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Output-Threshold Multiple-Relay-Selection Scheme for Cooperative Wireless Networks

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***Abstract*—An output-threshold multiple-relay-selection (OT-MRS) scheme for dual-hop multibranch cooperative wireless networks is proposed. In this scheme, the first L_c (nonordered) relays are sequentially selected out of L relays such that the SNR of the maximal ratio combined L_c relayed paths and the direct path exceeds a preset threshold. Analytical results for the performance bounds are derived. The numerical results verify the analyses and show that OT-MRS outperforms optimal single-relay selection and generalized-selection-combining-based multiple relay-selection for low to moderately high SNRs. The proposed scheme provides more flexibility in utilizing bandwidth and spatial diversity in cooperative wireless networks.**

***Index Terms*—Amplify-and-forward (AF) relaying, cooperative diversity, relay selection (RS).**

I. Introduction

Cooperative (relay) wireless networks achieve distributed spatial diversity, wider coverage, lower transmit power, and reduced interference [1], [2]. Their performance can further be enhanced by the use of relay selection (RS). RS schemes may be derived from classical adaptive diversity combining techniques. For example, a diversity combiner can add diversity branches until the cumulative output exceeds a threshold [3]–[6]. In this paper, we utilize this combiner to propose a new output-threshold multiple-relay-selection (OT-MRS) scheme.

A. Prior Related Research

The dual-hop multibranch cooperative network of Laneman and Wornell [7] uses all available multiple relays, which is henceforth called all-participate relaying (APR). Since APR has low spectral efficiency due to the use of multiple orthogonal channels, RS schemes have been considered [8]–[13]. They can broadly be divided into two categories, namely, single-RS (SRS) schemes and multiple-RS (MRS) schemes [13].

Among SRS schemes, the selection of the relay whose path has the maximum end-to-end SNR is the optimal scheme [10], [13]. This scheme achieves full diversity while maintaining higher throughput than others [13]. Suboptimal SRS schemes are also considered in the literature [8], [9], [13].

In [11], [13], and [14], several MRS schemes are proposed by generalizing the idea of SRS to allow for multiple relays to cooperate.

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The selection method of [11] involves minimizing the error probability under total energy constraints. Several selection methods of [13] involve the maximization of the received SNR subjected to per-relay power constraints. However, since the optimal selection rule of [13] has exponential complexity in the number of relays, several suboptimal schemes are proposed, which have linear complexity in the number of relays at the expense of performance loss. In [14], a generalized-selection-combining (GSC)-based MRS scheme is developed.

Apart from those RS schemes, incremental relaying [2] achieves higher spectral efficiencies over APR as the former effectively utilizes the degree of freedom of the channel with the aid of limited feedback from the destination. In [15], incremental relaying with the best RS scheme for amplify-and-forward (AF) relaying over fading channels is developed.

B. Motivation and Our Contribution

Although SRS schemes achieve higher spectral efficiencies than APR, their error rates and outages are higher. The complexity of the optimal MRS schemes proposed in [11] and [13] exponentially increases with the available number of relays. Although the GSC-based MRS scheme [14] achieves considerable performance gains, it requires channel estimation of all the relayed paths. In addition, the combined SNR may exceed the system requirements, and more relays than necessary may be selected. Thus, MRS schemes that offer a better tradeoff between error performance and spectral efficiency are desirable. Thus, in this paper, we develop an MRS scheme that uses orthogonal channeling, distributed AF relays, and maximal ratio combining (MRC) at the destination. The key concept of OT-MRS is threshold checking of the output SNR at the destination [4], [5]. The cumulative distribution function (cdf), probability distribution function (pdf), and moment-generating function (mgf) of an upper bound of the output SNR are derived. Lower bounds for the outage probability, average symbol error rate (SER), and average number of selected relays are also derived. The performance of OT-MRS is compared with that of the existing RS schemes. OT-MRS allows for tradeoffs among bandwidth, performance, and complexity for RS schemes.

The rest of this paper is organized as follows: Section II presents the system and the channel model. Section III describes the mode of operation of the proposed OT-MRS scheme in detail. Section IV presents a performance analysis. Section V contains the numerical results, and Section VI concludes this paper.

Notation: $\Gamma(z)$ is the Gamma function [16, 8.310.1]. $\gamma(\alpha, z)$ is the incomplete Gamma function [16, 8.350.1]. $\Gamma(\alpha, z)$ is the complementary incomplete gamma function [16, 8.350.2], and $Q(z)$ is the Q-function [17, 26.2.3]. $\mathcal{E}_\Lambda\{\cdot\}$ denotes the expected value over random variable Λ.

II. System and Channel Model

We consider a cooperative wireless network with $L+2$ terminals that include one source S, one destination D, and L AF relays $R_l|_{l=1}^{L}$ [see Fig. 1(a)]. Only single-antenna terminals are considered. As usual, source-to-destination communication takes place in two phases. In the first phase (the broadcast phase), S broadcasts to L relays and D. In the second phase (the relaying phase), RS is applied, i.e., only L_c $(1 \leq L_c \leq L)$ relays out of L relays are selected to forward the

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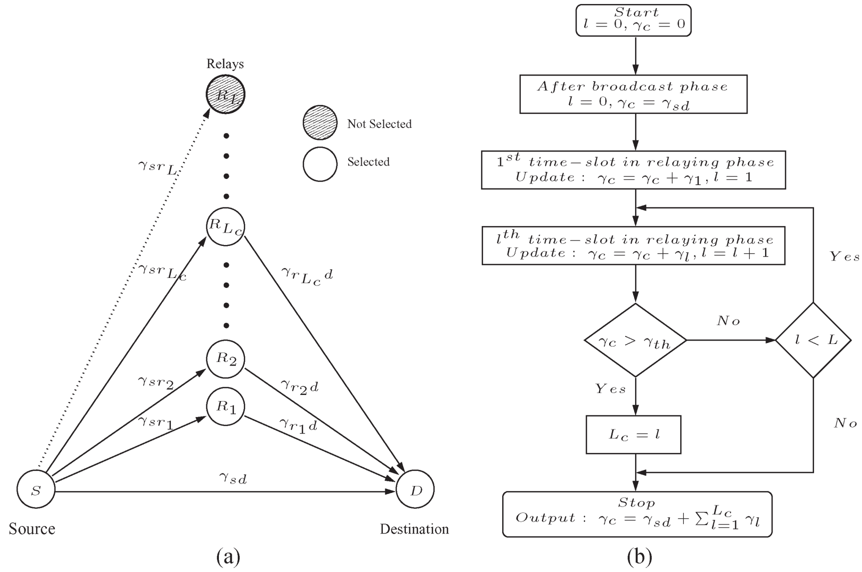

