PAPR Reduction Method Based on Parametric Minimum Cross Entropy for OFDM Signals

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***Abstract*—The partial transmit sequence (PTS) technique has received much attention in reducing the high peak to average power ratio (PAPR) of orthogonal frequency division multiplexing (OFDM) signals. However, the PTS technique requires an exhaustive search of all combinations of the allowed phase factors, and the search complexity increases exponentially with the number of sub-blocks. In this paper, a novel method based on parametric minimum cross entropy (PMCE) is proposed to search the optimal combination of phase factors. The PMCE algorithm not only reduces the PAPR significantly, but also decreases the computational complexity. The simulation results show that it achieves more or less the same PAPR reduction as that of exhaustive search.**

***Index Terms*—PTS, PAPR, OFDM, PMCE.**

I. Introduction

In various high-speed wireless communication systems, orthogonal frequency division multiplexing (OFDM) has been used widely due to its inherent robustness against multipath fading and resistance to narrowband interference [1]. However, one of the major drawbacks of OFDM signals is the high peak to average power ratio (PAPR) of the transmitted signal. Several solutions have been proposed in recent years, such as clipping [2], coding [3], selected mapping (SLM) [4], partial transmit sequence (PTS) [5] and others [6]. The PTS [5] technique is a distortionless technique based on combining signal subblocks which are phase-shifted by constant phase factors, which can reduce PAPR sufficiently. But the exhaustive search complexity of the optimal phase combination in PTS increases exponentially with the number of sub-blocks. Thus many suboptimal PTS techniques have been developed. the iterative flipping PTS (IPTS) in [7] has computational complexity linearly proportional to the number of subblocks. A neighborhood search is proposed in [8] by using gradient descent search. A suboptimal method in [9] is developed by modifying the problem into an equivalent problem of minimizing the sum of the phase-rotated vectors.

In this paper, we propose a novel phase optimization scheme, which can efficiently reduce the PAPR of the OFDM signals, based on the parametric minimum cross entropy (PMCE) method [11]. The proposed scheme can search for the nearly optimal combination of the initial phase factors. The simulation results show that this scheme can achieve a superior PAPR reduction performance, while requiring far less computational complexity than the existing techniques including the cross entropy approach [13].

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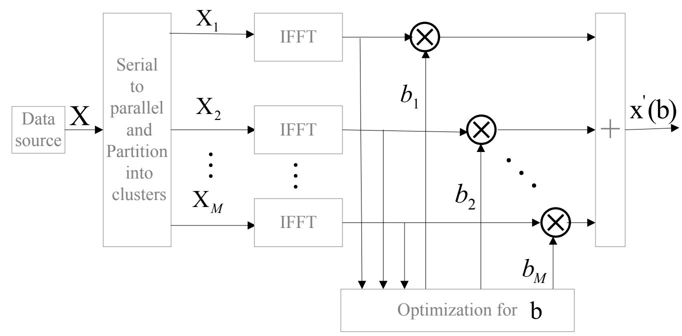

Fig. 1. Block diagram of the PTS technique.

II. OFDM System and PAPR

In an OFDM system, a high-rate data stream is split into N low-rate streams transmitted simultaneously by subcarriers. Each of the subcarriers is independently modulated by using a typical modulation scheme such as phase-shift keying (PSK) or quadrature amplitude modulation (QAM). The inverse discrete Fourier transform (IDFT) generates the ready-to-transmit OFDM signal. For an input OFDM block $\mathbf{X} = [X_0, \ldots, X_{N-1}]^T$, where N is the number of subcarriers, the discrete-time baseband OFDM signal $x(k)$ can therefore be expressed as

$$x(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{\frac{j2\pi nk}{LN}}, k = 0, 1, \cdots, LN-1, \quad (1)$$

where L is the oversampling factor. It was shown in [10] that the oversampling factor $L = 4$ is enough to provide a sufficiently accurate estimate of the PAPR of OFDM signals.

