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# OFDMA Uplink Frequency Offset Estimation via Cooperative Relaying

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Abstract—Frequency offset estimation for an Orthogonal Frequency-Division Multiple Access (OFDMA) uplink for amplify-and-forward (AF) relays and a new type of relay  $({\it R})$  called decode-and-compensate-and-forward (DcF) relays are studied. Multiple relays are considered, and the relay with the best  $S \rightarrow R$  channel is chosen to perform re-transmission, where S and R represent the source and relay nodes, respectively. Frequency offsets due to the mismatches between the transmitter and receiver oscillators are considered, and without considering the effect of Doppler Shift, both  $S \to D$  and  $S \to R \to D$ links have the same frequency offset, where D represents the destination. Thus, by using these two transmissions, D generates two frequency offset estimates, which are combined to minimize the mean square error (MSE). Power allocation between S and R can be adaptively adjusted to optimize the cooperative scheme in terms of frequency offset error variance. When channel state information (CSI) is available at each mobile node, a scheme where the relays adaptively switch between the cooperative and conventional (no relaying) transmissions is proposed to optimize the frequency offset estimation. Although the frequency offset estimation accuracy in the DcF mode is somewhat worse than the AF mode, both modes outperform the conventional transmission. However, DcF (or decode-and-forward (DF)) relays outperform AF relays in terms of channel capacity and bit error rate (BER).

Index Terms-Cooperative, frequency offset estimation, OFDMA.

#### I. INTRODUCTION

RTHOGONAL FREQUENCY-DIVISION MULTIPLE OACCESS (OFDMA), where each user employs a different set of orthogonal sub-carriers, eliminates multiple access interference (MAI) under perfect conditions [1], [2]. However, frequency offsets generate inter-carrier interference (ICI). Although the impact of frequency offset in OFDMA has been widely investigated [1]–[5], these conventional algorithms may perform poorly in fading, especially when the subcarriers allocated to one user are too few, and all these subcarriers are in a deep fade simultaneously.<sup>1</sup> The impact of fading can be alleviated by using cooperative relaying [6]–[11].

Manuscript received July 12, 2008; revised May 9, 2009; accepted July 1, 2009. The associate editor coordinating the review of this letter and approving it for publication was A. Hjorungnes.

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Digital Object Identifier 10.1109/TWC.2009.081525

<sup>1</sup>Frequency diversity can be exploited in [3], [4] by using interleaved carrier allocation scheme (CAS) or generalized CAS, provided that the number of subcarriers allocated to each user is large enough.

The relays can operate in either the conventional amplifyand-forward (AF) or the new decode-and-compensate-andforward (DcF) mode (proposed in this letter). The new method works when both  $S \to R \to D$  and  $S \to D$  links have the same frequency offset.<sup>2</sup> A DcF relay estimates the frequency offset between the source and itself and modifies the training sequence retransmitted to D. In this way, both training sequences transmitted over  $S \to D$  and  $S \to R \to D$  links have the same frequency offset. Thus, using these two transmissions, D can generate two frequency offset estimates, which can be linearly combined to minimize the mean square error (MSE). The Cramer-Rao Lower Bound (CRLB) is derived based on the received Signal-to-Interference-plus-Noise Ratio (SINR). Power allocation between S and R can be adaptively adjusted to optimize the cooperative scheme, and channel state information (CSI) is required. The transmission mode can adaptively switch between the cooperative and conventional (no relaying) transmissions to optimize the frequency offset estimation.

As in [1], perfect time synchronization is assumed in this letter. We consider only the training-sequence and/or pilotaided frequency offset estimation. Although the proposed scheme can improve the performance of any conventional training/pilot-based algorithm, training sequence design is not discussed in this letter.

The remainder of this letter is organized as follows. Section II proposes the cooperative OFDMA uplink signal model. The frequency offset estimation in the proposed cooperative scheme is analyzed in Section III, and the numerical results are given in Section IV. Finally, Section V concludes the letter.

*Notation*:  $(\cdot)^H$  denotes the conjugate transpose of a matrix. The imaginary unit is  $j = \sqrt{-1}$ . A circularly symmetric complex Gaussian variable with mean a and variance  $\sigma^2$  is denoted by  $z \sim C\mathcal{N}(a, \sigma^2)$ . The  $N \times N$  identity matrix is  $\mathbf{I}_N$ .  $\mathbf{x}[i]$  represents the *i*-th element of vector  $\mathbf{x}$ .  $[\mathbf{A}]_{ij}$  represents the ij-th element of matrix **A**. The mean and the variance are  $\mathbb{E}\{\bullet\}$  and  $\operatorname{Var}\{\bullet\}$ .

## II. COOPERATIVE OFDMA UPLINK SIGNAL MODEL

In OFDMA, complex data symbols are modulated from a signal constellation such as phase-shift keying (PSK) or

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<sup>&</sup>lt;sup>2</sup>In this letter, the frequency offset induced by only the mismatch between the transmitter and receiver oscillators is considered (no Doppler Shift). In the presence of Doppler Shift, this condition is not always true. Assume the frequency offsets for  $S \to D$  and  $S \to R \to D$  links are  $\varepsilon_{SD}$  and  $\varepsilon_{SRD}$ , respectively. Also assume that the oscillator frequency difference between Sand D is  $\Delta f_{SD}$ . The relationship between  $\varepsilon_{SD}$  and  $\varepsilon_{SRD}$  is  $\varepsilon_{SRD} =$  $\varepsilon_{SD} - f_{d_{SD}} + f_{d_{SR}} + f_{d_{RD}}$ , where  $f_{d_{SD}}$ ,  $f_{d_{SR}}$  and  $f_{d_{RD}}$  represent the Doppler Shift in  $S \to D$ ,  $S \to R$  and  $R \to D$  links, respectively. When  $f_{d_{SR}} + f_{d_{RD}} - f_{d_{SD}} \neq 0$ , we have  $\varepsilon_{SD} \neq \varepsilon_{SRD}$ .

quadrature amplitude modulation (QAM). The total number of subcarriers is assumed to be N, and each node is allocated  $N_u$  unique subcarriers. The subcarriers allocated to node a is denoted by the set  $G_a$ .  $\mathbf{F}_a$  is an  $N \times N_u$  matrix, which denotes the Inverse Discrete Fourier Transform (IDFT) matrix for the node a.  $\mathbf{F}_a$  is a submatrix of the  $N \times N$  IDFT matrix  $\mathbf{F}$  with  $[\mathbf{F}]_{mn} = \frac{1}{\sqrt{N}} e^{\frac{2\pi mn}{N}}, \quad 0 \le m, n \le N - 1$ . A cyclic-prefix (CP) is used to mitigate the Inter-Symbol-Interference (ISI).