Fig. 1. (a) System model. (b) Mode of operation of the OT-MRS scheme.

amplified version of the source signal to D. To facilitate orthogonal transmission in two phases, a time-division channel-allocation scheme with L_c timeslots is used [2]. The channels $S \rightarrow R_l$ and $R_l \rightarrow D$ are independent and identically distributed and undergo flat Rayleigh fading. Moreover, our model contains an independent flat Rayleigh fading direct channel from $S \rightarrow D$. The perfect channel-state information of only the selected relays is available to D. To limit the output power of relays when channel gains of $S \rightarrow R_l$ are low, we follow [2] and set the gain of the lth relay as $G_l = \sqrt{(P/P|h_l|^2 + N_{0,l})}$, where P is the average energy per symbol used at each terminal, h_l is the fading amplitude of $S \rightarrow R_l$, and $N_{0,l}$ is the variance of the zero-mean additive white Gaussian noise at the input of the lth relay's receiver. The combiner at D uses MRC with output threshold checking [4], [5].

Under this system and channel model, the instantaneous output SNR $\Gamma_i|_{i=1}^{L}$ at D with i active AF relays can be written as [18]–[20]

$$\Gamma_i = \gamma_{sd} + \sum_{l=1}^{i} \frac{\gamma_{sr_l}\gamma_{r_l d}}{\gamma_{sr_l} + \gamma_{r_l d} + 1} \tag{1}$$

where γ_{sd}, γ_{sr_l}, and $\gamma_{r_l d}$ $(l = 1, \ldots, i)$ are instantaneous SNRs of the channels $S \rightarrow D$, $S \rightarrow R_l$, and $R_l \rightarrow D$, respectively. For Rayleigh fading channels, γ_{sd}, γ_{sr_l}, and $\gamma_{r_l d}$ are independent exponential random variables with means $\bar{\gamma}_{sd}$, $\bar{\gamma}_{sr}$, and $\bar{\gamma}_{rd}$, respectively.

To analyze the system performance, the distribution of Γ_i (1) is required, which is not mathematically tractable. To overcome this problem and facilitate a comprehensive performance analysis, we thus replace Γ_i by a tight upper bound Γ_i^{ub} [18]–[20], i.e.,

$$\Gamma_i \leq \Gamma_i^{\mathrm{ub}} = \gamma_{sd} + \sum_{l=1}^{i} \gamma_l \tag{2}$$

where $\gamma_l = \min(\gamma_{sr_l}, \gamma_{r_l d})$. The performance metrics derived by using (2) serve as benchmarks for systems with practical relays. On the other hand, a tight lower bound for Γ_i is given by $\Gamma_i \geq \Gamma_i^{\mathrm{lb}} = \gamma_{sd} + \sum_{l=1}^{i}(1/2)\gamma_l$ [18]. The performance metrics derived by using Γ_i^{lb} serve as tight upper bounds. However, the theoretical development of the lower and upper bounds is the same. For the sake of brevity, we thus do not develop the upper bound results.

III. Proposed Relay-Selection Schemes

The OT-MRS scheme selects the first L_c $(1 \leq L_c \leq L)$ relays such that the combined SNR of the first L_c relayed paths and the direct path exceeds a preset threshold γ_{th}. This SNR threshold can be chosen to be the minimum required SNR for successful symbol decoding for a given modulation scheme. The flowchart in Fig. 1(b) illustrates the RS process. First, D receives the signal transmitted by S during the broadcast phase. Next, the first relay (labeled as R_1) forwards the amplified version of the source message to D in the first timeslot of the relaying phase. The combiner at D combines this signal with the signal received via the direct path. If the combiner output SNR exceeds the threshold at this point, no more relays are selected. Otherwise, the remaining relays $R_2, \ldots, R_{L-1}$ are selected in subsequent timeslots until the cumulative output SNR exceeds the threshold. In the worst case, all L relays are selected. However, in the best case, just the first relay is sufficient. Note that relays are not ordered based on SNR. This is how OT-MRS differs from the GSC approach [14].

IV. Performance Analysis

In this section, the cdf, pdf, and mgf of an upper bound of the output SNR are derived. The lower bounds for the outage probability, average SER, and average number of selected relays are also derived.

A. Statistics of the Output SNR

The instantaneous output SNR γ_c of the OT-MRS scheme can be written as follows:

$$\gamma_c = \begin{cases} \gamma_{sd} + \gamma_{R_1}, & \gamma_{sd} + \gamma_{R_1} \geq \gamma_{\mathrm{th}} \\ \gamma_{sd} + \sum_{l=1}^{L_c} \gamma_{R_l}, & \gamma_{sd} + \sum_{l=1}^{L_c} \gamma_{R_l} \geq \gamma_{\mathrm{th}} \\ & \text{and } 0 \leq \gamma_{sd} + \sum_{l=1}^{L_c - 1} \gamma_{R_l} < \gamma_{\mathrm{th}} \\ \gamma_{sd} + \sum_{l=1}^{L} \gamma_{R_l}, & \text{otherwise} \end{cases} \tag{3}$$

where $\gamma_{R_l} = \gamma_{sr_l}\gamma_{r_l d}/(\gamma_{sr_l} + \gamma_{r_l d} + 1)$ for $(l = 1, \ldots, L)$ is the arbitrarily ordered SNR of the relayed path via the lth relay R_l. A general expression for the cdf of the output SNR of OT-MRS can be written by using (3) as follows:

