Performance analysis of channel inversion over MIMO channels

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***Abstract*—The distribution of the received signal power Λ of a multiple-input multiple-output system, with the associated 2×2 complex Wishart matrix having n degrees of freedom, is investigated under the channel inversion power allocation scheme. Exact closed-form expressions are derived for the probability density function and the cumulative distribution function of Λ. The analysis covers systems with transmit-receive antennas $\{N_t, N_r| \min(N_t, N_r) = 2, \max(N_t, N_r) \geq 2\}$, and independent and identically distributed Rayleigh fading links between each antenna pair. We derive (i) the distribution of Shannon capacity of the multiple-input multiple-output channel (ii) average symbol error rate for a class of modulation schemes that includes Binary Phase Shift Keying.**

I. Introduction

Multiple-input multiple-output (MIMO) wireless technology [1] relies on multiple antenna nodes. Moreover, the use of special techniques to exploit the space-dimension results in diversity and/ or multiplexing advantages unparalleled in conventional single-antenna systems. As traditional systems that exploit the time and frequency dimensions become inadequate for meeting the ever increasing user demand for higher data rates, MIMO technology has been widely investigated and deployed (e.g. IEEE 802.11n standard).

The MIMO channel between a multiple-antenna transmitter and a multiple-antenna receiver may be treated and manipulated as a set of 'virtual' channels. With proper pre-processing at the transmitter and signal reconstruction at the receiver, these channels can be independently coded, modulated and power allocated for. This not only allows various technologies used in single-antenna systems to be adapted for MIMO, but also allows MIMO channel capacity to be maximized by using water-filling power allocation method.

The analysis of the MIMO channels dates back to Telatar [2], who derived the capacity of a MIMO channel with independent and identically distributed (i.i.d.) Rayleigh fading links between each transmitter-receiver antenna pair under equal power allocation across the virtual channels. Reference [2] is a pioneering work that initiated the use of random matrix theory [3] (e.g. distribution of eigenvalues in a Wishart matrix [4]) in analyzing MIMO channels and systems. But the extension of [2] for performance analysis under other power allocation schemes seems to be lacking. Thus, we seek to address the performance analysis of the channel inversion power allocation scheme. Moreover, this problem requires the distribution of the reciprocal of the trace of inverse Wishart matrix. Such distributions do not appear to be published in the vast literature on the eigenvalues of Wishart matrices.

'Channel inversion' [5] is the simplest power allocation scheme a transmitter may implement to exploit the channel-state-information (CSI). A transmitter can obtain CSI, typically through explicit feedback from the receiver or based on the reciprocity of wireless channels. When the channel is slow varying, CSI can be used to adjust the transmit parameters so that the quality-of-service criteria are met. The use of channel inversion strategy in MIMO systems has already been examined [6]–[9]; however, a comprehensive performance analysis has not been forthcoming.

Given a set of channels and a constraint on total power to be allocated among them, the capacity under channel inversion scheme is known to be inferior to that of the optimal 'water filling' solution. Nevertheless, channel inversion has other merits, making it viable for certain broadband applications.

- Unlike water filling, channel inversion can be implemented in a non-iterative manner; making it suitable for applications with tight delay constraints (e.g. [10, p.522]).
- Channel inversion maintains the distribution of received signal-to-noise ratio (SNR) identical for all the virtual channels. This allows the same modulation, coding schemes to be used in all of them to achieve capacity.
- Channel inversion, in general, requires only linear, unitary operations at the receiver for signal reconstruction [9]. This prevents noise enhancement which could arise if non-unitary operations are employed for signal reconstruction (e.g. zero forcing) at the receiver.
- For certain MIMO channels, channel inversion can be implemented through transmitter pre-processing only [7]. This simplifies the receiver greatly.
- When the number of transmit antennas are higher than the number of receiver antennas, channel inversions boils down the MIMO channels to parallel, non-interfering multiple-input single-output channels. This is appealing for multi-user MIMO downlink. 'Regularized channel inversion', a variant of channel inversion that accepts certain amount of interference so as to enhance channel capacity has been proposed [11], [12] in this regard.

