Observations of simulation results over Rician fading are similar but omitted by space limitations. The comparison in Fig. 6 shows that our trellis search algorithm performs slightly better than that of [6] over Rayleigh fading, as the branch metric in [6, eq. (23)] involves an approximation.

VI. Conclusion

We have proposed three pilot-based algorithms for detecting a long sequence over a slowly time-varying channel, which have better performance and lower complexity than the existing lattice and sphere decoding block-by-block detection algorithms.

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Performance Analysis of Joint Transmit and Receive Antenna Selection With Orthogonal Space-Time Coding

W. Zhang and C. Tellambura, *Senior Member, IEEE*

***Abstract*—This paper analyzes the performance of multiple-input–multiple-output (MIMO) systems with transmit and receive antenna selection (T-RAS). The average bit error rate (BER) and the average symbol error rate (SER) are derived by utilizing the characteristic function (CF) of the joint output signal-to-noise ratios (SNRs). Our approach can be used over not only independent but also arbitrary correlated channels. For the sake of brevity, this paper focuses on Nakagami-m fading channels. The simulation results are provided to validate the numerical calculations.**

***Index Terms*—Arbitrary correlated fading, characteristic function (CF), transmit and receive antenna selection (T-RAS).**

I. Introduction

Antenna selection for multiple-input–multiple-output (MIMO) systems—a promising low-complexity technology—has received much attention in the wireless community. Antenna-selection strategies may be classified as transmit antenna selection (TAS), receive antenna selection (RAS), and joint transmit and receive antenna selection (T-RAS). A variety of RAS and TAS techniques have extensively been investigated [1]–[6].

The performance analysis of antenna selection requires the statistical distribution of the maximum of SNRs of selected diversity branches. With T-RAS, these SNRs are not independent, even with independent spatial fading. Although order statistics is a well-established branch of statistics, surprisingly, few analytical results are available on order statistics of correlated random variables [7]. For this reason, the analysis is often made tractable by constraining antenna selection to be at either the transmitter or the receiver—but not both together [8].

To the best of our knowledge, only a few papers have analyzed T-RAS to date. Cai and Giannakis [9] analyze the error-rate performance for selecting one transmit antenna and an arbitrary number of receive antennas in independent Rayleigh fading. The error-rate performance for selecting two transmit antennas and one receive antenna in independent Rayleigh fading channels is analyzed in [10]. Thus, a general analysis of the joint selection of L_t out of N_t transmit antennas and L_r out of N_r receive antennas remains unavailable. In TAS or RAS, as aforementioned, the order statistics of independent fading gains can be employed to get the probability density function (pdf) of the output SNR. However, even for independent fading, T-RAS cannot be analyzed by making use of the order statistics of independent random variables.

This paper thus presents a framework for the performance analysis for the general T-RAS for an arbitrary number of transmit and receive antennas. To do so, we utilize an analytical framework due to Zhang and Lu [11] for the performance analysis of an arbitrary correlated multibranch selection combining. The key idea is based on using the

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The authors are with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6G 2V4, Canada (e-mail: wzhang@ece.ualberta.ca; chintha@ece.ualberta.ca).

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joint characteristic function (CF) of the branch SNRs to derive the statistics of the selection combiner output. The pdf of the output is given by a multidimensional integral. Although no closed-form expressions exist, the integral can readily be solved by using numerical methods. We adopt the same framework for the T-RAS performance analysis. We show that the joint CF can be obtained by using conventional results on the Hermitian quadratic forms of complex Gaussian variates. Our results are not restricted to independent MIMO channels but can also handle arbitrary correlated Rayleigh, Nakagami-m, or Rician fading channels [12].

This paper is organized as follows. The joint T-RAS system model is briefly described in Section II. An illustrative example is given in Section III. The joint CF of the outputs of T-RAS for Nakagami-m fading channels is given in Section III. Section IV derives the moment-generating function (MGF) of the output SNR, the average symbol error rate (SER), and the average bit error rate (BER). In Section V, the numerical results are presented to validate the theory and address the effects of various parameters on the BER performance, and the conclusions are presented in Section VI.