$$\begin{aligned} F_{\gamma_c}(x) &= \sum_{i=1}^{L} \Pr\left(\left[\gamma_c = \gamma_{sd} + \sum_{l=1}^{i} \gamma_{R_l}\right] \cap [\gamma_c \le x]\right) \\ &= \Pr(\gamma_{\text{th}} \le \Gamma_1 \le x) \\ &\quad + \sum_{i=2}^{L} \Pr\left([\gamma_{\text{th}} \le \Gamma_i \le x] \cap [0 \le \Gamma_{i-1} < \gamma_{\text{th}}]\right) \\ &\quad + \Pr\left([0 \le \Gamma_L \le x] \cap [0 \le \Gamma_{L-1} < \gamma_{\text{th}}]\right) \end{aligned} \tag{4}$$

where $\Pr(\cdot)$ is the probability assignment, and Γ_i is the combined SNR of the first i relayed paths and the direct path defined in (1). The first term of the second equality in (4) accounts for the event in which the combined SNR of the first relayed path and the direct path exceeds the threshold, i.e., $\Gamma_1 \ge \gamma_{\text{th}}$. The second term accounts for the event in which the i relays are required to be coherently combined with the direct path to form an output whose SNR exceeds the threshold. The third term corresponds to the worst case where the cumulative sum of the first $L-1$ relayed paths and the direct path SNRs does not exceed the threshold, and all L relays must be selected. After manipulations, we simplify (4) into a more mathematically tractable form, i.e.,

$$F_{\gamma_c}(x) = \begin{cases} F_{\Gamma_L}(x), & x<\gamma_{\text{th}} \\ F_{\Gamma_1}(x) - F_{\Gamma_1}(\gamma_{\text{th}}) \\ \quad + \sum_{i=2}^{L} \int_0^{\gamma_{\text{th}}} \int_{\gamma_{\text{th}}-\Gamma_{i-1}}^{x-\Gamma_{i-1}} f_{\Gamma_{i-1},\gamma_{R_i}}(\Gamma_{i-1},\gamma_{R_i}) d\gamma_{R_i} d\Gamma_{i-1} \\ \quad + \int_0^{\gamma_{\text{th}}} \int_0^{\gamma_{\text{th}}-\Gamma_{L-1}} f_{\Gamma_{L-1},\gamma_{R_L}}(\Gamma_{L-1},\gamma_{R_L}) d\gamma_{R_L} d\Gamma_{L-1}, & x \ge \gamma_{\text{th}} \end{cases} \tag{5}$$

where $F_{\Gamma_i}(x)$ is the cdf of the combined SNR of the first i relayed paths and the direct path, and $f_{\Gamma_i,\gamma_{R_i}}(\Gamma_i, \gamma_{R_i})$ is the joint pdf of Γ_i and γ_{R_i}. To evaluate the cdf of γ_c, one needs to find $f_{\Gamma_i,\gamma_{R_i}}(\Gamma_i, \gamma_{R_i})$. This can easily be obtained by using (1) and identifying the statistical independence of Γ_i and γ_{R_i} as follows:

$$f_{\Gamma_{i-1},\gamma_{R_i}}(\Gamma_{i-1}, \gamma_{R_i}) = f_{\Gamma_{i-1}}(\Gamma_{i-1}) f_{\gamma_{R_i}}(\gamma_{R_i}). \tag{6}$$

The pdf of Γ_{i-1}^{ub} in (2) can be written in closed form as in [20]

$$f_{\Gamma_{i-1}^{\text{ub}}}(x) = \beta_{sd,i-1} e^{-\frac{x}{\bar\gamma_{sd}}} + \sum_{l=1}^{i-1} \frac{\beta_{l,i-1}}{(l-1)!} x^{l-1} e^{-\frac{x}{\bar\gamma}} \tag{7}$$

where $\beta_{sd,k} = \bar\gamma_{sd}^{k-1}/(\bar\gamma_{sd} - \bar\gamma)^k$, $\beta_{l,k} = (-\bar\gamma_{sd})^{k-l}/\bar\gamma^{l-1}(\bar\gamma - \bar\gamma_{sd})^{k-l}$, and $\bar\gamma = \bar\gamma_{sr}\bar\gamma_{rd}/\bar\gamma_{sr} + \bar\gamma_{rd}$.

The cdf of Γ_{i-1}^{ub} can be found as

$$F_{\Gamma_{i-1}^{\text{ub}}}(x) = \beta_{sd,i-1}\bar\gamma_{sd}\left(1 - e^{-\frac{x}{\bar\gamma_{sd}}}\right) + \sum_{l=1}^{i-1} \frac{\beta_{l,i-1}}{(l-1)!} \bar\gamma^l \gamma\left(l, \frac{x}{\bar\gamma}\right). \tag{8}$$

By using (5)–(8), we derive the cdf of an upper bound of the output SNR γ_c^{ub}, which is obtained by replacing γ_c with γ_c^{ub} in (3) in closed form, i.e.,

$$F_{\gamma_c^{\text{ub}}}(x) = \begin{cases} \beta_{sd,L}\bar\gamma_{sd}\left(1 - e^{-\frac{x}{\bar\gamma_{sd}}}\right) \\ \quad + \sum_{l=1}^{L} \frac{\beta_{l,L}}{(l-1)!} \bar\gamma^l \gamma\left(l, \frac{x}{\bar\gamma}\right), & 0 \le x < \gamma_{\text{th}} \\ \beta_{sd,1}\bar\gamma_{sd} e^{-\frac{\gamma_{\text{th}}}{\bar\gamma_{sd}}} \left(1 - e^{-\frac{(x-\gamma_{\text{th}})}{\bar\gamma_{sd}}}\right) \\ \quad + \lambda e^{-\frac{\gamma_{\text{th}}}{\bar\gamma}} \left(1 - e^{-\frac{(x-\gamma_{\text{th}})}{\bar\gamma}}\right) + \kappa, & x \ge \gamma_{\text{th}} \end{cases}$$