In the following, S represents the source node and  $R_k, k \in$  $\{1, \dots, M\}$ , represent the k-th relay, where  $M \geq 1$  is the total number of relays. For each (S, D),  $\varepsilon_{SD} = \varepsilon_{SR} + \varepsilon_{RD}$ is satisfied, where  $\varepsilon_{ab}$  represents the frequency offset between nodes a and b. Our proposed DcF mode works as follows. If R can estimate  $\varepsilon_{SR}$  with a high accuracy based on a received training sequence, it can re-generate the training sequence by multiplying the k-th sample of the training sequence with  $e^{\frac{j2\pi\kappa\varepsilon_{SR}}{N}}$  ( $\hat{\varepsilon}_{SR}$  is an estimate of  $\varepsilon_{SR}$ ), and then forwards the resulting training sequence to D. Since the frequency offsets observed through  $S \to D$  and  $S \to R \to D$  are identical (i.e., equal to  $\varepsilon_{SD}$ ), node D has two copies of the same training sequence with the same frequency offset. In this way, the relay transmission helps the frequency offset estimation process at D. Since a two-time-slot period is required in the proposed scheme, it doubles the overhead requirement as compared to the conventional scheme. However, the power consumption in the proposed scheme is the same as the conventional one.

#### A. Channel Model

For a pair of nodes  $a, b \in \{S, R_1, \dots, R_M\}$ , a frequencyselective quasi-static fading channel with channel response  $\tilde{\mathbf{h}}_{ab} = [h_{ab}(0), h_{ab}(1), \dots, h_{ab}(L_m - 1)]^T$  is assumed. The maximum channel length for any (a, b) is  $L_m$ . The  $N_u \times N_u$  frequency-domain channel attenuation matrix is given by  $\mathbf{H}_{ab} = \text{diag} \{H_{ab}(g_1), H_{ab}(g_2), \dots, H_{ab}(g_{N_u})\}$ , where  $g_i \in G_b, 1 \le i \le N_u$ , and  $H_{ab}(n) = \sum_{d=0}^{L_m - 1} h_{ab}(d)e^{-\frac{j2\pi nd}{N}}$ is the channel attenuation at the *n*-th subcarrier. For each

 $1 \le k \le M$ , the following assumptions are made for the channel coefficients:

1. 
$$h_{R_kS}(i) \sim \mathcal{CN}(0, \sigma^2_{\text{Ssca}}(i))$$
, where  $\sum_{i=0}^{D_m} \sigma^2_{\text{Ssca}}(i) = 1$ ;  
 $H_{R_kS}(n) \sim \mathcal{CN}(0, 1)$ , where  $n \in G_S$ .

2. 
$$\{ h_{DR_k}(i), h_{DS}(i) \} \sim \mathcal{CN}(0, \sigma_{Lsca}^2(i)), \text{ where}$$
$$\sum_{\substack{L_m - 1 \\ i = 0 \\ \mathcal{CN}(0, \mathcal{L}_u).^3}} \sigma_{Lsca}^2(i) = \mathcal{L}_u < 1; \ \{ H_{DR_k}(n), H_{DS}(n) \} \sim$$

#### B. The First Time Slot

In the first time slot, the received signal at node D and relay  $R_k$  can be represented as

$$\mathbf{Y}_{D,1} = \underbrace{\mathbf{E}_{SD}\mathbf{F}_{S}\mathbf{H}_{SD}\boldsymbol{\Phi}_{S,1}}_{\mathbf{V}_{SD,1}}\mathbf{X}_{S,1}$$

<sup>3</sup>When modelling the wireless channel, path loss is usually modeled as  $\mathcal{L}_u = d_{zD}^{\theta}$ , where  $d_{zD}$  represents the distance between nodes z and D, and  $2 \leq \theta \leq 6$ .  $d_{zD}$  is some relative distance.  $\mathcal{L}_u$  accounts for the fact that the distance between the mobile nodes themselves is smaller than the distance between the mobile nodes and the destination (i.e., base station).

$$+\underbrace{\sum_{R_{k}\neq S} \underbrace{\mathbf{E}_{R_{k}D}\mathbf{F}_{R_{k}}\mathbf{H}_{R_{k}D}\Phi_{R_{k},1}}_{\mathbf{V}_{R_{k}D,1}}\mathbf{X}_{R_{k},1} + \mathbf{W}_{D,1},}_{\mathbf{\Xi}_{D,1}}$$
(1a)

$$\mathbf{Y}_{R_{k},1} = \underbrace{\mathbf{E}_{SR_{k}} \mathbf{F}_{S} \mathbf{H}_{SR_{k}} \Phi_{S,1}}_{\mathbf{V}_{SR_{k}}} \mathbf{X}_{S,1}$$

$$+ \underbrace{\underbrace{\sum_{R_{l} \neq S, R_{k}} \underbrace{\mathbf{E}_{R_{l}R_{k}} \mathbf{F}_{R_{l}} \mathbf{H}_{R_{l}R_{k}} \Phi_{R_{l},1}}_{\mathbf{V}_{R_{l}R_{k}}} \mathbf{X}_{R_{l},1} + \mathbf{W}_{R_{k}},$$

$$\underbrace{\mathbf{E}_{R_{k},1}}_{\mathbf{E}_{R_{k},1}} \mathbf{U}_{R_{k}} \mathbf{U}_{R_{k}}}_{\mathbf{E}_{R_{k},1}} \mathbf{U}_{R_{k}}} \mathbf{U}_{R_{k}} \mathbf{U}_{R_{k}} \mathbf{U}_{R_{k}}}$$

$$\underbrace{\mathbf{E}_{R_{k},1}}_{\mathbf{U}_{R_{k}}} \mathbf{U}_{R_{k}}} \mathbf{U}_{R_{k}} \mathbf{U}_{R_{k}}} \mathbf{U}_{R_{k}} \mathbf{U}_{R_{k}}} \mathbf{U}_{R_{k}} \mathbf{U}_{R_{k}} \mathbf{U}_{R_{k}} \mathbf{U}_{R_{k}}}$$

$$\underbrace{\mathbf{U}_{R_{k}}}_{\mathbf{U}_{R_{k}}} \mathbf{U}_{R_{k}}} \mathbf{U}_{R_{k}} \mathbf{U}_{R_{k}}} \mathbf{U}_{R_$$

