Relay Based Cooperative Spectrum Sensing in Cognitive Radio Networks

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***Abstract*—In this paper[1], we exploit cooperative spectrum sensing technique for applications in a relay based cognitive radio network. Relays are assigned in cognitive radio networks to transmit the primary user's signal to a cognitive coordinator. This research is focused on the detection of primary user in single or multiple cognitive relay scenarios. The performance of energy detector is analyzed for independent Rayleigh fading channels. False alarm and detection probabilities are derived theoretically with or without direct communication between the primary user and the cognitive coordinator. An upper bound is also given for detection probability. Our analysis is validated by numerical and simulation results.**

***Index Terms*—Cognitive radio, cooperative spectrum sensing, energy detection, relay.**

I. Introduction

Radio spectrum is an expensive and limited resource in wireless communications. Surprisingly it turns out that licensed users (termed *primary users*) rarely utilize all the assigned frequency bands at all the time. Consequently, spectrum holes exist, which are frequency bands not occupied by primary users at a certain time and a certain location. The resulting spectral inefficiency has motivated cognitive radio technology, an emerging novel concept in wireless access. Cognitive radio represents a much broader paradigm where many aspects of communication systems can be improved via cognition. The key features of a cognitive transceiver are radio environment awareness and spectrum intelligence [1]. Intelligence can be achieved through learning the spectrum environment and adapting transmission parameters. For instance, unlicensed users (termed *secondary users* or *cognitive users*) can first detect the activities of primary users, and get access to the spectrum if no primary activities are detected. So dynamic spectrum access [2], [3], [4], [5] can be achieved.

In cognitive radio networks, the primary users should be protected as much as possible. This task is usually fulfilled through spectrum sensing. Thus sensing accuracy is important for avoiding interference to primary users. Traditionally three methods can be used to perform spectrum sensing [6]: energy detector (non-coherent detection through received energy), matched filter (coherent detection through maximization of the signal-to-noise ratio), and cyclostationary feature detection (exploitation of the inherent periodicity of primary signals). Among them, the energy detector is the most popular method.

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To improve the spectrum sensing accuracy, cooperative sensing can help, benefiting from information exchange among secondary users. In [7], an optimal spectrum sensing framework is introduced by considering both spectrum efficiency and interference avoidance. The benefits of sensing cooperation in cognitive radio are illustrated in [8], [9] for both two-user and multiple-user networks. Reduction of detection time and increase of overall agility are observed.

In this paper, we propose a relay based cooperative spectrum sensing in cognitive radio networks. The idea is to utilize relay nodes to convey the signal transmitted from the primary user to a cognitive coordinator, which will make estimation of the presence or absence of primary activities. The cognitive coordinator uses an energy detector to make estimation. Cognitive relays are operated in an amplify-and-forward mode with variable-gain [10]. Note that instantaneous channel gain information is available for the channel from the primary user to each relay, and from each relay to the cognitive coordinator.

II. System Model

A. Channel Model

The wireless network is assumed to operate over independent and not necessarily identically distributed Rayleigh fading channels. h_{xy} is the fading coefficient for the $X \to Y$ link, and the magnitude of h_{xy} is with the probability density function (pdf) given [11] by $f_{|h_{xy}|}(t) = 2te^{-t^2},\ t \geq 0$, where $\mathbb{E}(|h_{xy}|^2) = 1$. Here $\mathbb{E}(\cdot)$ represents mathematical expectation. Additive white Gaussian noise (AWGN), denoted w_x at node X, is assumed, which is a circularly symmetric complex Gaussian random variable with mean zero and variance N_0 (i.e., $w_x \sim \mathcal{CN}(0, N_0)$).