where

$$\begin{aligned} \lambda &= \beta_{1,1}\bar\gamma + \sum_{i=2}^{L} \left(\beta_{sd,i-1}\left(\frac{\bar\gamma\bar\gamma_{sd}}{\bar\gamma - \bar\gamma_{sd}}\right)\left(1 - e^{-\frac{\gamma_{\text{th}}(\bar\gamma - \bar\gamma_{sd})}{\bar\gamma\bar\gamma_{sd}}}\right) \right. \\ &\qquad \left. + \sum_{l=1}^{i-1} \frac{\beta_{l,i-1}}{l!} \gamma_{\text{th}}^l \right) \\ \kappa &= \beta_{sd,L-1}\bar\gamma_{sd}\left(1 + \frac{1}{\bar\gamma - \bar\gamma_{sd}}\left(\bar\gamma_{sd} e^{-\frac{\gamma_{\text{th}}}{\bar\gamma_{sd}}} - \bar\gamma e^{-\frac{\gamma_{\text{th}}}{\bar\gamma}}\right)\right) \\ &\quad + \sum_{l=1}^{L-1} \beta_{l,L-1}\left(\frac{\bar\gamma^l}{(l-1)!}\gamma\left(l, \frac{\gamma_{\text{th}}}{\bar\gamma}\right) - \frac{\gamma_{\text{th}}^l}{l!} e^{-\frac{\gamma_{\text{th}}}{\bar\gamma}}\right). \end{aligned} \tag{9}$$

The pdf of γ_c^{ub} can easily be obtained by differentiating (9) as follows:

$$f_{\gamma_c^{\text{ub}}}(x) = \begin{cases} \beta_{sd,L} e^{-\frac{x}{\bar\gamma_{sd}}} + \sum_{l=1}^{L} \frac{\beta_{l,L}}{(l-1)!} x^{l-1} e^{-\frac{x}{\bar\gamma}}, & 0 \le x < \gamma_{\text{th}} \\ \beta_{sd,1} e^{-\frac{x}{\bar\gamma_{sd}}} + \frac{\lambda}{\bar\gamma} e^{-\frac{x}{\bar\gamma}}, & x \ge \gamma_{\text{th}}. \end{cases} \tag{10}$$

Moreover, the mgf of γ_c^{ub} can be derived from (10) as

$$\begin{aligned} M_{\gamma_c^{\text{ub}}}(s) = \mathcal{E}\{e^{-sx}\} &= \frac{\beta_{sd,L} + (\beta_{sd,1} - \beta_{sd,L}) e^{-\gamma_{\text{th}}\left(s + \frac{1}{\bar\gamma_{sd}}\right)}}{s + \frac{1}{\bar\gamma_{sd}}} \\ &\quad + \sum_{l=1}^{L} \frac{\beta_{l,L}}{(i-1)!\left(s + \frac{1}{\bar\gamma}\right)^l} \gamma\left(l, \gamma_{\text{th}}\left(s + \frac{1}{\bar\gamma}\right)\right) \\ &\quad + \frac{\frac{\lambda}{\bar\gamma} e^{-\gamma_{\text{th}}\left(s + \frac{1}{\bar\gamma}\right)}}{s + \frac{1}{\bar\gamma}}. \end{aligned} \tag{11}$$

B. SNR Outage Probability

The SNR outage probability P_{out} is defined as the probability that the instantaneous output SNR γ_c falls below the threshold γ_{th}. Thus, a lower bound for the SNR outage probability $P_{\text{out}}^{\text{lb}}$ of OT-MRS can immediately be obtained by using (9) as follows:

$$P_{\text{out}}^{\text{lb}} = \Pr\left(0 \le \gamma_c^{\text{ub}} \le \gamma_{\text{th}}\right) = F_{\gamma_c^{\text{ub}}}(\gamma_{\text{th}}). \tag{12}$$

C. Average Error Rate

Lower bounds for the average SER of various modulation schemes can be derived by integrating the conditional error probability (CEP) $P_e|\gamma$ over the pdf of the output SNR γ_c^{ub}. The CEP for coherent binary frequency-shift keying (C-BFSK) and M-ary pulse amplitude modulation can be expressed in the form of $P_e|\gamma = \zeta Q(\sqrt{\eta\gamma})$, where ζ and η are modulation-dependent constants. For example, $(\zeta = 1, \eta = 2)$ and $(\zeta = 1, \eta = 1)$ provide the exact bit error rate (BER) of the coherent binary phase-shift keying (C-BPSK) and C-BFSK, respectively. A lower bound of the average error rate for this case is given by the following closed-form expression (see the Appendix for details):

$$\begin{aligned}\bar{P}_e^{\mathrm{lb}} =& \frac{\zeta}{2}\beta_{sd,L}\bar{\gamma}_{sd}(1-\mu_{sd})+\zeta(\beta_{sd,1}-\beta_{sd,L})\bar{\gamma}_{sd}\\ &\times\left(Q(\sqrt{\eta\gamma_{\mathrm{th}}})e^{-\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{sd}}}-\mu_{sd}Q(\sqrt{2\nu_{sd}})\right)\\ &+\zeta\lambda\left(e^{-\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}}}Q(\sqrt{\eta\gamma_{\mathrm{th}}})-\mu Q(\sqrt{2\nu})\right)\\ &+\sum_{l=1}^{L}\zeta\beta_{l,L}\bar{\gamma}^l\left(\frac{1}{2}-Q(\sqrt{\eta\gamma_{\mathrm{th}}})\left(1-\frac{\gamma\left(l,\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}}\right)}{(l-1)!}\right)\right.\\ &\left.-\sum_{j=0}^{l-1}\frac{2^{j-1}\mu^{(2j+1)}\gamma\left(j+\frac{1}{2},\nu\right)}{\sqrt{\pi}j!(\eta\bar{\gamma})^j}\right)\end{aligned}\tag{13}$$

where $\mu_{sd} = \sqrt{\eta\bar{\gamma}_{sd}/(2+\eta\bar{\gamma}_{sd})}$, $\mu = \sqrt{(\eta\bar{\gamma}/2+\eta\bar{\gamma})}$, $\nu_{sd} = \gamma_{\mathrm{th}}(2+\eta\bar{\gamma}_{sd})/2\bar{\gamma}_{sd}$, and $\nu = \gamma_{\mathrm{th}}(2+\eta\bar{\gamma})/2\bar{\gamma}$.