The main contributions of our paper are as follows. We obtain closed-form expressions for the probability density function (pdf) and the cumulative distribution function (cdf)

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of the received SNR under channel-inversion for each virtual channel, for channel-matrices having a rank of 2. The distribution of the Shannon capacity and the average symbol error rate for a certain class of modulation schemes are derived. Numerical results are also provided.

The paper is organized as follows: Section II presents the system model and the mathematical formulation. In section III, the analytical results are verified against simulation results using two applications, on selected MIMO configurations. Section IV concludes the paper stating the possible extensions to our analysis. Proof of the results is annexed.

Notations: $P[\,\cdot\,]$ is the *probability assignment*; and $\mathcal{E}_X\{\,\cdot\,\}$ is the *expectation operator* with respect to random variable X. Given a random variable X, its pdf, cdf and complimentary cdf are given by $f_X(x)$, $F_X(x)$ and $\bar{F}_X(x)$. $K_\nu(\,\cdot\,)$ denotes the *Modified Bessel function of the second kind* [13, 9.6] of order ν; $\mathcal{G}^{m,n}_{p,\,q}\left(\cdot\,\middle|\,\begin{matrix} a_1,\ldots,a_n,a_{n+1},\ldots,a_p \\ b_1,\ldots,b_m,b_{m+1},\ldots,b_q \end{matrix}\right)$ represents the *Meijer G function* [14, p.419]. $\mathcal{Q}(\cdot)$ denotes the *Q-function* [13, 26.2.3], while $\Gamma(\,\cdot\,)$ denotes the Gamma function [13, 6.1]. $\log(\,\cdot\,)$ represents the natural logarithm. The Binomial coefficient $\frac{n!}{k!(n-k)!}$ is designated by $\binom{n}{k}$. The conjugate transpose of a matrix $\mathbf{A}$ is denoted $\mathbf{A}^H$.

II. System Model & Mathematical Formulation

Consider a MIMO system having respectively N_t and N_r transmit and receive antennas. All channels among all transmit-receive antenna pairs undergo i.i.d. Rayleigh fading. Resulting MIMO channel can be represented by a matrix $\mathbf{H}$ of i.i.d. complex Gaussian elements, and of dimension $N_t \times N_r$.

The complex Wishart matrix [2] $\mathbf{W}$, whose eigenvalues sufficiently characterize the MIMO channel, is the full-rank matrix defined either as $\mathbf{W} = \mathbf{H}\mathbf{H}^H$ or $\mathbf{W} = \mathbf{H}^H\mathbf{H}$.

We assume the minimum of number of transmit antennas and number of receive antennas to be 2 (Fig. 1). That is $\min(N_t, N_r) = 2$ and $n = \max(N_t, N_r) \geq 2$. It yields a 2×2 complex Wishart matrix $\mathbf{W}$ having n degrees of freedom.

Suppose also that the transmitter has full CSI; and that all the links are affected equally by additive circular-symmetric Gaussian noise having constant power N_0.

Eigenvalues $\{\lambda_1, \lambda_2\}$ of $\mathbf{W}$ represent the received SNR of the two virtual channels present in the system. Transmit power P is allocated using channel inversion (among λ_1 and λ_2, as g_1 and g_2 such that $g_1 + g_2 = P$). It makes $\lambda_1 g_1 = \lambda_2 g_2 = K$, a constant. Thus we have,

$$P = K\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right).$$

Let $\Lambda = \frac{K}{P}$. Then we get,

$$\Lambda = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}. \quad (1)$$

It is noteworthy that Λ is half of the Harmonic mean of λ_1 and λ_2. This connection increases the significance of this analysis outside of the present context.

