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JOINT BANDWIDTH AND POWER ALLOCATION IN WIRELESS MULTI-USER DECODE-AND-FORWARD RELAY NETWORKS

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ABSTRACT

The resource allocation problem in wireless multi-user decode-and-forward (DF) relay networks is considered. The conventional resource allocation schemes based on the equal distribution of bandwidth and/or power may not be efficient for the networks with constrained/limited power and bandwidth resources at both sources and relays. Therefore, joint bandwidth and power allocation schemes are proposed based on (i) the maximization of the sum capacity of all users (source-destination pairs); (ii) the maximization of the worst user capacity; (iii) the minimization of the total power consumptions for all users. It is shown that the proposed problem formulations can be transformed to equivalent convex optimization problems. Therefore, the joint bandwidth and power allocation problems can be efficiently solved. The performance improvements offered by the proposed schemes are demonstrated by simulations.

***Index Terms*—** Multiuser relay networks, bandwidth and power allocation, convex optimization.

1. INTRODUCTION

A critical issue in wireless networks is to efficiently allocate available resources to improve the network performance. Extensive research has been performed on the resource allocation in wireless relay networks. In [1], the power control scheme based on the average signal-to-noise ratio (SNR) optimization has been developed for the amplify-and-forward (AF) relay networks. Power allocation with the decode-and-forward (DF) protocol has been studied in [2] under the assumption that transmitters only know mean channel gains. However, [1] and [2] as well as most of the works on the resource allocation in wireless relay networks consider the case of a single user (source-destination pair).

Only few works have been conducted to investigate the resource allocation strategies in the setup of multiple users using relaying. In [3], time/bandwidth allocation strategies with constant power based on time division multiple access/frequency division multiple access (TDMA/FDMA) have been developed to optimize effective capacity. Power allocation to optimize the sum capacity for four different relay transmission strategies has been studied in [4]. In [5], an AF based strategy, where multiple sources share multiple relays using power control, has been developed. However, none of these works have considered the joint bandwidth and power allocation problem.

In this paper, the problem of joint bandwidth and power allocation for the DF relay networks with multiple source-destination pairs and multiple relays is considered. The joint bandwidth and power allocation is especially efficient for the networks, where both bandwidth and power are limited resources. The optimal joint bandwidth and power allocation schemes are derived to (i) maximize the sum capacity of all users; (ii) maximize the capacity of the worst user; (iii) minimize the total power consumption of all users. The corresponding problem formulations can be transformed to optimization problems that are proved to be convex. Therefore, the joint bandwidth and power allocation problem can be solved efficiently by using convex optimization techniques.

2. SYSTEM MODEL AND PROBLEM DESCRIPTION

Consider a wireless relay network where N source nodes S_i ($i = 1, ..., N$) transmit data to their corresponding destination nodes D_i ($i = 1, ..., N$). There are L relay nodes R_j, $j = 1, ..., L$, deployed to forward the data from the sources to the destinations. The total bandwidth $W^{\max}$ is available to assist the transmissions from the sources and the relays. The total bandwidth can be divided into distinct and nonoverlapping sub-channels with possibly different bandwidths, so that the sources and the relays share the available spectrum through frequency division to avoid interferences with each other. The relays are half-duplex due to the practical limitation that they can not transmit and receive simultaneously. A two-phased DF protocol is adopted, i.e., the relays receive and decode the transmitted data from the sources in the first phase, and re-encode and forward the data to the destinations in the second phase. It is assumed that the direct links between the sources and the destinations are blocked and thus are not available. To reduce the implementation complexity at the destinations,

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single relay assignment is adopted such that each source S_i is served by only one designated relay denoted as R_{S_i}. The set of sources assisted by R_j is denoted as $\mathcal{S}(R_j)$, i.e., $\mathcal{S}(R_j) = \{S_i | R_{S_i} = R_j\}$. Various relay selection methods, such as random selection, best selection, can be used in the single relay assignment.