D. Average Number of Selected Relays

The number of selected relays L_c fluctuates with the channel fading states. Accordingly, it is a discrete random variable with the range $1 \le L_c \le L$. This range results in interesting tradeoffs among bandwidth efficiency, power consumption, and performance. When L_c is equal to L, OT-MRS reverts to APR. On the other hand, when the number of selected relays is less than L, OT-MRS requires fewer orthogonal channels than that of APR and achieves higher bandwidth efficiency and lower power consumption. However, a low L_c results in a lower diversity gain than that of APR. To get more insight into such tradeoffs, we analyze the number of selected relays. The average number of selected relays $\bar{L}_c$ of OT-MRS can be obtained as follows:

$$\bar{L}_c = \sum_{l=1}^{L} l\ \Pr(L_c = l)\tag{14}$$

where $\Pr(L_c = l)$ is the probability that the selected number of relays is l. To obtain $\bar{L}_c$, $\Pr(L_c = l)$ is first derived by using the definition of γ_c given in (3), i.e.,

$$\begin{aligned}&\Pr(L_c = l)\\ &= \begin{cases}\Pr(\gamma_{sd}+\gamma_{R_1}\ge\gamma_{\mathrm{th}}), & l=1\\ \Pr\left(\left[\gamma_{sd}+\sum_{i=1}^{l}\gamma_{R_i}\ge\gamma_{\mathrm{th}}\right]\right.\\ \quad\left.\cap\left[\gamma_{sd}+\sum_{i=1}^{l-1}\gamma_{R_i}<\gamma_{\mathrm{th}}\right]\right), & l\in\{2,\ldots,L-1\}\\ \Pr\left(\gamma_{sd}+\sum_{i=1}^{L-1}\gamma_{R_i}<\gamma_{\mathrm{th}}\right), & l=L\end{cases}\\ &= \begin{cases}1-F_{\Gamma_1}(\gamma_{\mathrm{th}}), & l=1\\ \int_0^{\gamma_{\mathrm{th}}}\int_{\gamma_{\mathrm{th}}-\Gamma_{l-1}}^{\infty}f_{\Gamma_{l-1},\gamma_{R_l}}\\ \quad\times(\Gamma_{l-1},\gamma_{R_l})d\gamma_{R_l}d\Gamma_{l-1}, & l\in\{2,\ldots,L-1\}\\ F_{\Gamma_{L-1}}(\gamma_{\mathrm{th}}), & l=L.\end{cases}\end{aligned}\tag{15}$$

By substituting (7) and (8) into (15), and after some manipulations, a lower bound for $\Pr(L_c = l)$ can be derived as

$$\begin{aligned}&\Pr(L_c = l)\\ &\ge\begin{cases}\beta_{sd,1}\bar{\gamma}_{sd}e^{-\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{sd}}}+\beta_{1,1}\bar{\gamma}e^{-\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}}}, & l=1\\ \beta_{sd,l-1}\left(\frac{\bar{\gamma}\bar{\gamma}_{sd}}{\bar{\gamma}-\bar{\gamma}_{sd}}\right)e^{-\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}}}\\ \times\left(1-e^{-\frac{\gamma_{\mathrm{th}}(\bar{\gamma}-\bar{\gamma}_{sd})}{\bar{\gamma}\bar{\gamma}_{sd}}}\right)\\ +\sum_{i=1}^{l-1}\frac{\beta_{i,l-1}}{i!}\gamma_{\mathrm{th}}^i e^{-\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}}}, & l\in\{2,\ldots,L-1\}\\ \beta_{sd,L-1}\bar{\gamma}_{sd}\left(1-e^{-\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{sd}}}\right)\\ +\sum_{i=1}^{L-1}\frac{\beta_{i,L-1}}{(i-1)!}\bar{\gamma}^i\gamma\left(i,\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}}\right), & l=L.\end{cases}\end{aligned}\tag{16}$$

By substituting (16) into (14), a lower bound of the average number of selected relays $\bar{L}_c$ can be obtained as

$$\begin{aligned}\bar{L}_c^{\mathrm{lb}} =& L\beta_{sd,L-1}\bar{\gamma}_{sd}+(\beta_{sd,1}-L\beta_{sd,L-1})\bar{\gamma}_{sd}e^{-\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{sd}}}\\ &+\beta_{1,1}\bar{\gamma}e^{-\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}}}+\sum_{i=1}^{L-1}\frac{L\beta_{i,L-1}}{(i-1)!}\bar{\gamma}^i\gamma\left(i,\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}}\right)\\ &+\sum_{l=2}^{L-1}\left(l\beta_{sd,l-1}\left(\frac{\bar{\gamma}\bar{\gamma}_{sd}}{\bar{\gamma}-\bar{\gamma}_{sd}}\right)\right.\\ &\times\left(1-e^{-\frac{\gamma_{\mathrm{th}}(\bar{\gamma}-\bar{\gamma}_{sd})}{\bar{\gamma}\bar{\gamma}_{sd}}}\right)e^{-\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}}}\\ &\left.+\sum_{i=1}^{l-1}\frac{l\beta_{i,l-1}}{i!}\gamma_{\mathrm{th}}^i e^{-\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}}}\right).\end{aligned}\tag{17}$$

V. Numerical Results

In this section, numerical results are presented to illustrate and compare various performance metrics of OT-MRS. Monte Carlo simulation results see are provided as an independent verification of the analytical results (Section IV).

Fig. 2 shows the exact simulation results for the average BER of BPSK of OT-MRS for six- and ten-relay cooperative networks. The average BER of BPSK via $\zeta = 1$ and $\eta = 2$ in (13) is plotted as a function of the normalized average SNR per branch $(\bar{\gamma}^* = \bar{\gamma}/\gamma_{\mathrm{th}})$. The figure shows that the analytical BER lower bound (13) is tight for the SNRs significantly higher than the threshold. As $\bar{\gamma}^*$ increases, the BER curves experience a sudden kink. This kink is because OT-MRS selects only a single relay at high SNRs, which limits the diversity order to 2.