where  $\mathbf{Y}_{D,1}$  and  $\mathbf{Y}_{R_{k,1}}$  are  $N \times T$  matrices, and  $\mathbf{E}_{ab} = \text{diag}\left\{1, e^{\frac{j2\pi\varepsilon_{ab}}{N}}, \cdots, e^{\frac{j2\pi\varepsilon_{ab}(N-1)}{N}}\right\}$ .<sup>4</sup> We assume that the oscillators of the mobile nodes should be calibrated with the oscillator of the base station and, therefore, that each  $\varepsilon_{zD}$ ,  $z \in \{S, R_1, \cdots, R_M\}$ , can be approximated as an independent and identically distributed (i.i.d.) random variable (RV) with mean zero and variance  $\sigma_{\epsilon}^2$  (but not necessarily Gaussian). Since  $\varepsilon_{SD} = \varepsilon_{SR_k} + \varepsilon_{R_kD}$  holds, we have  $\operatorname{Var} \{ \varepsilon_{zD} \} = \sigma_{\epsilon}^2$  and  $\operatorname{Var} \{ \varepsilon_{SR_k} \} = \operatorname{Var} \{ \varepsilon_{SD} - \varepsilon_{R_kD} \} =$  $2\sigma_{\epsilon}^2, z \in \{S, R_1, \cdots, R_M\}$ . Identical power is allocated to each pilot subcarrier. We also assume that  $\alpha N_u P$  is allocated to node S in the first time slot, and that in the second time slot, the relay uses the remaining power, i.e.,  $(1-\alpha)N_uP$ , where P represents the average power of each subcarrier, and  $0 < \alpha <$ 1. Therefore,  $\mathbf{\Phi}_{S,1} = \mathbf{\Phi}_{R_l,1} = \sqrt{\alpha \bar{P}} \mathbf{I}_{N_u}$  are  $N_u \times N_u$  diagonal matrices with each diagonal entry representing the transmit power of one subcarrier of nodes S and  $R_l$ , respectively, in the first time slot.  $\mathbf{X}_{a,1} = [\mathbf{x}_{a,1}(0), \cdots, \mathbf{x}_{a,1}(T-1)]$ , which is an  $N_u \times T$  matrix  $(T = 1, 2, \cdots)$ , represents the transmit matrix of node a, and we assume that  $[\mathbf{X}_{a,1}]_{mn} \sim \mathcal{CN}(0,1)$ .  $\mathbf{W}_a$  and  $\mathbf{W}_{D,1}$  are  $N_u \times T$  matrices of additive white Gaussian noise (AWGN) with  $\{\mathbf{W}_a[m], \mathbf{W}_{D,1}[m]\} \sim \mathcal{CN}(0, \sigma_w^2)$ .

Based on the interference analysis in [12], the effective SINR at nodes D and  $R_k$  is given by

$$\gamma_{SD,1} = \frac{\mathbb{E}\left\{ \operatorname{trace}\left\{\mathbf{F}_{S}^{H}\mathbf{V}_{SD,1}\mathbf{X}_{S,1}\mathbf{X}_{S,1}^{H}\mathbf{V}_{SD,1}^{H}\mathbf{F}_{S}\right\}\right\}}{\mathbb{E}\left\{\operatorname{trace}\left\{\mathbf{F}_{S}^{H}\mathbf{\Xi}_{D,1}\mathbf{\Xi}_{D,1}^{H}\mathbf{F}_{S}\right\}\right\}}$$
$$= \frac{\alpha\bar{P}\beta_{1}\cdot\mathbb{E}\{\nu_{SD}\}}{\frac{\mathcal{L}_{u}\pi^{2}N_{u}\sigma_{e}^{2}\alpha\bar{P}}{3} + N_{u}\sigma_{w}^{2}} = \frac{\mathcal{L}_{u}\alpha\bar{P}\beta_{1}}{\frac{\mathcal{L}_{u}\pi^{2}\sigma_{e}^{2}\alpha\bar{P}}{3} + \sigma_{w}^{2}}, \quad (2a)$$
$$\gamma_{SR_{k},1} = \frac{\mathbb{E}\left\{\operatorname{trace}\left\{\mathbf{F}_{S}^{H}\mathbf{V}_{SR_{k},1}\mathbf{X}_{S,1}\mathbf{X}_{S,1}^{H}\mathbf{V}_{SR_{k},1}^{H}\mathbf{F}_{S}\right\}\right\}}{\mathbb{E}\left\{\operatorname{trace}\left\{\mathbf{F}_{S}^{H}\mathbf{\Xi}_{R_{k},1}\mathbf{\Xi}_{R_{k},1}^{H}\mathbf{F}_{S}\right\}\right\}}$$

$$= \frac{\alpha P \beta_2 \cdot \mathbb{E}\{\nu_{SR_k}\}}{\frac{2\pi^2 N_u \sigma_e^2 \alpha \bar{P}}{3} + N_u \sigma_w^2} = \frac{\alpha P \beta_2}{\frac{2\pi^2 \sigma_e^2 \alpha \bar{P}}{3} + \sigma_w^2}, \qquad (2b)$$

where 
$$\nu_{Sb} = \text{trace} \{ \mathbf{H}_{Sb} \mathbf{H}_{Sb}^H \}, \ \beta_1 = \left( 1 - \frac{\pi^2 \sigma_{\epsilon}^2}{3} + \frac{\pi^4 \sigma_{\epsilon}^4}{20} \right)$$

<sup>4</sup>The symbol  $\varepsilon_{ab}$  represents the normalized frequency offset (the frequency offset normalized to a subcarrier spacing of OFDM symbols) between the nodes *a* and *b*. Since the initial phase is independent of the frequency offsets and channel coefficient, and also because the estimation of the initial phase is beyond the discussion of this letter, we assume that initial phase has been estimated and corrected.

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Fig. 1. Channel outage probability as a function of  $N_u$  in OFDMA.



Fig. 2. PDF of  $\nu_{max}$  in cooperative OFDMA uplink transmission.

and 
$$\beta_2 = \left(1 - \frac{2\pi^2 \sigma_\epsilon^2}{3} + \frac{\pi^4 \sigma_\epsilon^4}{5}\right).$$

In this letter, we assume that the subcarriers allocated to each node are not contiguous, and the distance between any two neighboring subcarriers is assumed to be large enough to make the correlation between them be negligible. Therefore, each  $\nu_{Sb}$  is a central *chi-square* RV with  $2N_u$  degrees of freedom [13, page 41]. From [13, page 42], for a threshold  $T_h > 0$ , the probability that  $\nu_{Sb} < T_h$  is given by

$$P_r\{\nu_{Sb} < N_u \cdot T_h\} = 1 - e^{-N_u \cdot T_h} \sum_{k=0}^{N_u - 1} \frac{(N_u \cdot T_h)^k}{k!}.$$
 (3)

For a larger  $N_u$ , a smaller outage probability  $P_r\{\nu_{Sb} < N_u \cdot T_h\}$  is obtained, i.e., a more robust wireless channel is obtained, as shown in Fig. 1.