B. Cooperative Scheme

We consider a relay-based spectrum sensing. A number, n, of cognitive relays (named $r_1, r_2, ..., r_n$) are added in the cognitive radio network, as shown in Fig. 1. As the primary user starts using the band, cognitive radios receive the signal of the primary user. Therefore in the first phase, all cognitive relays listen to the primary user signal. Instead of making individual hard decision about the presence of the primary user, relay-based cognitive radios simply amplify and retransmit the noisy version of the received signals to the cognitive coordinator in the second phase. Each communication between cognitive relay and cognitive coordinator occurs in orthogonal channels to avoid the inter-channel interference. In orthogonal

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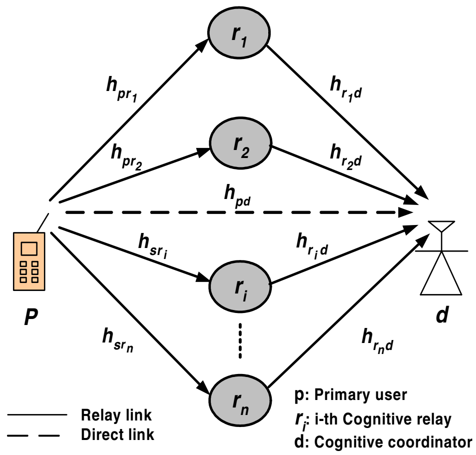

Fig. 1. Illustration of a cooperative network with cognitive relays.

channels, the cognitive coordinator receives independent signals from the primary user and cognitive relays based on time division multiple access (TDMA). The cognitive coordinator is equipped with the energy detector which compares the received signal strength with a pre-defined threshold. Based on the decision, the cognitive coordinator informs the cognitive radios of the presence or absence of primary user's activities. It is assumed that the primary user is not affected by cognitive radio transmission.

In this research, we consider a dual-hop topology. Although a multi-hop scheme can cover a larger area, it takes longer time to relay the primary user's signal to the cognitive coordinator. Therefore, detection time would be increased. Further, errors could be propagated from one hop to another.

In the following, the cases with a single cognitive relay and multiple cognitive relays are discussed, respectively.

C. Single Cognitive Relay

In the case with a single relay denoted r, we have three nodes, i.e., the primary user, the cognitive relay, and the cognitive coordinator. The cognitive relay continuously monitors the signal received from the primary user. The received signal by the cognitive relay, denoted y_{pr}, is given by $y_{pr} = \theta x h_{pr} + w_r$, where θ denotes the primary activity indicator, which is equal to 1 at the presence of primary activity, or equal to 0 otherwise, x is the transmitted signal from the primary user, h_{pr} is the channel gain between the primary user and relay, and w_r is the noise signal at the cognitive relay. The cognitive relay acts as a variable gain amplify-and-forward relay (AF), which is more practical than decode-and-forward or blind/semi-blind relay operation. The cognitive relay has a transmission power constraint E_r. Therefore, the amplification factor, β_r is given by $|\beta_r|^2 = \frac{E_r}{\theta^2 E_p |h_{pr}|^2 + N_0}$, where E_p is transmitted signal power from the primary user. Thus, the received signal at the cognitive coordinator, denoted y_{rd}, is given by

$$\begin{aligned} y_{rd} &= \sqrt{\beta_r} y_{pr} h_{rd} + w_d \\ &= \theta \sqrt{\beta_r} h_{pr} h_{rd} x + \sqrt{\beta_r} h_{rd} w_r + w_d \\ &= \theta h x + w, \end{aligned} \qquad (1)$$

where h_{rd} is the channel gain between the relay and the cognitive coordinator, w_d is the noise signal at the cognitive coordinator, $h = \sqrt{\beta_r} h_{pr} h_{rd}$, and $w = \sqrt{\beta_r} h_{rd} w_r + w_d$ is the total effective noise at the cognitive coordinator, which can be modeled as $w|_{h_{pr}, h_{rd}} \sim \mathcal{CN}\big(0, (\beta_r |h_{rd}|^2 + 1) N_0\big)$.

The received signal at the cognitive coordinator follows a binary hypothesis:

$$y_{rd} = \begin{cases} w & : H_0 \qquad \theta = 0, \\ hx + w & : H_1 \qquad \theta = 1. \end{cases} \qquad (2)$$

As described in [12], [13], the received signal is first pre-filtered by an ideal bandpass filter with center frequency f_c and bandwidth W in order to normalize the noise variance. The output of this filter is then squared and integrated over a time interval T to finally produce a measure of the energy of the received waveform. The output of the integrator, denoted Y, acts as the test statistic. The pdf of Y is given [13] by

$$f_Y(y) = \begin{cases} \frac{1}{2^u \Gamma(u)} y^{u-1} e^{-\frac{y}{2}} & : H_0 \\ \frac{1}{2} \left(\frac{y}{2\gamma}\right)^{\frac{u-1}{2}} e^{-\frac{2\gamma + y}{2}} I_{u-1}(\sqrt{2\gamma y}) & : H_1 \end{cases}$$

where $\Gamma(\cdot)$ is the gamma function, $I_v(\cdot)$ is the v^{th} order modified Bessel function of the first kind, and $u = TW$ where T and W are chosen to restrict u to an integer value.