For brevity, we only present the model for the user i. Let P_{S_i} and $P_{R_{S_i}}$ denote the transmit power of S_i and R_{S_i}, respectively, allocated to support the transmission from the source S_i. In the first phase, a channel of bandwidth W_{S_i} is allocated to S_i. Thus, the received SNR at R_{S_i} can be found as

$$\gamma_{R_{S_i}} = \frac{P_{S_i} h_{S_i R_{S_i}}}{W_{S_i} N_0} \tag{1}$$

where $h_{S_i R_{S_i}}$ is the channel gain of the link $S_i - R_{S_i}$ and $W_{S_i} N_0$ is the power of additive white Gaussian noise (AWGN) over the bandwidth W_{S_i}. The channel gain $h_{S_i R_{S_i}}$ results from such effect as the path loss, shadowing, and fading. Due to the fact that the power spectral density (PSD) of AWGN is constant over all frequencies with the constant value N_0, the noise power in each sub-channel is linearly increasing with the sub-channel bandwidth.

In the second phase, a channel of bandwidth $W_{R_{S_i}}$ is allocated to R_{S_i} to forward the data from S_i. Then, the received SNR at D_i is given by

$$\gamma_{D_i} = \frac{P_{R_{S_i}} h_{R_{S_i} D_i}}{W_{R_{S_i}} N_0}. \tag{2}$$

It can be seen from (1) and (2) that channels with larger bandwidth introduce higher noise power and thus reduce the SNRs.

Channel capacity gives the upper bound on the achievable rate of a link. Given the SNR $\gamma_{R_{S_i}}$ of the link $S_i - R_{S_i}$, its capacity can be written as

$$C_{S_i R_{S_i}} = W_{S_i} \log(1+\gamma_{R_{S_i}}) = W_{S_i} \log\left(1 + \frac{P_{S_i} h_{S_i R_{S_i}}}{W_{S_i} N_0}\right). \tag{3}$$

Note that W_{S_i} can be seen as the raw data rate, while $\log(1 + \gamma_{R_{S_i}})$ can be seen as the data success rate. Thus, $C_{S_i R_{S_i}}$ denotes the amount of data transmitted without errors over the link $S_i - R_{S_i}$ per unit time. Similarly, the capacity of the link $R_{S_i} - D_i$ can be written as

$$C_{R_{S_i} D_i} = W_{R_{S_i}} \log(1+\gamma_{D_i}) = W_{R_{S_i}} \log\left(1 + \frac{P_{R_{S_i}} h_{R_{S_i} D_i}}{W_{R_{S_i}} N_0}\right). \tag{4}$$

Then, the capacity of user i can be found as

$$C_{S_i D_i} = \min\{C_{S_i R_{S_i}}, C_{R_{S_i} D_i}\}. \tag{5}$$

It can be seen from (3) that for fixed W_{S_i}, the capacity $C_{S_i R_{S_i}}$ is a concave increasing function of P_{S_i}. It can be also seen that $C_{S_i R_{S_i}}$ is a concave increasing function of W_{S_i} for fixed P_{S_i}, although $\gamma_{R_{S_i}}$ is a linear decreasing function of W_{S_i}. Moreover, it can be verified that $C_{S_i R_{S_i}}$ is a concave function of P_{S_i} and W_{S_i} jointly. It can further be noted from (3), (4), and (5) that if equal bandwidth is allocated to W_{S_i} and $W_{R_{S_i}}$, $C_{S_i R_{S_i}}$ and $C_{R_{S_i} D_i}$ can be unequal due to the power limits on P_{S_i} and $P_{R_{S_i}}$, and the end-to-end capacity $C_{S_i D_i}$ is constrained by the minimum of $C_{S_i R_{S_i}}$ and $C_{R_{S_i} D_i}$. In addition, since all users share the same available bandwidth, the equal bandwidth allocation for all links can be inefficient. Therefore, the joint allocation of the bandwidth and the power is desirable.