## C. The Second Time Slot

In the second time slot, we assume that all nodes have CSI. Only one relay is chosen for re-transmission. The active relay  $R_s$  may be chosen by maximizing the composite channel gains:

$$R_s = \arg \max_{R_1, \cdots, R_M} \left\{ \nu_{SR_1}, \cdots, \nu_{SR_M} \right\}.$$
(4)

It was proven by [14] that the opportunistic relaying strategy by using the "best" relay to perform re-transmission is optimal in terms of outage probability. The Probability Density Function (PDF) of  $\nu_{SR_k}$  is given by  $f(\nu) = \frac{1}{(N_u - 1)!} \nu^{N_u - 1} e^{-\nu}$ . The PDF of  $\nu_{max} = \max\{\nu_{SR_1}, \cdots, \nu_{SR_M}\}$  is therefore  $f(\nu_{max}) = M\left(1 - e^{-\nu_{max}} \sum_{m=0}^{N_u - 1} \frac{\nu_{max}^m}{m!}\right)^m \frac{\nu_{max}^{N_u - 1} e^{-\nu_{max}}}{(N_u - 1)!}$ , and the expectation of  $\nu_{max}$  is derived as

$$\bar{\nu}_{\max} = \int_{0}^{\infty} \nu_{\max} f(\nu_{\max}) d\nu_{\max}$$

$$= M \sum_{k=0}^{M-1} \binom{M-k}{k}$$

$$\cdot \sum_{n=0}^{(N_u-1)(M-k)} \frac{g_k^{(n)}(0)}{(M-k)^2(M-k-1)!},$$
(5)

where  $g_k(x) = \left(\sum_{m=0}^{N_u-1} \frac{x^m}{m!}\right)^{m-\kappa}$ , and  $g_k^{(n)}(0) = d^{(n)} q_k(x)$ 

 $\frac{d^{(n)}g_k(x)}{\displaystyle \frac{dx^n}{x=0}}\Big|_{x=0} \text{ represents the }n\text{-order derivative of }g_k(x) \text{ with }$ 

The PDF of  $\nu_{\text{max}}$  as a function of M is shown in Fig.2, where we assume that N = 1024 and  $N_u = 16$ . Fig. 2 shows that the peak of  $f(\nu_{\text{max}})$  shifts to the right as M increases, and that, accordingly,  $\bar{\nu}_{\text{max}}$  increases as M increases. For example, when M = 2,  $\bar{\nu}_{\text{max}} = 18.2$ ; if we increase M to 8, we have  $\bar{\nu}_{\text{max}} = 22.1$ , and  $\bar{\nu}_{\text{max}} = 23.8$  if we further increase M to 16. For a constant  $N_u$ , since a larger M implies a higher spatial diversity gain, we can finally improve the robustness of the training sequence transmission and, as a result, reduce the frequency offset estimation error.

1) AF Mode: In the AF mode,  $R_s$  simply re-transmits the received training sequence to D. The received training sequence at node D in the second time slot is

$$\mathbf{Y}_{D,2}^{AF} = \rho_{SR_s} \mathbf{E}_{R_s D} \mathbf{F}_S \mathbf{H}_{R_s D} \boldsymbol{\Phi}_{R_s,2} \mathbf{F}_S^H \mathbf{Y}_{R_s,1} + \sum_{R_k \neq S, R_s} \mathbf{E}_{R_k D} \mathbf{F}_{R_k} \mathbf{H}_{R_k D} \boldsymbol{\Phi}_{R_k,2} \mathbf{X}_{R_k,2} + \mathbf{W}_{D,2} = \underbrace{\rho_{SR_s} \mathbf{E}_{R_s D} \mathbf{F}_S \mathbf{H}_{R_s D} \boldsymbol{\Phi}_{R_s,2} \mathbf{F}_S^H \mathbf{V}_{SR_s}}_{\mathbf{V}_{SRD,2}^{AF}} \mathbf{X}_{S,1} + \tilde{\mathbf{W}}_{D,2}^{AF},$$
(6)

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where we have 
$$\begin{split} & \text{where we have } V_{D,2}^{\text{AF}} = \\ & \rho_{SR_s} \mathbf{E}_{R_s D} \mathbf{F}_S \mathbf{H}_{R_s D} \boldsymbol{\Phi}_{R_s, 2} \mathbf{F}_S^H \sum_{R_l \neq S, R_s} \mathbf{V}_{R_l R_s} \mathbf{X}_{R_l, 1} = \\ & \rho_{SR_s} \mathbf{E}_{R_s D} \mathbf{F}_S \mathbf{H}_{R_s D} \boldsymbol{\Phi}_{R_s, 2} \mathbf{F}_S^H \mathbf{W}_{R_s} = \\ & \sum_{R_k \neq S, R_s} \mathbf{E}_{R_k D} \mathbf{F}_{R_k} \mathbf{H}_{R_k D} \boldsymbol{\Phi}_{R_k, 2} \mathbf{X}_{R_k, 2} + \mathbf{W}_{D, 2} \end{split}$$

 $\Phi_{R_s,2} = \sqrt{\frac{(1-\alpha)P}{M}} \mathbf{I}_{N_u}$  represents the power consumed in  $R_s$  for re-transmission in the second time slot, and  $\rho_{SR_s} = \sqrt{\frac{N_u}{\alpha \bar{\nu}_{\max} \bar{P} + N_u \sigma_w^2}}$  represents the amplifying

From [12], the received SINR is given by

$$\begin{split} \gamma_{2}^{\text{AF}} &= \frac{\mathbb{E}\left\{ \text{trace}\left\{\mathbf{F}_{S}^{H}\mathbf{V}_{SRD,2}^{\text{AF}}\mathbf{X}_{S,1}\mathbf{X}_{S,1}^{H}\left(\mathbf{V}_{SRD,2}^{\text{AF}}\right)^{H}\mathbf{F}_{S}\right\}\right\}}{\mathbb{E}\left\{ \text{trace}\left\{\mathbf{F}_{S}^{H}\tilde{\mathbf{W}}_{D,2}^{\text{AF}}\left(\tilde{\mathbf{W}}_{D,2}^{\text{AF}}\right)^{H}\mathbf{F}_{S}\right\}\right\}} \\ &= \frac{\mathcal{L}_{u}\alpha(1-\alpha)\bar{P}^{2}\beta_{1}\beta_{2}\mathbb{E}\{\bar{\nu}_{\max}\nu_{R_{s}D}\}}{\frac{2\mathcal{L}_{u}\alpha\bar{P}\pi^{2}N_{u}\sigma_{\epsilon}^{2}}{3}+N_{u}\sigma_{w}^{2}+\left(\frac{\mathcal{L}_{u}(1-\alpha)\bar{P}\pi^{2}N_{u}\sigma_{\epsilon}^{2}}{3}+N_{u}\sigma_{w}^{2}\right)\xi_{SR_{s}}}, \end{split}$$
(7)

