Receive Antenna Selection for Unitary Space-Time Modulation over Semi-Correlated Ricean Channels

Mahdi Ramezani, *Student Member, IEEE,* Mahdi Hajiaghayi, *Student Member, IEEE,* Chintha Tellambura, *Senior Member, IEEE,* and Masoud Ardakani, *Senior Member, IEEE*

***Abstract*—Receive antenna selection for unitary space-time modulation (USTM) over semi-correlated Ricean fading channels is analyzed (this work generalizes that of Ma and Tepedelenlioğlu for the independent and identically distributed (i.i.d.) Rayleigh fading case). The antenna selection rule is that the receive antennas with the largest signal powers are chosen. For single antenna selection, we derive the maximum likelihood decoding for the correlated Ricean case. We also derive the Chernoff bound on the pairwise error probability for the high signal-to-noise ratio (SNR) region and obtain the coding gain and diversity order. Our results show that even when there are transmitter side correlations and a line of sight component, receive antenna selection with USTM preserves the full diversity order if the USTM constellation is of full rank. We also give an approximation to the distribution function of a quadratic form of non-zero mean complex Gaussian variates (from Nabar *et al.*) at the high SNR region. Based on this approximation, a closed-form expression for the coding gain is also obtained and compared with that of the i.i.d. Rayleigh case. We also analyze the case of multiple receive antenna selection and derive the coding gain and diversity order. We show that USTM constellations, which have been proposed for the i.i.d. Rayleigh channel, can be used with the correlated Ricean channel as well.**

***Index Terms*—Space-time codes, unitary space-time modulation, correlated channel, Ricean channel, cumulative distribution function (CDF), Chernoff bound, antenna selection, pairwise error probability, diversity.**

I. Introduction

MULTIPLE-input multiple-output (MIMO) systems with multiple antennas at the transmit and receiver ends of a wireless link can improve reliability and capacity [1], [2]. However, as the number of antennas and/or the Doppler spread increases, channel estimation results in added complexity and a significant overhead of pilot symbols. These drawbacks may be overcome by the use of unitary space-time modulation (USTM), which does not use channel estimates, but achieves full capacity and full diversity order for fast fading MIMO channels [3]–[6].

Paper approved by C. Tepedelenlioğlu, the Editor for Transmission Systems of the IEEE Communications Society. Manuscript received December 10, 2008; revised August 26, 2009.

M. Ramezani, C. Tellambura, and M. Ardakani are with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB, Canada, T6G2V4 (e-mail: {ramezani, chintha, ardakani}@ece.ualberta.ca).

M. Hajiaghayi is with the Department of Electrical and Computer Engineering, University of Toronto, Toronto, ON M5S 3G4 Canada (e-mail: mahdih@comm.utoronto.ca).

This work was supported in part by the Natural Sciences and Engineering Research Council (NSERC) and Alberta Ingenuity Funding.

Digital Object Identifier 10.1109/TCOMM.2010.02.080664

Most USTM studies consider that all available antennas are utilized for signal transmission and reception [7]–[9]. In practice, however, each transmit-receive antenna pair requires a radio frequency (RF) chain, along with an increase in the complexity of signal processing [10]. A promising low-complexity solution is the selection of a subset of all available transmit/receive antenna pairs. Antenna selection (AS) algorithms have been designed and analyzed extensively [8], [11]–[17].

Receive antenna selection (RAS) based on a maximum-norm (power) criterion for independent and identically distributed (i.i.d.) Rayleigh fading channels was developed by Ma and Tepedelenlioğlu [18]. With this selection method, a subset of receive antennas whose received signal powers are the largest is chosen [18] and this method requires no channel state information (CSI) at the receiver. By analyzing the pairwise error probability (PEP), they showed that the diversity gain with AS in this case is preserved for unitary space-time codes with full spatial diversity, the same as for the case with the CSI at the receiver.

However, [18] considers only the i.i.d. fading case. In reality, insufficient antenna spacing, angle spread or the lack of rich scattering may cause spatial correlation among antennas, particularly at the transmit side [19], [20]. Moreover, channel measurements show that in some propagation environments, a line of sight (LOS) component is present [21], [22]. In such cases, the Ricean distribution is used to model the channel, and the mean of the channel matrix is not zero.

This paper considers semi-correlated Ricean fading channels with spatial correlation among the transmit antennas. The correlations and LOS components are long-term statistics of the channel, which do not vary during the transmission. We first investigate the selection of one receive antenna based on the maximum received signal power. The maximum likelihood (ML) detection rule based on this antenna selection criteria is derived. This rule has considerably less complexity than that of the full complexity system, where all receive antennas and all of the signal points in the constellation are searched. The Chernoff bound on the PEP is derived. We show that the proposed system achieves the full diversity order just like the full complexity system. Surprisingly, the effect of the correlation and the LOS components on the coding gain (compared to the i.i.d. Rayleigh channel [18]) can be described within a factor which depends only on the long-term statistics of the channel. Therefore, since in a full diversity system, the code design process is usually based on the maximization of the coding gain, the rich unitary codes for the i.i.d. Rayleigh channel reported in [18] can be used for the correlated Ricean fading channel as well. The results are also extended to the multiple antenna selection scenario. It is shown that the effect of the correlation and the LOS components on the coding gain for no antenna selection (utilizing all receive antennas) is described by the same factor as in the single antenna selection case.

This paper is organized as follows. In Section II, we describe the system model and present the USTM and differential USTM. In Section III, the ML detection and antenna selection rule for selecting single antenna at the receiver is discussed. Performance analysis using the Chernoff bound on the PEP is presented in Section IV. Multiple antenna selection is considered in Section V, followed by the simulation results in Section VI. We conclude the paper in Section VII.

Notation: The real part, Hermitian, transpose, trace, determinant and Frobenius norm of a matrix $\mathbf{A}$ are denoted by $\Re(\mathbf{A})$, $\mathbf{A}^\dagger$, $\mathbf{A}'$, $\mathrm{Tr}\,(\mathbf{A})$, $|\mathbf{A}|$, $\|\mathbf{A}\|_{\mathrm{F}}$, respectively. All vectors are column vectors and the all-one vector is denoted by $\mathbf{1}$. Also, a circularly symmetric complex Gaussian random vector with mean vector $\mathbf{m}$ and covariance matrix $\mathbf{\Sigma}$ is denoted by $\mathbf{x} \sim \mathcal{CN}(\mathbf{m}, \mathbf{\Sigma})$. E denotes the expectation and the set of complex numbers and nonnegative real numbers are represented by $\mathbb{C}$ and $\mathbb{R}_+$. We write a function $f(x)$ as $o(x)$ if it falls off faster than x when x goes to infinity, i.e., $\lim_{x\to\infty} f(x)/x = 0$. For an $\epsilon \in [0,1]$, $\bar{\epsilon}$ stands for $1-\epsilon$. The notation $\overset{J}{\max}_n X_n$ means the J largest X_n's. Also, we use the notation $\mathcal{I}_N(k) = \{m_1, \ldots, m_N \in \{0,1,\ldots,k\} | \sum_{i=1}^N m_i = k\}$ to simplify some indices. The cardinality of $\mathcal{I}_N(k)$ is $|\mathcal{I}_N(k)| = \binom{N+k-1}{N-1}$.

II. System Model and the USTM Scheme

We consider a MIMO system with M transmit and N receive antennas operating over flat Ricean-fading channels. The channel coefficient h_{ij} between the ith transmit antenna and the jth receive antenna is assumed to be constant for T ($T > M$) symbol periods. The received matrix signal is [3]

$$\mathbf{Y} = \sqrt{\frac{\rho}{M}}\mathbf{S}\mathbf{H} + \mathbf{W} \tag{1}$$

where $\mathbf{Y} = [\mathbf{y}_1, \ldots, \mathbf{y}_N]$ is a $T \times N$ complex received signal matrix, $\mathbf{S}$ is a $T \times M$ complex transmitted signal matrix, $\mathbf{H}$ is an $M \times N$ channel matrix, and $\mathbf{W}$ denotes a $T \times N$ additive noise matrix with i.i.d. $\mathcal{CN}(0,1)$ elements.

Let $s_{t,i}$ be the data symbol transmitted from the ith antenna at time t. At each time slot $t = 1, 2, \ldots, T$, the transmitted signal is normalized to have a unit average power over the M transmit antennas, i.e., $\frac{1}{M}\sum_{i=1}^M E|s_{t,i}|^2 = 1$, so that ρ is the average SNR at each receive antenna, regardless of the number of transmit antennas.

The channel model $\mathbf{H}$ is assumed to be correlated Ricean, i.e., it consists of a fixed component and a random component. The channel may be represented as [22], [23]

$$\mathbf{H} = \sqrt{\frac{K}{K+1}}\bar{\mathbf{H}} + \sqrt{\frac{1}{K+1}}\mathbf{R}_T^{1/2}\mathbf{H}_w \tag{2}$$

where the first and second terms are the mean (LOS) and the diffuse of the communication channel, respectively. The Ricean factor K indicates the relative strength of the LOS component over the diffuse component, indicating the link quality [24]. Also, the $M \times N$ matrix $\mathbf{H}_w$ has i.i.d. $\mathcal{CN}(0,1)$ elements. The $M \times M$ positive Hermitian matrix $\mathbf{R}_T$ denotes the spatial transmit correlation with all diagonal entries 1. In (2), $\bar{\mathbf{H}}$, K and $\mathbf{R}_T$ are long-term statistics of the channel known to the receiver and remain fixed for each period of signal transmission. All N receive antennas experience the same LOS component, i.e., all columns of $\bar{\mathbf{H}}$ are identical and denoted by $\bar{\mathbf{h}}$, hereafter.