The total end-to-end signal-to-noise radio (SNR), denoted γ, is given [10] by

$$\gamma = \frac{\gamma_{pr} \gamma_{rd}}{\gamma_{pr} + \gamma_{rd} + 1}, \qquad (3)$$

where $\gamma_{pr} = |h_{pr}|^2 E_p / N_0$ and $\gamma_{rd} = |h_{rd}|^2 E_r / N_0$ are SNRs of the links from the primary user to the cognitive relay and from the cognitive relay to the cognitive coordinator, respectively.

D. Multiple Cognitive Relays

In the case with multiple cognitive relays, we have n cognitive relays between the primary user and the cognitive coordinator, as shown in Fig. 1. h_{pr_i}, h_{pd} and $h_{r_i d}$ denote the channel gains between the primary user and the i^{th} cognitive relay r_i, between the primary user and the cognitive coordinator, and between the i^{th} cognitive relay r_i and the cognitive coordinator, respectively. All cognitive relays simultaneously receive primary user's signal through independent fading channels. Each cognitive relay (say relay r_i) amplifies the received primary signal by an amplification factor β_{r_i} given as $|\beta_{r_i}|^2 = \frac{E_{r_i}}{\theta^2 E_p |h_{pr_i}|^2 + N_0}$ and forwards to the cognitive coordinator. All the cognitive relays use mutually orthogonal channels to forward the received primary signal. Such orthogonal channels can be realized by using TDMA technology. The received signals at the cognitive coordinator can then be considered as independent copies through orthogonal channels. Therefore, the maximal ratio combining (MRC) at the cognitive coordinator can be implemented, and after an integrator, the final test statistic Y is obtained. The total end-to-end SNR is given by $\gamma = \sum_{i=1}^{n} \frac{\gamma_{pr_i} \gamma_{r_i d}}{\gamma_{pr_i} + \gamma_{r_i d} + 1}$ where γ_{pr_i} and $\gamma_{r_i d}$ are SNRs of the links from the primary user to the cognitive relay r_i and from the cognitive relay r_i to the cognitive coordinator, respectively.

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III. Detection Analysis

A. Energy Detector

At the cognitive coordinator, the test statistic Y is compared with the predefined threshold value λ. The probabilities of detection (P_d) and false alarm (P_f) can be generally evaluated by $Pr(Y > \lambda | H_1)$ and $Pr(Y > \lambda | H_0)$ respectively to yield [13]

$$P_f = \frac{\Gamma(u, \frac{\lambda}{2})}{\Gamma(u)} \tag{4}$$

and

$$P_d = Q_u(\sqrt{2\gamma}, \sqrt{\lambda}), \tag{5}$$

where $Q_u(\cdot,\cdot)$ is the generalized Marcum-Q function and $\Gamma(\cdot,\cdot)$ is the upper incomplete gamma function which is defined by the integral form $\Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}dt$ and $\Gamma(a,0) = \Gamma(a)$. Probability of false alarm P_f can easily be calculated using (4).

B. Average Detection Probability

Generalized Marcum-Q function can be written as a circular contour integral within the contour radius $r \in [0, 1)$. Therefore (5) can be re-written [14] as

$$P_d = \frac{e^{-\frac{\lambda}{2}}}{j2\pi} \oint_C \frac{e^{(\frac{1}{z}-1)\gamma + \frac{\lambda}{2}z}}{z^u(1-z)} dz, \tag{6}$$

where Δ is a circular contour of radius $r \in [0, 1)$. The moment generating function (MGF) of γ is $M_\gamma(s) = \mathbb{E}(e^{-s\gamma})$. Thus, the average detection probability is given by

$$\bar{P}_d = \frac{e^{-\frac{\lambda}{2}}}{j2\pi} \oint_C M_\gamma\left(1 - \frac{1}{z}\right) \frac{e^{\frac{\lambda}{2}z}}{z^u(1-z)} dz. \tag{7}$$

