Joint Frequency Offset and Channel Estimation Methods for Two-Way Relay Networks

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***Abstract*—In this paper, we study the problem of joint carrier frequency offset (CFO) and channel estimation for two-way relay network (TWRN) that comprises two source terminals and one relay node. We build up the signal model, from which we identify the CFO and channels at the two source terminals. As the very first attempt to discuss the joint CFO and the channel estimation for TWRN, we consider relay node that purely amplifies and forwards, which is also known as the *repeater*. The new model is different from the traditional ones in that the unknown CFO is combined with only part of the channel parameters. We then propose two joint estimation methods, i.e., the approximate maximum-likelihood (ML) method and the nulling-based method. The Cramér-Rao Bounds (CRB) of both methods are derived in closed-form. Simulations are then provided to corroborate the proposed studies.**

I. Introduction

Recently, two-way relay network (TWRN) becomes a popular topic mainly due to its capability of enhancing the overall communication rate between two source terminals via relay nodes in between [1], [2]. By utilizing "network coding"-like manner, the collision of the information signals that happens at relays can be perfectly resolved at source terminals because of the natural *a prior* knowledge of their own signals. As a consequence, two-way relaying is sometimes referred to as physical layer networking coding (PLNC) [3] or analog network coding (ANC) [4].

Both the *amplify-and-forward* (AF) and the *decode-and-forward* (DF) relaying protocols in one way relay network (OWRN) were extended to the half-duplex TWRN in [5]. In [6], the distributed space-time code for both AF and DF TWRN were developed, and in [7] the optimal mapping functions at the relay nodes that could minimize the transmission bit-error rate (BER) were derived. In [8], the authors designed the optimal beamforming at the multi-antenna relay that can maximize the capacity for AF-based TWRN.

Most existing works in TWRN [5]–[8] assumed perfect synchronization and channel state information (CSI) at the relay node and/or the source terminals. Although the traditional methods can be readily applied to DF based TWRN, it is necessary to re-visit these two issues for AF based TWRN, of which the first and the second transmission phases are mixed with each other.

In this paper, we look into the problem of joint carrier frequency offset (CFO) and channel estimation for TWRN. Moreover, we will consider the frequency selective environment. We first build up the signal model, from which we recognize the CFO and the channels that should be estimated at both source terminals. We then propose two different joint estimation methods. The first one is intrigued from the optimal estimation criterion, i.e., maximum-likelihood (ML), but only a suboptimal substitute is found practical. The second method comes form the observation that the mixed signals can be separated by nulling out one of them. From numerical results we find that the nulling-based method has similar performance as sub-ML but it requires less computational complexity. Closed-formed Cramér-Rao Bounds (CRB) are also derived for both methods.

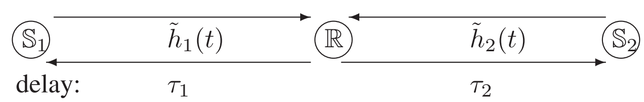

Fig. 1. System configuration for two-way relay network.

II. Problem Formulation

Consider a wireless network with two terminals $\mathbb{S}_i$, $i = 1, 2$ that intend to communicate with each other via one relay $\mathbb{R}$, as shown in Fig. 1. Each node has only one antenna that cannot transmit and receive simultaneously. The channel between each node pair is assumed to be quasi-stationary, which is constant within one frame[1] but may vary from frame to frame.

One round of data communication between $\mathbb{S}_i$'s is divided into two phases: In Phase I (uplink phase), signal blocks are sent from $\mathbb{S}_i$'s, which are then superposed at $\mathbb{R}$; In Phase II, the mixed signal is amplified and sent out from $\mathbb{R}$ (downlink phase), and $\mathbb{S}_i$ will remove its own signal part from the received signals. As in most joint CFO and channel estimation works [9], we assume that perfect time synchronization among nodes is obtained, i.e., signals from $\mathbb{S}_i$'s during the first phase can arrive at $\mathbb{R}$ simultaneously, and all three nodes perfectly detect the starting point of each frame.

[1]One frame contains many data blocks, and one data block contains many symbols.