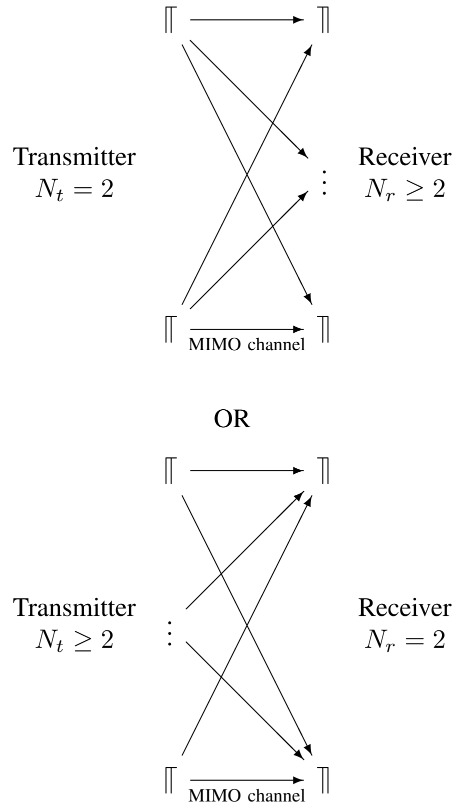

Fig. 1. System Model

We know that the joint pdf of the eigenvalues of $\mathbf{W}$ is given [2] for $\lambda_1, \lambda_2 \geq 0$ by

$$f_{\lambda_1,\lambda_2}(\lambda_1,\lambda_2) = K_1 e^{-(\lambda_1+\lambda_2)}(\lambda_1-\lambda_2)^2\lambda_1^{n-2}\lambda_2^{n-2}, \quad (2)$$

$$\text{where, } K_1 = \frac{1}{2\left\{\Gamma(n+1)\,\Gamma(n-1) - (\Gamma(n))^2\right\}}.$$

The distribution of Λ can be found using (1) and (2).

Theorem 1: the pdf of Λ

Let λ_1, λ_2 be the eigenvalues of a 2×2 complex Wishart matrix having n degrees of freedom. The pdf of Λ in (1) is given by:

$$f_\Lambda(x) = 2K_1 x^{2(n-1)} e^{-2x} \sum_{k=0}^{2n} \binom{2n}{k} \left((n-k-2x)K_{k-n}(2x) + 2xK_{k+1-n}(2x)\right).$$

Proof: See appendix. ■

Theorem 2: the cdf of Λ

Let λ_1, λ_2 be the eigenvalues of a 2×2 complex Wishart matrix having n degrees of freedom. The

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cdf of Λ in (1) is given by:

$$F_\Lambda(x) = \frac{\sqrt{\pi}K_1}{4^{2n-1}} \sum_{k=0}^{2n} \binom{2n}{k} \Bigg[2(n-k)\mathcal{G}_{2,3}^{2,1}\left(4x \middle| \begin{matrix} 1, 2n-0.5 \\ 3n-k-1, n+k-1, 0 \end{matrix}\right) - \mathcal{G}_{2,3}^{2,1}\left(4x \middle| \begin{matrix} 1, 2n+0.5 \\ 3n-k, n+k, 0 \end{matrix}\right) + \mathcal{G}_{2,3}^{2,1}\left(4x \middle| \begin{matrix} 1, 2n+0.5 \\ 3n-k-1, n+k+1, 0 \end{matrix}\right) \Bigg]$$

Proof: See appendix. ∎

III. Results

Given the distribution of Λ, one may obtain various performance analysis results. Here, we present two applications of the results derived in the previous section.

A. The distribution of Shannon capacity

The Shannon capacity of this MIMO channel is given by a random variable C defined as

$$C = \sum_{k=1}^{2} \log_2\left(1 + \frac{g_i\lambda_i}{N_0}\right) = 2\log_2\left(1 + \frac{P}{N_0}\Lambda\right), \tag{3}$$

where P denotes the transmit SNR (transmit power normalized w.r.t. the noise variance), i.e. we assume $N_0 = 1$ without any loss of generality.

The pdf and cdf of capacity C is readily obtainable as follows.

$$F_C(c) = \mathcal{P}\left[2\log_2(1+P\Lambda) \le c\right] = \mathcal{P}\left[\Lambda \le \frac{2^{c/2}-1}{P}\right] = F_\Lambda\left(\frac{2^{c/2}-1}{P}\right) \tag{4}$$

$$f_C(c) = \frac{\mathrm{d}}{\mathrm{d}c}F_\Lambda\left(\frac{2^{c/2}-1}{P}\right) = \frac{\log(2)\,2^{c/2}}{2P} f_\Lambda\left(\frac{2^{c/2}-1}{P}\right) \tag{5}$$

Given a target information rate R, $F_C(R) = P[C \le R]$ gives a useful measure know as the information outage probability.