where  $\xi_{SR_s} = \mathcal{L}_u \alpha \bar{P} \beta_2 \bar{\nu}_{max} + \frac{2\mathcal{L}_u \alpha \bar{P} \pi^2 N_u \sigma_{\epsilon}^2}{3} + N_u \sigma_w^2$ . 2) *DcF Mode:* In this mode, the relay first identifies the

received training sequence, and then re-generates this training sequence and re-transmits it. After performing the demodulating and decoding, the node  $R_s$  should estimate  $\varepsilon_{SR_s}$ ; i.e.,  $\hat{\varepsilon}_{SR_s,1} = \varepsilon_{SR_s} + e_{SR_s,1}$  ( $e_{SR_s,1}$  represents the estimation error of  $\hat{\varepsilon}_{SR_s,1}$ ), and then uses  $\hat{\varepsilon}_{SR_s,1}$  to pre-compensate for the frequency offset between S and  $R_s$  to re-generate the training sequence. The received training sequence at D is

$$\mathbf{Y}_{D,2}^{\text{DcF}} = \underbrace{\mathbf{E}_{R_s D} \mathbf{F}_S \mathbf{H}_{R_s D} \boldsymbol{\Phi}_{R_s,2} \mathbf{F}_S^H \hat{\mathbf{E}}_{SR_s} \mathbf{F}_S}_{\mathbf{V}_{SRD,2}^{\text{DcF}}} \mathbf{X}_S + \underbrace{\sum_{R_k \neq S, R_s} \mathbf{E}_{R_k D} \mathbf{F}_{R_k} \mathbf{H}_{R_k D} \boldsymbol{\Phi}_{R_k,2} \mathbf{X}_{R_k,2} + \mathbf{W}_{D,2}}_{\tilde{\mathbf{W}}_{D,2}^{\text{DcF}}},$$
(8)

The average SINR in node D is

$$\gamma_{2}^{\text{DcF}} = \frac{\mathbb{E}\left\{\text{trace}\left\{\mathbf{K}^{\text{DcF}}\right\}\right\}}{\mathbb{E}\left\{\text{trace}\left\{\mathbf{F}_{S}^{H}\tilde{\mathbf{W}}_{D,2}^{\text{DcF}}\left(\tilde{\mathbf{W}}_{D,2}^{\text{DcF}}\right)^{H}\mathbf{F}_{S}\right\}\right\}} = \frac{(1-\alpha)\bar{P}\beta_{1}\cdot\mathbb{E}\{\nu_{R_{s}D}\}}{\frac{\mathcal{L}_{u}\pi^{2}N_{u}\sigma_{e}^{2}(1-\alpha)\bar{P}}{3} + N_{u}\sigma_{w}^{2}},$$
(9)

### **III. FREQUENCY OFFSET ESTIMATION IN THE COOPERATIVE SCHEME**

The training sequences received in first and the second time slots can be used by D to estimate frequency offset. The first time slot frequency offset  $\varepsilon_{SD}$  can be estimated as  $\hat{\varepsilon}_{SD,1} =$  $\varepsilon_{SD} + e_{SD,1}$ , where  $e_{SD,1}$  is an estimation error. From [1],

[15], [16], for an unbiased estimator  $\hat{\varepsilon}_{SD,1}$ , the Cramer-Rao Lower Bound (CRLB) can be represented as

$$\operatorname{Var}\left\{e_{SD,1}\middle|\nu_{SD}\right\} \ge \frac{1}{\mathcal{A}_T \cdot \gamma_{SD,1}},\tag{10}$$

where  $A_T$  is a positive coefficient specified by the structure of the training sequence  $X_S$ . For example, if the training sequence proposed in [15] is used, it can be shown that  $\mathcal{A}_T = 4\pi^2 N_u$ . Similarly, the frequency offset estimation at node  $R_s$  can be represented as  $\hat{\varepsilon}_{SR_s,1} = \varepsilon_{SR_s} + e_{SR_s,1}$ . For an unbiased estimator, the CRLB is

$$\operatorname{Var}\left\{e_{SR_{s},1}\middle|\nu_{SR_{s}}\right\} \geq \frac{1}{\mathcal{A}_{T} \cdot \gamma_{SR_{s},1}}.$$
(11)

The frequency offset estimation in the second time slot depends on the relaying mode. Let us use  $\hat{\varepsilon}_{SR_sD,2} = \varepsilon_{SD} + \varepsilon_{SD}$  $e_{SR_sD,2}$  to represent the estimate of  $\varepsilon_{SD}$  in the second time slot. The CRLB for the AF mode is given by

$$\operatorname{Var}\left\{e_{SR_sD,2}\Big|\nu_{SR_s},\nu_{R_sD};\operatorname{AF}\right\} \geq \frac{1}{\mathcal{A}_T \cdot \gamma_2^{\operatorname{AF}}}.$$
 (12)

In the DcF mode, the estimation error  $e_{SR_s,1}$  will be accumulated and propagated to the final result, and the CRLB is given by

$$\operatorname{Var}\left\{e_{SR_{s}D,2}\middle|\nu_{SR_{s}},\nu_{R_{s}D};\operatorname{DcF}\right\} = \operatorname{Var}\left\{e_{SR_{s},1}\middle|\nu_{SR_{s}}\right\} + \operatorname{Var}\left\{e_{R_{s}D,2}\middle|\nu_{R_{s}D}\right\} \\ \geq \frac{1}{\mathcal{A}_{T}}\left(\frac{1}{\gamma_{SR_{s},1}} + \frac{1}{\gamma_{2}^{\operatorname{DcF}}}\right).$$
(13)

Since the frequency offset estimates in both the first and second time slots are conditionally unbiased with mean  $\varepsilon_{SD}$ , these two estimates can be combined to improve the estimation accuracy. From [17, chapter 6], the estimation results in a twotime-slot period can be combined to be

$$\hat{\varepsilon}_{SD} = \lambda_1 \hat{\varepsilon}_{SD,1} + \lambda_2 \hat{\varepsilon}_{SR_sD,2} = \varepsilon_{SD} + \underbrace{\lambda_1 e_{SD,1} + \lambda_2 e_{SR_sD,2}}_{e_{SD}},$$
(14)

