# Performance Analysis of Partial Relay Selection With Feedback Delay

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***Abstract*—We analyze the impact of outdated channel state information due to feedback delay on the performance of amplify-and-forward relays with the $k$th worst partial relay selection scheme. In our analysis, new expressions for the system's outage probability and the average bit error rate are derived. The effects of the rank of the relay chosen, the average SNR imbalance, and the correlation between the delayed and current signal-to-noise ratio (SNR) on the system performance are investigated. Additionally, simple and accurate outage and average BER approximations are also derived to quantify the performance at high SNR. We also give simulation results to support the theoretical study.**

***Index Terms*—Amplify-and-forward, error performance, feedback delay, outage probability, partial relay selection.**

## I. Introduction

RELAY deployment in wireless networks offers a variety of significant performance benefits, including hotspot throughput improvements and cellular signal coverage enhancements. Relay selection for multi-relay systems is a highly active topic in the literature [1]–[9]. For instance, [2] considers an amplify-and-forward (AF) selection scheme where a single relay is activated for signal forwarding according to the instantaneous global (two hop) channel state information of the network. However, in some applications such as resource constrained ad hoc and sensor networks, monitoring the connectivity among all links can limit the network lifetime. Such challenges have motivated the development of partial relay selection (PRS) schemes, which require only (single hop) information [5].

Several other works [6]–[9] have also analyzed the performance of the PRS scheme first proposed in [5]. In [6] and [7], by deriving closed-form expressions for the outage probability, probability density function, average bit error rate (BER) and moments of the end-to-end (e2e) signal-to-noise ratio (SNR), the performance of PRS with fixed-gain relaying has been studied. In [8], asymptotic lower and upper capacity bounds of a fixed-gain PRS scheme in which selection is performed using only feedback from eligible relays having a better channel quality have been investigated. In [9], tight closed-form approximations for the ergodic capacity of the PRS scheme with fixed-gain relaying over independent Nakagami-$m$ fading channels have been derived.

When PRS is implemented in real time, there is a possibility that outdated channel state information (CSI) could be used for relay selection due to feedback delay. In other words, selected relay's CSI at the instant of actual data transmission may not be the best among the relay set. As a result, the system performance is adversely affected. The current literature has addressed not this practical issue.

In this letter, we investigate the impact of feedback delay on the performance of the PRS scheme. The best relay might be unavailable to forward the source signal; hence based on the source-relay link channel, we consider PRS with the $k$th worst relay selection. Thus, the PRS scheme of [5] based on the best source-relay link may be treated as a special case of our general analysis. We derive closed-form expressions for the system's outage probability and the average BER applicable for a range of modulation schemes. To gain valuable insights, we also present simple high SNR outage and BER approximations. These expressions clearly reveal the impact of the number of relays, feedback delay and the SNR imbalance between source-relay and relay-destination links on the system performance.

## II. System Model

We consider an AF dual-hop transmission network with one source, $S$, one destination, $D$, and $N_r \geq 1$ relays. $S$ has no direct link with $D$, and the transmission is performed only through relays. $S$ periodically monitors the quality of its connectivity with the relays via transmission of a local feedback. We assume that this feedback link has a time delay of $T_d$. Based on the outdated CSI, $S$ selects a single relay, $R_{(k)}$, with the $k$th worst $S-R$ link. After that the communication takes place in two time slots. During the first time slot, $S$ transmits its signal, $x(t)$, with an average power normalized to unity to the selected relay. In the second time slot, the relay amplifies the received signal and the output is transmitted to $D$.

The received signal at the selected relay can be written as

$$y_k(t) = \sqrt{P_s} h_{S,R_{(k)}}(t) x(t) + n_{R_{(k)}}(t) \tag{1}$$

where $P_s$ is the transmit power at $S$, the complex channel between $S$ and $R_{(k)}$ is $h_{S,R_{(k)}}(t)$ and $n_{R_{(k)}}(t)$ is the additive white Gaussian noise (AWGN) satisfying $E\left(|n_{R_{(k)}}(t)|^2\right) = \sigma_R^2$. Since a variable gain relay is assumed, the relay multiplies the received signal by a gain, $G = \sqrt{P_r/P_s|h_{S,R_{(k)}}(t)|^2 + \sigma_R^2}$ and the output is transmitted to $D$. The received signal at $D$ is given by

$$y_D(t) = h_{R_{(k)},D}(t) G y_k(t) + n_D(t) \tag{2}$$

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where $h_{R_{(k)},D}(t)$ is the complex channel between $R_{(k)}$ and $D$ and $n_D(t)$ is the AWGN satisfying $E\left(|n_D(t)|^2\right) = \sigma_D^2$. After some manipulations, it can be shown that the e2e SNR, $\gamma_{eq1}$, is given by