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Denote the channel from $\mathbb{S}_i$ to $\mathbb{R}$ as $\tilde{h}_i(t)$. For TWRN, it is more interesting to consider time division multiplexing (TDD) links [1]– [8];[2] namely, the same frequency band is used for both uplink and downlink. In this case, the channels from $\mathbb{R}$ to $\mathbb{S}_i$'s are still $\tilde{h}_i(t)$'s. Denote the oscillator frequencies at $\mathbb{S}_1$, $\mathbb{S}_2$, and $\mathbb{R}$ as f_1, f_2, and f_0, respectively. The ideal case that $f_1 = f_2$ cannot be achieved in practical implementation. The difference between these two values, although could be kept small, may still affect the performance of the transmission. Moreover, the initial phases of the three oscillators are taken as $\theta_0 = \theta_1 = \theta_2 = 0$ for simplicity.

Assume that the baseband discrete data sequence from $\mathbb{S}_i$ is $s_i[m]$, and the baseband continuous signal, after pulse shaping, is

$$s_i(t) = \sum_{m=-\infty}^{+\infty} s_i[m]p(t - mT_s), \quad i = 1, 2, \tag{1}$$

where T_s is the symbol period. The complex expression of the passband signal sent by $\mathbb{S}_i$ is

$$\tilde{s}_i(t) = \sum_{m=-\infty}^{+\infty} s_i[m]p(t - mT_s)e^{j2\pi f_i t}. \tag{2}$$

Suppose the delay between $\mathbb{S}_i$ and $\mathbb{R}$ is τ_i. With the assumption of perfect synchronization, the signal blocks from both $\mathbb{S}_i$'s arrive at $\mathbb{R}$ simultaneously, as shown in Fig. 2. The received signal in $\mathbb{R}$ is

$$\begin{aligned}\tilde{r}_r(t) &= \sum_{i=1}^{2} \tilde{h}_i(t) * \tilde{s}_i(t) + \tilde{n}_r(t) \qquad (3)\\ &= \sum_{i=1}^{2} \left((\tilde{h}_i(t)e^{-j2\pi f_i t}) * s_i(t) \right) e^{j2\pi f_i t} + \tilde{n}_r(t),\end{aligned}$$

where $*$ denotes the continuous-time convolution, and $\tilde{n}_r(t)$ denotes the passband noise with power spectrum σ_n^2. Note that, the delay information is involved in $\tilde{h}_i(t)$. For example in tapped delay line model, $\tilde{h}_i(t) = \sum_{j=0}^{L} a_{ij}\delta(t - \Delta_{ij})$ with $\Delta_{i0} = \tau_i$.

If the relay node is equipped with a high-frequency (HF) amplifier, then, the relay can directly amplify $\tilde{r}_r(t)$ by a factor α, and the re-constructed passband signal is

$$\tilde{r}_s(t) = \alpha \tilde{r}_r(t). \tag{4}$$

Otherwise, $\tilde{r}_r(t)$ should be down-converted and the intermediate-frequency (IF) amplifier is applied. Suppose the down-conversion is made by $e^{-j2\pi f_0 t}$. It is then natural that the same oscillator is used for up-conversion after the amplification by a factor α, i.e. $e^{j2\pi f_0 t}$. Then the resulted signal is the same as that in (4).

Remark: A more general situation is when relay down-convert the received signal to baseband and then apply specific digital signal processing techniques to improve the overall system performance, e.g., beamforming [8] and resource allocation [10]. In such cases, the frequency offsets between

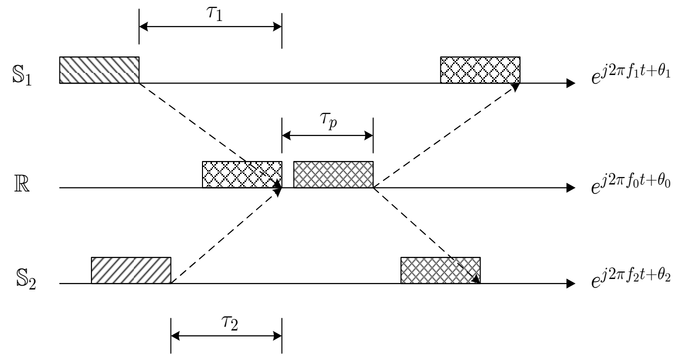

Fig. 2. Illustration of channel delay and signal processing delay in two-way relay transmission.

source nodes and the relay node have to be counted in (4). Such a general discussion can be found in the journal version of this paper [11]. However, here, we adopt relay as a *repeater* that works with the actual physical signal, which does not attempt to interpret the data being transmitted.