Notation: Bold symbols denote matrices or vectors. $(\cdot)^T$ and $(\cdot)^H$ denote transpose and complex conjugate transpose, respectively. $\mathbb{E}\{x\}$ is the expected value. $\mathbf{A}_{m,n}$ is the (m,n)th entry of the matrix $\mathbf{A}$. $\det(\mathbf{A})$ and $\mathrm{vec}(\mathbf{A})$ denote the determinant and vectorization of the matrix $\mathbf{A}$. $\|\mathbf{A}\|_F^2$ is the Frobenius norm of $\mathbf{A}$. $\mathbf{A}\otimes\mathbf{B}$ denotes the Kronecker product of matrices $\mathbf{A}$ and $\mathbf{B}$. A circularly complex Gaussian variable z with mean μ and variance σ^2 is denoted by $z\sim\mathcal{CN}(\mu,\sigma^2)$, and $\mathrm{j}=\sqrt{-1}$. An $N\times M$ MIMO system is a system with N transmit antennas and M receive antennas.

II. System and Channel Model

We consider an $N_t\times N_r$ MIMO system. The MIMO channel is assumed to be slow fading. The receiver knows the channel gains and selects a subset of L_t transmit antennas and L_r receive antennas. The selection information is conveyed to the transmit side via a feedback link. The number of feedback bits required is $\log_2[\binom{N_t}{L_t}]$. Orthogonal space-time block code (OSTBC) signal matrices are sent over the subset of selected transmit antennas.

Let $\mathbf{H}=[h_{k,l}]$ be the $N_r\times N_t$ channel matrix. Each element's amplitude $|h_{k,l}|$ $(1\le k\le N_r, 1\le l\le N_t)$ follows the Nakagami-m fading model. Both cases, where the $h_{k,l}$'s are independent or are correlated with each other, are treated in this paper. Correlation may be caused by realistic conditions such as the insufficient angular spread induced by the scattering environment or closely spaced antenna elements.

Let the $L_r\times L_t$ channel matrix after antenna selection be $\widetilde{\mathbf{H}}$. The received signals can be expressed by

$$\mathbf{Y}=\sqrt{\frac{E_s}{L_t}}\widetilde{\mathbf{H}}\mathbf{X}+\mathbf{N} \tag{1}$$

where E_s is the energy of the transmitted symbol, $\mathbf{Y}$ is the $L_r\times T$ received signal matrix, $\mathbf{X}$ represents the $L_t\times T$ transmitted signal matrix, and $\mathbf{T}$ are the block symbol periods. The elements of $\mathbf{N}$ are independent identically distributed Gaussian random variables denoted by $\mathcal{CN}(0,N_0)$, where N_0 is the noise variance.

To maximize the total received signal power for OSTBC transmission, the subset of transmit and receive antennas that yields the largest channel norm should be selected. There are $N=\binom{N_t}{L_t}\binom{N_r}{L_r}$ possible selections of transmit and receive antennas. The jth channel matrix (after antenna selection) out of the N possible antenna subsets, which is denoted as $\widetilde{\mathbf{H}}_j$ $(1\le j\le N)$, is a submatrix of $\mathbf{H}$ formed by selecting certain L_t columns and L_r rows from $\mathbf{H}$. With OSTBC transmission, the maximum-likelihood decoder (with perfect channel information) decomposes the MIMO system to Q independent single-input–single-output (SISO) channels [13]. In the case of the jth antenna subset selection, it can easily be shown that these equivalent SISO channels have the following effective SNR per symbol [14]:

$$\gamma_j=\frac{E_s}{R_sL_tN_0}\|\widetilde{\mathbf{H}}_j\|_F^2=\frac{\rho}{R_sL_t}\|\widetilde{\mathbf{H}}_j\|_F^2=a\|\widetilde{\mathbf{H}}_j\|_F^2 \tag{2}$$

$j=1,\ldots,N$, where R_s is the symbol rate (in symbols per second), $\rho=E_s/N_o$ is the transmit SNR, and $a=E_s/R_sL_tN_0$. The receive SNR of T-RAS with OSTBC transmission is then equal to

$$\gamma=\max\{\gamma_1,\ldots,\gamma_N\}. \tag{3}$$

The receiver may inform the transmitter about its selection. Doing so requires a feedback link. Interesting research questions arise when the reliability and finite capacity of the feedback link are taken into consideration, but these issues are beyond the scope of this paper.