$$\gamma_{eq1} = \frac{\tilde{\gamma}_{1(k)}\gamma_2}{\tilde{\gamma}_{1(k)} + \gamma_2 + 1} \tag{3}$$

where $\tilde{\gamma}_{1(k)} = |h_{S,R_{(k)}}|^2\eta_1$, $\gamma_2 = |h_{R_{(k)},D}(t)|^2\eta_2$ with $\eta_1 = P_s/\sigma_R^2$ and $\eta_2 = P_r/\sigma_D^2$. Let $\gamma_{1(1)} \leq \gamma_{1(2)} \leq \cdots \leq \gamma_{1(N_r)}$ be the order statistics obtained by arranging $\gamma_{1(\ell)}$ for $\ell = 1, \ldots, N_r$ in an increasing order of magnitude. Note that $\tilde{\gamma}_{1(k)}$ is now the delayed version of $\gamma_{1(k)}$ by a time delay of $T_d$.

## III. Performance Analysis

### A. Outage Probability

The outage probability, $P_o$, defined as the probability that $\gamma_{eq1}$ drops below a predefined SNR threshold $\gamma_T$, is an important quality of service (QoS) measure. Therefore, to study the system's outage probability, the cumulative distribution function (cdf) of the e2e SNR is required. To this end, we define a new random variable (RV), $Z$, as

$$Z = \frac{\tilde{\gamma}_{1(k)}\gamma_2}{\tilde{\gamma}_{1(k)} + \gamma_2 + c} \tag{4}$$

where $c \geq 0$ is a constant.

The cdf of $Z$ can be written as

$$F_Z(z) = \int_0^\infty \Pr\left(\frac{x\gamma_2}{x + \gamma_2 + c} < \gamma_T\right) f_{\tilde{\gamma}_{1(k)}}(x)dx \tag{5}$$

where $\Pr(\cdot)$ denotes the probability and $f_{\tilde{\gamma}_{1(k)}}(x)$ is the probability density function of $\tilde{\gamma}_{1(k)}$. After applying some algebraic manipulations to (5), it can be shown that

$$F_Z(\gamma_T) = 1 - \int_0^\infty f_{\tilde{\gamma}_{1(k)}}(\gamma_T + x) \times \left(1 - F_{\gamma_2}\left(\gamma_T + \frac{\gamma_T^2 + c\gamma_T}{x}\right)\right) dx \tag{6}$$

where $F_{\gamma_2}(x) = 1 - e^{-x/\eta_2}$ is the cdf of $\gamma_2$. In order to simplify (6) we require an expression for $f_{\tilde{\gamma}_{1(k)}}(x)$.

According to the principles of concomitants or induced order statistics, the pdf of $\tilde{\gamma}_{1(k)}$, denoted by $f_{\tilde{\gamma}_{1(k)}}(x)$, is given by

$$f_{\tilde{\gamma}_{1(k)}}(x) = \int_0^\infty f_{\tilde{\gamma}_{1(k)}|\gamma_{1(k)}}(x|y) f_{\gamma_{1(k)}}(y)dy \tag{7}$$

where $f_{\tilde{\gamma}_{1(k)}|\gamma_{1(k)}}(x|y) = f_{\tilde{\gamma}_{1(\ell)},\gamma_{1(\ell)}}(x,y)/f_{\gamma_{1(\ell)}(y)}$ is the pdf of $\tilde{\gamma}_{1(k)}$ conditioned on $\gamma_{1(k)}$. Since $\tilde{\gamma}_{1(\ell)}$ and $\gamma_{1(\ell)}$ are two correlated exponentially distributed RVs, their joint pdf is given by

$$f_{\tilde{\gamma}_{1(\ell)},\gamma_{1(\ell)}}(x,y) = \frac{1}{(1-\rho)\eta_1^2} e^{-x+y/(1-\rho)\eta_1} I_0\left(\frac{2\sqrt{\rho xy}}{(1-\rho)\eta_1}\right) \tag{8}$$

where $I_0(x)$ is the modified Bessel function of the first kind.