Validity of the expressions (5) and (4) are verified respectively in Fig. 2 and 3, for the cases: $n \in \{2, 3, 5\}$. They show how well the analytical results compare against corresponding Monte-Carlo simulation results obtained using 100000 sample points. The results agree with the fact that information outage probability decreases with increasing n, even when the rank of the channel matrix remains at 2.

Moreover the *ergodic capacity* $\mathcal{E}_C\{C\}$ of the channel can be numerically computed as

$$\mathcal{E}_C\{C\} = \int_0^\infty c f_C(c)\,\mathrm{d}c = 2\int_0^\infty \log_2(1+Px) f_\Lambda(x)\,\mathrm{d}x,$$

for any given P.

Fig. 4 compares the numerical values thus obtained for ergodic capacity against Monte-Carlo simulation results, for the cases: $n \in \{2, 3, 5\}$. As expected, the ergodic capacity increases logarithmically with the transmit SNR (i.e., appears

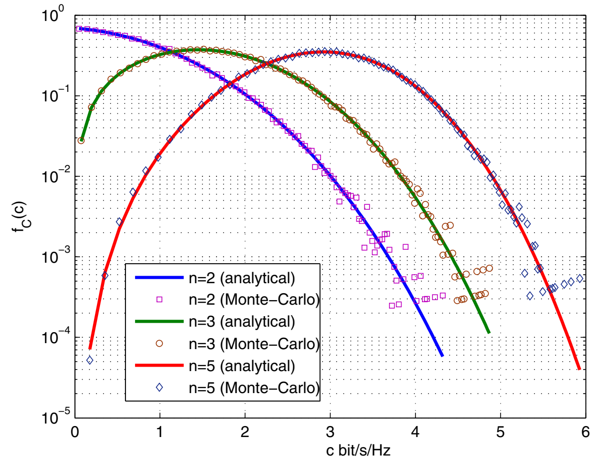

Fig. 2. PDF of capacity C, for $P = 1$

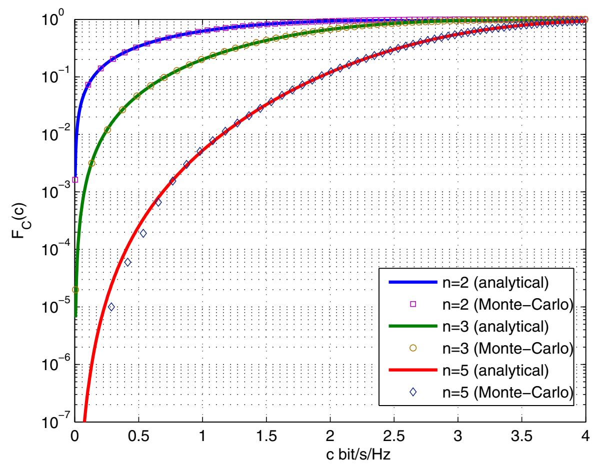

Fig. 3. CDF of capacity C, for $P = 1$

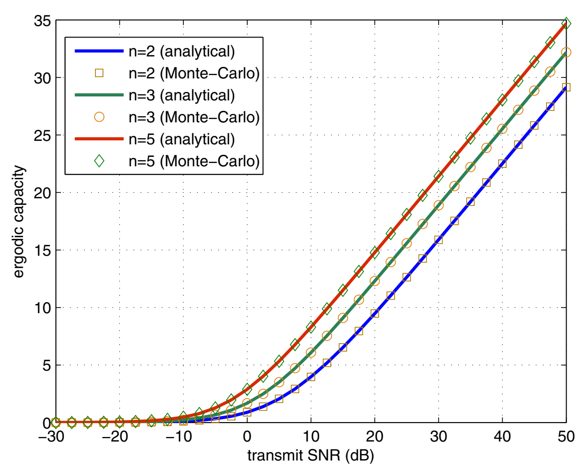

Fig. 4. ergodic capacity

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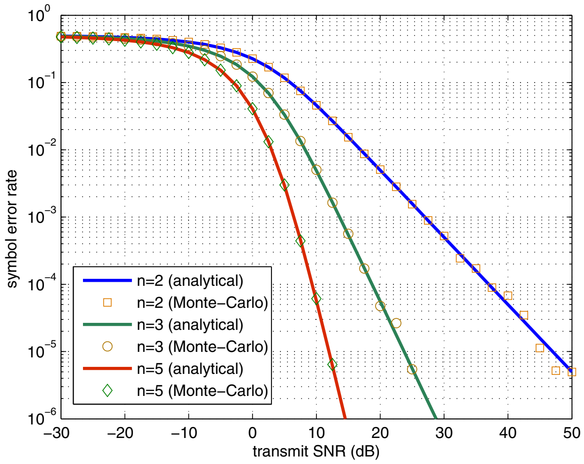

Fig. 5. average symbol error rate (for BPSK: $\mu = \nu = 1$)

as a straight line at high transmit SNR, when SNR is given in dB); and increases (although not-linearly) with n.

B. Average symbol error rate

Average bit error rate is another performance criteria that may be derived from the distribution of Λ.

Suppose now, that data transmitted over the MIMO system analyzed in Section II is modulated using a modulation scheme $\mathcal{M}$ whose symbol error rate expression is given exactly or approximately by

$$P_s = \mathcal{E}_\Lambda\left\{\mu\mathcal{Q}\left(\sqrt{2\nu\Lambda P}\right)\right\}, \tag{6}$$