Therefore, it is important to know the MGF of γ. Based on the closed-form MGF for Nakagami-m fading [15], we can derive the MGF of γ in (3), denoted $M_\gamma(s)$, with a single relay over Rayleigh fading as

$$\begin{aligned} M_\gamma(s) = \quad & 1 - s\sum_{k=0}^{2}\binom{2}{k}(-1)^{2-k}\frac{d^{2-k}}{dt^{2-k}}\left[\frac{e^{3t/2}}{\gamma_{pr_i}\gamma_{r_id}}\right. \\ & \cdot\Gamma\left(-1, \frac{t-\sqrt{t^2-\frac{4}{\gamma_{pr_i}\gamma_{r_id}}}}{2}\right) \\ & \left.\left.\cdot\Gamma\left(-1, \frac{t+\sqrt{t^2-\frac{4}{\gamma_{pr_i}\gamma_{r_id}}}}{2}\right)\right]\right|_{t=s+\frac{\gamma_{pr_i}+\gamma_{r_id}}{\gamma_{pr_i}\gamma_{r_id}}}. \end{aligned} \tag{8}$$

A closed-form solution for the exact $\bar{P}_d$ seems analytically intractable. However, efficient numerical algorithms are available to evaluate a circular contour integral. We use Mathematica software that provides adaptive algorithms to recursively partition the integration region. With a high precision level, the numerical algorithm can provide an efficient and accurate solution for (7).

C. Upper Bound of $\bar{P}_d$

Now we proceed to derive an upper bound for the average detection probability $\overline{P_d^{up}}$. The total SNR γ can be upper bounded by γ_{up} as

$$\gamma \le \gamma_{\text{up}} = \sum_{i=1}^{n} \gamma_i^{\min}, \tag{9}$$

where $\gamma_i^{\min} = \min(\gamma_{pr_i}, \gamma_{r_id})$. Therefore, MGF of γ_{up} can be written as

$$M_{\gamma_{\text{up}}}(s) = \prod_{i=1}^{n} M_{\gamma_i^{\min}}(s), \tag{10}$$

for independent links. $M_{\gamma_{\text{up}}}(s)$ can be derived as

$$M_{\gamma_{\text{up}}}(s) = \prod_{i=1}^{n} \frac{\bar{\gamma}_{pr_i} + \bar{\gamma}_{r_id}}{\bar{\gamma}_{pr_i}\bar{\gamma}_{r_id}} \frac{1}{\left(s + \frac{\bar{\gamma}_{pr_i}+\bar{\gamma}_{r_id}}{\bar{\gamma}_{pr_i}\bar{\gamma}_{r_id}}\right)}, \tag{11}$$

where $\bar{\gamma}_{pr_i} = \frac{\mathbb{E}(|h_{pr_i}|^2)E_p}{N_0}$ and $\bar{\gamma}_{r_id} = \frac{\mathbb{E}(|h_{r_id}|^2)E_{r_i}}{N_0}$ are average SNRs for links from the primary user to cognitive relay r_i and from cognitive relay r_i to the cognitive coordinator, respectively. We refer readers to the Appendix for the details. Substituting (11) to (7), $\overline{P_d^{up}}$ can be re-written as

$$\overline{P_d^{up}} = \frac{e^{-\frac{\lambda}{2}}}{j2\pi} \oint_C g(z)dz, \tag{12}$$

where
$g(z) = \frac{e^{\frac{\lambda}{2}z}}{z^{u-n}(1-z)} \prod_{i=1}^{n} \frac{1-\Delta_i}{z-\Delta_i}$, with $\Delta_i = \frac{\gamma_{pr_i}\gamma_{r_id}}{\gamma_{pr_i}+\gamma_{r_id}+\gamma_{pr_i}\gamma_{r_id}}$.

Since the Residue Theorem [16] in complex analysis is a powerful tool to evaluate line integrals of functions over closed curves and can often be used to compute real integrals as well, it is used in this research to evaluate the integral in (12). Two cases need to be considered.