The pdf $f_{\gamma_{1(k)}}(y)$ is given by $f_{\gamma_{1(k)}}(y) = N_r!/(k-1)!(N_r-k)! \; [F_{\gamma_{1(\ell)}}(y)]^{k-1}[1 - F_{\gamma_{1(\ell)}}(y)]^{N_r-k} f_{\gamma_{1(\ell)}}(y)$ where $f_{\gamma_{1(\ell)}}(y) = 1/\eta_1 e^{-y/\eta_1}$ and $F_{\gamma_{1(\ell)}}(y) = 1 - e^{-y/\eta_1}$. Following the approach in [10] and simplifying yields

$$f_{\tilde{\gamma}_{1(k)}}(x) = k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m}{\eta_1}\binom{k-1}{m} \times \frac{1}{(N_r-k+m)(1-\rho)+1} \times e^{-(N_r-k+m+1)x/((N_r-k+m)(1-\rho)+1)\eta_1}. \tag{9}$$

Now substituting (9) into (6), the integral can be solved in closed-form using [11, eq. (3.478.4)]. Therefore, $F_Z(\gamma_T)$ is given by (10) shown at the top of next page where $K_1(x)$ is the first order modified Bessel function of the second kind. Finally, the exact outage probability follows by substituting $c = 1$ in (10):

$$F_Z(\gamma_T) = 1 - 2k\binom{N_r}{k}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m} \times \sqrt{\frac{\gamma_T^2 + c\gamma_T}{(N_r-k+m+1)((N_r-k+m)(1-\rho)+1)\eta_1\eta_2}} \times e^{-(N_r-k+m+1/((N_r-k+m)(1-\rho)+1)\eta_1+1/\eta_2)\gamma_T} \times K_1\left(2\sqrt{\frac{(N_r-k+m+1)(\gamma_T^2+c\gamma_T)}{((N_r-k+m)(1-\rho)+1)\eta_1\eta_2}}\right). \tag{10}$$

### B. Outage Probability at High SNR

Although the outage probability in (10) is exact and valid for arbitrary SNRs,(10) hardly gives direct insights. For example, it is interesting to know how system and network parameters such as $N_r$, $\rho$ and SNR imbalance influence the system's outage performance. Since such insight can not be *directly* obtained from (10), we now develop a simple outage probability expression valid for high SNR.

In the asymptotic SNR regime, we have $\eta_1, \eta_2 \to \infty$. Applying a Bessel function approximation for small arguments of $x$, $0 < x \ll \sqrt{\nu+1}$, given by

$$K_\nu(x) \simeq \frac{2^{\nu-1}\Gamma(\nu)}{x^\nu} \tag{11}$$

in (10), we immediately write

$$F_{eq1}(\gamma_T) = 1 - k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m}{N_r-k+m+1}\binom{k-1}{m} \times e^{-(N_r-k+m+1/((N_r-k+m)(1-\rho)+1)\eta_1+1/\eta_2)\gamma_T}. \tag{12}$$

Let $\eta_2 = \mu\eta_1$ and $F_{eq1}(\gamma_T)$ can be re-expressed as

$$F_{eq1}(x) = 1 - k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m}{N_r-k+m+1}\binom{k-1}{m} \times e^{-(N_r-k+m+1/(N_r-k+m)(1-\rho)+1+1/\mu)x} \tag{13}$$

where $x = \gamma_T/\eta_1$. Using the McLaurin series representation for the exponential function in (13) one gets

$$F_{eq1}(x) = 1 - k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m}{N_r-k+m+1}\binom{k-1}{m} \times \sum_{p=0}^{\infty}(-1)^p\frac{\left(\frac{N_r-k+m+1}{(N_r-k+m)(1-\rho)+1}+\frac{1}{\mu}\right)^p}{p!}x^p. \quad (14)$$

Simplifying further and collecting the first order terms only yield an outage probability approximation given by

$$F_{eq1}(x) = k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m}{N_r-k+m+1}\binom{k-1}{m} \times \left(\frac{N_r-k+m+1}{(N_r-k+m)(1-\rho)+1}+\frac{1}{\mu}\right)x. \quad (15)$$

Finally, substituting $x = \gamma_T/\eta_1$, the outage probability can be written as

$$F_{eq1}(\gamma_T) = k\binom{N_r}{k}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m} \times \frac{\gamma_T}{((N_r-k+m)(1-\rho)+1)\eta_1}+\frac{\gamma_T}{\mu\eta_1}. \quad (16)$$

It follows from (16) that in the special cases of $\rho = 0$ and $\rho = 1$, the outage of best relay selection $(k = N_r)$ becomes

$$F_{eq1}(\gamma_T) = \begin{cases} \left(1+\frac{1}{\mu}\right)\frac{\gamma_T}{\eta_1} & \rho = 0, \\ \frac{\gamma_T}{\mu\eta_1} & \rho = 1. \end{cases} \quad (17)$$

### C. Average BER

We now derive expressions for the system's average BER. For many modulation formats (see below) used in wireless applications, it can be expressed as

$$P_b = \alpha E\left[Q\left(\sqrt{\beta\gamma_{eq1}}\right)\right] \quad (18)$$

where $\alpha, \beta > 0$, and $Q(x) = 1/\sqrt{2\pi}\int_x^{\infty} e^{-y^2/2}dy$ is the Gaussian $Q$-function. For binary phase shift keying (BPSK) $(\alpha, \beta) = (1, 2)$, quadrature phase shift keying (QPSK) $(\alpha, \beta) = (1, 1)$ gives the exact BER and for $M$-PSK $(\alpha, \beta) = 1/\log_2 M, \log_2 M \sin^2 \pi/M$ can be used to approximate the BER.