III. Characteristic Function of Transmit and Receive Antenna Selection

A. Joint CF of the Output of T-RAS

The joint CF of the N possible output SNRs γ_j is defined as

$$\phi(t_1,\ldots,t_N)=\mathbb{E}\left\{e^{\mathrm{j}t_1\gamma_1+\cdots+\mathrm{j}t_N\gamma_N}\right\} \tag{4}$$

where the γ_j's $(1\le j\le N)$ are the output SNRs of each possible antenna selection defined in (2). The main difficulty in evaluating (4) is that each γ_j does not correspond to one single fading channel gain but is related to the $L_t\times L_r$ summation of the square norm of the elements of the jth channel matrix $\widetilde{\mathbf{H}}_j$.

To find the joint CF of joint antenna selection, the first step is to define the N possible antenna subset selections, that is, one has to determine which transmit and receive antennas are included in each antenna subset selection. To describe the jth antenna subset selection, we use $\mathbf{p}_j$ and $\mathbf{q}_j$, which are two binary vectors. The length of $\mathbf{q}_j$ is N_r, with the nth element $\mathbf{q}_{j_n}\in\{0,1\}$, where 0 indicates noninclusion, and 1 indicates selection. The length of $\mathbf{p}_j$ is N_t, with the mth element $\mathbf{q}_{j_m}\in\{0,1\}$. For example, $\mathbf{q}_{j_i}=1$ and $\mathbf{p}_{k_m}=1$ indicate that the ith receive antenna and the kth transmit antenna are selected. The N corresponding output SNRs γ_j can easily be derived by using (2). By observing that γ_j is the linear function of $|h_{k,i}|^2(1\le k\le N_r, 1\le i\le N_t)$, the output SNRs γ_j are then substituted into (4) to obtain

$$\phi(\mathbf{t})=\mathbb{E}\left\{e^{a\mathrm{j}\left(|h_{1,1}|^2\sum\limits_{j\in\mathbb{O}_{1,1}}t_j+\cdots+|h_{N_r,N_t}|^2\sum\limits_{j\in\mathbb{O}_{N_r,N_t}}t_j\right)}\right\} \tag{5}$$

where the vector $\mathbf{t}=\{t_1,\ldots,t_N\}$ is used for brevity, and the exponent in (5) is expressed as a linear function of $|h_{k,i}|^2(1\le k\le N_r, 1\le i\le N_t)$ instead of a linear function of γ_j's $(1\le j\le N)$. In (5), $a=E_s/N_oL_tR_s$, and $\mathbb{O}_{k,i}$ is a set of indices indicating that $t_j(1\le j\le N)$ is included in the summation of the coefficient of $|h_{k,i}|^2$. For $|h_{k,i}|^2$, the included t_j's should satisfy the condition that the jth antenna subset selection includes the ith transmit antenna and the kth receive antenna. As for the two index vectors for jth antenna selection, i.e., $\mathbf{q}_j$ and $\mathbf{p}_j$, the elements correspond to the kth receive antenna, and the ith transmit antenna should be equal to 1. $\mathbb{O}_{k,i}$ is thus described as $\mathbb{O}_{k,i}\stackrel{\triangle}{=}\{j:\mathbf{q}_{j_k}=1,\mathbf{p}_{j_i}=1\}$. The number of t_j's in the summation of the coefficient of each $|h_{k,i}|^2$ is $\binom{N_t-1}{L_t-1}\binom{N_r-1}{L_r-1}$. By defining the vector $\mathbf{h}$ as

$$\mathbf{h} = \mathrm{vec}[\mathbf{H}] \tag{6}$$

the exponential in (5) can be rewritten as the Hermitian quadratic form of $\mathbf{h}$ as

$$\phi(\mathbf{t}) = \mathbb{E}\left\{e^{\mathbf{h}^H\mathbf{Q}(\mathbf{t})\mathbf{h}}\right\} \tag{7}$$

where $\mathbf{Q}(\mathbf{t})$ is the diagonal matrix, with the diagonal elements being coefficients of $|h_{k,i}|^2$, i.e.,

$$\mathbf{Q}(\mathbf{t}) = \mathrm{diag}\left\{a\mathrm{j}\sum_{j\in\mathbb{O}_{1,1}} t_j, \ldots, a\mathrm{j}\sum_{j\in\mathbb{O}_{N_r,N_t}} t_j\right\}. \tag{8}$$