3. JOINT BANDWIDTH AND POWER ALLOCATION

Sum Capacity Based Allocation

In the applications without delay constraints, high data rate from any user is preferable, so it is desirable to allocate the resources to maximize the overall network performance. Then the joint bandwidth and power allocation problem to maximize the sum capacity can be mathematically formulated as

$$\max_{P_{S_i}\geq 0, W_{S_i}\geq 0, P_{R_{S_i}}\geq 0, W_{R_{S_i}}\geq 0} \sum_{i=1}^{N} C_{S_i D_i} \tag{6a}$$

$$\text{subject to} \sum_{S_i \in \mathcal{S}(R_j)} P_{R_{S_i}} \leq P_{R_j}^{\max},\ j = 1, ..., L \tag{6b}$$

$$\sum_{i=1}^{N} W_{S_i} \leq W^{\max} \tag{6c}$$

$$\sum_{i=1}^{N} W_{R_{S_i}} \leq W^{\max} \tag{6d}$$

$$P_{S_i} \leq P_{S_i}^{\max},\ i = 1, ..., N \tag{6e}$$

$$\text{or} \sum_{i=1}^{N} P_{S_i} \leq P^{\max}. \tag{6f}$$

The constraints (6b) mean that the total power allocated to forward the data from all the sources assisted by R_j is limited by $P_{R_j}^{\max}$, while the constraints (6e) limit the peak power at S_i by $P_{S_i}^{\max}$. The constraints (6c) and (6d) indicate that the total bandwidth of the channels allocated to the sources and the relays is limited. Note that in some applications the total power limits on the sources $P^{\max}$ are used instead of the individual power limits, e.g., an access point allocates its total available power for downlink transmissions to its users via relays. In these case, the constraint (6f) replaces the constraints (6e).

Introducing new variables t_i $(i = 1, ..., N)$, the optimization problem (6a)–(6f) can be equivalently rewritten in a standard form as

$$\min_{P_{S_i}\geq 0, W_{S_i}\geq 0, P_{R_{S_i}}\geq 0, W_{R_{S_i}}\geq 0, t_i \geq 0} -\sum_{i=1}^{N} t_i \tag{7a}$$

$$\text{subject to}\ t_i - C_{S_i R_{S_i}} \leq 0,\ i = 1, ..., N \tag{7b}$$

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$$t_i - C_{R_{S_i}D_i} \leq 0, \; i = 1, ..., N \quad (7c)$$

$$\text{the constraints (6b), (6c), (6d), and (6e) or (6f).} \quad (7d)$$

The following proposition is in order.

Proposition 1: *The optimization problem (7a)–(7d) is convex.*

Proof: Note that the objective function (7a) is a linear function of the variables t_i, so it is convex. Since $C_{S_i R_{S_i}}$ is a concave function of P_{S_i} and W_{S_i}, $-C_{S_i R_{S_i}}$ is a convex function of the variables. Therefore, (7b) and (7c) are convex constraints. Moreover, (7d) is a set of linear constraints and, thus, is convex. Therefore, the optimization problem (7a)–(7d) is convex. □

Therefore, standard convex optimization techniques can be applied to solve (7a)–(7d) efficiently.

Max-Min Capacity Based Allocation

Fairness among users is also a major issue for resource allocation. If fairness issue has to be taken into account, the achievable rate of the worst user(s) is commonly used as the network performance measure. Therefore, the joint bandwidth and power allocation problem can be mathematically formulated as

$$\max_{P_{S_i} \geq 0, W_{S_i} \geq 0, P_{R_{S_i}} \geq 0, W_{R_{S_i}} \geq 0} \quad \min_i C_{S_i D_i} \quad (8a)$$

$$\text{subject to the constraints (7d).} \quad (8b)$$

Similar to the previous case, the problem (8a)–(8b) can be equivalently written in a standard form which can be verified to be a convex problem. Thus, the optimal solution can be efficiently obtained.