The PAPR of $x(k)$ is defined as the ratio of the maximum instantaneous power to the average power; that is

$$PAPR = \frac{\max\limits_{0 \le n < LN} |x(k)|^2}{E[|x(k)|^2]}. \quad (2)$$

III. PTS Techniques

The structure of the PTS method is shown in Fig. 1. The input data block $\mathbf{X}$ is partitioned into M disjoint sub-blocks $\mathbf{X}_m, m = 1, 2, \ldots M$ such that $\mathbf{X} = \sum_{m=1}^{M} \mathbf{X}_m$. The sub-blocks

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are combined in the time domain to minimize the PAPR. The L-times oversampled time-domain signal of $\mathbf{X}_m$ is denoted as $\mathbf{x}_m, m = 1, 2, \ldots M$, which are obtained by taking an IDFT of length NL on $\mathbf{X}_m$ concatenated with $(L-1)N$ zeros. Each $\mathbf{x}_m$ is multiplied by a phase-weighting factor $b_m = e^{j\phi_m}$, where $\phi_m \in [0, 2\pi)$ for $m = 1, 2, \ldots M$. The goal of the PTS approach is to find an optimal phase-weighted combination to minimize the PAPR. The combined transmitted signal in the time domain can then be expressed as

$$\mathbf{x}^{'}(\mathbf{b}) = \sum_{i=1}^{M} b_i \mathbf{x}_i, \tag{3}$$

where $\mathbf{x}^{'}(\mathbf{b}) = [x_1^{'}(\mathbf{b}), x_2^{'}(\mathbf{b}), \cdots, x_{NL}^{'}(\mathbf{b})]$.

In general, the selection of the phase factors is limited to a set with a finite number of elements to reduce the search complexity. The set of allowed phase factors is

$$\mathbf{P} = \{e^{j2\pi\ell/W} | \ell = 0, 1, \ldots, W-1\}, \tag{4}$$

where W is the number of allowed phase factors. Thus, W^M sets of phase factors are searched for the optimal set of phase factors. The search complexity increases exponentially with M, the number of sub-blocks.

IV. Minimize PAPR Using Parametric Minimum Cross Entropy (PMCE) Method

The Parametric Minimum Cross Entropy Method (PMCE) was first proposed by Rubinstein [11] to solve rare event probability estimation and counting problems. It is a parametric method to solve the well known Kullback Minimum Cross Entropy (MinxEnt) problem [12]. The PMCE algorithm first casts a deterministic optimization problem into an associate rare-event probability estimation, then solves the resulting program to obtain an optimally marginal distributions derived from the optimal joint MinxEnt distribution. This method finds the optimal parameters of the importance sampling distribution to efficiently estimate the desired quantity. For an accurate understanding of PMCE, the readers are referred to [11].

The minimum PAPR for PTS method is relative to the following problem:

$$\begin{aligned} \text{Minimize } F(\mathbf{b}) =& \frac{\max |x^{'}(\mathbf{b})|^2}{E[|x^{'}(\mathbf{b})|^2]}, \\ s.t. \qquad & \mathbf{b} \in \{e^{j\phi_m}\}^M, \end{aligned} \tag{5}$$

where $\phi_m \in \{\frac{2\pi k}{W} | k = 0, 1, \ldots, W-1\}$. The phase factor $\mathbf{b} = \{-1, 1\}^M$ is chosen in this paper and generated by using $\mathbf{b} = 1 - 2\mathbf{c}$ from a binary vector $\mathbf{c} = \{c_i\}_{i=0}^{M-1}$. Thus minimization of (5) is translated into the following problem:

$$\begin{aligned} \text{Minimize } F(\mathbf{c}) =& \frac{\max |x^{'}(1-2\mathbf{c})|^2}{E[|x^{'}(1-2\mathbf{c})|^2]}, \\ s.t. \qquad & \mathbf{c} \in \{0, 1\}^M. \end{aligned} \tag{6}$$

Each element of **c** can be modeled as an independent Bernoulli random variable with the probability mass function $P(c_i = 1) = p_i$, $P(c_i = 0) = 1 - p_i$, for $i = 0, 1, \ldots, M-1$. Then the probability distribution of **c** is

$$f(\mathbf{c}, \mathbf{p}) = \prod_{i=0}^{M-1} p_i^{c_i} (1-p_i)^{1-c_i}. \tag{7}$$

In order to solve (6) by using PMCE, we first randomize the deterministic problem by $f(\mathbf{c}, \mathbf{p})$ for $\mathbf{p} \in [0, 1]^M$ and $\mathbf{c} \in \{0, 1\}^M$. That is to associate (6) with the problem of estimating the probability $P\{F(\mathbf{c}) \leq \gamma\}$ for a given PAPR threshold γ.