(8) where  $\lambda_1$  and  $\lambda_2$  are two non-negative coefficients, and  $\lambda_1 + \lambda_2 = 1$ .  $e_{SR_s}$  = diag  $\left\{ 1, e^{\frac{j2\pi\hat{\epsilon}_{SR_s,1}}{N}}, \cdots, e^{\frac{j2\pi\hat{\epsilon}_{SR_s,1}(N-1)}{N}} \right\}$ .

$$\operatorname{Var}\left\{e_{SD}\middle|\nu_{SD},\nu_{SR_{s}},\nu_{R_{s}D}\right\} = \lambda_{1}^{2}\operatorname{Var}\left\{e_{SD,1}\middle|\nu_{SD}\right\} + \lambda_{2}^{2}\operatorname{Var}\left\{e_{SR_{s}D,2}\middle|\nu_{SR_{s}},\nu_{R_{s}D}\right\}.$$
(15)

In the AF mode, for a given  $\alpha$  and  $(\nu_{SD}, \nu_{SR_s}, \nu_{R_sD})$ , the optimal  $\lambda_1$  and  $\lambda_2$ ,<sup>5</sup> i.e.,  $\lambda_{1,opt}^{AF} = \frac{\gamma_{SD,1}}{\gamma_{SD,1} + \gamma_2^{AF}}$  and  $\lambda_{2,opt}^{AF} =$ 

where  $\mathbf{K}^{\text{DcF}} = \mathbf{F}_{S}^{H} \mathbf{V}_{SRD,2}^{\text{DcF}} \hat{\mathbf{E}}_{SR_{s}} \mathbf{X}_{S} \mathbf{X}_{S}^{H} \hat{\mathbf{E}}_{SR_{s}}^{H} \left( \mathbf{V}_{SRD,2}^{\text{DcF}} \right)^{H} \mathbf{F}_{S}$ . <sup>5</sup>In the linear combining estimation, "optimal" means the values of  $\lambda_{1}$  and  $\lambda_{2}$  that minimize the variance error Var  $\left\{ e_{SD} \middle| \nu_{SD}, \nu_{SR_{s}}, \nu_{R_{s}D} \right\}$ . The optimal  $\lambda_1$  and  $\lambda_2$  for the AF and DcF modes can be calculated by taking partial derivative to (12) and (13), respectively, and set in the set of the results to zero. By resolving these equations, the "optimal"  $\lambda_1$  and  $\lambda_2$  in terms of minimized linearly combined variance error are  $\lambda_{1,opt}^{AF} = \frac{\gamma_{SD,1}}{\gamma_{SD,1} + \gamma_2^{AF}}$  and  $\lambda_{2,opt}^{AF} = \frac{\gamma_{SD,1}}{\gamma_{SD,1} + \gamma_2^{AF}}$ 

$$\frac{\gamma_2}{\gamma_{SD,1} + \gamma_2^{\text{AF}}} \text{ for the AF mode, or } \lambda_{1,\text{opt}}^{\text{DcF}} = \frac{1}{1 + \frac{\gamma_{SD,1}(\gamma_{SR_s,1} + \gamma_2^{\text{DcF}})}{\gamma_{SR_s,1}\gamma_2^{\text{DcF}}}} \text{ and}$$
$$\lambda_{2,\text{opt}}^{\text{DcF}} = \frac{\gamma_{SD,1}(\gamma_{SR_s,1} + \gamma_2^{\text{DcF}})}{\gamma_{SR_s,1}\gamma_2^{\text{DcF}} + \gamma_{SD,1}(\gamma_{SR_s,1} + \gamma_2^{\text{DcF}})}, \text{ for the DcF mode.}$$

 $\sqrt{AF}$ , are used to minimize the variance of  $e_{SD}$  as

$$\operatorname{Var}\left\{e_{SD} \left| 0 < \alpha < 1; \nu_{SD}, \nu_{SR_s}, \nu_{R_sD}; \lambda_{1,\text{opt}}^{\text{AF}}, \lambda_{2,\text{opt}}^{\text{AF}}\right\}\right\} \\ \geq \frac{1}{\mathcal{A}_T\left(\gamma_{SD,1} + \gamma_2^{\text{AF}}\right)}.$$
(16)

and

In the DcF mode, when  $\lambda_{1,\text{opt}}^{\text{DcF}} = \frac{1}{1 + \frac{\gamma_{SD,1}(\gamma_{SR_{s},1} + \gamma_{2}^{\text{DcF}})}{\gamma_{SR_{s},1}\gamma_{2}^{\text{DcF}}}}$   $\lambda_{2,\text{opt}}^{\text{DcF}} = \frac{\gamma_{SD,1}(\gamma_{SR_{s},1} + \gamma_{2}^{\text{DcF}})}{\gamma_{SR_{s},1}\gamma_{2}^{\text{DcF}} + \gamma_{SD,1}(\gamma_{SR_{s},1} + \gamma_{2}^{\text{DcF}})}$ , the mini-variance of  $e_{SD}$  can be obtained as the minimum

$$\operatorname{Var}\left\{e_{SD}\left|0<\alpha<1;\nu_{SD},\nu_{SR_{s}},\nu_{R_{s}D};\lambda_{1,\text{opt}}^{\text{DcF}},\lambda_{2,\text{opt}}^{\text{DcF}}\right\}\right.$$

$$\geq \frac{1}{\mathcal{A}_{T}\left(\gamma_{SD,1}+\frac{\gamma_{SR_{s},1}\gamma_{2}^{\text{DcF}}}{\gamma_{SR_{s},1}+\gamma_{2}^{\text{DcF}}}\right)}.$$
(17)

Both (16) and (17) are functions of  $\alpha$  and can be minimized by using the optimal  $\alpha^6$ . However, the optimal  $\alpha$ depends on whether or not the base station sends channel state information (CSI) to the mobile nodes. With feedback, the mobile nodes can adaptively optimize  $\alpha$  based on the current  $(\nu_{SD}, \nu_{SR_s}, \nu_{R_sD})$  values. If the base station does not feedback CSI,  $\alpha$  can be optimized based on only the statistical information of  $(\nu_{SD}, \nu_{SR_s}, \nu_{R_sD})$ .