Since Hermitian quadratic forms in complex Gaussian variables have been studied in great detail, the expectation required in (7) is available for many different scenarios that involve the practical cases of interest. Note that the form of $\mathbf{Q}(\mathbf{t})$ depends on the arrangement order of the N subset antenna selections. Once the N possible antenna subset selections are defined, the form of $\mathbf{Q}(\mathbf{t})$ is unique.

B. Systematic Way to Construct $\mathbf{Q}(\mathbf{t})$

The matrix $\mathbf{Q}(\mathbf{t})$ can be computed by the following procedure. First, define the N possible antenna subset selections, and then, the associated SNRs γ_j are readily given by (2). Second, substitute the N corresponding γ_j's into the general CF function in (4). The diagonal elements of $\mathbf{Q}(\mathbf{t})$ correspond to the coefficient of $|h_{k,i}|^2$. To systematically construct matrix $\mathbf{Q}(\mathbf{t})$, we use the two selection index vectors $\mathbf{p}_j$ and $\mathbf{q}_j$ of the ith antenna subset selection. Next, we find the t_j's in $\mathbb{O}_{k,i}$ for each $h_{k,i}$ in (7) by taking the j where the kth element of $\mathbf{p}_j$ and the ith element of $\mathbf{q}_j$ are equal to 1. Finally, we sum up all t_j and multiply with $a\mathrm{j}$. An example is given next to illustrate how to determine $\mathbf{Q}(\mathbf{t})$.

1) Example: Consider selecting two out of three transmit antennas and one out of two receive antennas. The number of possible antenna selections is $\binom{3}{2}\binom{2}{2} = 6$. Based on (2), all the possible SNRs $\{\gamma_1, \ldots, \gamma_6\}$ and corresponding antenna selection index vectors are given as

$$\begin{aligned}
\gamma_1 &= a\left(|h_{1,1}|^2 + |h_{1,2}|^2\right), \quad \mathbf{q}_1 = [1,0], \quad \mathbf{p}_1 = [1,1,0]\\
\gamma_2 &= a\left(|h_{1,1}|^2 + |h_{1,3}|^2\right), \quad \mathbf{q}_2 = [1,0], \quad \mathbf{p}_2 = [1,0,1]\\
\gamma_3 &= a\left(|h_{1,2}|^2 + |h_{1,3}|^2\right), \quad \mathbf{q}_3 = [1,0], \quad \mathbf{p}_3 = [0,1,1]\\
\gamma_4 &= a\left(|h_{2,1}|^2 + |h_{2,2}|^2\right), \quad \mathbf{q}_4 = [0,1], \quad \mathbf{p}_4 = [1,1,0]\\
\gamma_5 &= a\left(|h_{2,1}|^2 + |h_{2,3}|^2\right), \quad \mathbf{q}_5 = [0,1], \quad \mathbf{p}_5 = [1,0,1]\\
\gamma_6 &= a\left(|h_{2,2}|^2 + |h_{2,3}|^2\right), \quad \mathbf{q}_6 = [0,1], \quad \mathbf{p}_6 = [0,1,1].
\end{aligned} \tag{9}$$