A. Correlation models

The transmit correlation matrix can take several forms. The *exponential correlation* model [25] is often used to describe the correlation coefficients among antennas and may hold for the practical case of an equi-spaced linear array of antennas. The correlation matrix and corresponding (distinct) eigenvalues of this model are given by [25], [26]

$$[\mathbf{R}_T]_{ij} = r^{|i-j|}, \quad |r| \le 1, \tag{3}$$

and $\lambda_i = (1-r^2)/(1-2r\cos\theta_i + r^2)$, $i = 1, 2, \ldots, M$, respectively, where r is the correlation coefficient of the neighboring antennas and θ_is are the solutions of $\sin\left(\frac{M+1}{2}\theta\right) = r\sin\left(\frac{M-1}{2}\theta\right)$ and $\cos\left(\frac{M+1}{2}\theta\right) = r\cos\left(\frac{M-1}{2}\theta\right)$.

Constant correlation matrix is another practical model that is frequently used for an array of three antennas placed on an equilateral triangle or for closely spaced antennas [27]. The correlation matrix is

$$[\mathbf{R}_T]_{ij} = \begin{cases} 1 & i = j \\ r & i \ne j. \end{cases}$$

$\mathbf{R}_T$ has only two eigenvalues $\lambda_1 = 1 + r(M-1)$ and $\lambda_2 = 1 - r$ of order one and $M-1$, respectively.

Our analysis holds for an arbitrary correlation matrix. The above correlation models are used only for simulation purposes.

B. USTM constellation

The USTM constellation $\mathcal{V}$ contains L unitary signals as [3]

$$\mathcal{V} = \{\mathbf{S}_\ell,\ \ell \in \{0,1,\ldots,L-1\} | \mathbf{S}_\ell = \sqrt{T}\mathbf{\Phi}_\ell,\ \mathbf{\Phi}_\ell^\dagger\mathbf{\Phi}_\ell = \mathbf{I}_M\},$$

where the $T \times M$ unitary matrix $\mathbf{\Phi}_\ell$ has M orthonormal columns. To transmit a data sequence of integers $z_1, z_2, \ldots$ with $z_\tau \in \{0, 1, \ldots, L-1\}$, each z_τ is mapped to a distinct unitary matrix $\mathbf{\Phi}_{z_\tau}$ and $\mathbf{S}_\tau = \sqrt{T}\mathbf{\Phi}_{z_\tau}$ from the constellation $\mathcal{V}$ is transmitted. The rate of the constellation $\mathcal{V}$ is $R = \frac{1}{T}\log_2 L$ bits per channel use. Finally, note that at high SNR, USTM is capacity achieving provided that $T > M$ [3].

In differential USTM [5], a unitary codebook of $M \times M$ matrices $\{\mathbf{V}_0, \ldots, \mathbf{V}_{L-1}\}$ is used. Each z_τ is mapped to the corresponding $\mathbf{V}_{z_\tau}$. The transmitted signal used in (1) will be of the form $\mathbf{S} = [\mathbf{S}_{\tau-1}'\ \mathbf{S}_\tau']'$ where $\mathbf{S}_\tau = \mathbf{V}_{z_\tau}\mathbf{S}_{\tau-1}$, $\tau = 1, 2, \ldots$, and $\mathbf{S}_0 = \mathbf{I}_M$. From [5], differential USTM can be viewed as a special case of general USTM by defining an equivalent $T \times M$ unitary matrix $\mathbf{\Phi}_{z_\tau}$ of the form $\mathbf{\Phi}_{z_\tau} = \frac{1}{\sqrt{2}}[\mathbf{I}_M \mathbf{V}_{z_\tau}']'$, where $T = 2M$. As a result, only USTM is discussed in the remained of this paper.

III. Antenna Selection and ML decoding

We first consider the case in which only one receive antenna ($J=1$), say the $\hat{n}$th antenna, is selected. By using the system model (1), for the transmission of $\mathbf{S}_\ell \in \mathcal{V}$, the mean of the received signal at the selected antenna can be written as

$$\bar{\mathbf{y}} = \sqrt{\eta K}\boldsymbol{\Phi}_\ell \bar{\mathbf{h}}, \tag{4}$$

where $\eta = \frac{\rho T}{M(K+1)}$. The use of η instead of ρ ensures notational brevity.

The received signal at the selected antenna is $\mathbf{y}_{\hat{n}} \sim \mathcal{CN}(\bar{\mathbf{y}}, \mathbf{R}_\ell)$ where $\mathbf{R}_\ell$ is a $T \times T$ matrix indicating the receive covariance matrix. The probability density function of $\mathbf{y}_{\hat{n}}$ conditioned on the transmission of $\mathbf{S}_\ell = \sqrt{T}\boldsymbol{\Phi}_\ell$ is

$$p_{\mathbf{y}_{\hat{n}}}(\mathbf{y}|\boldsymbol{\Phi}_\ell) = \frac{1}{\pi^T |\mathbf{R}_\ell|} \exp\left(-(\mathbf{y}-\bar{\mathbf{y}}_\ell)^\dagger \mathbf{R}_\ell^{-1}(\mathbf{y}-\bar{\mathbf{y}}_\ell)\right). \tag{5}$$

For single antenna selection, the optimal selection rule coupled with the maximum a posteriori (MAP) decoding rule is

$$\hat{n} = \arg\max_{n=1,2,\ldots,N} \max_{\mathbf{S}_\ell \in \mathcal{V}} p_{\mathbf{y}_n}(\mathbf{S}_\ell|\mathbf{y}), \tag{6}$$

which has high complexity because it needs the receiver to search over all receive antennas and all signals $\mathcal{V}$ [18]. As a result, a simple antenna selection rule is called for. The simple maximum-power selection rule, where the antenna with the largest received signal norm is chosen [18], fits the bill. This simple rule eliminates the need for the full search required in (6). Since the antenna whose received power is the largest among all receive antennas is selected [18], we find

$$\hat{n} = \arg\max_{n=1,2,\ldots,N} \|\mathbf{y}_n\|^2. \tag{7}$$

This rule can be implemented by the use of simple analog circuits before the analog/digital converter at the receiver [18].

In Section IV, we will show that by using the maximum-norm criteria over the correlated Ricean channels, the full diversity can be achieved.

A. Decoding rule

The maximum-norm antenna selection rule (7) is used throughout this paper. Once a receive antenna is selected, the output of the selected antenna is used by the receiver for signal detection. The resulting ML decoding rule for the selected antenna is given by

$$\boldsymbol{\Phi}_{\mathrm{ML}} = \arg\max_{\boldsymbol{\Phi}_\ell \in \mathcal{V}} p_{\mathbf{y}_{\hat{n}}}(\mathbf{y}|\boldsymbol{\Phi}_\ell)$$

where $\boldsymbol{\Phi}_\ell$ corresponds to $\mathbf{S}_\ell$. In order to expand and simplify this decoding rule, we need several properties of the receive correlation matrix that appears in the pdf (5).

By using the received signal model (1), the receive covariance matrix can be written as

$$\mathbf{R}_\ell = \mathbf{I}_T + \eta \boldsymbol{\Phi}_\ell \mathbf{R}_T \boldsymbol{\Phi}_\ell^\dagger. \tag{8}$$

This result is due to the i.i.d. columns of the received signal matrix $\mathbf{Y}$. From (8), it is clear that the receive covariance matrix $\mathbf{R}_\ell$ is full rank and has T nonzero eigenvalues; i.e., $T-M$ eigenvalues are unity and the rest are $\{1+\eta\lambda_i\}_{i=1}^M$, where λ_i's are the eigenvalues of $\mathbf{R}_T$. The determinant of $|\mathbf{R}_\ell|$ is then given by

$$|\mathbf{R}_\ell| = \prod_{i=1}^M (1+\eta\lambda_i). \tag{9}$$

This result shows that the determinant of the receive correlation matrix is independent of the transmit signal matrix $\boldsymbol{\Phi}_\ell$.

Let the eigenvalue decomposition of the spatial correlation matrix be $\mathbf{R}_T = \mathbf{U}\mathbf{D}\mathbf{U}^\dagger$, where $\mathbf{D} = \mathbf{diag}\{\lambda_i\}_{i=1}^M$, and $\mathbf{U}$ is unitary. Define

$$\boldsymbol{\Upsilon} = \mathbf{I}_M - \left(\mathbf{I}_M + \eta \mathbf{R}_T\right)^{-1}.$$

By using Woodbury's identity, we find

$$\begin{aligned}
\mathbf{R}_\ell^{-1} &= \mathbf{I}_T - \boldsymbol{\Phi}_\ell \left[\mathbf{I}_M + (\eta\mathbf{R}_T)^{-1}\right]^{-1} \boldsymbol{\Phi}_\ell^\dagger \\
&= \mathbf{I}_T - \boldsymbol{\Phi}_\ell \mathbf{U}\left[\mathbf{I}_M + (\eta\mathbf{D})^{-1}\right]^{-1} \mathbf{U}^\dagger \boldsymbol{\Phi}_\ell^\dagger \\
&= \mathbf{I}_T - \boldsymbol{\Phi}_\ell \boldsymbol{\Phi}_\ell^\dagger + \boldsymbol{\Phi}_\ell \left(\mathbf{I}_M + \eta \mathbf{R}_T\right)^{-1} \boldsymbol{\Phi}_\ell^\dagger \\
&= \mathbf{I}_T - \boldsymbol{\Phi}_\ell \boldsymbol{\Upsilon} \boldsymbol{\Phi}_\ell^\dagger.
\end{aligned} \tag{10}$$