Using integration by parts it can be shown that (18) can be reexpressed as

$$P_b = \frac{\alpha}{\sqrt{2\pi}}\int_0^{\infty} F_{\gamma_{eq1}}\left(\frac{t^2}{\beta}\right)e^{-t^2/2}dt. \quad (19)$$

As far as we know, (7) does not have a closed-form solution. To overcome this challenge, we consider, $\gamma_{eq2} = \tilde{\gamma}_{1(k)}\gamma_2/\tilde{\gamma}_{1(k)} + \gamma_2$, since it is a tight upper bound for $\gamma_{eq1}$ in the regimes of medium-to-high SNR. We provide extensive simulation results in Section IV to complement the bounds. Thus, after substituting (10) with $c = 0$ into (7) we obtain (20), shown at the bottom of the page. The integral in (20) can be solved with the help of [11, eq. (6.621.3)], to yield (21), shown at the bottom of the page.

### D. Average BER at High SNR

Substituting (16) into (7) the average BER at high SNR can be written as

$$P_b^{\infty} = \frac{\alpha}{\sqrt{2\pi}\eta_1\beta}\left(k\binom{N_r}{k}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m} \times \frac{1}{(N_r-k+m)(1-\rho)+1}+\mu^{-1}\right) \times \int_0^{\infty} w^2 e^{-w^2/2}dw. \quad (22)$$

Simplifying the integral in (22) with the help of [11, eq. (3.381.4)] yields

$$P_b^{\infty} \approx \frac{\alpha}{2}\left(k\binom{N_r}{k}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m} \times \frac{1}{(N_r-k+m)(1-\rho)+1}+\mu^{-1}\right)(\beta\eta)^{-1}. \quad (23)$$

## IV. Numerical Results

Analytical results derived were compared with simulations done by using MATLAB. We used the Cholesky decomposition method to obtain correlated circularly Gaussian RVs [12]. By taking the square of the absolute value of those RVs, correlated

$$P_b \approx \frac{\alpha}{2} - \frac{\alpha k}{\beta}\sqrt{\frac{2}{\pi\eta_1\eta_2}}\binom{N_r}{k} \times \sum_{m=0}^{k-1}\frac{(-1)^m\binom{k-1}{m}}{\sqrt{(N_r-k+m+1)((N_r-k+m)(1-\rho)+1)}} \times \int_0^{\infty} t^2 e^{-(N_r-k+m+1/((N_r-k+m)(1-\rho)+1)\beta\eta_1+1/\beta\eta_2+1/2)t^2} \times K_1\left(\frac{2t^2}{\beta}\sqrt{\frac{(N_r-k+m+1)}{((N_r-k+m)(1-\rho)+1)\eta_1\eta_2}}\right). \quad (20)$$

$$P_b \approx \frac{\alpha}{2} - \frac{3\pi\alpha k}{\sqrt{2}\beta^2\eta_1\eta_2}\binom{N_r}{k} \times \sum_{m=0}^{k-1}\frac{(-1)^m\binom{k-1}{m}}{\left(\frac{N_r-k+m+1}{((N_r-k+m)(1-\rho)+1)\beta\eta_1}+\frac{1}{\beta\eta_2}+\frac{1}{2}+\frac{2}{\beta}\sqrt{\frac{(N_r-k+m+1)}{((N_r-k+m)(1-\rho)+1)\eta_1\eta_2}}\right)^{2.5}} \times \frac{{}_2F_1\left(\frac{5}{2},\frac{3}{2},2;\frac{\frac{N_r-k+m+1}{((N_r-k+m)(1-\rho)+1)\beta\eta_1}+\frac{1}{\beta\eta_2}+\frac{1}{2}-\frac{2}{\beta}\sqrt{\frac{(N_r-k+m+1)}{((N_r-k+m)(1-\rho)+1)\eta_1\eta_2}}}{\frac{N_r-k+m+1}{((N_r-k+m)(1-\rho)+1)\beta\eta_1}+\frac{1}{\beta\eta_2}+\frac{1}{2}+\frac{2}{\beta}\sqrt{\frac{(N_r-k+m+1)}{((N_r-k+m)(1-\rho)+1)\eta_1\eta_2}}}\right)}{((N_r-k+m)(1-\rho)+1)}. \quad (21)$$