By substituting (9) into (4), the exponential in (4) becomes

$$\begin{aligned}
\sum_{k=1}^{6}\mathrm{j}t_k\gamma_k = a\big\{&|h_{1,1}|^2\mathrm{j}(t_1+t_2) + |h_{1,2}|^2\mathrm{j}(t_1+t_3)\\
&+ |h_{1,3}|^2\mathrm{j}(t_2+t_3) + |h_{2,1}|^2\mathrm{j}(t_4+t_5)\\
&+ |h_{2,2}|^2\mathrm{j}(t_4+t_6) + |h_{2,3}|^2\mathrm{j}(t_5+t_6)\big\}
\end{aligned} \tag{10}$$

where $a = \rho/2$. Thus, the diagonal matrix $\mathbf{Q}(\mathbf{t})$ can readily be obtained as

$$\begin{aligned}
\mathbf{Q}(\mathbf{t}) = \mathrm{diag}\big\{&a\mathrm{j}(t_1+t_2), a\mathrm{j}(t_4+t_5), a\mathrm{j}(t_1+t_3)\\
&a\mathrm{j}(t_4+t_6), a\mathrm{j}(t_2+t_3), a\mathrm{j}(t_5+t_6)\big\}.
\end{aligned} \tag{11}$$

The matrix $\mathbf{Q}(\mathbf{t})$ can also easily be constructed by determining the t_j's of each element of $\mathbf{Q}(\mathbf{t})$ directly from the two selection index vectors of each antenna-subset selection. Take the $\mathbf{Q}_{1,1}(\mathbf{t})$ that corresponds to $h_{1,1}$ as an example. In the definition of six antenna-subset selections, the subset selections that satisfy $\mathbf{q}_j = 1, \mathbf{p}_{j_1} = 1$ are the first and second antenna-subset selections. $\mathbf{Q}_{1,1}(\mathbf{t})$ is then given as $a\mathrm{j}(t_1+t_2)$.

C. Nakagami-m Fading Channels

The Nakagami-m channel model has widely been adopted in wireless communications due to its good accuracy and versatility [15] and includes the classical Rayleigh fading as a special case. The fading parameter m can take on any positive real value from 0.5 to infinity. Smaller values of m correspond to more severe fading, and the case of $m = 1$ reverts to Rayleigh fading.

We consider the case when the parameters m are equal for all $N_t \times N_r$ channels. For arbitrary correlated Nakagami-m fading channels with integer m, the joint CF had been obtained by using the central Wishart distribution with [16, eq. (12)]

$$\phi(\mathbf{t}) = \det\left(\mathbf{I} - \frac{\mathbf{R}}{m}\mathbf{Q}(\mathbf{t})\right)^{-m} = \det\left(\mathbf{I} - \boldsymbol{\Psi}\mathbf{Q}(\mathbf{t})\right)^{-m} \tag{12}$$

where $\mathbf{R}$ is defined in (6), and $(\mathbf{R}/m) = \boldsymbol{\Psi}$.

IV. Performance Analysis

Having derived the joint CF, we now adapt Zhang and Lu's [11] analytical framework (which was originally developed for RAS) for the problem at hand. The idea is to express the joint pdf of γ_j as the multidimensional Fourier transform of the joint CF. The cumulative distribution function and pdf of the maximum SNR γ among all possible γ_j's are given as [11, eqs. (8) and (9)]

$$F_\gamma(\gamma) = \frac{1}{(2\pi)^N}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\phi(\mathbf{t})\prod_{k=1}^{N}\frac{1-e^{-\mathrm{j}t_k\gamma}}{\mathrm{j}t_k}\,d\mathbf{t} \tag{13}$$

where $\mathbf{t} = (t_1, \ldots, t_N)$, and

$$\begin{aligned}
f_\gamma(\gamma) = \frac{1}{(2\pi)^N}&\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\phi(\mathbf{t})\prod_{k=1}^{N}(\mathrm{j}t_k)^{-1}\\
&\times\sum_{l=1}^{N}(-1)^{l+1}\sum_{b_1+\cdots+b_N=l}\frac{\mathrm{j}T_N}{\exp(\mathrm{j}\gamma T_N)}\,d\mathbf{t}
\end{aligned} \tag{14}$$

where $N = \binom{N_t}{L_t}\cdot\binom{N_r}{L_r}$, $T_N = b_1t_1 + \cdots + b_Nt_N$, and $b_1, \ldots, b_N$ are binary variables that take the values of 0 or 1. In (13), $\phi(\mathbf{t})$ is the joint CF for γ_j, which has already been derived in the previous section.