where P is the transmit SNR; μ and ν are constants dependent on the modulation scheme (e.g. $\mu = \nu = 1$ for BPSK).

Theorem 3: average symbol error rate

Let λ_1, λ_2 be the eigenvalues of a 2×2 complex Wishart matrix of n degrees of freedom. Average symbol error rate (6) is given by:

$$\begin{aligned} P_s = \frac{\mu K_1}{2(4^{2n-1})} \sum_{k=0}^{2n} \binom{2n}{k} & \\ \Bigg[2(n-k)\mathcal{G}_{3,3}^{2,2}\left(\frac{4}{\nu P}\,\middle|\, \begin{matrix} 0.5, 1, 2n-0.5 \\ 3n-k-1, n+k-1, 0 \end{matrix}\right) & \\ -\mathcal{G}_{3,3}^{2,2}\left(\frac{4}{\nu P}\,\middle|\, \begin{matrix} 0.5, 1, 2n+0.5 \\ 3n-k, n+k, 0 \end{matrix}\right) & \\ +\mathcal{G}_{3,3}^{2,2}\left(\frac{4}{\nu P}\,\middle|\, \begin{matrix} 0.5, 1, 2n+0.5 \\ 3n-k-1, n+k+1, 0 \end{matrix}\right)\Bigg] & \end{aligned}$$

Proof: See appendix. ■

Fig. 5 illustrates for the cases: $n \in \{2, 3, 5\}$, how the analytical results for symbol error rate of BPSK compares with corresponding Monte-Carlo simulation results obtained with 100000 sample points. A reduction of symbol error rate and an increase of diversity order is observed as n increases.

IV. Conclusion

The received signal power over each virtual channel in a MIMO system, with the associated 2×2 complex Wishart matrix having n degrees of freedom, has been examined under the channel inversion power allocation scheme. Closed-form results have been derived for its pdf and the cdf. The analysis covered MIMO systems with transmit-receive antennas $\{N_t, N_r| \min(N_t, N_r) = 2, n = \max(N_t, N_r) \geq 2\}$, and i.i.d. Rayleigh fading links between each antenna pair. As applications of the results, we deduced (i) the distribution of Shannon capacity; and (ii) average symbol error rate for a class of modulation schemes.

The analysis may be extended to cover other performance criteria; and possibly to MIMO systems described by a rank 3 channel matrix.