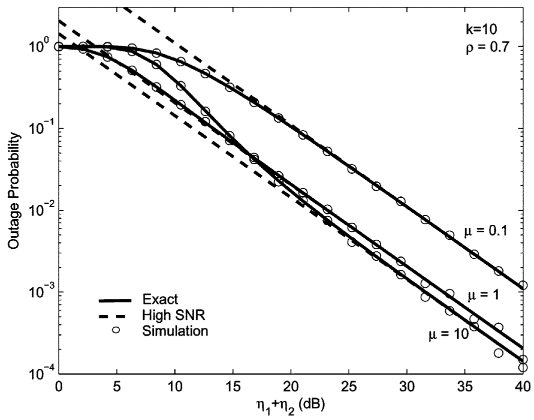


Fig. 1. Effect of average SNR imbalance on the outage probability.

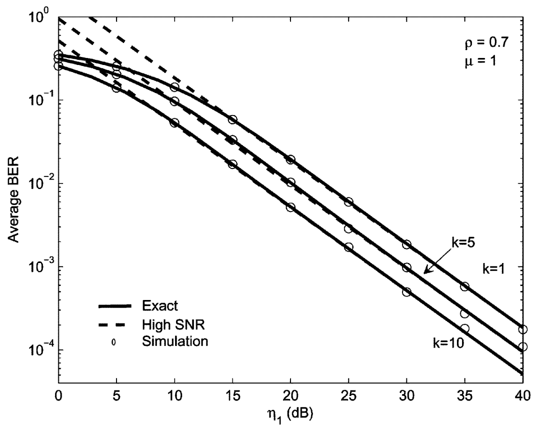


Fig. 2. Average BER of QPSK for different $k$.

exponential RVs required for the simulations were generated. In all cases $N_r = 10$. Figs. 1–3 show the influence of the rank of the relay chosen, $k$, the SNR imbalance, $\mu$, and $\rho$ on the outage probability and the average BER. In Fig. 1 we present the impact of $\mu$ on system's outage probability. We assume that $k = 10$ (best relay is chosen), $\rho = 0.7$ and $\gamma_T = 1$. Interestingly, compared to the case of $\mu = 1$, a stronger $\eta_2$ $(\mu = 10)$ increases the outage when $\eta_1 + \eta_2 < 15$ dB. Simulation results excluded to avoid clutter also showed that for large $k$ values $(k \geq 5)$, the performance is more sensitive to the average SNR of $R - D$ link, while for lower $k$ vales $(k < 5)$ the performance is more sensitive to that of $S - R$ link. This performance gap reduces as $\rho \to 0$. In Fig. 2, the average BER curves when the best $(k = 10)$, $k = 5$ and worst $(k = 1)$ relay is chosen are presented. Additionally, as $\rho \to 1$, the performance gap between the best and worse relay selection cases widens. As expected, the curves overlap as $\rho \to 0$. Finally, in Fig. 3 the effect of $\rho$ on system's average BER is illustrated. We observe that high correlation results in better performance, and the performance variation with $\rho$ reduces with $\mu$. A high $\mu$, reduces the strength of the $S - R$ link and system's performance becomes less sensitive to $\rho$.

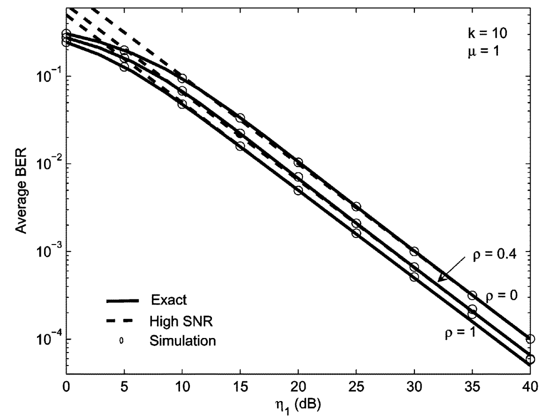


Fig. 3. Average BER of BPSK for different $\rho$.

## V. Conclusions

In this paper, we analyzed the effect of feedback delay on the network performance of partial relay selection. We derived analytic results for the outage probability and the average BER, when the $k$th worst relay is chosen by using outdated channel information. We also derived the high SNR approximations for those performance parameters. An excellent agreement between analytical and Monte Carlo simulation results was observed.

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