A. BER Analysis

With the derived joint CF, the pdf of the output SNR γ of T-RAS can be obtained from (14), and the average error probability can be expressed as the same expression as in [11], i.e.,

$$\overline{P}_e = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \phi(\mathbf{t}) w(\mathbf{t})\, d\mathbf{t} \tag{15}$$

with

$$w(\mathbf{t}) = \frac{1}{(2\pi)^N} \int_0^{\infty} P_e(\gamma) \prod_{k=1}^{N} (\mathrm{j}t_k)^{-1} \times \sum_{l=1}^{N} (-1)^{l+1} \sum_{b_1+\cdots+b_N=l} \frac{\mathrm{j}T_N}{\exp(\mathrm{j}\gamma T_N)}\, d\gamma \tag{16}$$

where $P_e(\gamma)$ is the conditional error probability for either the BER or the SER, given instantaneous output SNR γ. In (15), the first factor $\phi(\mathbf{t})$ in the integrand is the joint CF of γ_j depending solely on the channel characters and the number of selected and available transmit and receive antennas. The second factor $w(\mathbf{t})$ is the weighting function, which depends only on the modulation scheme. For a given modulation scheme operating over a specified environment, these two factors need to be determined, and (15) yields the average error probability.

The conditional BER of M-ary square amplitude modulation (M-QAM) with Gray mapping can be represented as a sum of $(\sqrt{M}-1)$ Q functions as [17], [18]

$$P_{e|\mathrm{BER}}(\gamma) = \sum_{i=1}^{\sqrt{M}-1} a_i Q\left(\sqrt{b_i \gamma}\right) \tag{17}$$

where the coefficients a_i and b_i depend on the constellation size M. Inserting (17) into (16) yields (similar to [19, eq. (12)])

$$w(\mathbf{t}) = \sum_{i=1}^{\sqrt{M}-1} \frac{a_i}{2(2\pi)^N} \prod_{k=1}^{N} (\mathrm{j}t_k)^{-1} \times \left(1 + \sum_{l=1}^{N} (-1)^l \sum_{b_1+\cdots+b_N=l} \sqrt{\frac{b_i}{b_i + 2\mathrm{j}T_N}}\right). \tag{18}$$

B. SER Analysis

By defining $P_e(\gamma)$ as the conditional SER on the output SNR, the average SER can be derived. The SER can also be calculated by the well-known MGF-based approach [20]. From the pdf of γ in (14), the MGF of the output SNR can be derived as

$$\begin{aligned} M_\gamma(s) &= \int_0^{\infty} e^{-s\gamma} f_\gamma(\gamma)\, d\gamma \\ &= \frac{1}{(2\pi)^N} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \phi(\mathbf{t}) \prod_{k=1}^{N} (\mathrm{j}t_k)^{-1} \times \sum_{l=1}^{N} (-1)^{l+1} \sum_{b_1+\cdots+b_N=l} \frac{\mathrm{j}T_N}{s + \mathrm{j}T_N}\, d\mathbf{t}. \end{aligned} \tag{19}$$

With the MGF of γ, the SER of M-ary phase-shift keying and M-QAM can be calculated (see [20]).

V. Numerical Results

The performance analysis derived in Section IV is validated first. The 4-ary QAM constellation is used for all numerical examples.

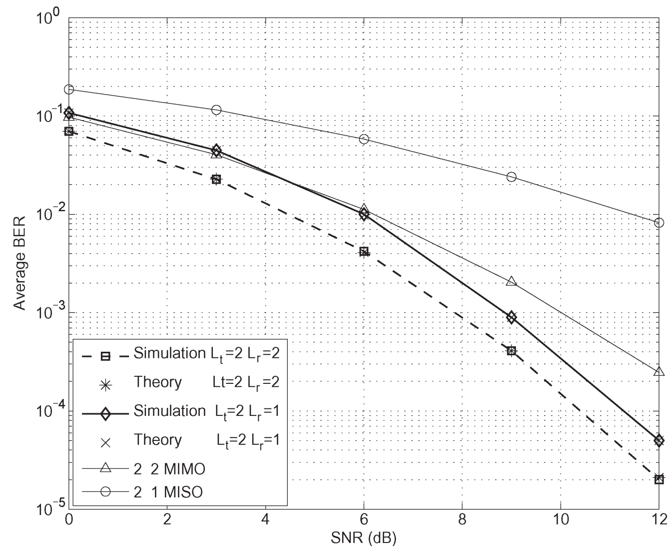

Fig. 1. Average BER versus transmit SNR over independent Rayleigh fading channels.