Fig. 3 plots the average number of selected relays $\bar{L}_c$ as a function of the normalized output threshold $(\gamma_{\mathrm{th}}^* = \gamma_{\mathrm{th}}/\bar{\gamma})$ for four-, six-, eight-, and ten-relay cooperative networks. Clearly, $\bar{L}_c$ is highly dependent on both γ_{th} and $\bar{\gamma}$. As γ_{th} significantly increases beyond $\bar{\gamma}$, $\bar{L}_c$ decreases from 1 to L, as expected.

In Fig. 4, the average BER of OT-MRS is compared with that of the best RS [10], GSC-based MRS (which selects best five out of ten relays) [14], and APR [2]. The APR scheme is included as a benchmark. The average BER of fixed L_n out of L RS is also plotted as a comparison between a fixed-RS scheme and an adaptive-RS scheme (OT-MRS). In low to moderate SNRs, OT-MRS performs

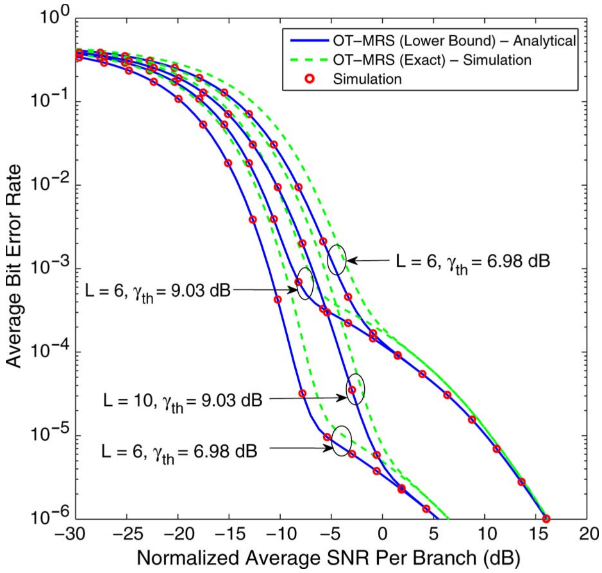

Fig. 2. Average BER of BPSK as a function of the normalized average SNR per branch ($\bar{\gamma}^* = \bar{\gamma}/\gamma_{\rm th}$).

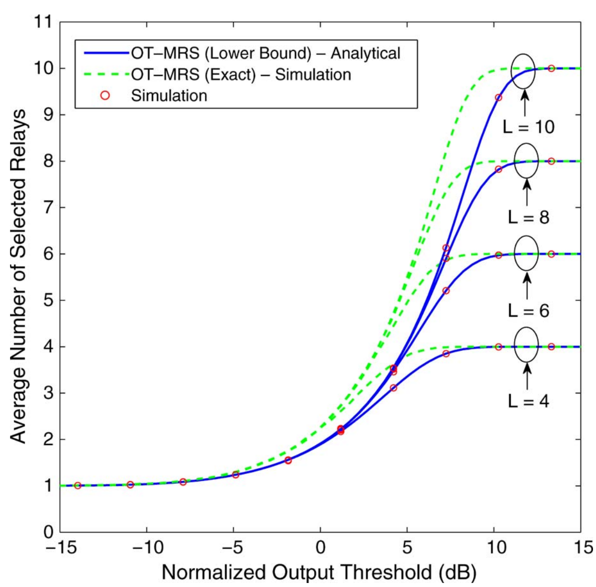

Fig. 3. Average number of selected relays $\bar{L}_c$ versus normalized output threshold $\gamma_{\rm th}^* = \gamma_{\rm th}/\bar{\gamma}$.

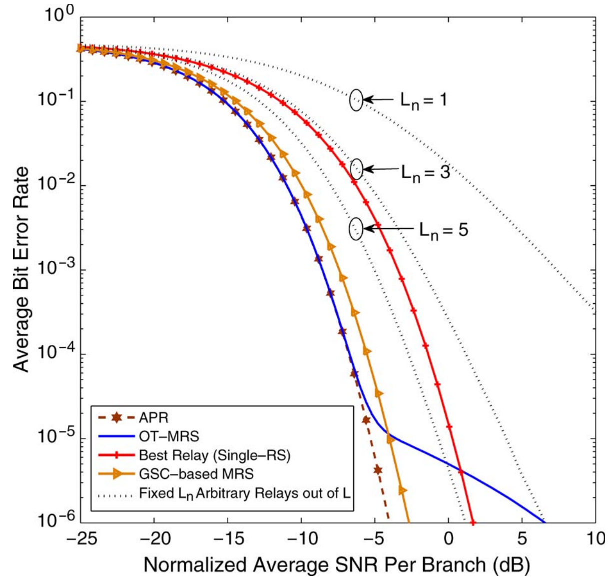

Fig. 4. Comparison of the average BER of BPSK of several RS schemes for a ten-relay cooperative network. The GSC-based MRS scheme selects the best five relays out of ten available relays. $L = 10$.

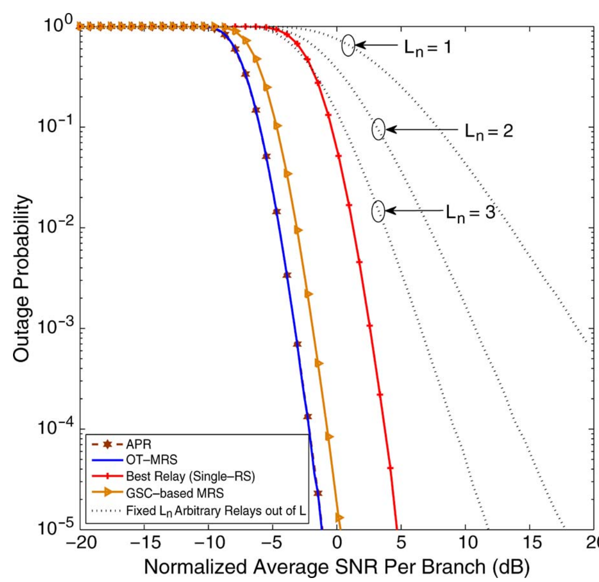

Fig. 5. Comparison of the SNR outage probability of several RS schemes. The GSC-based MRS scheme selects the best five relays out of ten available relays. $\bar{\gamma}^* = \bar{\gamma}/\gamma_{\rm th}$. $L = 10$.

identical to APR and outperforms the other RS schemes. As expected, OT-MRS loses diversity gain for the SNRs significantly higher than the threshold. The diversity order is limited to 2 in this region because only one relay is selected in this case. However, it still provides the required output SNR. Moreover, it has the advantages of fewer channel-estimation requirements and no relay ordering. The GSC-based MRS scheme outperforms OT-MRS for higher values of $\bar{\gamma}^*$. This is not surprising since the former selects the best five relays compared with an arbitrary relay in OT-MRS. However, this performance gain in high $\bar{\gamma}^*$ values is achieved at the cost of additional bandwidth, channel estimation, and power requirements, compared with the OT-MRS scheme.