Note that the eigenvalues of $\mathbf{R}_\ell$ and thus $|\mathbf{R}_\ell|$ are independent of the transmitted signal $\boldsymbol{\Phi}_\ell$. Therefore, the determinant (5) remains the same for all elements of the USTM signal constellation. Using (10), we can simplify the ML decoding rule as

$$\begin{aligned}
\boldsymbol{\Phi}_{\mathrm{ML}} &= \arg\min_{\boldsymbol{\Phi}_\ell \in \mathcal{V}} \left\{\mathbf{y}^\dagger \mathbf{R}_\ell^{-1}\mathbf{y} - 2\Re\{\mathbf{y}^\dagger \mathbf{R}_\ell^{-1} \bar{\mathbf{y}}_\ell\} + \bar{\mathbf{y}}_\ell^\dagger \mathbf{R}_\ell^{-1} \bar{\mathbf{y}}_\ell\right\} \\
&= \arg\max_{\boldsymbol{\Phi}_\ell \in \mathcal{V}} \left\{\|\mathbf{y}^\dagger \boldsymbol{\Phi}_\ell \boldsymbol{\Upsilon}^{\frac{1}{2}}\|^2 + 2\sqrt{\eta K}\Re\{\mathbf{y}^\dagger \boldsymbol{\Phi}_\ell (\mathbf{I}_M - \boldsymbol{\Upsilon})\bar{\mathbf{h}}\}\right\}
\end{aligned} \tag{11}$$

where the first term corresponds to the ML detection rule for the USTM with RAS for i.i.d. Rayleigh channel ($K=0$ and $\mathbf{R}_T = \mathbf{I}_M$) [18], i.e.,

$$\boldsymbol{\Phi}_{\mathrm{ML}} = \arg\max_{\boldsymbol{\Phi}_\ell \in \mathcal{V}} \|\mathbf{y}^\dagger \boldsymbol{\Phi}_\ell\|.$$

Note that in order to extract the most likely transmitted signal using (11), the LOS component $\bar{\mathbf{h}}$, the K factor, the transmit correlation matrix $\mathbf{R}_T$, and average SNR ρ (hence η and Υ), which are in the channel model and assumed to be fixed over a long time, should be known at the receiver. Because of the size of the search space, the detection rule of (11) has considerably lower complexity compared to the full complexity detection given in (6).

IV. Performance Analysis

We now evaluate the performance of the USTM with RAS for the correlated Ricean fading channels in terms of the Chernoff bound on the pairwise error probability (PEP). First, we derive the diversity order and coding gain of the system. Then, the cumulative distribution function (cdf) of the instantaneous power at the selected antenna, which is needed for performance analysis, will be derived.

A. Coding gain and diversity order derivation using the Chernoff bound

By using the theory of order statistics [28], the Chernoff bound on the PEP of mistaking $\mathbf{\Phi}_\ell$ for $\mathbf{\Phi}_{\ell'}$ based on the selection of the antenna with the maximum instantaneous power is expressed as [18], [29]:

$$P_{\text{CB}}(\mu) = \frac{N}{2}\int_{\mathbb{C}^T} \frac{F^{N-1}(\mathbf{y}^\dagger\mathbf{y})e^{-\xi(\mu|\mathbf{y})}}{|\pi\mathbf{R}_\ell|^{\bar{\mu}}|\pi\mathbf{R}_{\ell'}|^{\mu}}\, d\mathbf{y}$$

where $\mu \in (0,1)$ is a free parameter that is chosen to minimize $P_{\text{CB}}(\mu)$ and $\xi(\mu|\mathbf{y})$ is

$$\mu(\mathbf{y}-\bar{\mathbf{y}}_{\ell'})^\dagger\mathbf{R}_{\ell'}^{-1}(\mathbf{y}-\bar{\mathbf{y}}_{\ell'}) + \bar{\mu}(\mathbf{y}-\bar{\mathbf{y}}_{\ell})^\dagger\mathbf{R}_{\ell}^{-1}(\mathbf{y}-\bar{\mathbf{y}}_{\ell}).$$

Also, $F(\cdot)$ denotes the cdf of $\|\mathbf{y}\|^2$, and $\mathbf{R}_\ell$ and $\mathbf{R}_{\ell'}$ are receive covariance matrices conditioned on the transmission of $\mathbf{\Phi}_\ell$ and $\mathbf{\Phi}_{\ell'}$, respectively.

By (4) and (10), one can expand $\xi(\mu|\mathbf{y})$ and obtain

$$\xi(\mu|\mathbf{y}) = \mathbf{y}^\dagger\mathbf{\Omega}(\mu,\rho)\mathbf{y} - 2\Re\{\mathbf{y}^\dagger\mathbf{\Xi}(\mu,\rho)\} + \Delta(\rho)$$

where

$$\begin{aligned}\mathbf{\Omega}(\mu,\rho) &= \mu\mathbf{R}_{\ell'}^{-1} + \bar{\mu}\mathbf{R}_{\ell}^{-1}\\ &= \mathbf{I}_T - \mu\mathbf{\Phi}_{\ell'}\mathbf{\Upsilon}\mathbf{\Phi}_{\ell'}^\dagger - \bar{\mu}\mathbf{\Phi}_{\ell}\mathbf{\Upsilon}\mathbf{\Phi}_{\ell}^\dagger,\\ \mathbf{\Xi}(\mu,\rho) &= \mu\mathbf{R}_{\ell'}\bar{\mathbf{y}}_{\ell'}^\dagger + \bar{\mu}\mathbf{R}_{\ell}\bar{\mathbf{y}}_{\ell}^\dagger\\ &= \frac{\sqrt{\eta K}}{1+\eta}\big(\mu\mathbf{\Phi}_{\ell'} + \bar{\mu}\mathbf{\Phi}_{\ell}\big)\bar{\mathbf{h}}, \qquad (12)\\ \Delta(\rho) &= \mu\bar{\mathbf{y}}_{\ell'}^\dagger\mathbf{R}_{\ell'}^{-1}\bar{\mathbf{y}}_{\ell'} + \bar{\mu}\bar{\mathbf{y}}_{\ell}^\dagger\mathbf{R}_{\ell}^{-1}\bar{\mathbf{y}}_{\ell}\\ &= \eta K\bar{\mathbf{h}}^\dagger\big(\mathbf{I}_M + \eta\mathbf{R}_T\big)^{-1}\bar{\mathbf{h}}.\end{aligned}$$

In order to obtain the diversity order and coding gain, the Chernoff bound must be further simplified. At the high SNR region, the Chernoff bound on the PEP may be approximated as

$$P_{\text{CB}} = (G_c\rho)^{-G_d} + o(\rho^{-G_d}), \quad \text{as } \rho\to\infty \qquad (13)$$

where G_d and G_c are the diversity order and coding gain, respectively.

From (12), it is clear that $\lim_{\rho\to\infty}\mathbf{\Xi}(\mu,\rho) = 0$. Let $\mathbf{\Omega}(\mu) = \lim_{\rho\to\infty}\mathbf{\Omega}(\mu,\rho)$ and $\Delta = \lim_{\rho\to\infty}\Delta(\rho)$. At high SNR, one gets

$$\mathbf{\Omega}(\mu) = \mathbf{I}_T - \mu\mathbf{\Phi}_{\ell'}\mathbf{\Phi}_{\ell'}^\dagger - \bar{\mu}\mathbf{\Phi}_{\ell}\mathbf{\Phi}_{\ell}^\dagger, \qquad (14a)$$

$$\Delta = K\bar{\mathbf{h}}^\dagger\mathbf{R}_T^{-1}\bar{\mathbf{h}}. \qquad (14b)$$

Consider codes that can achieve full spatial diversity. Such codes satisfy [3] for $\forall\ell\neq\ell'$,

$$\text{rank}\{\mathbf{I}_M - \mathbf{\Phi}_\ell^\dagger\mathbf{\Phi}_{\ell'}\mathbf{\Phi}_{\ell'}^\dagger\mathbf{\Phi}_\ell\} = M. \qquad (15)$$

Considering the singular values of $\mathbf{\Phi}_\ell^\dagger\mathbf{\Phi}_{\ell'}$ as $\sigma\big(\mathbf{\Phi}_\ell^\dagger\mathbf{\Phi}_{\ell'}\big) = \{d_m\}_{m=1}^M$, one can see that (15) results in $d_m \in [0,1)$, $m = 1,2,\ldots,M$.

Now, we prove that $\mathbf{\Omega}(\mu)$ given in (14a) is full rank for any $\mu\in(0,1)$, suggesting that the eigenvalues of $\mathbf{\Omega}(\mu,\rho)$ are all nonzero and tend to eigenvalues of $\mathbf{\Omega}(\mu)$ at high SNR.