Suppose the processing delay at $\mathbb{R}$ is τ_p. The passband signal sent out by relay is then

$$\tilde{r}_s(t - \tau_p) = \alpha \sum_{i=1}^{2} \tilde{h}_i(t) * \tilde{s}_i(t - \tau_p) + \alpha \tilde{n}_r(t - \tau_p) \tag{5}$$

and will be broadcasted to both $\mathbb{S}_i$'s immediately.

Due to symmetry, we only look into the demodulation at node $\mathbb{S}_1$. The data received at $\mathbb{S}_1$ is

$$\begin{aligned}\tilde{y}(t) &= \tilde{h}_1(t) * \tilde{r}_s(t - \tau_p) + \tilde{n}_1(t),\\ &= \alpha \sum_{i=1}^{2} \tilde{h}_1(t) * \tilde{h}_i(t) * \tilde{s}_i(t - \tau_p) + \alpha \tilde{h}_1(t) * \tilde{n}_r(t - \tau_p)\\ &\qquad + \tilde{n}_1(t)\\ &= \alpha \sum_{i=1}^{2} \left(((\tilde{h}_1(t) * \tilde{h}_i(t))e^{-j2\pi f_i t}) * s_i(t - \tau_p) \right)\\ &\qquad \times e^{j2\pi f_i (t-\tau_p)} + \alpha \tilde{h}_1(t) * \tilde{n}_r(t - \tau_p) + \tilde{n}_1(t), \qquad (6)\end{aligned}$$

where $\tilde{n}_1(t)$ is the noise at $\mathbb{S}_1$ and is assumed to have power spectrum σ_n^2.

Without loss of generality, we assume that $\mathbb{S}_1$ down-converts $\tilde{y}(t)$ to baseband by $e^{-j2\pi f_1 t}$,[3] and the resulted signal is

$$\begin{aligned}y(t) &= \tilde{y}(t)e^{-j2\pi f_1 t}\\ &= \alpha \left(\left((\tilde{h}_1(t) * \tilde{h}_1(t))e^{-j2\pi f_1 t} \right) * s_1(t - \tau_p) \right) e^{j\phi_1}\\ &+ \alpha \left(\left((\tilde{h}_1(t) * \tilde{h}_2(t))e^{-j2\pi f_2 t} \right) * s_2(t - \tau_p) \right) e^{j2\pi v t + j\phi_2}\\ &+ \alpha \left(\tilde{h}_1(t) * \tilde{n}_r(t - \tau_p) \right) e^{-j2\pi f_1 t} + \tilde{n}_1(t)e^{-j2\pi f_1 t}, \qquad (7)\end{aligned}$$

where $v \triangleq f_2 - f_1$ is the frequency offset observed at $\mathbb{S}_1$ and ϕ_i's represent the corresponding unknown phase offsets.

[2]Otherwise, channels from each direction can be estimated separately, which is a rather trivial problem.

[3]There may be additional phase, which can nevertheless be absorbed into channel factor later.