1) When $u > n$: There are $(u - n)$ poles at origin and n poles for Δ_i's ($i = 1, .., n$) in radius $r \in [0, 1)$. Therefore, $\overline{P_d^{up}}$ can be derived as

$$\overline{P_d^{up}} = e^{-\frac{\lambda}{2}} \left(\text{Res}\,(g; 0) + \sum_{i=1}^{n} \text{Res}\big(g; \Delta_i\big) \right) \tag{13}$$

where $\text{Res}\,(g; 0)$ and $\text{Res}\big(g; \Delta_i\big)$ denote the residue of the function $g(z)$ at origin and Δ_i, respectively.

2) When $u \le n$: There are n poles at Δ_i's ($i = 1, .., n$) in radius $r \in [0, 1)$. Therefore, $\overline{P_d^{up}}$ can be derived as

$$\overline{P_d^{up}} = e^{-\frac{\lambda}{2}} \sum_{i=1}^{n} \text{Res}\big(g; \Delta_i\big). \tag{14}$$

We refer readers to the Appendix for the details of the derivation of $\text{Res}(g; \cdot)$.

D. Incorporation with the Direct Link

In preceding subsections, the cognitive coordinator receives only signals coming from cognitive relays. Actually it can also receive the signal of the primary user when the primary user starts to utilize its band. The detection of primary activities becomes reliable if the cognitive coordinator is close to the

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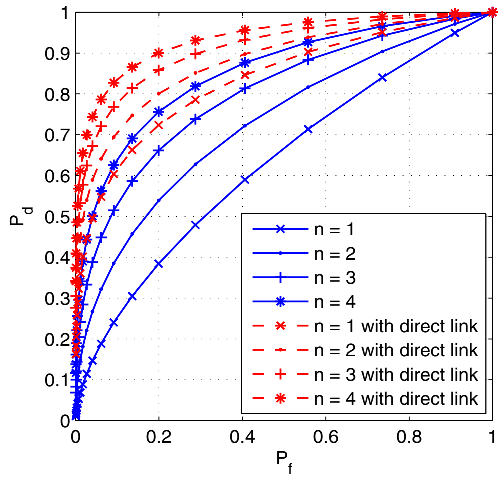

Fig. 2. Variation of P_d with P_f for different number of cognitive relays.

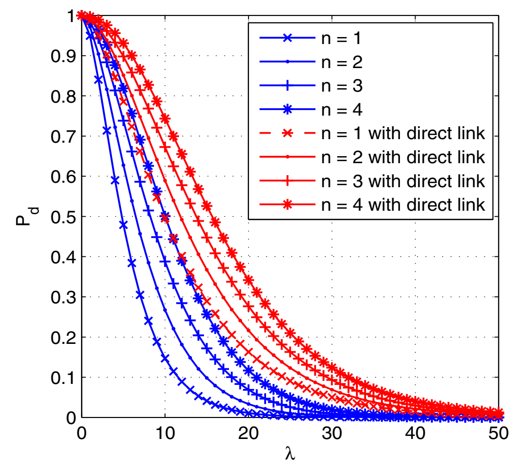

Fig. 3. Variation of P_d with λ for different number of cognitive relays.

primary user, benefiting from the strong direct link. Then the total SNR at the cognitive coordinator can be given as

$$\gamma^{\dagger} = \gamma_d + \sum_{i=1}^{n} \gamma_{r_i}, \tag{15}$$

where $\gamma_d = \frac{|h_{pd}|^2 E_p}{N_0}$ is the direct path SNR, and $\gamma_{r_i} = \frac{\gamma_{r_i d}\gamma_{pr_i}}{\gamma_{r_i d}+\gamma_{pr_i}+1}$ is the relayed path SNR from relay r_i. Therefore corresponding MGF of $\gamma^{\dagger}$, assuming independent fading channels, can be written as $M_{\gamma^{\dagger}}(s) = M_{\gamma_d}(s)\prod_{i=1}^{n} M_{\gamma_{r_i}}(s)$ where $M_{\gamma_d}(s)$ is given by $\frac{1}{(1+\bar{\gamma}_d s)}$, $\overline{\gamma_d} = \mathbb{E}(\gamma_d)$, and $M_{\gamma_{r_i}}(s)$ is from (8). As in preceding subsections, we can find accurate average detection probability using numerical integration.