The idea of the PMCE algorithm is to iteratively generate the sequences γ_j and $\mathbf{p}_j$, which converge to the optimal tuple γ^* and $\mathbf{p}^*$ in the sense of minimal cross entropy [11]. Then the optimal $\mathbf{c}^*$ can be obtained from $\mathbf{p}^*$ by $f(\mathbf{c}, \mathbf{p})$. More specifically, we initialize the PMCE algorithm by setting $\mathbf{p} = \mathbf{p}_0$, and choosing a $\rho \in (0, 1)$ (called rarity parameter in PMCE [11]) such that the probability of the event $\{F(\mathbf{c}) \leq \gamma\}$ is around ρ. Each iteration of the PMCE consists of two main phases [11]:

1) For a given $\mathbf{p}_{j-1}$, randomly generate a set of samples $\mathbf{c}_1^{j-1}, \cdots, \mathbf{c}_J^{j-1}$ from $f(\mathbf{c}, \mathbf{p}_{j-1})$, and then calculate the PAPRs $F(\mathbf{c}_1^{j-1}), \cdots, F(\mathbf{c}_J^{j-1})$. Sort $F(\mathbf{c}_1^{j-1}), \cdots, F(\mathbf{c}_J^{j-1})$ in an increasing order and denote it as $F_{(1)}^{j-1}, \cdots, F_{(J)}^{j-1}$. Assign

$$\gamma_j = \frac{1}{\lceil \rho J \rceil} \sum_{k=1}^{\lceil \rho J \rceil} F_{(k)}^{j-1}, \tag{8}$$

where $\lceil \cdot \rceil$ is the ceiling function.

2) The $\mathbf{p}_j = (p_{j,0}, \cdots, p_{j,M-1})$ is updated as

$$p_{j,i} = \frac{\sum_{k=1}^{J} I_{\{\mathbf{c}_{k,i}^{j-1}=1\}} \exp\left(-F(\mathbf{c}_k^{j-1})\lambda_j\right)}{\sum_{k=1}^{J} \exp\left(-F(\mathbf{c}_k^{j-1})\lambda_j\right)}, \tag{9}$$

where the indicator function $I_{\{x=1\}} = 1$ if $x = 1$ and 0 otherwise, and the parameter λ_j are obtained from the solution of the following equation [11]

$$\gamma_j = \frac{\sum_{k=1}^{J} F(\mathbf{c}_k^{j-1}) \exp\left(-F(\mathbf{c}_k^{j-1})\lambda_j\right)}{\sum_{k=1}^{J} \exp\left(-F(\mathbf{c}_k^{j-1})\lambda_j\right)}. \tag{10}$$

In order to prevent a fast convergence to a local optimum, instead of directly using (9), we use a smoothed version [11]

$$\hat{\mathbf{p}}_j = \alpha \mathbf{p}_j + (1-\alpha)\mathbf{p}_{j-1}, \tag{11}$$

where α $(0 < \alpha < 1)$ is called a smoothing parameter.

It is important to note that Eq. (9) is similar to the standard CE heuristic formula (8) in [13], with the only difference that the indicator function in the CE updating formula $I_{\{F(\mathbf{c}_k^{j-1}) \leq \gamma\}}$ is replaced by $\exp\left(-F(\mathbf{c}_k^{j-1})\lambda_j\right)$. Eq. (9) is preferable to the standard CE formula (8) in [13], because PMCE uses the entire set of samples, whereas the standard CE only uses the "elite" samples while updating **p**. A nearly optimal solution $\mathbf{c}^*$ that results in lower PAPR will be generated by the PMCE method.

Our proposed PMCE PAPR-reduction algorithm can thus be summarized as follows.

1) *Initialize* $\hat{\mathbf{p}}_0 = [0.5, 0.5, 0.5, \ldots, 0.5]$, ρ, *and* α.
2) *Generate* J *samples* $\mathbf{c}_1^{j-1}, \ldots, \mathbf{c}_J^{j-1}$ *from the density* $f(\mathbf{c}, \hat{\mathbf{p}}_{j-1})$ *and compute their PAPR* $F(\mathbf{c}_k^{j-1})$ *for* $k = 1, \cdots, J$.
3) *Compute* γ_j *by (8), and use (10) to find* λ_j.
4) *Update* $\mathbf{p}_j$ *by (9).*
5) *Obtain the smoothed* $\hat{\mathbf{p}}_j$ *by (11).*