Fig. 5 presents a comparison of the SNR outage probability of the RS schemes. OT-MRS outperforms all the other RS schemes in all SNRs apart from APR. OT-MRS and APR perform identically. This happens because OT-MRS sequentially selects relays until the cumulative output SNR exceeds the preset threshold $\gamma_{\rm th}$, and an outage event only occurs when the cumulative output SNR of all L relays is less than $\gamma_{\rm th}$.

In Fig. 6, the average BER of the RS schemes is plotted as a function of the normalized output threshold $\gamma_{\rm th}^*$. This figure shows the dependence of the average BER of OT-MRS on $\gamma_{\rm th}^*$. When the output threshold is significantly higher than the average SNR per branch, OT-MRS selects all L relays and performs identically to APR. Fig. 6 reveals that OT-MRS adaptively utilizes the wireless resources to improve system performance.

VI. Conclusion

This paper has developed an OT-MRS scheme for cooperative relay networks. The cdf, pdf, and mgf of an upper bound of the output

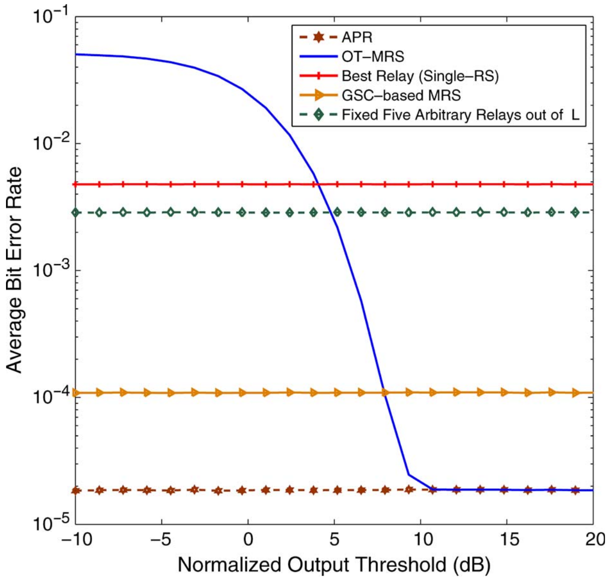

Fig. 6. Comparison of the BPSK BER of several RS schemes against the normalized output threshold $\bar{\gamma}_{\mathrm{th}}^*$. The GSC-based MRS scheme selects the best five relays out of ten available relays. $\bar{\gamma}_{\mathrm{th}}^* = \gamma_{\mathrm{th}}/\bar{\gamma}$. $\bar{\gamma} = 3$ dB. $L = 10$.

SNR were derived. Lower bounds for the outage probability, average SER, and average number of selected relays were also derived. The numerical and simulation results for the OT-MRS scheme were compared with those of the optimal SRS and GSC-based MRS schemes. OT-MRS outperforms these schemes for low to moderate SNRs, provides flexibility to adapt in fading, and enables tradeoffs among bandwidth, performance, and complexity in cooperative relay networks.

Appendix

The lower bound for the average SER can be derived as follows:

$$\begin{aligned}\bar{P}_e^{\mathrm{lb}} &= \int_0^\infty \zeta Q(\sqrt{\eta x}) f_{\gamma_c}(x)dx = \int_0^{\gamma_{\mathrm{th}}} \zeta Q(\sqrt{\eta x}) \\ &\quad \times \left(\beta_{sd,L} e^{-\frac{x}{\bar{\gamma}_{sd}}} + \sum_{l=1}^{L} \frac{\beta_{l,L}}{(l-1)!} x^{l-1} e^{-\frac{x}{\bar{\gamma}}}\right) dx \\ &\quad + \int_{\gamma_{\mathrm{th}}}^\infty \zeta Q(\sqrt{\eta x}) \left(\beta_{sd,1} e^{-\frac{x}{\bar{\gamma}_{sd}}} + \frac{\lambda}{\bar{\gamma}} \exp - \frac{x}{\bar{\gamma}}\right) dx \\ &= \zeta \beta_{sd}(L) I_1 + \zeta \sum_{l=1}^{L} \beta_{l,L} I_l + \zeta \beta_{sd,1} I_2 + \frac{\zeta\lambda}{\bar{\gamma}} I_3 \qquad (18)\end{aligned}$$

where λ is defined in (9). Here, I_1, I_2, I_3, and I_l are given by

$$I_1 = \int_0^{\gamma_{\mathrm{th}}} Q(\sqrt{\eta x}) e^{-\frac{x}{\bar{\gamma}_{sd}}} dx \quad I_2 = \int_{\gamma_{\mathrm{th}}}^\infty Q(\sqrt{\eta x}) e^{-\frac{x}{\bar{\gamma}_{sd}}} dx,$$

$$I_3 = \int_{\gamma_{\mathrm{th}}}^\infty Q(\sqrt{\eta x}) e^{-\frac{x}{\bar{\gamma}}} dx \qquad I_l = \int_0^{\gamma_{\mathrm{th}}} \frac{Q(\sqrt{\eta x})}{(l-1)!} x^{l-1} e^{-\frac{x}{\bar{\gamma}}} dx.$$