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Define

$$a(t) = \alpha(\tilde{h}_1(t) * \tilde{h}_1(t))e^{-j2\pi f_1 t}e^{j\phi_1}$$
$$b(t) = \alpha(\tilde{h}_1(t) * \tilde{h}_2(t))e^{-j2\pi f_2 t}e^{j\phi_2}$$
$$h_1(t) = \alpha\tilde{h}_1(t)e^{-j2\pi f_1 t}.$$

The baseband signal (7) is rewritten as

$$y(t) = a(t) * s_1(t-\tau_p) + b(t) * s_2(t-\tau_p)e^{j2\pi vt} + h_1(t) * (\tilde{n}_r(t-\tau_p)e^{-j2\pi f_1 t}) + \tilde{n}_1(t)e^{-j2\pi f_1 t}. \quad (8)$$

Note that the noise after being multiplied by $e^{-j2\pi f_1 t}$ has the same statistics as before. Moreover, with perfect time synchronization, the effect of the delay τ_p can be ingored, i.e., we can always consider $y(t+\tau_p)$ as the received signal, which does not change the statistics of the noise. We then obtain the equivalent baseband discrete signal model (after matched filter) as

$$y[m] = a[m] \otimes s_1[m] + b[m] \otimes s_2[m]e^{j2\pi vmT_s} + h_1[m] \otimes n_r[m] + n_1[m], \quad (9)$$

where $\otimes$ denotes the discrete-time convolution, and each term represents the discrete sample of its corresponding continuous signal.

If $a[m]$ is known at $\mathbb{S}_1$, the first term on the right-hand side (RHS) of (9) can be removed, leaving only the signal component from $\mathbb{S}_2$. Furthermore, if v is estimated, the effect of the CFO can be immediately removed from the remaining signal. Finally, the maximum-likelihood (ML) detection can be achieved once $b[m]$ is known. Clearly, the task of the channel estimation and frequency synchronization at $\mathbb{S}_1$ is to estimate $a[m]$, $b[m]$ and v.

Let $(L+1)T_s$ be the maximum channel lengths of $h_i(t)$. Then, the length of $a(t)$ and $b(t)$ can be bounded by $(2L+1)T_s$. Denote the corresponding channel vectors by $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{h}_1$, respectively. Suppose $\mathbb{S}_i$ sends the data sequence $\mathbf{s}_i = \big[s_i[-2L-1], \ldots, s_i[0], s_i[1] \ldots, s_i[N-1]\big]^T$. Following the traditional approach [9], we only keep the received symbols from $y[0]$ to $y[N-1]$ in order to remove the possible interference from the previous and the next data symbols. These N symbols are stacked into a vector $\mathbf{y}$ whose matrix expressions is given by

$$\mathbf{y} = \mathbf{S}_1\mathbf{a} + \mathbf{\Gamma}\mathbf{S}_2\mathbf{b} + \mathbf{H}_1\mathbf{n}_r + \mathbf{n} \quad (10)$$

where

$$\mathbf{\Gamma} = \mathrm{diag}\Big(1, e^{j2\pi vT_s}, ..., e^{j2\pi v(N-1)T_s}\Big),$$

$$\mathbf{H}_1 = \alpha \begin{bmatrix} h_1[L] & \ldots & h_1[0] & \ldots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ldots & h_1[L] & \ldots & h_1[0] \end{bmatrix}.$$

Moreover, $\mathbf{S}_i$ is the $N \times (2L+1)$ Toeplitz matrix with first column $\big[s_i[0], \ldots, s_i[N-1]\big]^T$ and the first row $\big[s_i[0], \ldots, s_i[-2L-1]\big]$. For simplicity, the noise variances are set to be unit at all three nodes. The problem becomes estimating $\mathbf{a}$, $\mathbf{b}$, v from the received vector $\mathbf{y}$.

III. Approximated ML Estimation

The ML estimates of the parameters are obtained by maximizing the probability density function (pdf) of $\mathbf{y}$:

$$p(\mathbf{y}|\mathbf{\Theta}) = \frac{1}{\pi^N \det(\mathbf{H}_1\mathbf{H}_1^H + \mathbf{I})} \times \exp\big\{(\mathbf{y} - \mathbf{S}_1\mathbf{a} - \mathbf{\Gamma}\mathbf{S}_2\mathbf{b})^H(\mathbf{H}_1\mathbf{H}_1^H + \mathbf{I})^{-1}(\mathbf{y} - \mathbf{S}_1\mathbf{a} - \mathbf{\Gamma}\mathbf{S}_2\mathbf{b})\big\}$$

where $\mathbf{\Theta}$ denotes the unknown-parameter set. Unfortunately, the direct ML estimation requires exhaustive search over multiple unknown variables, which is time-consuming and non-practical. We will then provide an approximated ML (AML) algorithm that is practically implementable.