V. Appendix

This section presents the proofs of theorems 1 through 3.

Proof: Theorem 1: the pdf of Λ

$$\begin{aligned} F_\Lambda(x) &= \mathcal{P}\left[\frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2} \leq x\right] = \mathcal{P}\left[\lambda_1(\lambda_2 - x) \leq \lambda_2 x\right] \\ &= \int_0^x \underbrace{\bar{F}_{\lambda_1|\lambda_2}\left(\frac{\lambda_2 x}{\lambda_2 - x}\,\middle|\,\lambda_2\right)}_{\doteq 1} f_{\lambda_2}(\lambda_2)\,\mathrm{d}\lambda_2 \\ &\quad + \int_x^\infty F_{\lambda_1|\lambda_2}\left(\frac{\lambda_2 x}{\lambda_2 - x}\,\middle|\,\lambda_2\right) f_{\lambda_2}(\lambda_2)\,\mathrm{d}\lambda_2 \\ \bar{F}_\Lambda(x) &= \int_x^\infty \bar{F}_{\lambda_1|\lambda_2}\left(\frac{\lambda_2 x}{\lambda_2 - x}\,\middle|\,\lambda_2\right) f_{\lambda_2}(\lambda_2)\,\mathrm{d}\lambda_2 \end{aligned}$$

Differentiating it with respect to x, we get:

$$\begin{aligned} f_\Lambda(x) &= \int_x^\infty \frac{\lambda_2^2}{(\lambda_2 - x)^2} f_{\lambda_1|\lambda_2}\left(\frac{\lambda_2 x}{\lambda_2 - x}\,\middle|\,\lambda_2\right) f_{\lambda_2}(\lambda_2)\,\mathrm{d}\lambda_2 \\ &= \int_x^\infty \frac{\lambda_2^2}{(\lambda_2 - x)^2} f_{\lambda_1,\lambda_2}\left(\frac{\lambda_2 x}{\lambda_2 - x}, \lambda_2\right)\mathrm{d}\lambda_2 \\ &= \int_0^\infty \left(\frac{t+x}{t}\right)^2 f_{\lambda_1,\lambda_2}\left(\frac{x(t+x)}{t}, (t+x)\right)\mathrm{d}t \quad (7) \end{aligned}$$

Substituting (2) in (7), and using the Binomial expansion and [15, (3.471.9, 8.471.1)] we get:

$$\begin{aligned} f_\Lambda(x) &= K_1 \int_0^\infty \left(\frac{t+x}{t}\right)^2 e^{-\frac{(t+x)^2}{t}} \\ &\qquad \left(\frac{(t+x)^2(t-x)^2}{t^2}\right)\left(\frac{x(t+x)^2}{t}\right)^{n-2}\mathrm{d}t \\ &= K_1 x^{n-2} e^{-2x} \\ &\qquad \sum_{k=0}^{2n}\binom{2n}{k} x^{2n-k} \int_0^\infty \frac{(t-x)^2}{t^{n+2-k}} e^{-\left(t+\frac{x^2}{t}\right)}\mathrm{d}t \\ &= 2K_1 x^{2(n-1)} e^{-2x} \sum_{k=0}^{2n}\binom{2n}{k} \\ &\qquad \left((n-k-2x)K_{k-n}(2x) + 2xK_{k+1-n}(2x)\right) \quad (8) \end{aligned}$$

■

Proof: Theorem 2: the cdf of Λ

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Consider the following integral, which can be simplified into a single *Meijer G function* [14, p.419], first by using [15, 9.34.4]; then by applying [16, 07.34.21.0002.01] after making the substitution $u \doteq 2t$.