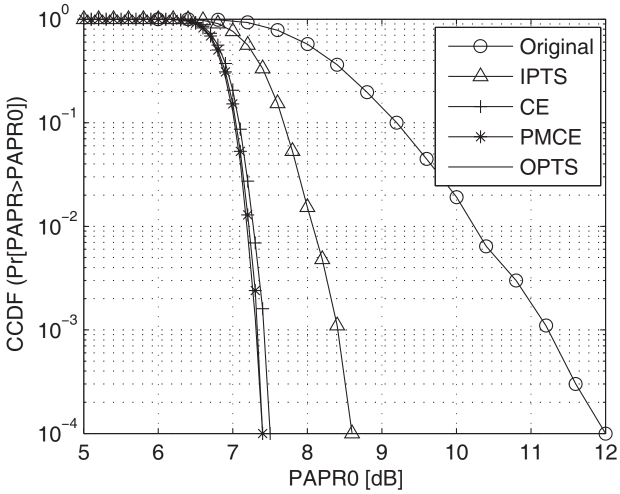

Fig. 2. Comparison of PAPR reduction by different methods.

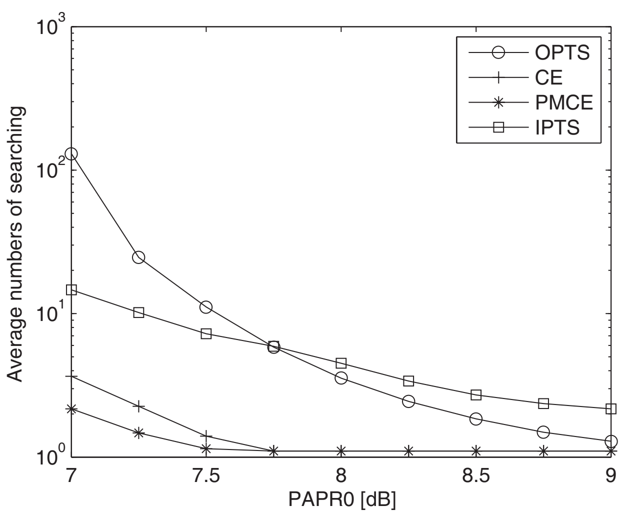

Fig. 3. Average numbers of searching for different methods with thresholds.

6) *If* $\mathbf{0} < \hat{\mathbf{p}}_j < \mathbf{1}$ *for some* j*, return to step 2. Otherwise, output the optimal solution* $\mathbf{c}^* = 1 - 2\mathbf{p}^*$ *and stop.*

V. Simulation Results

In our simulation, quadrature PSK (QPSK) modulation with $N = 256$ sub-carriers is used. In order to obtain the complementary cumulative distribution function (CCDF) $\Pr(PAPR > PAPR_0)$, 10^5 random OFDM symbols are generated. The transmitted signal is oversampled by a factor of $L = 4$ for accurate PAPR [10].

In Fig. 2, the CCDF for the sub-blocks of $M = 8$ using random partition is shown. In the PMCE algorithm, $\rho = 0.1$, $\alpha = 0.6$ and the sample numbers $n = 40$. When CCDF$=10^{-4}$, the PAPR of the conventional OFDM is 12 dB. The PAPR of IPTS with $(M-1)W = 7 \cdot 2 = 14$ searches is 8.6 dB. The PAPRs of PMCE and CE with 22 searches are 7.4 dB and 7.5 dB respectively. The PAPR of the optimal PTS (OPTS) with $2^8 = 256$ searches is 7.4 dB. Compared to the OPTS technique, PMCE thus offers more or less the same PAPR reduction with lower complexity and obtains the nearly optimal phase factors.

In Fig. 3, we compare the average number of searchers of OPTS, PMCE, CE and IPTS for the thresholds $T = 7, 7.25, 7.5, 7.75, 8, 8.25, 8.5, 8.75, 9$. Here, these algorithms are terminated whenever a phase factor that leads to a PAPR below the threshold T is found. Fig. 3 reveals that the PMCE has lower complexity than OPTS and IPTS for all thresholds. For the thresholds between 7.75 dB and 9 dB, PMCE and CE has the same complexity. For the thresholds between 7 dB and 7.75 dB, PMCE has less searching complexity than CE. Fig. 3 shows that PMCE achieves a low PAPR and decreases the computational complexity.

VI. Conclusion

In this paper, we propose a PMCE-based PTS algorithm. The algorithm finds a nearly optimal combination of phase factors for OFDM signals, with significantly reduced computational complexity. Simulation results show that our method outperforms the existing methods both in the CCDF of PAPR and the computational complexity.

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