## A. Without CSI Feedback from the Base Station

In this case, the optimal  $\alpha$  should minimize the expected variance error as

$$\alpha_{\rm WF}^{\rm AF} = \arg\min_{0<\alpha<1} \mathbb{E}_{\nu} \left\{ \frac{1}{\mathcal{A}_T \left( \gamma_{SD,1} + \gamma_2^{\rm AF} \right)} \right\}, \quad (\rm AF)$$
(18a)

$$\alpha_{\rm WF}^{\rm DcF} = \arg\min_{0<\alpha<1} \mathbb{E}_{\nu} \left\{ \frac{1}{\mathcal{A}_T \left( \gamma_{SD,1} + \frac{\gamma_{SR_s,1}\gamma_2^{\rm DcF}}{\gamma_{SR_s,1} + \gamma_2^{\rm DcF}} \right)} \right\}, \ (\rm DcF)$$
(18b)

for the AF and DcF modes, respectively, where the expectation is performed with respect to  $(\nu_{SD}, \nu_{SR_s}, \nu_{R_sD})$ . By taking the partial derivative of (18) with respect to  $\alpha$  and setting the result to zero, the optimal  $\alpha$  for either the AF or DcF mode can be achieved by resolving the function, if the closed-form solution is available. Actually, we can also find the optimal  $\alpha$ that maximizes (18) numerically. In this letter, the numerical method is used.

#### B. With CSI Feedback from the Base Station

The optimal  $\alpha$  derived in Section III-A does not change as the current channel changes, so that neither  $\alpha_{WF}^{AF}$  nor  $\alpha_{WF}^{DcF}$  is always optimal in each channel realization. If the base station sends the current CSI to the mobile nodes, a lower variance error can be achieved by adaptively optimizing  $\alpha$  based on the current CSI. For a given ( $\nu_{SD}$ ,  $\nu_{SR_s}$ ,  $\nu_{R_sD}$ ), the adaptively



Fig. 3. Adaptive cooperation in OFDMA uplink frequency offset estimation.

optimized  $\alpha$  is given by

$$\alpha_{\rm FB}^{\rm AF} = \arg\min_{0<\alpha<1} \left\{ \frac{1}{\mathcal{A}_T \left(\gamma_{SD,1} + \gamma_2^{\rm AF}\right)} \right\}, \ (\rm AF)$$
(19a)  
$$\alpha_{\rm FB}^{\rm DcF} = \arg\min_{0<\alpha<1} \left\{ \frac{1}{\mathcal{A}_T \left(\gamma_{SD,1} + \frac{\gamma_{SR_s,1}\gamma_2^{\rm DcF}}{\gamma_{SR_s,1} + \gamma_2^{\rm DcF}}\right)} \right\}, \ (\rm DcF)$$
(19b)

for the AF and DcF modes, respectively.

# C. Adaptive Switching Between Cooperative and Conventional Non-Cooperative Transmissions

A hybrid cooperative scheme is proposed to adaptively optimize the transmission, as shown in Fig. 3. The basic idea is: for a given  $(M, \sigma_{\epsilon}^2, \text{SNR})$ , if there is an  $\alpha$   $(0 < \alpha < 1)$ to make the cooperative transmission outperform the conventional transmission or vice versa, the transmitter switches accordingly. Since the base station may either send CSI to the mobile nodes or not, the cooperative transmission may be performed in two cases: (1) if the base station does not feedback CSI, the second switch should be switched to "21" to perform "Without Feedback" cooperation.  $\alpha_{WF}^{AF}$  (or  $\alpha_{WF}^{DcF}$ ) will be used for the AF (or DcF) mode; and (2) if the base station sends the CSI to the mobile nodes, the second switch should be switched to "22" to perform "With Feedback" cooperation, and  $\alpha_{\rm FB}^{\rm AF}$  (or  $\alpha_{\rm FB}^{\rm DcF}$ ) should be used for the AF (or DcF) mode in this case. An information-sharing scheme should be performed between S and  $R_s$  to guarantee that an identical  $\alpha$  will be used by them in the same transmission, but how to perform this information-sharing scheme is beyond the scope of this letter.

#### **IV. NUMERICAL RESULTS**

An OFDMA uplink system with IDFT length of 1024 is simulated. A CP of length 64 is used. An equal allocation of subcarriers per node is made. The number of relays is M.

<sup>&</sup>lt;sup>6</sup>Since the relaying AF channel is no longer Gaussian (see [18]), the linear combined estimator is not minimum-variance unbiased (MVU) estimator, we cannot actually find the "optimal"  $\alpha$  and  $\lambda$ . Here, "optimal" means the "minimized" linear combining error.

Frequency offsets are assumed to be i.i.d. RVs with mean zero and variance  $\sigma_{\epsilon}^2$ . The algorithm proposed in [15] is used in this letter to verify the performance improvement obtained by using cooperative relays<sup>7</sup>, i.e., T = 2. The training sequence is known to all the nodes.

Fig. 4 compares the variance of errors of the proposed cooperative scheme as a function of  $\alpha$  and that of the conventional estimation ( $\alpha = 1$ ), where SNR=20 dB and M = 16. When  $\sigma_{\epsilon}^2 = 10^{-2}$ , the AF and DcF cooperative schemes outperforms conventional estimation when  $0.11 < \alpha < 1$  and  $0.65 < \alpha < 1$ , respectively. When  $\sigma_{\epsilon}^2 = 10^{-3}$ , the corresponding ranges of  $\alpha$  are  $0.13 < \alpha < 1$  and  $0.56 < \alpha < 1$ , respectively. For each  $\alpha$ , the AF mode always outperforms the DcF mode. From Fig. 4, the optimal  $\alpha$ , i.e.,  $\alpha_{WF}^{AF}$  and  $\alpha_{WF}^{DcF}$ , can easily be found. For example, when  $\sigma_{\epsilon}^2 = 10^{-2}$ , we have  $\alpha_{WF}^{AF} = 0.53$  (with a variance error  $2.85 \times 10^{-5}$ ) and  $\alpha_{WF}^{DcF} = 0.81$  (with a variance error  $3.42 \times 10^{-5}$ ).

Since the power allocation between S and  $R_s$  can be adaptively optimized in each transmission if the CSI is available to the nodes, the performance gains can then be further improved. For example, in Table I, with SNR=20 dB and  $\sigma_{\epsilon}^2 = 10^{-1}$ , the performance improvement of AF (or DcF) mode over the conventional scheme is 2.2 dB (or 0.4 dB) if the base station does not send CSI. If it does, the performance advantage increases to 4.9 dB and 1.7 dB for the AF and DcF modes, respectively.