$$\int_0^x t^{\mu} K_{\nu}(t)\, e^{-t} \mathrm{d}t = \sqrt{\pi} \int_0^x t^{\mu}\, \mathcal{G}_{1,2}^{2,0}\left(2t \middle| \begin{array}{c} 0.5 \\ -\nu, \nu \end{array}\right) \mathrm{d}t$$
$$= \frac{\sqrt{\pi}}{2^{\mu+1}} \int_0^{2x} u^{\mu}\, \mathcal{G}_{1,2}^{2,0}\left(u \middle| \begin{array}{c} 0.5 \\ -\nu, \nu \end{array}\right) \mathrm{d}u$$
$$= \frac{\sqrt{\pi}}{2^{\mu+1}} \mathcal{G}_{2,3}^{2,1}\left(2x \middle| \begin{array}{c} 1, \mu+1.5 \\ \mu-\nu+1, \mu+\nu+1, 0 \end{array}\right) \quad (9)$$

Although relative uncommonly used in the wireless literature, Meijer G function is directly available in the key computational environments including Mathematica, Maple and MATLAB. Hence, our results can easily be evaluated at high precision.

Now let's consider the cdf of Λ.

$$F_{\Lambda}(x) = \int_0^x f_{\Lambda}(t)\, \mathrm{d}t$$
$$= 2K_1 \sum_{k=0}^{2n} \binom{2n}{k} \Bigg((n-k) \int_0^x t^{2n-2} e^{-2t} K_{k-n}(2t)\, \mathrm{d}t$$
$$-2 \int_0^x t^{2n-1} e^{-2t} K_{k-n}(2t)\, \mathrm{d}t$$
$$+2 \int_0^x t^{2n-1} e^{-2t} K_{k+1-n}(2t)\, \mathrm{d}t \Bigg) \quad (10)$$

Applying the result of eqn. (9), in (10), we get:

$$F_{\Lambda}(x) = \frac{\sqrt{\pi} K_1}{4^{2n-1}} \sum_{k=0}^{2n} \binom{2n}{k}$$
$$\Bigg[2(n-k) \mathcal{G}_{2,3}^{2,1}\left(4x \middle| \begin{array}{c} 1, 2n-0.5 \\ 3n-k-1, n+k-1, 0 \end{array}\right)$$
$$-\mathcal{G}_{2,3}^{2,1}\left(4x \middle| \begin{array}{c} 1, 2n+0.5 \\ 3n-k, n+k, 0 \end{array}\right)$$
$$+\mathcal{G}_{2,3}^{2,1}\left(4x \middle| \begin{array}{c} 1, 2n+0.5 \\ 3n-k-1, n+k+1, 0 \end{array}\right) \Bigg] \quad (11)$$

■

Proof: Theorem 3: average symbol error rate

Eqn. (6) can be simplified as follows using integration by parts and the *Leibniz's rule for differentiation*.

$$P_s = \int_0^{\infty} \mu \mathcal{Q}\left(\sqrt{2\nu x P}\right) \mathrm{d}F_{\Lambda}(x)$$
$$= \mu \int_0^{\infty} \frac{e^{-\nu P x}}{\sqrt{2\pi}} \cdot \frac{\sqrt{2\nu P}}{2\sqrt{x}} F_{\Lambda}(x)\, \mathrm{d}x \quad (12)$$

Consider the integral

$$\mathbb{I}(q,\alpha,\beta,\gamma) = \int_0^{\infty} x^{-0.5} e^{-qx} \mathcal{G}_{2,3}^{2,1}\left(4x \middle| \begin{array}{c} 1, \alpha \\ \beta, \gamma, 0 \end{array}\right) \mathrm{d}x$$
$$= \mathcal{L}\left\{ x^{-0.5} \mathcal{G}_{2,3}^{2,1}\left(4x \middle| \begin{array}{c} 1, \alpha \\ \beta, \gamma, 0 \end{array}\right) \right\}\Bigg|_{s \doteq q}$$

where q, α, β, γ are positive reals; and $\mathcal{L}\{\,.\,\}$ denotes the Laplace transform. It can be solved using [17, 4.23 (34)] to get

$$\mathbb{I}(q,\alpha,\beta,\gamma) = q^{-0.5} \mathcal{G}_{3,3}^{2,2}\left(\frac{4}{q} \middle| \begin{array}{c} 0.5, 1, \alpha \\ \beta, \gamma, 0 \end{array}\right) \quad (13)$$

Substituting (11) in (12), and using (13) completes the proof. ■

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