In this case, $g(z)$ in (12) can be written as

$$g(z) = \frac{e^{\frac{\lambda}{2}z}}{z^{u-n-1}(1-z)} \left(\frac{1-\Delta}{z-\Delta}\right) \prod_{i=1}^{n} \frac{1-\Delta_i}{z-\Delta_i}. \tag{16}$$

where $\Delta = \frac{\bar{\gamma}_d}{1+\bar{\gamma}_d}$. When $u > (n+1)$, there are $(u-n-1)$ poles at origin, one pole at Δ and n poles for Δ_i's $(i = 1, .., n)$ in radius $r \in [0, 1)$. Further, a tight upper bound of the detection probability, denoted $\overline{P_d^{\dagger,up}}$, can be derived in closed-form as

$$\overline{P_d^{\dagger,up}} = e^{-\frac{\lambda}{2}} \left(\text{Res}\,(g;0) + \text{Res}\,(g;\Delta) + \sum_{i=1}^{n} \text{Res}\big(g;\Delta_i\big) \right). \tag{17}$$

Where Res $(g;\Delta)$ denotes the residue at Δ of the function $g(z)$ in (16). For $g(z)$ given in (16), Res $(g;0)$ and Res$\big(g;\Delta_i\big)$ can be calculated as previous. Calculation of Res $(g;\Delta)$ is given in the Appendix. When $u \leq (n+1)$, there are one pole at Δ and n poles for Δ_i's $(i = 1, .., n)$ in radius $r \in [0, 1)$. Similarly, $\overline{P_d^{\dagger,up}}$ can be derived in closed-form for this case.

IV. Numerical and Simulation Results

This section provides analytical and simulation results. Note that in all figures in this section, numerical results are presented by curves, while simulation results are represented by discrete marks on the curves. For simulations, the links from the primary user to cognitive relays and from cognitive relays to the cognitive coordinator are independent and identically Rayleigh faded with average SNR being 5 dB. The detection threshold (λ) varies from 0 to 50. The value of u is set to be 2.

Fig. 2 shows the numerical and simulation results regarding how the detection probability P_d changes with the false alarm probability P_f. For the numerical results, the integral formula (7) is used for different cases. Clearly, the numerical results match perfectly with their simulation counterparts, confirming the accuracy of the analysis. As the number of cognitive relays increases, the detection probability also increases. Fig. 2 also shows that a direct path gives a major impact on the detection probability.

Fig. 3 shows the impact of the detection threshold λ on the detection probability P_d. When the detection threshold λ is at a small region (e.g., from 0 to 10 in Fig. 3), the detection probability P_d dramatically decreases when λ increases. Similar to the observation in Fig. 2, more cognitive relays or a direct path tends to increase the detection probability.

Fig. 4 shows the upper bound of detection probability derived in Section III. C. It can be seen that the upper bound is not tight when there is a single cognitive relay. The upper bound is much tighter when a strong direct path can be utilized. This is because, in the upper bound derivation for $\gamma^{\dagger}$ given in (15), the approximation is only for the relay paths, not for the strong direct path.

V. Conclusion

We have studied the relay based spectrum sensing with an energy detector for a cognitive radio network with Rayleigh fading channels. The analysis covers the detection probability and the false alarm probability. The MGF of received SNR of the primary user's signal is utilized to analyze the detection

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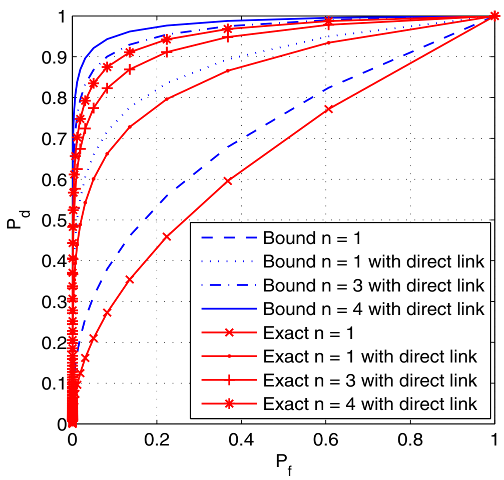

Fig. 4. Upper bound and exact plot between P_d and P_f for different number of cognitive relays.

probability. It is shown that the detection probability increases when the number of cognitive relays increases. Furthermore, direct path communication between the primary user and the cognitive coordinator has a major impact on the detection probability by introducing spatial diversity. A closed-form upper bound expression of the detection probability is derived. This bound becomes tight when there is a strong direct path between the primary user and the cognitive coordinator.