Let $\mathbf{\Phi} = [\sqrt{\mu}\mathbf{\Phi}_{\ell'}\ \sqrt{\bar{\mu}}\mathbf{\Phi}_\ell]$ be a $T\times 2M$ matrix. Note that $\mathbf{\Omega}(\mu) = \mathbf{I}_T - \mathbf{\Phi}\mathbf{\Phi}^\dagger$, and

$$\mathbf{\Phi}^\dagger\mathbf{\Phi} = \begin{bmatrix}\mu\mathbf{I}_M & \sqrt{\mu\bar{\mu}}\mathbf{\Phi}_{\ell'}^\dagger\mathbf{\Phi}_\ell\\ \sqrt{\mu\bar{\mu}}\mathbf{\Phi}_\ell^\dagger\mathbf{\Phi}_{\ell'} & \bar{\mu}\mathbf{I}_M\end{bmatrix}.$$

It is true that

$$|\mathbf{I}_T - \mathbf{\Phi}\mathbf{\Phi}^\dagger| = |\mathbf{I}_{2M} - \mathbf{\Phi}^\dagger\mathbf{\Phi}|.$$

Thus, in order to obtain $|\mathbf{\Omega}(\mu)|$, it is enough to find the eigenvalues of $\mathbf{\Phi}^\dagger\mathbf{\Phi}$, i.e., $\lambda(\mathbf{\Phi}^\dagger\mathbf{\Phi})$. Using the block matrix determinant lemma which is

$$\begin{vmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{C} & \mathbf{D}\end{vmatrix} = |\mathbf{A}||\mathbf{D}-\mathbf{C}\mathbf{A}^{-1}\mathbf{B}|,$$

provided that $\mathbf{A}$ is invertible, we have

$$\begin{aligned}&\big|\mathbf{\Phi}^\dagger\mathbf{\Phi} - \lambda\mathbf{I}_{2M}\big|\\ &= (\mu-\lambda)^M\Big|(\bar{\mu}-\lambda)\mathbf{I}_M - \frac{\mu\bar{\mu}}{\mu-\lambda}\mathbf{\Phi}_\ell^\dagger\mathbf{\Phi}_{\ell'}\mathbf{\Phi}_{\ell'}^\dagger\mathbf{\Phi}_\ell\Big|\\ &= \Big|(\mu-\lambda)(\bar{\mu}-\lambda)\mathbf{I}_M - \mu\bar{\mu}\,\mathbf{diag}\{d_m^2\}_{m=1}^M\Big| = 0\end{aligned}$$

and hence,

$$\lambda(\mathbf{\Phi}^\dagger\mathbf{\Phi}) = \frac{1}{2}\Big(1\pm\sqrt{1-4\mu\bar{\mu}(1-d_m^2)}\Big),\ m = 1,2,\ldots,M.$$

Finally, we arrive at

$$\begin{aligned}|\mathbf{\Omega}(\mu)| &= |\mathbf{I}_{2M} - \mathbf{\Phi}^\dagger\mathbf{\Phi}|\\ &= \prod_{m=1}^{2M}\big(1-\lambda(\mathbf{\Phi}^\dagger\mathbf{\Phi})\big) = \prod_{m=1}^{M}\mu\bar{\mu}(1-d_m^2). \qquad (16)\end{aligned}$$

That is, for any $\mu\in(0,1)$, $\mathbf{\Omega}(\mu)$ is full rank. Thus, the eigenvalues of $\mathbf{\Omega}(\mu,\rho)$ are all nonzero and tend to eigenvalues of $\mathbf{\Omega}(\mu)$ at high SNR.

As shown in the Appendix, at high SNR $F(x)$ can be written as

$$F(x) = \eta^{-M}\Psi(x) + o(\rho^{-M}) \qquad (17)$$

where M is the number of transmit antennas and $\Psi(x)$ is a function depending on the channel model. Using the Lebesgue's dominated convergence theorem [30], at high SNR the Chernoff bound on the PEP becomes

$$P_{\text{CB}}(\mu) = \frac{\eta^{-MN}Ne^{-\Delta}}{2\pi^T|\mathbf{R}_T|}\int_{\mathbb{C}^T} e^{-\mathbf{y}^\dagger\mathbf{\Omega}(\mu)\mathbf{y}}\Psi^{N-1}(\mathbf{y}^\dagger\mathbf{y})\, d\mathbf{y} + o(\rho^{-MN})$$

where from (9), we know that $|\mathbf{R}_\ell|$ does not depend on the transmitted $\mathbf{\Phi}_\ell$, and at high SNR $|\mathbf{R}_\ell| \to \eta^M|\mathbf{R}_T|$. Now, consider the eigenvalue decomposition of $\mathbf{\Omega}(\mu)$ as $\mathbf{Q}\,\mathbf{diag}\{\sigma_i\}_{i=1}^T\mathbf{Q}^\dagger$. Let $x_i = |v_i|^2$, $i = 1,2,\ldots,T$, where v_i is the ith element of $\mathbf{yQ}$. Since for an entire function $f(s)$

$$\frac{1}{\pi}\int_{\mathbb{C}} f(|s|^2)ds = \int_0^\infty f(t)dt,$$

we can see that

$$\frac{1}{\pi^T}\int_{\mathbb{C}^T} e^{-\mathbf{y}^\dagger\mathbf{\Omega}(\mu)\mathbf{y}}\Psi^{N-1}(\mathbf{y}^\dagger\mathbf{y})\, d\mathbf{y} = \int_{\mathbb{R}_+^T} e^{-\mathbf{\Sigma}'\mathbf{x}}\Psi^{N-1}(\mathbf{1}'\mathbf{x})\, d\mathbf{x}$$

where $\mathbf{\Sigma}$ is a column vector of the eigenvalues of $\mathbf{\Omega}(\mu)$.

Finally, the Chernoff bound is obtained as

$$P_{\text{CB}}(\mu) = \frac{\eta^{-MN}Ne^{-\Delta}}{2|\mathbf{R}_T|}\int_{\mathbb{R}_+^T} e^{-\mathbf{\Sigma}'\mathbf{x}}\Psi^{N-1}(\mathbf{1}'\mathbf{x})\, d\mathbf{x} + o(\rho^{-MN}) \qquad (18)$$

which, according to (13), gives the full diversity order and coding gain as

$$G_c = \frac{T}{M(K+1)} \left(\frac{Ne^{-\Delta}}{2|\mathbf{R}_T|} \int_{\mathbb{R}_+^T} e^{-\mathbf{\Sigma}'\mathbf{x}} \Psi^{N-1}(\mathbf{1}'\mathbf{x})\, d\mathbf{x} \right)^{-1/G_d}. \tag{19}$$

Therefore, similar to the full complexity system, USTM with the simple maximum-norm RAS criteria achieves full diversity order over the semi-correlated Ricean fading channels. Thus, the good news is that RAS in this case achieves the full diversity order while reducing the overall complexity. It is worth noticing that the union bound on the probability of error would be

$$P_{\text{UB}} = \inf_{\mu \in [0,1]} \frac{1}{L} \sum_{\ell=0}^{L-1} \sum_{\substack{\ell'=0 \\ \ell' \neq \ell}}^{L-1} P_{\text{CB}}(\mu)$$

which means that the full diversity is preserved.

The i.i.d. Rayleigh case where $\mathbf{R}_T = \mathbf{I}_M$ and $K = 0$ is well studied in [18]. Surprisingly, $\Psi(x)$ given by Theorem 3 in the Appendix is the same as in the i.i.d. Rayleigh channel case, except for a multiplicative factor ($\hat{f}_0$, see the Appendix), which only depends on the LOS component and the transmit spatial correlation. Let $\Psi_R(x)$ be the corresponding $\Psi(x)$ function for the i.i.d. Rayleigh case; i.e., $\Psi_R(x) = \Psi(x)/\hat{f}_0$. The coding gains for the i.i.d. Rayleigh channel given in [18] and for the correlated Ricean channel given in (19) are

$$G_c^R = \frac{T}{M} \left(\frac{N}{2} \int_{\mathbb{R}_+^T} e^{-\mathbf{\Sigma}'\mathbf{x}} \Psi_R^{N-1}(\mathbf{1}'\mathbf{x})\, d\mathbf{x} \right)^{-1/G_d}$$

and

$$G_c = \frac{T}{M(K+1)} \left(\frac{N\hat{f}_0}{2} \int_{\mathbb{R}_+^T} e^{-\mathbf{\Sigma}'\mathbf{x}} \Psi^{N-1}(\mathbf{1}'\mathbf{x})\, d\mathbf{x} \right)^{-1/G_d},$$

respectively. Therefore, the ratio of the coding gains taking the correlation and LOS components into account can be expressed as

$$\begin{aligned} \Gamma = \frac{G_c}{G_c^R} &= \frac{\left(\hat{f}_0^N\right)^{-1/G_d}}{K+1} \\ &= \frac{1}{K+1} \exp\left(\frac{K}{M} \bar{\mathbf{h}}^\dagger \mathbf{R}_T^{-1} \bar{\mathbf{h}} \right) |\mathbf{R}_T|^{1/M} \end{aligned} \tag{20}$$

where M is the number of transmit antennas. By setting $K = 0$ (correlated Rayleigh) and/or $\mathbf{R}_T = \mathbf{I}_M$ (Ricean), one can see the effect of the correlation and/or LOS component on the coding gain. In Section VI, the numerical evaluations of Γ will be given.

