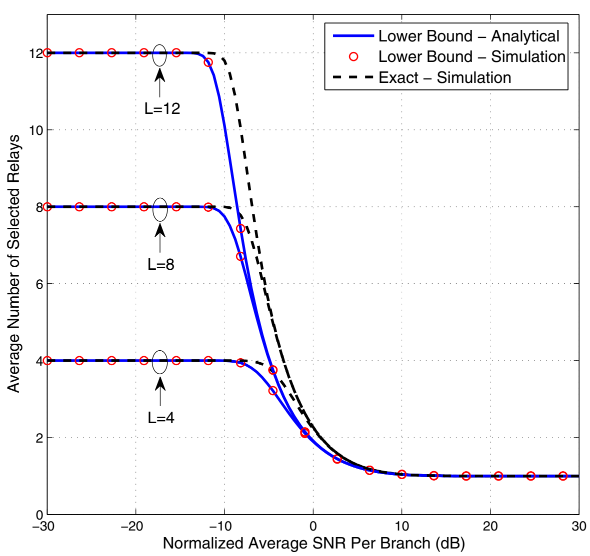

Fig. 3. The average number of selected relays vs. the normalized average SNR per branch $\bar{\gamma}^* = \bar{\gamma}/\gamma_{th}$

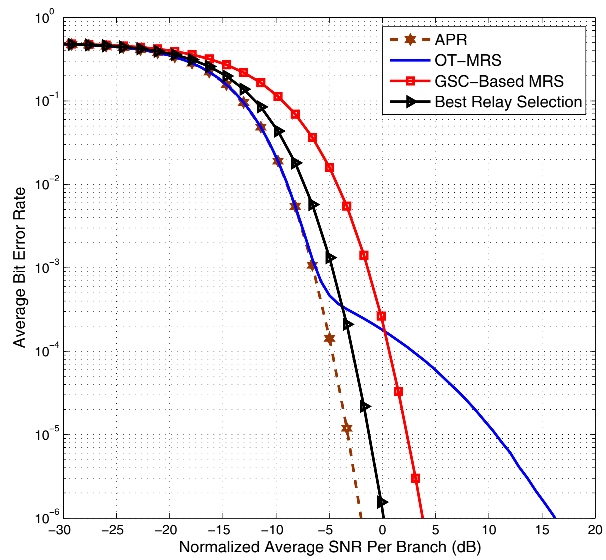

Fig. 5. A comparison of the average BER of BPSK of several relay selection schemes. $L = 10$ for OT-MRS. The GSC-based MRS scheme selects best four relays out of 10.

all three OT-MRS schemes perform identically regardless of the available number of relays, since the they select one relay arbitrarily. In the low-to-moderate SNR regime, the OT-MRS scheme with higher number of relays performs better than the others. The figure shows that the analytically derived lower bound for the average BER is tight in the high SNR regime. Moreover, Monte-Carlo simulation points verify the correctness of our analytical results.

Fig.3 plots the average number of selected relays $\bar{L}_c$ against $\bar{\gamma}^*$ for 4-relay, 8-relay, and 12-relay cooperative networks. The average number of selected relays is highly dependent on both γ_{th} and $\bar{\gamma}$. As $\bar{\gamma}^*$ increases from low SNR regime to high SNR regime, $\bar{L}_c$ decreases from L to 1 as expected. In moderate $\bar{\gamma}^*$ regime, $\bar{L}_c$ changes dramatically.

Fig.4 shows the ergodic capacity C against $\bar{\gamma}^*$. The rate of increase of C is low in the low SNR regime whereas in the moderate-to-high SNR regime, it shows a steep increase with $\bar{\gamma}^*$. The approximated C given in (21) is closer to its exact value in the low SNR regime than in the high SNR regime.

In Fig.5, the average BER of BPSK is compared with that of the best relay selection, GSC-based MRS scheme (which selects best 4 out of 10 relays) and APR. As expected, the proposed scheme performs better than the best relay selection in the low-to-moderate SNR regime. The OT-MRS scheme has a poor BER performance in the high SNR regime because only one relay is selected arbitrarily in the very high SNR regime. However, this scheme has additional

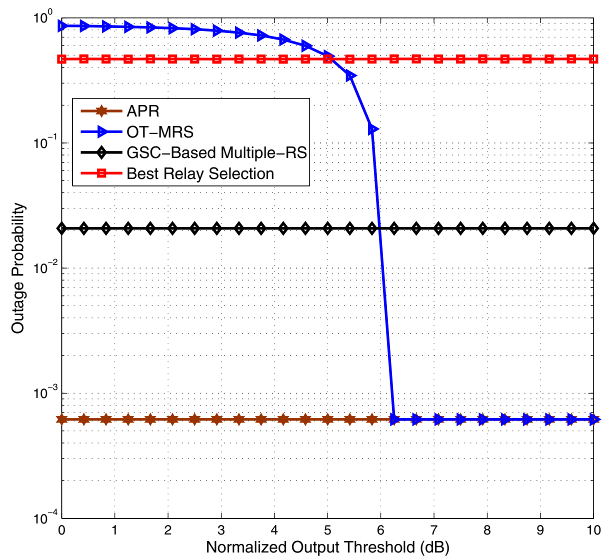

Fig. 6. The outage probability comparison of several relay selection schemes against normalized output threshold $\gamma_{th}^* = \gamma_{th}/\bar{\gamma}$. The GSC-based MRS scheme selects best four relays out of 10. $\bar{\gamma} = 3\,\text{dB}$, $\gamma_0 = 6\,\text{dB}$ and $L = 10$.

advantages in terms of less channel estimation requirements, lower power consumption and no relay ordering compared to other schemes. The GSC-based MRS scheme outperforms the OT-MRS scheme in the high SNR regime, since the GSC-based scheme selects the best four relays always. However, this performance gain in the high SNR regime is achieved at the cost of additional bandwidth, channel estimation and power requirements compared to the OT-MRS scheme.

In Fig. 6, the outage probability of several relay selection schemes is plotted against the normalized output threshold $\gamma_{th}^* = \gamma_{th}/\bar{\gamma}$. This figure shows the dependence of the outage probability of the OT-MRS scheme on γ_{th}^*. In the very high γ_{th}^* regime, the OT-MRS scheme selects all L relays and perform identically to APR. As expected, in very low γ_{th}^* regime, only one arbitrary relay is selected and thus, OT-MRS exhibits a high average BER. However, in low-to-moderate γ_{th}^* regime the number of selected relays varies adaptively. Fig. 6 clearly reveals that the OT-MRS scheme utilizes the wireless resources adaptively to improve the system performance.

VI. Conclusion

An adaptive OT-MRS scheme for dual-hop multi-branch cooperative networks was developed. Numerical results and Monte-Carlo simulation results were provided. The numerical results showed that the proposed scheme outperforms the corresponding optimal single relay selection scheme and GSC-based MRS scheme for low-to-moderately high SNRs. The proposed relay selection method provides flexibility to utilize system resources adaptively in fading environments.

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