A. Algorithm

When the number of the channel taps is large, $\mathbf{H}_1\mathbf{H}_1^H$ can be approximated by

$$\mathbf{H}_1\mathbf{H}_1^H \approx \sum_{l=0}^{L} \sigma_{h,l}^2 \mathbf{I}, \quad (11)$$

where $\sigma_{h,l}^2$ is the variance of $h_1[l]$. Hence, each noise element in $\mathbf{H}_1\mathbf{n}_r$ is approximately Gaussian with variance $\sum_{l=0}^{L}\sigma_{h,l}^2$. Then the ML estimation can be directly conducted for $\mathbf{a}$, $\mathbf{b}$, and v from[4]

$$\{\widehat{\mathbf{a}}, \widehat{\mathbf{b}}, \widehat{v}\} = \arg\min_{\mathbf{a},\mathbf{b},v} \|\mathbf{y} - \mathbf{S}_1\mathbf{a} - \mathbf{\Gamma}\mathbf{S}_2\mathbf{b}\|^2. \quad (12)$$

Denote $\mathbf{C} = [\mathbf{S}_1, \mathbf{\Gamma}\mathbf{S}_2]$ and $\mathbf{d} = [\mathbf{a}^T, \mathbf{b}^T]^T$. As long as $N > 4L+2$, $\mathbf{d}$ can be estimated as

$$\widehat{\mathbf{d}} = (\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H\mathbf{y}. \quad (13)$$

Substituting (13) back to (12), we can estimate CFO as

$$\begin{aligned} \widehat{v} &= \arg\min_v \ \|\mathbf{y} - \mathbf{C}\widehat{\mathbf{d}}\|^2 \\ &= \arg\max_v \ \mathbf{y}^H\mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H\mathbf{y} \\ &= \arg\max_v \ g(v). \end{aligned} \quad (14)$$

Since there is only one unknown variable, the grid searching method can be immediately applied to find $\hat{v}$. Moreover, $g(v)$ is a polynomial of v, so the rooting method suggested in [13] can be applied to improve the efficiency. Once $\hat{v}$ is found, the estimates of $\mathbf{a}$, $\mathbf{b}$ can be obtained from (13).

B. Cramér-Rao Bound of CFO Estimation

In our considered system, there are three variable, i.e., v, $\mathbf{a}$, $\mathbf{b}$. Following [12], the Fisher Information Matrix (FIM) is obtained as:

$$\mathbf{F} = \frac{2}{\sum_l \sigma_{h,l}^2 + 1} \begin{bmatrix} F_{11} & \mathbf{r}^T & \mathbf{s}^T \\ \mathbf{r} & \mathbf{K} & \mathbf{V}^T \\ \mathbf{s} & \mathbf{V} & \mathbf{N} \end{bmatrix}, \quad (15)$$

[4]We can immediately draw the same result if we consider the least square (LS) criterion.

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where

$$F_{11} = \mathbf{b}^H\mathbf{S}_2^H\mathbf{D}^2\mathbf{S}_2\mathbf{b}, \quad \mathbf{D} = 2\pi T_s \text{diag}\{0, 1, \ldots, (N-1)\},$$

$$\mathbf{r} = \begin{bmatrix} -\Im(\mathbf{S}_1^H\mathbf{D}\mathbf{\Gamma}\mathbf{S}_2\mathbf{b}) \\ \Re(\mathbf{S}_1^H\mathbf{D}\mathbf{\Gamma}\mathbf{S}_2\mathbf{b}) \end{bmatrix}, \quad \mathbf{s} = \begin{bmatrix} -\Im(\mathbf{S}_2^H\mathbf{D}\mathbf{S}_2\mathbf{b}) \\ \Re(\mathbf{S}_2^H\mathbf{D}\mathbf{S}_2\mathbf{b}) \end{bmatrix},$$