V. Multiple Antenna Selection

This section develops performance analysis of multiple-receive-antenna selection. We may select J antennas, denoted by $\hat{n}_1, \ldots, \hat{n}_J$, whose received-signal norms are the largest [18]; i.e.,

$$[\hat{n}_1, \ldots, \hat{n}_J] = \arg \max_{n=1,2,\ldots,N}^{J} \|\mathbf{y}_n\|^2.$$

We can show that the ML decoder will be the same as (11) by using a $\text{Tr}(\cdot)$ operator and replacing the norm $\|\cdot\|$, the vectors $\mathbf{y}$, and $\bar{\mathbf{h}}$ with the Frobenius norm $\|\cdot\|_{\text{F}}$, $\hat{\mathbf{Y}} = [\mathbf{y}_{\hat{n}_1}, \ldots, \mathbf{y}_{\hat{n}_J}]$, and $\bar{\mathbf{H}}_J = [\bar{\mathbf{h}}_{\hat{n}_1}, \ldots, \bar{\mathbf{h}}_{\hat{n}_J}]$, respectively. Therefore, the ML decoding rule is

$$\begin{aligned} \mathbf{\Phi}_{\text{ML}} = \arg \max_{\mathbf{\Phi}_\ell \in \mathcal{V}} \Big\{ &\|\hat{\mathbf{Y}}^\dagger \mathbf{\Phi}_\ell \mathbf{\Upsilon}^{\frac{1}{2}}\|_{\text{F}}^2 + 2\sqrt{\eta K} \\ &\times \text{Tr}\left\{ \Re\{ \hat{\mathbf{Y}}^\dagger \mathbf{\Phi}_\ell (\mathbf{I}_M - \mathbf{\Upsilon}) \bar{\mathbf{H}}_J \} \right\} \Big\}. \end{aligned}$$

In order to calculate the Chernoff bound on the PEP, we use the order statistics theory [28] to find the cdf of the J largest instantaneous power at the receive antennas. Unlike what we did with the case of single antenna selection, we now have to confine the integration range to the region in which the order of selected antenna is preserved [18]. For the sake of simplicity, we can still integrate over the whole space and get a looser bound on the PEP as in [18]. In the case of multiple antenna selection, we have to replace Δ in the single antenna case by

$$\Delta_J = K \sum_{i=1}^{J} \bar{\mathbf{h}}_i^\dagger \mathbf{R}_T^{-1} \bar{\mathbf{h}}_i = J\Delta$$

where the last equality results because the columns of $\bar{\mathbf{H}}$ are all identical.

Therefore, from (17) and [18], we have

$$\begin{aligned} P_{\text{CB}}(\mu) \leq \eta^{-MN} &\frac{\binom{N}{J} e^{-J\Delta}}{2J\pi^{TJ} |\mathbf{R}_T|^J} \int_{\mathbb{C}^{TJ}} e^{-\sum_{i=1}^{J} \mathbf{y}_i^\dagger \mathbf{\Omega}(\mu) \mathbf{y}_i} \\ &\times \sum_{i=1}^{J} \Psi^{N-J}(\|\mathbf{y}_i\|^2)\, d\mathbf{Y} + o(\rho^{-MN}). \end{aligned} \tag{21}$$

which shows that we can achieve the full diversity order as we did in the full complexity system. In (21), we implicitly assumed that $d\mathbf{Y} = \prod_{i=1}^{J} d\mathbf{y}_i$. We can see that for $J = 1$, the bound in (21) gets tight and coincides with (18).

Also, forcing $J = N$ tightens the bound and is equivalent to the Chernoff bound on the PEP for the full complexity system, i.e.,

$$\begin{aligned} P_{\text{CB}}(\mu) &= \eta^{-MN} \frac{e^{-N\Delta}}{2|\mathbf{R}_T|^N |\mathbf{\Omega}(\mu)|^N} + o(\rho^{-MN}) \\ &= \frac{1}{2} \eta^{-MN} \hat{f}_0^N \left(\prod_{m=1}^{M} \mu\bar{\mu}(1 - d_m^2) \right)^{-N} + o(\rho^{-MN}) \end{aligned}$$

where $\hat{f}_0$ is defined in Theorem 2, and the last equality results from (16). In order to get the minimum of the Chernoff bound, we should choose $\mu = \frac{1}{2}$, since the term $\mu\bar{\mu}$ appears in the denominator. Moreover, from [3, Theorem 5], for the full complexity system over the i.i.d. Rayleigh channel, we have

$$\begin{aligned} P_{\text{CB}}^R &= \frac{1}{2} \prod_{m=1}^{M} \left(1 + \frac{(\rho T/M)^2 (1 - d_m^2)}{4(1 + \rho T/M)} \right)^{-N} \\ &= \frac{1}{2} \left(\frac{\rho T}{M} \right)^{-MN} \left(\prod_{m=1}^{M} \frac{1}{4}(1 - d_m^2) \right)^{-N} + o(\rho^{-MN}). \end{aligned}$$

Similar to how the coding gain ratio was defined in (20), the same ratio can be defined as

$$\Gamma_{\text{full}} = \frac{G_c^{\text{full}}}{G_c^{R,\text{full}}} = \frac{\left(\hat{f}_0^N\right)^{-1/G_d}}{K+1} = \Gamma$$

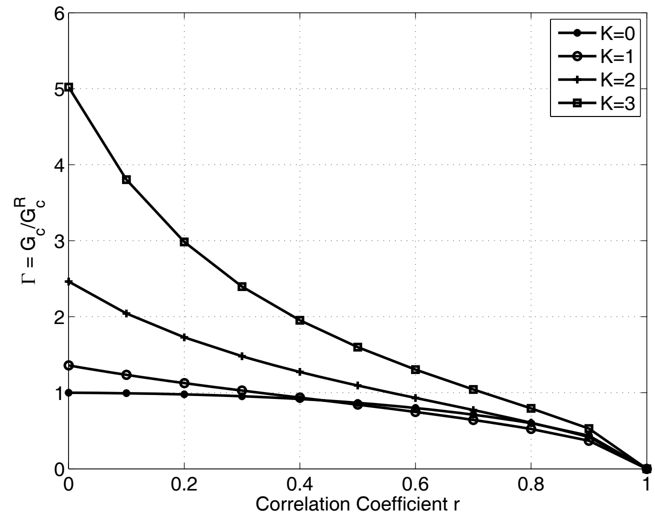

Fig. 1. The ratio of the coding gains for a 2×2 system over a correlated Ricean channel (constant correlation and an all–one $\bar{\mathbf{H}}$) and the i.i.d. Rayleigh channel.

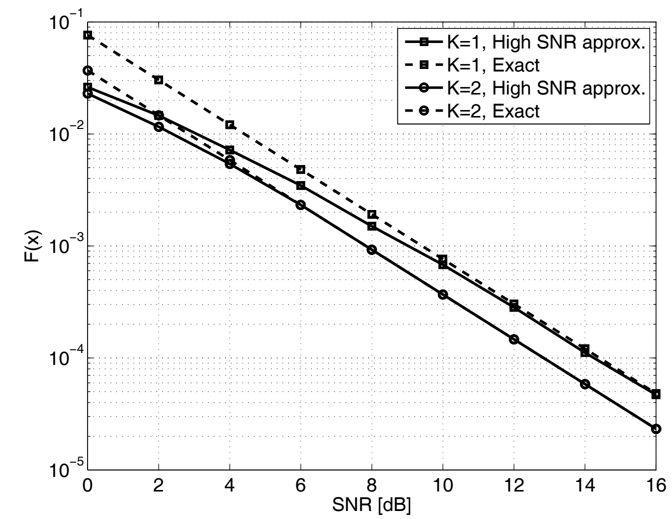

Fig. 2. The comparison between the exact form and high SNR approximation of $F(x)$ at $x = 2$ for a 2×2 system over a correlated Ricean channel (constant correlation with $r = 0.3$ and an all–one $\bar{\mathbf{H}}$).

where Γ is the ratio of the coding gains of the correlated Ricean and i.i.d. Rayleigh channels, which employ USTM with RAS.

VI. Simulation Results

A. A simple 2×2 MIMO system

We consider a simple 2×2 USTM system with $T = 4$, over a channel with constant transmit correlation matrix, which is equal to the exponential correlation matrix (3), and a LOS component with an all–one 2×2 $\bar{\mathbf{H}}$. The following quantities are used to simplify the results:

$$\mathcal{M}_j^T(\mu) = \int_{\mathbb{R}_+^T} e^{-\mathbf{\Sigma}'\mathbf{x}} \frac{(\mathbf{1}'\mathbf{x})^j}{j!}\, d\mathbf{x} = |\mathbf{\Omega}(\mu)|^{-1} \sum_{\mathcal{I}_T(j)} \prod_{t=1}^{T} \sigma_t^{-m_t}$$

$$\mathcal{N}_j^T(\mu) = \int_{\mathbb{R}_+^T} e^{-\mathbf{\Sigma}'\mathbf{x}} \gamma_j(\mathbf{1}'\mathbf{x})\, d\mathbf{x} = |\mathbf{\Omega}(\mu)|^{-1} - |\mathbf{I}_T + \mathbf{\Omega}(\mu)|^{-1} \sum_{p=0}^{j-1} \sum_{\mathcal{I}_T(p)} \prod_{t=1}^{T} (\sigma_t + 1)^{-m_t}$$

where $\mathbf{\Sigma} = [\sigma_1, \ldots, \sigma_T]'$ is a positive vector comprising the eigenvalues of $\mathbf{\Omega}(\mu)$ defined in (14a).

From Theorem 2 and Theorem 3 in the Appendix, we get

$$F(x) = \frac{\exp(-\frac{2K}{1+r})}{4\rho^2(1-r^2)} \left(2\gamma_1(x) + \gamma_2(x) - 2x + \frac{x^2}{2} \right) + o(\rho^{-2}).$$

Therefore, the Chernoff bound on the PEP will be

$$P_{\text{CB}}(\mu) = \frac{(K+1)^4 \exp(-\frac{4K}{1+r})}{16\rho^4(1-r^2)^2} \big(2\mathcal{N}_1^4(\mu) + \mathcal{N}_2^4(\mu) - 2\mathcal{M}_1^4(\mu) + \mathcal{M}_2^4(\mu) \big) + o(\rho^{-4}). \quad (22)$$

In order to determine the optimum μ to minimize P_{CB}, one can run a computer search to find μ_{opt} over the interval [0,1]. However, according to the results of Section IV, $\Psi(x)$ and therefore P_{CB} are proportional to those of the i.i.d. Rayleigh case studied in [18]. Thus, following the same lines as in [18], we conclude that (22) is a monotonically decreasing function of $\mu\bar{\mu}$, which is minimized by $\mu_{\text{opt}} = \frac{1}{2}$.