Table II evaluates the performance of the proposed scheme as a function of M when the base station sends CSI to the mobile nodes. In this simulation, for each M, the variances of estimation error for  $\alpha_{FB}^{AF}$  (for the AF mode) and  $\alpha_{FB}^{DcF}$ (for the DcF mode) are evaluated. In either the AF or DcF mode, the variance decreases monotonically with M. As mentioned above for Table I, a performance advantage over the conventional scheme can also be achieved in the cooperative scheme with feedback. The AF mode still outperforms the DcF mode. For example, when  $\sigma_{\epsilon}^2 = 10^{-2}$  and M = 16, the variance error achieved in the DcF mode is  $1.098 \times 10^{-5}$ , but that achieved in the AF mode is  $4.56 \times 10^{-6}$ . We can explain this finding as follows: In the interference-limited cooperative transmission, the interference due to the frequency offset in  $S \to R_s$  link is twice that of either the  $S \to D$  or  $R_s \to D$ link. In the DcF relaying mode,  $R_s$  should estimate  $\varepsilon_{SR_s}$ , and the estimation error will be accumulated and propagated to the final result. When the frequency offset is large, the error in  $R_s$  will dominate the overall variance error, as given by (17). However, this error propagation from  $R_s$  to D can be mitigated in the AF mode.

The bit error rate (BER) performance of the proposed cooperative scheme with CSI feedback from the base station is evaluated in Fig. 5. The subcarrier modulation is either QPSK or 16QAM. The effect of channel estimation error on BER is also considered, and Least-Square (LS) channel estimation proposed in [19] is applied in this simulation. The AF mode always outperforms the DcF mode in terms of BER if there

Fig. 4. Cooperative frequency offset estimation as a function of  $\alpha$  without feedback from the base station.

10

뜠 10

Fig. 5.



10 E<sub>b</sub>/N<sub>0</sub> (dB)

are frequency offset errors, and this gap increases as the SNR increases.<sup>8</sup> In both relaying modes, the BER performance improves with M. However, the performance improvement in the AF mode is more than that in the DcF mode. We can explain this finding as follows: in the AF mode, frequency offset estimation error is dominated by the effective SINR of both the  $S \to D$  and  $S \to R_s \to D$  links, and more relays reduce the SINR degradation due to the frequency offset error. Whereas the frequency offset estimation error in the DcF mode is dominated by the estimation error accumulated in the relay, and hence more relays yield diminishing returns on the amount of SINR improvement.

<sup>8</sup>If an identical frequency offset is considered, DcF (or decode-and-forward (DF)) mode always outperforms AF mode in terms of BER and outage capacity. However, the DcF mode has a higher frequency offset error than the AF mode, and as a result, the BER performance in the DcF mode becomes worse than the AF mode by applying the same frequency offset estimation algorithm.



<sup>&</sup>lt;sup>7</sup>Actually, the proposed analysis can be used for any practical algorithms, and the conclusion always holds. As long as the frequency offset variance error of an algorithm can be represented as  $\frac{1}{A_T \cdot \text{SINR}}$ , any frequency recovery method can be used as an example, and we can always derive the variance error of cooperative transmissions.

#### TABLE I

PERFORMANCE IMPROVEMENT IN THE PROPOSED COOPERATIVE SCHEME WITH AND WITHOUT FEEDBACK FROM THE BASE STATION OVER THE CONVENTIONAL NON-COOPERATIVE ALGORITHM

Without Feedback; $M = 16$											
SNR		20	dB		30 dB						
$\sigma_{\epsilon}^2$	$10^{-4}$	$10^{-3}$	$10^{-2}$	$10^{-1}$	$10^{-4}$	$10^{-3}$	$10^{-2}$	$10^{-1}$			
AF	1.19	1.21	1.669	2.217	1.533	2.072	3.473	3.236			
DcF	0.492	0.438	0.401	0.412	0.454	0.529	0.427	0.471			
			With F	eedback:	M = 16						
SNR		20	With Fo dB	eedback:	<i>M</i> = 16	30	dB				
$\frac{\text{SNR}}{\sigma_{\epsilon}^2}$	$10^{-4}$	$\frac{20}{10^{-3}}$	With Formula $dB$ $10^{-2}$	eedback:	M = 16 $10^{-4}$	$\frac{30}{10^{-3}}$	$\frac{dB}{10^{-2}}$	$10^{-1}$			
$\frac{\text{SNR}}{\sigma_{\epsilon}^2}$	$\frac{10^{-4}}{25.35}$	$20 \\ 10^{-3} \\ 20.17$	With Formula $\frac{10^{-2}}{10.48}$	eedback: $\frac{10^{-1}}{4.95}$	M = 16 $10^{-4}$ 19.97	$30 \\ 10^{-3} \\ 10.58$	$\frac{dB}{10^{-2}}$ 5.56	$\frac{10^{-1}}{3.48}$			

TABLE II Performance Improvement in the Proposed Cooperative Scheme with Feedback as a function of M

Transmission Mode	Variance Errors: $N = 1024$ , $N_u = 8$ , $\mathcal{L}_u = 10^{-1}$ , SNR=20 dB, $\sigma_{\epsilon} = 10^{-2}$							
Conventional	М	2	4	8	16	32		
Transmission	Error	$5.15 \times 10^{-5}$	$5.15 \times 10^{-5}$	$5.15 \times 10^{-5}$	$5.15 \times 10^{-5}$	$5.15 \times 10^{-5}$		
AF	М	2	4	8	16	32		
Relaying	Error	$4.603 \times 10^{-6}$	$4.585 \times 10^{-6}$	$4.572 \times 10^{-6}$	$4.562 \times 10^{-6}$	$4.554 \times 10^{-6}$		
DF	М	2	4	8	16	32		
Relaying	Error	$1.134 \times 10^{-5}$	$1.121 \times 10^{-5}$	$1.109 \times 10^{-5}$	$1.1089 \times 10^{-5}$	$1.1087 \times 10^{-5}$		

#### V. CONCLUSION

This letter discussed improved OFDMA uplink frequency offset estimation by using cooperative relaying. A new DcF relaying mode was developed. The idea is to adjust for the frequency offset in the  $S \to R$  so that both  $S \to R \to D$ link and  $S \rightarrow D$  link have the same frequency offset. The training sequence received in both  $S \rightarrow D$  and  $S \rightarrow R \rightarrow D$ links can then be used to estimate the frequency offset, and the two estimates can be linearly combined to minimize the variance error. Further improvement is possible by adjusting the power allocation ratio  $\alpha$  between the source and the relay. When CSI is available, the relays can adaptively switch between the cooperative and conventional modes, and this proposed adaptation yields gains 4.95 dB and 1.75 dB over the conventional non-cooperative scheme for the AF and DcF modes, respectively, in low SINR. Although the DcF (or DF) mode outperforms the AF mode in terms of channel capacity and bit error rate (BER) with the same frequency offset, the AF mode obtains the performance advantage over the DcF mode by considering the frequency offset estimation, because the frequency offset error in the DcF mode is higher than that in the AF mode. At high SNR, the AF mode outperforms the DcF mode by about 4.1 dB for QPSK, and this gap is 7 dB for 16QAM.

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