I_1, I_2, and I_3 can be evaluated as in [20]. I_l can be evaluated by using $\int_0^x t^{l-1} \exp -t/\bar{\gamma} dt = \bar{\gamma}^l \gamma(l, x/\bar{\gamma})$ and integration by parts as follows:

$$\begin{aligned}I_l &= \frac{\bar{\gamma}^l}{(l-1)!} Q(\sqrt{\eta\gamma_{\mathrm{th}}}) \gamma\left(l, \frac{\gamma_{\mathrm{th}}}{\bar{\gamma}}\right) + 0.5\bar{\gamma}^l \left(1 - 2Q(\sqrt{\eta\gamma_{\mathrm{th}}})\right) \\ &\quad + \sum_{j=0}^{l-1} \frac{2^{j-1}\bar{\gamma}^{l-j}}{\sqrt{\pi}\eta^j j!} \left(\frac{\eta\bar{\gamma}}{2+\eta\bar{\gamma}}\right)^{j+0.5} \gamma\left(j+0.5, \frac{\gamma_{\mathrm{th}}}{2\bar{\gamma}}(2+\eta\bar{\gamma})\right). \end{aligned} \qquad (19)$$

The desired result (13) can be obtained by using the identities $\gamma((1/2), x) = \sqrt{\pi}(1 - 2Q(\sqrt{2x}))$ and $\Gamma(a, x) + \gamma(a, x) = \Gamma(x)$ [17].

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Distributed Self-Concatenated Coding for Cooperative Communication

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***Abstract*—In this paper, we propose a power-efficient distributed binary self-concatenated coding scheme using iterative decoding (DSECCC-ID) for cooperative communications. The DSECCC-ID scheme is designed with the aid of binary extrinsic information transfer (EXIT) charts. The source node transmits self-concatenated convolutional coded (SECCC) symbols to both the relay and destination nodes during the first transmission period. The relay performs SECCC-ID decoding, where it may or may not encounter decoding errors. It then reencodes the information bits using a recursive systematic convolutional (RSC) code during the second transmission period. The resultant symbols transmitted from the source and relay nodes can be viewed as the coded symbols of a three-component parallel concatenated encoder. At the destination node, three-component DSECCC-ID decoding is performed. The EXIT chart gives us an insight into operation of the distributed coding scheme, which enables us to significantly reduce the transmit power by about 3.3 dB in signal-to-noise ratio (SNR) terms, as compared with a noncooperative SECCC-ID scheme at a bit error rate (BER) of 10^{-5}. Finally, the proposed system is capable of performing within about 1.5 dB from the two-hop relay-aided network's capacity at a BER of 10^{-5}, even if there may be decoding errors at the relay.**

***Index Terms*—Cooperation diversity, distributed coding, extrinsic information transfer (EXIT) charts, iterative decoding, self-concatenated coding.**

I. Introduction

Digital communication exploiting multiple-input–multiple-output (MIMO) wireless channels have recently attracted considerable attention. The wireless communication systems of future generations are required to provide reliable transmissions at high data rates to offer a variety of multimedia services to commercial wireless products and networks. Space–time coding schemes [1], which employ multiple transmitters and receivers, are among the most efficient techniques designed to achieve a high diversity gain, provided that the associated MIMO channels [2], [3] experience independent fading. Utilizing cooperative techniques eliminates the correlation of the signals when using multiple antennas at the mobiles, which is imposed by the limited affordable element spacing.

Although full-duplex relaying and the associated capacity theorem derived for the discrete memoryless relay channel model have been proposed in [4], practical cooperative diversity schemes were only proposed much later in [5]–[8]. In practice, each mobile collaborates with a single or a few partners for the sake of reliably transmitting both its own information and that of its partners jointly, which emulates a virtual MIMO scheme. The most popular collaborative protocols used between the source, relay, and destination nodes are amplify-and-forward, demodulate-and-forward, and decode-and-forward (DAF) schemes [9]. We derive the theoretical lower and upper bounds on the continuous-input–continuous-output memoryless channel (CCMC) capacity, as well as of the discrete-input–continuous-output memoryless channel (DCMC) [3], [4], [10], [11] capacity (constrained information rate) for independent and uniformly distributed (i.u.d.) sources.

The philosophy of concatenated coding schemes was proposed by Forney in [12]. Turbo codes, which were developed in [13], constitute a class of error correction codes based on two or more parallel concatenated convolutional codes that are used as constituent codes. They are high-performance codes capable of operating near the Shannon limit. Since their invention, they have found diverse applications in bandwidth-limited communication systems, where the maximum achievable information rate has to be supported in the presence of transmission errors due to both the ubiquitous additive white Gaussian noise (AWGN) and channel fading. Various bandwidth-efficient turbo codes were proposed in [14]–[16]. Serially concatenated convolutional codes [17] have been shown to yield a performance comparable, and in some cases superior, with turbo codes. Iteratively decoded self-concatenated convolutional codes (SECCC-ID) proposed by Benedetto *et al.* [18] constitute another attractive family of iterative-detection-aided schemes. The SECCC arrangement is a low-complexity scheme using a single encoder and a single decoder. The extrinsic information transfer (EXIT) chart [19]-based analysis of the iterative decoder provides an insight into its decoding convergence behavior, and hence, it is helpful for finding the best constituent coding schemes for creating near-capacity SECCCs both for AWGN and Rayleigh fading channels [20]. EXIT charts constitute a semianalytical tool used to predict the SNR value, where an infinitesimally low bit error ratio (BER) can be achieved without performing time-consuming bit-by-bit decoding, which employs a high number of decoding iterations.

Distributed coding [21] constitutes another attractive cooperative diversity technique, where joint signal design and coding are invoked at the source and relay nodes. Distributed turbo codes [22], [23] have also been proposed for cooperative communications, although typically under the simplifying assumption of having a perfect communication link between the source and relay nodes. These are half-duplex relay-aided systems, where the source transmits to both the relay and the destination during the first transmission period, and after decoding the information from the source, the relay reencodes it and sends it to the destination in the second transmission period. Hence, half-duplex systems do not suffer from multiaccess interference, which results in a simplified receiver structure at the cost of halving the spectral efficiency. As a more realistic design alternative, a

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