$$\mathbf{K} = \begin{bmatrix} \Re(\mathbf{S}_1^H\mathbf{S}_1) & -\Im(\mathbf{S}_1^H\mathbf{S}_1) \\ \Im(\mathbf{S}_1^H\mathbf{S}_1) & \Re(\mathbf{S}_1^H\mathbf{S}_1) \end{bmatrix},$$

$$\mathbf{V} = \begin{bmatrix} \Re(\mathbf{S}_2^H\mathbf{\Gamma}^H\mathbf{S}_1) & -\Im(\mathbf{S}_2^H\mathbf{\Gamma}^H\mathbf{S}_1) \\ \Im(\mathbf{S}_2^H\mathbf{\Gamma}^H\mathbf{S}_1) & \Re(\mathbf{S}_2^H\mathbf{\Gamma}^H\mathbf{S}_1) \end{bmatrix},$$

$$\mathbf{N} = \begin{bmatrix} \Re(\mathbf{S}_2^H\mathbf{S}_2) & -\Im(\mathbf{S}_2^H\mathbf{S}_2) \\ \Im(\mathbf{S}_2^H\mathbf{S}_2) & \Re(\mathbf{S}_2^H\mathbf{S}_2) \end{bmatrix}.$$

The CRB of CFO is the upper-left block in FIM^{-1}, which can be explicitly calculated as:

$$\text{CRB}_1(v) = \frac{\sum_l \sigma_{h,l}^2 + 1}{2}[F_{11} - \mathbf{t}_1^T\mathbf{Q}_1^{-1}\mathbf{t}_1]^{-1}, \tag{16}$$

where

$$\mathbf{t}_1 = \begin{bmatrix} \mathbf{r} \\ \mathbf{s} \end{bmatrix}, \quad \mathbf{Q}_1 = \begin{bmatrix} \mathbf{K}, \mathbf{V}^T \\ \mathbf{V}, \mathbf{N} \end{bmatrix}.$$

IV. Nulling Based Method

A. Algorithm

Since $\mathbf{S}_1$ is of size $N \times (2L+1)$, it is possible to find an $N \times (N - 2L - 1)$ matrix $\mathbf{J}$ such that $\mathbf{J}^H\mathbf{S}_1 = \mathbf{0}$. We suggest an orthonormal matrix $\mathbf{J}$ with $\mathbf{J}^H\mathbf{J} = \mathbf{I}$ because orthogonal training together with orthogonal nulling is proved to be optimal for 2×1 multi-input single-output (MISO) system, which is mathematically similar to the two-way signal model if no CFO exists. With the non-zero CFO, the optimality of orthogonal nulling is still unknown but the orthonormal nulling can be treated as a good one for small enough CFO value. Moreover, it will be seen later that this choice of $\mathbf{J}$ yields the best performance.

Left-multiply both sides of (10) with $\mathbf{J}^H$, we obtain:

$$\mathbf{J}^H\mathbf{y} = \mathbf{0} + \underbrace{\mathbf{J}^H\mathbf{\Gamma}\mathbf{S}_2}_{\mathbf{G}}\mathbf{b} + \mathbf{J}^H(\mathbf{H}\mathbf{n}_\mathbf{r} + \mathbf{n}), \tag{17}$$

where $\mathbf{G}$ is defined as the corresponding term. The AML estimate of $\mathbf{b}$ can be immediately found as

$$\hat{\mathbf{b}} = (\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H\mathbf{y}. \tag{18}$$

Similar as before, CFO is estimated as

$$\hat{v} = \arg\max_{v} \ \mathbf{y}^H\mathbf{J}\mathbf{G}(\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H\mathbf{J}^H\mathbf{y}, \tag{19}$$

and $\hat{\mathbf{b}}$ is obtained from (18).

Finally, the least square (LS) estimation of channel $\mathbf{a}$ is obtained from

$$\hat{\mathbf{a}} = (\mathbf{S}_1^H\mathbf{S}_1)^{-1}\mathbf{S}_1^H(\mathbf{y} - \hat{\mathbf{\Gamma}}\mathbf{S}_2\hat{\mathbf{b}}), \tag{20}$$

where $\hat{\mathbf{\Gamma}} = \text{diag}\{1, e^{j2\pi\hat{v}T_s}, \ldots, e^{j2\pi\hat{v}(N-1)T_s}\}$.