The ratio of coding gains for the correlated Ricean channel and i.i.d. Rayleigh channel (see (20)) is

$$\Gamma = \frac{e^{\frac{K}{1+r}} \sqrt{1-r^2}}{K+1}.$$

Fig. 1 shows Γ versus r for different values of K–factor. The higher the K–factor, the larger the ratio. The point $K = 0$ and $r = 0$ correspond to the i.i.d. Rayleigh channel. The coding gains is quite sensitive to r and highly correlated transmit antennas may result in gains even lower than those in i.i.d. Rayleigh case.

B. Code selection

Since our results show that USTM with RAS achieves the full diversity order over the correlated Ricean channels, codes can be designed based on the maximization of the coding gain. We consider an elegant class of unitary codes called parametric codes [31]. These codes perform better than the differential Alamouti codes [32] and diagonal cyclic group codes [5]. In [18], the authors report the best parametric codes with the maximum coding gain. From Section IV, we know that the coding gains for the correlated Ricean channel and i.i.d. Rayleigh channels are proportional via a factor which depends only on the long-term stationary parameters. Thus, the same optimal codes given in [18] can be utilized for the correlated Ricean case.

C. Results

Consider the 2×2 MIMO system described earlier in this section. Fig. 2, compares the exact and high SNR approximation of $F(x)$ at $x = 2$ for different K–factors. As SNR increases, it can be seen that $F(x)$ decreases as ρ^{-2} and the approximation becomes more accurate.

For a constellation of rate $R = 2$, i.e., $L = 16$, by using the optimal parametric codes [18], we compare the effects of

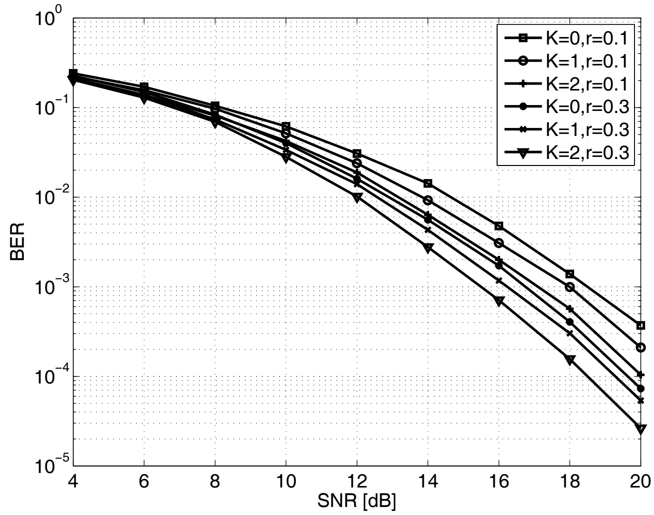

Fig. 3. The BER comparison for a 2×2 system employing a constellation of rate $R = 2$ over a correlated Ricean channel (constant correlation with $r = 0.1$ and $r = 0.3$, and an all–one $\bar{\mathbf{H}}$) with different K–factors.

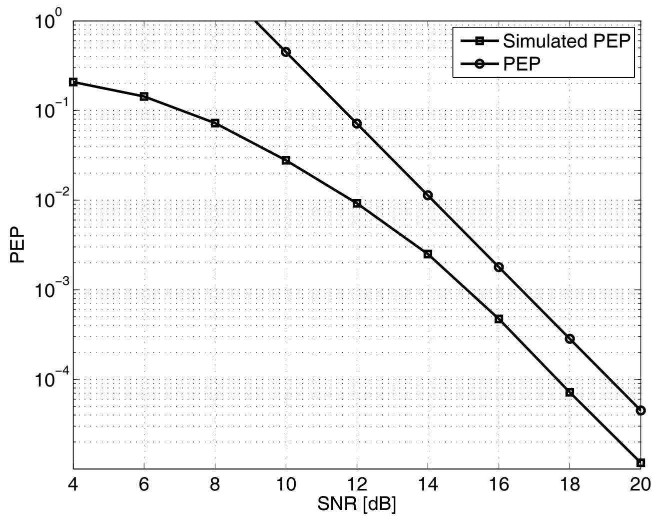

Fig. 4. The comparison between the simulated PEP and the PEP upper bound for a pair of codewords from the optimal parametric codebook for a 2×2 system over a correlated Ricean channel ($r = 0.1$, $K = 2$ and an all–one $\bar{\mathbf{H}}$).

K–factor and correlation r when a single antenna is selected at the receiver. Fig. 3 shows the real bit error rate (BER) versus SNR for $r = 0.1$ and $r = 0.3$ for different K–factors. As we expected from Fig. 1 at high SNR, the larger the K–factor, the larger the coding gain. Note that in all cases the full diversity order is achieved. Also, for a pair of codewords from the optimal parametric codes reported in [18], Fig. 4 shows the simulated PEP versus the high SNR approximation. We can see that as SNR increases, PEP falls off as ρ^{-4} and the approximation gets more accurate.

In Fig. 5, we compare the performance in terms of the number of antennas selected at the receiver. Clearly, when $N = 1$, the diversity is less than the case with two antennas. When $N = 2$ (no matter how many antenna is selected), the full diversity is achieved. The coding gain for $N = 2$ when one antenna is selected, i.e., $J = 1$, is between that of the full complexity system ($J = 2$) and that of the system with one antenna.

VII. Conclusion

The performance of receive antenna selection for USTM over the semi-correlated Ricean fading channels were analyzed (this work generalizes that of Ma and Tepedelenlioğlu for the independent and identically distributed (i.i.d.) Rayleigh fading case). Instead of performing the full complexity ML detection with all the receive antennas, we considered both the single antenna and multiple antenna selection scenarios based on the maximum received instantaneous power. The distribution function of a quadratic form of non-zero mean complex Gaussian variates was approximated at the high SNR region. It was shown that if the unitary codebook is of the full rank then the USTM system achieves the full diversity order as does the full complexity system. The coding gain expression was obtained and compared with the i.i.d. Rayleigh channel. To demonstrate the effect of correlation and LOS components, computer simulation results were provided for different channel parameters and antenna setups.

Appendix
Approximate $F(x)$ in the High SNR Region

The cdf of the instantaneous power received at each antenna is required to compute the Chernoff bound and coding gain given in (18) and (19), respectively. In this appendix, we will show that for the channel model given in (2), the cdf can be written as

$$F(x) = \eta^{-M}\Psi(x) + o(\rho^{-M})$$

where $\Psi(x)$ depends on the channel model parameters.

In fact, the cdf of $\|\mathbf{y}\|^2$ is the cdf of a noncentral quadratic form over a complex circularly symmetric Gaussian random vector; i.e., $\mathbf{y} \sim \mathcal{CN}(\bar{\mathbf{y}}, \mathbf{R}_\ell)$. We will use the results of [33] and [34]. In [33], the cdf is obtained for the real Gaussian vector, while [34] extends the results to the complex case. In the following, we first state the results of [34] by Theorem 1. Then, based on these results, we characterize the behavior of $F(x)$ at the high SNR region, when this function is applied to our problem, by using Theorem 2.

Theorem 1 (Nabar et al. [34]): Consider a $P \times 1$ complex circularly symmetric Gaussian vector with mean $\bar{\mathbf{x}}$ and a covariance matrix $\mathbf{R}$ whose eigenvalue decomposition is $\mathbf{R} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^\dagger$ where $\mathbf{\Lambda} = \mathbf{diag}\{\kappa_i\}_{i=1}^{P}$. The cdf of the quadratic form $\mathbf{x}^\dagger\mathbf{x}$ over $\mathbf{x} \sim \mathcal{CN}(\bar{\mathbf{x}}, \mathbf{R})$ is

$$F(x) = \sum_{k=0}^{\infty} f_k \frac{(x-\delta)^{\mathrm{rank}(\mathbf{R})+k}}{(\mathrm{rank}(\mathbf{R})+k)!} u(x-\delta)$$

where $u(x)$ is the unit step function,

$$\delta = \|\bar{\mathbf{x}}\|^2 - \sum_{j=1}^{\mathrm{rank}(\mathbf{R})} \frac{|q_j|^2}{\kappa_j},$$

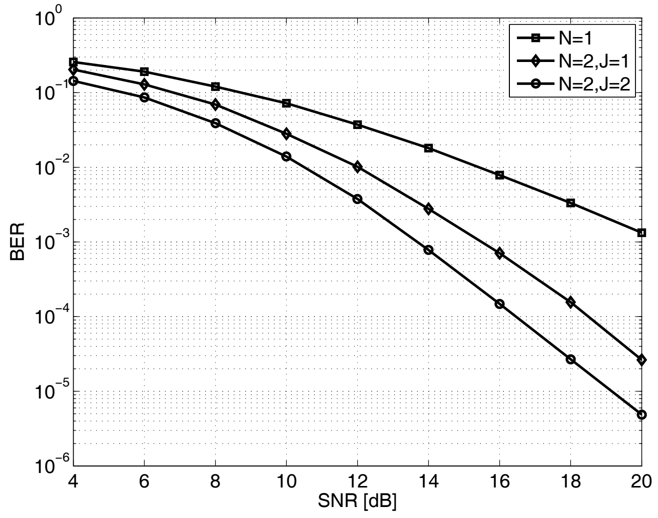

Fig. 5. The BER comparison for a 2×2 system employing a constellation of rate $R = 2$ over a correlated Ricean channel (constant correlation with $r = 0.3$, an all–one $\bar{\mathbf{H}}$, and $K = 2$) with different number of selected antennas.