Power Minimization Based Allocation

Another widely considered design objective is the minimization of the total power consumption of all sources and relays. The minimization is performed under the constraint that the capacity requirements of all users are satisfied. The corresponding joint bandwidth and power allocation problem can be written as

$$\min_{P_{S_i} \geq 0, W_{S_i} \geq 0, P_{R_{S_i}} \geq 0, W_{R_{S_i}} \geq 0} \quad \sum_{i=1}^{N} (P_{S_i} + P_{R_{S_i}}) \quad (9a)$$

$$\text{subject to } C_{S_i D_i}^{\min} - C_{S_i D_i} \leq 0, \; i = 1, ..., N \quad (9b)$$

$$\text{the constraint (7d)} \quad (9c)$$

where $C_{S_i D_i}^{\min}$ is the minimum acceptable capacity for user i. The constraint (9b) means that the capacity achieved by user i should be no less than the given capacity threshold. Similar to the previous two cases, it can be verified that the problem (9a)–(9c) is convex and, therefore, can be efficiently solved.

4. SIMULATION RESULTS

Consider a wireless network which consists of four users (source-destination pairs) and two relay nodes. Each source is assisted by one randomly assigned designated relay, and each relay assists two sources. The path loss and the Rayleigh fading are present in each link. The path loss gain is given by $g = (1/d)^2$, where d is the distance between two transmission ends. The Rayleigh fading gain is given by $h = x^2 + y^2$, where $x \sim N(0, \sigma^2/2)$ and $y \sim N(0, \sigma^2/2)$ are two independent Gaussian random variables. The sources and destinations are randomly distributed inside a rectangular area with diagonal vertexes (0,0) and (10,10), while the relays are located at the fixed points (5,3) and (5,7). We set $P_{S_i}^{\max} = 20$ $(i = 1, ..., N)$, $P_{R_j}^{\max} = 40$ $(j = 1, ..., L)$, $W^{\max} = 10$, and $\sigma^2 = 5$ as default values if no changes are indicated otherwise. The noise power N_0 equals to 1. All results are averaged over 1000 instances of random channel realizations.

The following resource allocation schemes are compared to each other: the proposed optimal joint bandwidth and power allocation (OBPA), the equal bandwidth with optimal power allocation (EBOPA), and the equal bandwidth and power allocation (EBPA). Software package TOMLAB has been to solve the corresponding convex optimization problems.

In Fig. 1, the performance of the sum capacity based resource allocation is shown versus the power limits at the relays $P_{R_j}^{\max}$ $(j = 1, ..., L)$ and the total available bandwidth $W^{\max}$, correspondingly. It can be seen that the OBPA scheme achieves about 30% to 50% better performance than the other two schemes over all the ranges of $P_{R_j}^{\max}$ $(j = 1, ..., L)$ and $W^{\max}$. The performance improvement is higher when the available power or bandwidth is larger. The observed significant performance improvement for the OBPA can be partly attributed to the fact that the sum capacity based joint bandwidth and power allocation can lead to a highly unbalanced resource allocation, while the bandwidth is always equally allocated in the EBOPA and both bandwidth and power are always equally allocated in the EBPA.

Fig. 2 demonstrates the performance of the max-min capacity based resource allocation versus the power limits at the relays $P_{R_j}^{\max}$ $(j = 1, ..., L)$ and the total available bandwidth $W^{\max}$. The performance of the OBPA is about 10% to 30% higher than that of the EBOPA for all the values of $P_{R_j}^{\max}$ $(j = 1, ..., L)$ and $W^{\max}$. The improvement provided by the OBPA, in this case, is not as significant as that in Fig. 1 that can be attributed to the fact that the max-min capacity based allocation results in a relatively balanced resource distribution, while the EBOPA and the EBPA are balanced allocation schemes.