Compared to the direct AML, one advantage of the nulling based method is the reduced complexity during the grid searching, where $\mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H$ or $\mathbf{G}(\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H$ needs to be computed each time for one grid of v. Clearly, nulling based method has less complexity.

B. CRB of CFO Estimation

The FIM is calculated as:

$$\mathbf{F} = \frac{2}{\sum_l \sigma_{h,l}^2 + 1}\begin{bmatrix} F'_{11} & \mathbf{t}_2^T \\ \mathbf{t}_2 & \mathbf{Q}_2 \end{bmatrix}, \tag{21}$$

where

$$F'_{11} = \mathbf{b}^H\mathbf{S}_2^H\mathbf{D}\mathbf{\Gamma}^H\mathbf{J}\mathbf{J}^H\mathbf{D}\mathbf{\Gamma}\mathbf{S}_2\mathbf{b},$$

$$\mathbf{t}_2 = \begin{bmatrix} -\Im(\mathbf{S}_2^H\mathbf{\Gamma}^H\mathbf{J}\mathbf{J}^H\mathbf{D}\mathbf{\Gamma}\mathbf{S}_2\mathbf{b}) \\ \Re(\mathbf{S}_2^H\mathbf{\Gamma}^H\mathbf{J}\mathbf{J}^H\mathbf{D}\mathbf{\Gamma}\mathbf{S}_2\mathbf{b}) \end{bmatrix},$$

$$\mathbf{Q}_2 = \begin{bmatrix} \Re(\mathbf{G}^H\mathbf{G}) & -\Im(\mathbf{G}^H\mathbf{G}) \\ \Im(\mathbf{G}^H\mathbf{G}) & \Re(\mathbf{G}^H\mathbf{G}) \end{bmatrix}.$$

The CRB of frequency offset estimation is:

$$\text{CRB}_2(v) = \frac{\sum_l \sigma_{h,l}^2 + 1}{2}[F'_{11} - \mathbf{t}_2^T\mathbf{Q}_2^{-1}\mathbf{t}_2]^{-1}. \tag{22}$$

V. Simulation Results

We consider both $\mathbf{h}_i$'s as 3-tap channels, and each tap has unit variance. The normalized CFO vT_s is as large as 0.2. The estimation mean square error (MSE) is chosen as the figure of merit, defined by

$$\text{MSE}(v) = \frac{1}{10000}\sum_{i=1}^{10000}(\hat{v}_i - v)^2,$$

$$\text{MSE}(\mathbf{x}) = \frac{1}{10000}\sum_{i=1}^{10000}\frac{1}{3}(\hat{\mathbf{x}}_i - \mathbf{x})^2,$$

where $\mathbf{x}$ represents either $\mathbf{a}$ or $\mathbf{b}$, and 10000 is the number of Monte-Carlo runs. In all examples, we fix $N = 16$, and set training sequences $\mathbf{s}_1$ and $\mathbf{s}_2$ to be orthogonal to each other.

A. Comparing AML and nulling with orthonormal $\mathbf{J}$

We first examine the performance of both CFO estimation and channel estimation from AML and nulling based method. The estimation MSEs versus SNR for $\mathbf{a}$ are given in Fig.3, where we do not present the estimation of $\mathbf{b}$ for the conciseness of the figure. The CRB derived in (16) and (22) are also displayed in the same figure. The following observations are found:

- Numerically, both the CFO and channel estimation accuracy are the same for direct AML method and nulling based method with orthonormal $\mathbf{J}$. However, the latter is computationally more efficient in terms of grid search in CFO estimation.
- It is seen that the performance of both methods approach their corresponding CRBs at high SNR, and this finding proves the effectiveness of the proposed methods. In fact, the gaps between CRB and the numerical results at lower SNR is due to the fact that some Monte-Carlo runs gives the ambiguous CFO estimation and deteriorates the overall performance. This phenomenon is sometimes known as *outlier* effect [13].