$\mathbf{q} = [q_1, \ldots, q_P]' = \mathbf{\Lambda}^{\frac{1}{2}}\mathbf{V}^\dagger\bar{\mathbf{x}}$, and the coefficients f_k are obtained recursively by

$$f_0 = \frac{1}{|\mathbf{R}_\ell|}\exp\left(-\sum_{j=1}^{P}\frac{|q_j|^2}{\kappa_j^2}\right),\ f_k = \frac{1}{k}\sum_{r=0}^{k-1} f_r h_{k-r},$$
$$h_k = (-1)^k\sum_{j=1}^{P}\left(\frac{1}{\kappa_j^k} - \frac{k|q_j|^2}{\kappa_j^{k+2}}\right).$$

Now, we use Theorem 1 to investigate the behavior of $F(x)$ when the SNR gets large. Let us start with the eigenvalue decomposition of the received covariance matrix as $\mathbf{R}_\ell = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^\dagger$ where $\mathbf{\Lambda} = \mathbf{diag}\{\kappa_i\}_{i=1}^T$. From (8), we know that the eigenvalues of $\mathbf{R}_\ell$ are 1 of order $T - M$ and $\{1 + \eta\lambda_i\}_{i=1}^M$, where $\{\lambda_i\}_{i=1}^M$ are the eigenvalues of the spatial correlation matrix $\mathbf{R}_T$. Let $\mathbf{q} = [q_1, \ldots, q_T]' = \mathbf{\Lambda}^{\frac{1}{2}}\mathbf{V}^\dagger\bar{\mathbf{y}}$. In Theorem 2, we show that at high SNR, the dominant term of $F(x)$ in terms of SNR is η^{-M}.

We first state the following lemma, which will be useful in the proof of Theorem 2.

Lemma 1: For $\mathbf{R}_\ell^{-1}$ given in (10) and any positive integer m, we have

$$\mathbf{R}_\ell^{-m} = \mathbf{I}_T - \mathbf{\Phi}_\ell\big[\mathbf{I}_M - (\mathbf{I}_M + \eta\mathbf{R}_T)^{-m}\big]\mathbf{\Phi}_\ell^\dagger.$$

Proof: We use the binomial expansion as follows:

$$\begin{aligned}\mathbf{R}_\ell^{-m} &= \mathbf{I}_T - \mathbf{\Phi}_\ell\mathbf{\Phi}_\ell^\dagger + \sum_{p=0}^{m}\binom{m}{p}(-1)^p\mathbf{\Phi}_\ell\mathbf{\Upsilon}^p\mathbf{\Phi}_\ell^\dagger \\ &= \mathbf{I}_T - \mathbf{\Phi}_\ell\mathbf{\Phi}_\ell^\dagger + \mathbf{\Phi}_\ell\left[\sum_{p=0}^{m}\binom{m}{p}(-1)^p\mathbf{\Upsilon}^p\right]\mathbf{\Phi}_\ell^\dagger.\end{aligned}$$

■

The following theorem approximates the cdf as an infinite series at the high SNR region.

Theorem 2: At high SNR, the cdf of $\mathbf{y}^\dagger\mathbf{y}$ over $\mathbf{y} \sim \mathcal{CN}(\bar{\mathbf{y}}, \mathbf{R}_\ell)$ is $F(x) = \eta^{-M}\Psi(x) + o(\rho^{-M})$ where

$$\Psi(x) = \sum_{k=0}^{\infty}\hat{f}_k\frac{x^{T+k}}{(T+k)!},\ x \geq 0.$$

The coefficients $\hat{f}_k$ are

$$\hat{f}_k = (-1)^k\binom{T-M+k-1}{k}\hat{f}_0, \quad \hat{f}_0 = \frac{e^{-\Delta}}{|\mathbf{R}_T|}$$

and Δ is defined in (14b).

Proof: According to Theorem 1, since $\mathbf{R}_\ell$ is full rank, $\delta = \|\bar{\mathbf{y}}\|^2 - \mathbf{q}^\dagger\mathbf{\Lambda}^{-1}\mathbf{q} = 0$. Also, the exponent of the numerator of f_0 is

$$\mathbf{q}^\dagger\mathbf{\Lambda}^{-2}\mathbf{q} = \bar{\mathbf{y}}^\dagger\mathbf{R}_\ell^{-1}\bar{\mathbf{y}} = \Delta(\rho)$$

where $\Delta(\rho)$ is defined in Section IV-A. Therefore, we arrive at

$$f_0 = e^{-\Delta(\rho)}\prod_{i=1}^{M}(1 + \eta\lambda_i)^{-1} = \eta^{-M}\hat{f}_0 + o(\rho^{-M}) \tag{23}$$

where $\hat{f}_0 = \frac{e^{-\Delta}}{|\mathbf{R}_T|}$ and Δ is defined in (14b). Notice that the factor η^{-M} has appeared in (23), and since the coefficient f_k is calculated recursively, only the constant term of h_k is taken into account. Hence, we have

$$\begin{aligned}(-1)^k h_k &= \mathrm{Tr}(\mathbf{\Lambda}^{-k}) - k\mathbf{q}^\dagger\mathbf{\Lambda}^{-(k+2)}\mathbf{q} \\ &= T - M + \eta^{-k}\,\mathrm{Tr}(\mathbf{R}_T^{-k}) + o(\rho^{-k}) \\ &\qquad - k\bar{\mathbf{y}}^\dagger\mathbf{R}_\ell^{-(k+1)}\bar{\mathbf{y}} \\ &\overset{(a)}{=} T - M + \eta^{-k}\,\mathrm{Tr}(\mathbf{R}_T^{-k}) - k\eta K\bar{\mathbf{h}}^\dagger \\ &\qquad \times(\mathbf{I}_M + \eta\mathbf{R}_T)^{-(k+1)}\bar{\mathbf{h}} + o(\rho^{-k}) \\ &= T - M + \eta^{-k}\,\mathrm{Tr}(\mathbf{R}_T^{-k}) - k\eta^{-k}K \\ &\qquad \times\bar{\mathbf{h}}^\dagger\mathbf{R}_T^{-(k+1)}\bar{\mathbf{h}} + o(\rho^{-k}) \\ &= T - M + \eta^{-k}d_k + o(\rho^{-k})\end{aligned}$$

where $d_k = \mathrm{Tr}(\mathbf{R}_T^{-k}) - kK\bar{\mathbf{h}}^\dagger(\mathbf{I}_M + \eta\mathbf{R}_T)^{-(k+1)}\bar{\mathbf{h}}$, and (a) results from Lemma 1. Thus, the constant term in h_k is $\hat{h}_k = (-1)^k(T - M)$.

For $k = 1, 2$, we have

$$f_1 = f_0h_1 = -\eta^{-M}(T - M)\hat{f}_0 + o(\rho^{-M})$$

and

$$\begin{aligned}f_2 &= \frac{1}{2}(f_0h_2 + f_1h_1) \\ &= \eta^{-M}\frac{1}{2}(T - M)(T - M + 1)\hat{f}_0 + o(\rho^{-M}).\end{aligned}$$

By induction, it can be shown that $f_k = \eta^{-M}\hat{f}_k + o(\rho^{-M})$ where $\hat{f}_k$ is given in Theorem 2.

Therefore, at high SNR, $F(x)$ is

$$F(x) = \eta^{-M}\underbrace{\sum_{k=0}^{\infty}\hat{f}_k\frac{x^{T+k}}{(T+k)!}}_{\Psi(x)} + o(\rho^{-M}).$$

■

In the sequel, we invoke the *regularized incomplete Gamma function* defined as

$$\gamma_n(x) = \int_0^x \frac{u^{n-1}e^{-u}}{(n-1)!}du = \sum_{j=0}^{\infty} \frac{(-1)^j x^{j+n}}{j!(n-1)!(j+n)}.$$

To avoid infinite summations in the calculation of the coding gain, we propose a closed-form finite summation for $\Psi(x)$ by the following theorem.

Theorem 3: The function $\Psi(x)$ given in Theorem 2 can be written as

$$\Psi(x) = \sum_{k=1}^{T-M} g_k \gamma_k(x) + \sum_{k=1}^{M} t_k \frac{x^k}{k!} \tag{24}$$

where

$$g_k = (-1)^M \binom{T-k-1}{T-k-M} \hat{f}_0, \; t_k = (-1)^{M+k} \binom{T-k-1}{T-M-1} \hat{f}_0.$$

Proof: For a fixed k, one can compare the coefficient of $x^k/k!$ from $\Psi(x)$ given in Theorem 2 and the one resulting from the two summations given in (24) and show that they are equal for every k. ■

Acknowledgment

The authors would like to thank Payam Dehghani for his helpful suggestion in Section IV-A. Also, we would like to thank Mohsen Eslami, the Editor and anonymous reviewers for their constructive comments.

References

[1] V. Tarokh, A. Naguib, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance criteria in the presence of channel estimation errors, mobility, and multiple paths," *IEEE Trans. Commun.*, vol. 47, no. 2, pp. 199-207, Feb. 1999.

[2] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs Tech. J.,*, vol. 1, pp. 41-59, 1996.