Fig.3 shows the total power cost of the sources and relays versus the available bandwidth $W^{\max}$ for the power minimization based resource allocation. In this simulation, we set the capacity threshold $C_{S_i D_i}^{\min}$ equal to 1. Note that the total power

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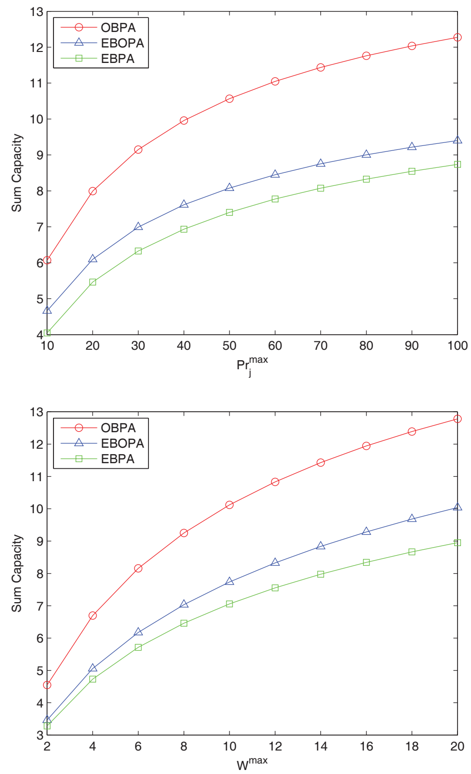

Fig. 1. Sum capacity based resource allocation.

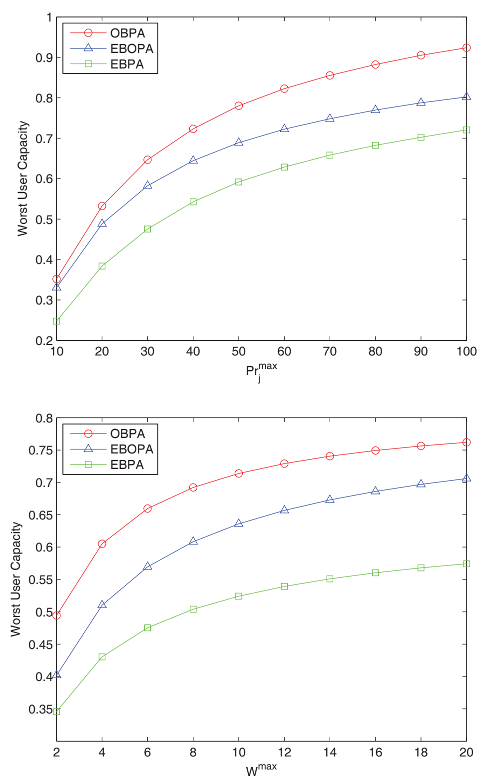

Fig. 2. Max-min capacity based resource allocation.

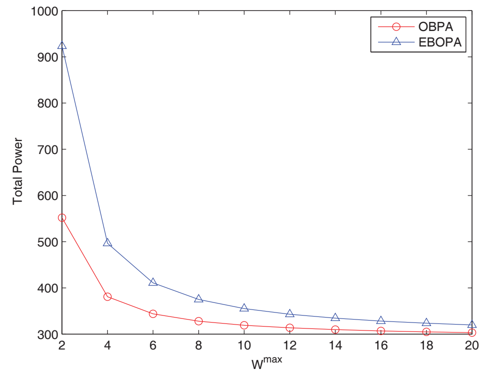

Fig. 3. Power minimization based resource allocation.

cost of the OBPA is alway about 10% to 30% less than that of the EBOPA, and the total power difference between the two tested schemes is larger when $W^{\max}$ is smaller. This shows that more power is saved when the available bandwidth is limited due to the flexible bandwidth allocation in the OBPA.

5. CONCLUSION

The problem of joint bandwidth and power allocation in wireless multi-user DF relay networks with limited user and relay powers and bandwidth is studied. The optimal bandwidth and power allocation schemes have been proposed to (i) maximize the sum capacity of all users; (ii) maximize the capacity of the worst user; (iii) minimize the total power consumption of all users. The problems under consideration are shown to be convex and, therefore, their optimal solutions can be efficiently found. Simulation results demonstrate the advantages of the proposed schemes.

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