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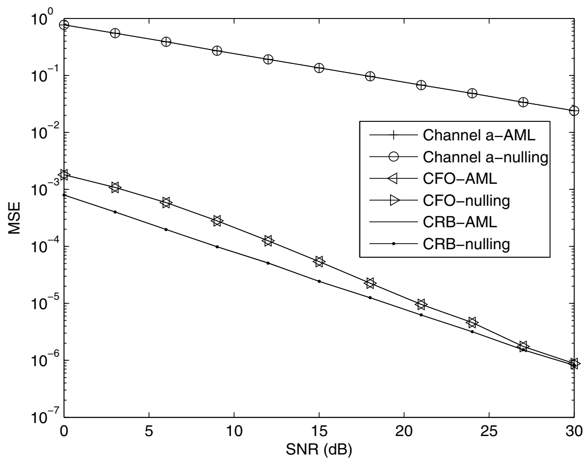

Fig. 3. MSEs of CFO and channel estimation versus SNR for AML and nulling based method with orthonormal **J**

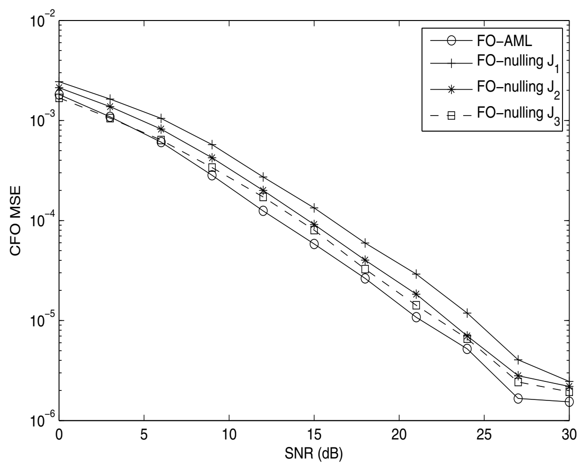

Fig. 4. MSEs of CFO estimation versus SNR for AML and nulling based method with random **J**

B. Comparing AML and nulling with random J

It is then interesting to examine the effect of random $\mathbf{J}$ on the nulling based estimation. In this example, we randomly select three different $\mathbf{J}_k$, $k = 1, 2, 3$, where $\mathbf{J}_1$ has the largest condition number and $\mathbf{J}_3$ has the smallest condition number. We then compare their corresponding CFO and channel estimation results with that from the direct AML method. The estimation MSEs of CFO and channel estimation are separately drawn in Fig. 4 and Fig. 5. Clearly, the random nulling can greatly affect the performance of the estimation. Roughly speaking, the larger the condition number is, the worse the performance of the nulling method will be. However, a firm conclusion should be made after rigorous computation, which is one of our task in the future work.

VI. Conclusions

In this paper, we formulate the signal model for two-way relay networks with frequency synchronization errors in a frequency selective environment. We developed two joint CFO

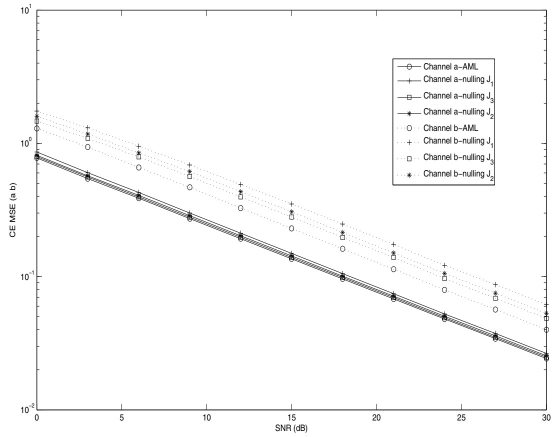

Fig. 5. MSEs of channel estimation versus SNR for AML and nulling based method with random **J**

and channel estimations methods, i.e., AML and nulling-based methods. With orthonormal nulling, both the methods yield the same performance through numerical examples. However, if nulling is based on random matrix, the performance degrades severely. The CRBs of CFO estimation are derived, with which the numerical results verify the effectiveness of the proposed methods.

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