[3] B. M. Hochwald and T. L. Marzetta, "Unitary space-time modulation for multiple-antenna communications in rayleigh flat fading," *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 543-564, Mar. 2000.

[4] B. L. Hughes, "Differential space-time modulation," *IEEE Trans. Inf. Theory*, vol. 46, no. 7, pp. 2567-2578, Nov. 2000.

[5] B. M. Hochwald and W. Sweldens, "Differential unitary space-time modulation," *IEEE Trans. Commun.*, vol. 48, no. 12, pp. 2041-2052, Dec. 2000.

[6] C. Shan, A. Nallanathan, and P. Y. Kam, "A new class of signal constellations for differential unitary space-time modulation (DUSTM)," *IEEE Commun. Lett.*, vol. 8, no. 1, pp. 1-3, Jan. 2004.

[7] J. Wang, M. K. Simon, and K. Yao, "On the optimum design of differential unitary space-time modulation," in *IEEE Global Telecommn. Conf. (GLOBECOM)*, vol. 4, Dec. 2003, pp. 1968-1972.

[8] I. Bahceci, T. M. Duman, and Y. Altunbasak, "Antenna selection for multiple-antenna transmission systems: performance analysis and code construction," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2669-2681, Oct. 2003.

[9] M. Hajiaghayi and C. Tellambura, "Unitary signal constellations for differential space-time modulation," *IEEE Commun. Lett.*, vol. 11, no. 1, pp. 25-27, Jan. 2007.

[10] N. Mehta and A. Molish, *MIMO System Technology for Wireless Communications*. CRC Press, 2006, ch. antenna selection in MIMO systems, pp. 147-173.

[11] R. W. Heath Jr. and D. J. Love, "Multimode antenna selection for spatial multiplexing systems with linear receivers," *IEEE Trans. Signal Process.*, vol. 53, pp. 3042-3056, Aug. 2005.

[12] K. T. Phan and C. Tellambura, "Receive antenna selection based on union-bound minimization using convex optimization," *IEEE Signal Process. Lett.*, vol. 14, no. 9, pp. 609-612, Sept. 2007.

[13] S. Sanayei and A. Nosratinia, "Antenna selection in MIMO systems," *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 68-73, Oct. 2004.

[14] D. A. Gore and A. J. Paulraj, "MIMO antenna subset selection with space-time coding," *IEEE Trans. Signal Process.*, vol. 50, no. 10, pp. 2580-2588, Oct. 2002.

[15] A. F. Molisch, M. Z. Win, Y.-S. Choi, and J. H. Winters, "Capacity of MIMO systems with antenna selection," *IEEE Trans. Wireless Commun.*, vol. 4, no. 4, pp. 1759-1772, July 2005.

[16] A. Gorokhov, D. Gore, and A. Paulraj, "Performance bounds for antenna selection in MIMO systems," in *Proc. IEEE Int. Conf. Commun.(ICC)*, vol. 5, May 2003, pp. 3021-3025.

[17] D. A. Gore and A. J. Paulraj, "MIMO antenna subset selection with space-time coding," *IEEE Trans. Signal Process.*, vol. 50, no. 10, pp. 2580-2588, Oct. 2002.

[18] Q. Ma and C. Tepedelenlioğlu, "Antenna selection for unitary space-time modulation," *IEEE Trans. Inf. Theory*, vol. 51, no. 10, pp. 3620-3631, Oct. 2005.

[19] X. Cai and G. B. Giannakis, "Differential space-time modulation with eigen-beamforming for correlated MIMO fading channels," *IEEE Trans. Signal Process.*, vol. 54, no. 4, pp. 1279-1288, Apr. 2006.

[20] S. A. Jafar and A. Goldsmith, "Multiple-antenna capacity in correlated Rayleigh fading with channel covariance information," *IEEE Trans. Wireless Commun.*, vol. 4, no. 3, pp. 990-997, May 2005.

[21] D. S. Baum, D. Gore, R. Nabar, S. Panchanathan, K. V. S. Hari, V. Erceg, and A. J. Paulraj, "Measurement and characterization of broadband MIMO fixed wireless channels at 2.5 GHz," in *Proc. IEEE Int. Conf. Personal Wireless Commun. (ICPWC)*, Hyderabad, Dec. 2000, pp. 203-206.

[22] R. N. A. Paulraj and D. Gore, *Introduction to Space-Time Wireless Communications*, 1st ed. Cambridge University Press, 2003.

[23] M. R. Bhatnagar, A. Hjørungnes, and L. Song, "Precoded differential orthogonal space-time modulation over correlated ricean MIMO channels," *IEEE J. Sel. Areas Signal Proc.*, vol. 2, no. 2, pp. 124-134, Apr. 2008.

[24] C. Tepedelenlioğlu, A. Abdi, and G. B. Giannakis, "The Ricean k factor: estimation and performance analysis," *IEEE Trans. Wireless Commun.*, vol. 2, no. 4, pp. 799-810, July 2003.

[25] B. Holter and G. E. Oien, "On the amount of fading in MIMO diversity systems," *IEEE Trans. Wireless Commun.*, vol. 4, no. 5, pp. 2498-2507, Sept. 2005.

[26] S. Kotz and J. W. Adams, "Distribution of sum of identically distributed exponentially correlated Gamma-variables," *Annals Mathematical Statistics*, vol. 35, no. 1, pp. 277-283, Mar. 1964.

[27] M. S. Alouini, A. Abdi, and M. Kaveh, "Sum of Gamma variates and performance of wireless communication systems over Nakagami-fading channels," *IEEE Trans. Veh. Commun.*, vol. 50, no. 6, pp. 1471-1480, Nov. 2001.

[28] H. A. David, *Order Statistics*, 2nd ed. New York: Wiley, 1981.

[29] A. Dogandzic, "Chernoff bounds on pairwise error probabilities of space-time codes," *IEEE Trans. Inf. Theory*, vol. 49, no. 5, pp. 1327-1336, May 2003.

[30] H. L. Royden, *Real Analysis*. Prentice-Hall, 1988.

[31] X.-B. Liang and X.-G. Xia, "Unitary signal constellations for differential space-time modulation with two transmit antennas: parametric codes, optimal designs, and bounds," *IEEE Trans. Inf. Theory*, vol. 48, no. 8, pp. 2291-2322, Aug. 2002.

[32] V. Tarokh and H. Jafarkhani, "A differential detection scheme for transmit diversity," *IEEE J. Sel. Areas Commun.*, vol. 18, no. 7, pp. 1169-1174, July 2000.

[33] A. M. Mathahi and S. B. Provost, *Quadratic Forms in Random Variables*. New York: Marcell Dekker, 1992.

[34] R. U. Nabar, H. Bolcskei, and A. J. Paulraj, "Diversity and outage performance in space-time block coded Ricean MIMO channels," *IEEE Trans. Wireless Commun.*, vol. 4, no. 5, pp. 2519-2532, Sept. 2005.

Mahdi Ramezani received the B.Sc. degree in Electrical Engineering from the Iran University of Science and Technology in 2006. He joined the Informatics Circle of Research Excellence (*i*CORE) Wireless Communications Laboratory at the University of Alberta, Canada in September 2006 and received his M.Sc. in Electrical and Computer Engineering in 2008. He is currently a Ph.D. student at the University of Alberta, Canada. He is the recipient of *i*CORE Ph.D. Scholarship in ICT. His research interests include information theory and modern coding theory.

Mahdi Hajiaghayi received the B.Sc degrees in Electrical Engineering and Petroleum Engineering concurrently from Sharif University of Technology, Tehran, Iran in 2005, and the M.Sc. degree in Electrical and Computer Engineering from the University of Alberta, Canada in 2007. He is currently pursuing the Ph.D. degree in Electrical and Computer Engineering at the University of Toronto, Canada. Mahdi is the Ted Rogers Scholarship recipient and a member of IEEE Communications Society.

Chintha Tellambura (SM'02) received the B.Sc. degree (with first-class honors) from the University of Moratuwa, Moratuwa, Sri Lanka, in 1986, the M.Sc. degree in electronics from the University of London, London, U.K., in 1988, and the Ph.D. degree in electrical engineering from the University of Victoria, Victoria, BC, Canada, in 1993.

He was a Postdoctoral Research Fellow with the University of Victoria (1993-1994) and the University of Bradford (1995-1996). He was with Monash University, Melbourne, Australia, from 1997 to 2002. Presently, he is a Professor with the Department of Electrical and Computer Engineering, University of Alberta. His research interests include Diversity and Fading Countermeasures, Multiple-Input Multiple-Output (MIMO) Systems and Orthogonal Frequency Division Multiplexing (OFDM) systems.

Prof. Tellambura is an Associate Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS and Area Editor for Wireless Communications Systems and Theory in the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. He was Chair of the Communication Theory Symposium in Globecom'05 held in St. Louis, MO.

Masoud Ardakani (SM'09) received the B.Sc. degree from Isfahan University of Technology in 1994, the M.Sc. degree from Tehran University in 1997 and the Ph.D. degree from the University of Toronto in 2004, all in Electrical Engineering. He was a Postdoctoral fellow at the University of Toronto from 2004 to 2005. He is currently an Associate Professor of Electrical and Computer Engineering and Alberta Ingenuity New Faculty at the University of Alberta, where he holds an Informatics Circle of Research Excellence (*i*CORE) Junior Research Chair in wireless communications. His research interests are in the general area of digital communications, codes defined on graphs and iterative decoding techniques.

He serves as an Associate Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS and the IEEE COMMUNICATION LETTERS.