Fig. 1 depicts the simulation and theory results of the average BER in T-RAS MIMO systems over independent Rayleigh fading. The Alamouti space-time code is employed. System 1 (TAS) selects $L_t = 2$ out of $N_t = 3$ transmit antennas and $L_r = 2$ out of two receive antennas, and System 2 (T-RAS) chooses $L_t = 2$ out of three transmit antennas and $L_r = 1$ out of two receive antennas. The integrals in (15) are approximated with a truncated multidimensional Riemann sum. For example, the threefold integral takes about 10 min for an accuracy of up to the 10^{-6} level when the transmit SNR $\rho = 12$ dB. For the same accuracy level, the Monte Carlo simulation needs random samples on the order of 10^6.

In Fig. 1, the theoretical results of the both systems match extremely well with the simulation results. The BERs of 2×2 MIMO and 2×1 multiple-input–single-output systems are plotted for comparison. Both System 1 and System 2 achieve full diversity order. System 1 gains 2 dB over the 2×2 MIMO system with one more added available transmit antenna. System 2 incurs 1-dB performance loss to System 1 by selecting only one receive antenna from two available receive antennas, with the advantage of lower complexity at the receiver side. However, with the joint T-RAS scheme, System 2 gains 1 dB over the 2×2 MIMO system with fewer active receive antennas. This 1-dB gain is due to transmit antenna selection.

The BER of antenna selection in a MIMO system with four available transmit antennas and three available receive antennas is shown in Fig. 2. OSTBC is taken from [21, eq. (41)]. The theoretical results again match well with the simulation results.

VI. Conclusion

This paper has analyzed the performance of MIMO systems with joint T-RAS. The main technical challenge is the correlation between the different subset antenna selections, arising not only from antenna selection but from the spatial correlation (if any) as well. The solution is to derive the pdf of the maximum output SNR as a function of the joint CF of all possible output SNRs. The joint CF can be related to a quadratic form in the complex Gaussian variables and has a closed-form expression in many cases. The pdf is then given by an N-dimensional integral, where N is the number of all possible output SNRs. Important performance measures, including the average BER and the average SER, have been derived. The simulation results illustrate the effect of the antenna array configuration and the operating

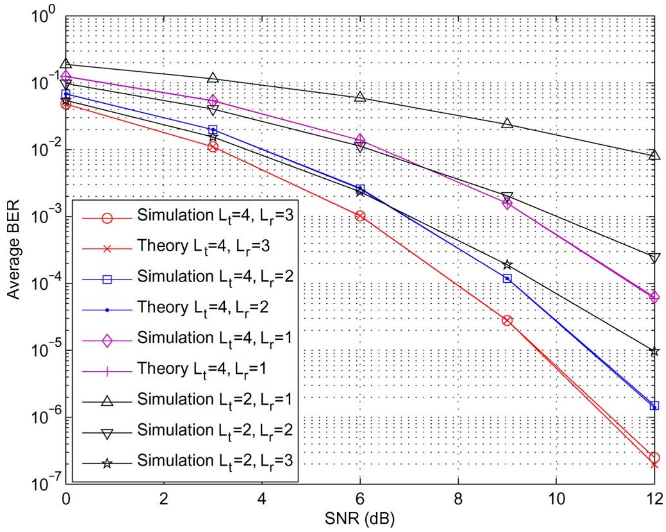

Fig. 2. Average BER versus transmit SNR over independent Rayleigh fading channels with $N_t = 4$ and $N_r = 3$.

environment on the average BER performance through the correlation coefficient. Our framework can be applied in a wide range of channel models such as correlated Rayleigh, Nakagami-m, and Rician fading channels. Moreover, RAS and TAS can be obtained as special cases of T-RAS. Future extensions may include analyzing the impact of feedback errors on the overall system performance.

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