Appendix

A. MGF of $\gamma_i^{\min}$

The cumulative distribution function (cdf) of $\gamma_i^{\min} = \min(\gamma_{pr_i}, \gamma_{r_id})$ is given by

$$\begin{aligned} F_{\gamma_i^{\min}}(t) &= 1 - \Pr(\gamma_{pr_i} > t, \gamma_{r_id} > t) \\ &= F_{\gamma_{pr_i}}(t) + F_{\gamma_{r_id}}(t) - F_{\gamma_{pr_i},\gamma_{r_id}}(t,t) \\ &= 1 - e^{-\left(\frac{\bar{\gamma}_{pr_i}+\bar{\gamma}_{r_id}}{\bar{\gamma}_{pr_i}\bar{\gamma}_{r_id}}\right)t} \end{aligned} \quad (18)$$

where $F_{\gamma_{pr_i}}(t)$ and $F_{\gamma_{r_id}}(t)$ are the cdf of γ_{pr_i} and γ_{r_id}, and $F_{\gamma_{pr_i},\gamma_{r_id}}(t,t)$ is the joint cdf of γ_{pr_i} and γ_{r_id}.

By the first-order derivative of $F_{\gamma_i^{\min}}(t)$, the pdf of $\gamma_i^{\min}$ is given by

$$f_{\gamma_i^{\min}}(t) = \left(\frac{\bar{\gamma}_{pr_i}+\bar{\gamma}_{r_id}}{\bar{\gamma}_{pr_i}\bar{\gamma}_{r_id}}\right) e^{-\left(\frac{\bar{\gamma}_{pr_i}+\bar{\gamma}_{r_id}}{\bar{\gamma}_{pr_i}\bar{\gamma}_{r_id}}\right)t}. \quad (19)$$

Therefore, the MGF of $\gamma_i^{\min}$, $M_{\gamma_i^{\min}}(s) = \mathbb{E}(e^{-\gamma_i^{\min}s})$ is given by

$$M_{\gamma_i^{\min}}(s) = \left(\frac{\bar{\gamma}_{pr_i}+\bar{\gamma}_{r_id}}{\bar{\gamma}_{pr_i}\bar{\gamma}_{r_id}}\right) \frac{1}{\left(s + \frac{\bar{\gamma}_{pr_i}+\bar{\gamma}_{r_id}}{\bar{\gamma}_{pr_i}\bar{\gamma}_{r_id}}\right)}. \quad (20)$$

B. Calculation of Residue $Res(g;\cdot)$

If $g(z)$ has the Laurent series representation, i.e., $g(z) = \sum_{i=-\infty}^{\infty} a_n(z-z_0)^n$ for all z, the coefficient a_{-1} of $(z-z_0)^{-1}$ is the residue of $g(z)$ at z_0 [16]. In eq. (13), Res$(g;0)$ can be evaluated as

$$\mathrm{Res}\,(g;0) = \frac{1}{(u-n-1)!}\left(\frac{d^{u-n-1}}{dz^{u-n-1}}\frac{e^{\frac{\lambda}{2}z}}{(1-z)}\prod_{i=1}^{n}\frac{1-\Delta_i}{z-\Delta_i}\right)\Bigg|_{z=0}.$$

Similarly, Res$(g;\Delta_j)$ in eq. (13) can be evaluated as

$$\mathrm{Res}\,(g;\Delta_i) = \frac{e^{\frac{\lambda}{2}\Delta_i}}{\Delta_i^{u-n}(1-\Delta_i)}\prod_{j=1,i\neq j}^{n}\frac{1-\Delta_j}{(\Delta_i-\Delta_j)}.$$

Res$(g;\Delta)$ in eq. (17) can be evaluated as

$$\mathrm{Res}\,(g;\Delta) = \frac{e^{\frac{\lambda}{2}\Delta}}{\Delta^{u-n-1}(1-\Delta)}\prod_{i=1}^{n}\frac{1-\Delta_i}{\Delta-\Delta_i}.$$

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