ON THE ENERGY DETECTION OF UNKNOWN DETERMINISTIC SIGNAL OVER NAKAGAMI CHANNELS WITH SELECTION COMBINING

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ABSTRACT

Blind sensing for identifying unused frequency bands is of particular interest in cognitive radio and ultra wide-band applications. Energy detection is one such method proposed to identify the presence of an unknown band-limited deterministic signal. In this paper, by using an alternative series representation of the Marcum-Q function, the exact average detection probability over the Nakagami-*m* fading channel is derived. Moreover, we formulate the decision variable of a selection diversity combined energy detector and derive the exact average detection and false alarm probabilities.

***Index Terms*—** Energy Detection, Cognitive Radio, Selection Combining, Nakagami Fading

1. INTRODUCTION

The detection of unoccupied frequency bands is of interest in emerging technologies such as cognitive radio and ultra wide-band. Blind sensing of frequency spectrum by using an energy detector is one attractive method being proposed for this purpose. The ability to detect with less priori knowledge and the simple structure of the energy detector motivate the further exploration of the detector performance.

Blind detection of an unknown band-limited deterministic signal by using an energy detector is introduced in [1]. Kostylev [2] extends the analysis of [1] to Rayleigh, Rician and Nakagami fading channels. However, the result over Nakagami channel is limited to an integral form expression [2]. In [3] and [4], the detection probability over Nakagami channel is derived for an integer-valued shape parameter (m). The result for Rician fading is limited to unity time bandwidth product in the decision variable (u). Further, detection over maximal ratio, selection and switch and stay diversity receivers over i.i.d. Rayleigh fading branches is considered in [3]. The detector performance over Rayleigh fading with square-law combining schemes is given in [4]. In [5] detection over selection and maximal ratio combining is discussed assuming branch fading to be independent and identically distributed (i.i.d.) Rayleigh.

Careful study of prior work reveals that a consistent decision variable formulation is not available for diversity combining and energy detection. Further, all the available results are limited to the Rayleigh fading channel. This is mainly due to the difficulty that arises in evaluating the related integrals of Marcum-Q function. In our work presented in [6] the energy detector performance is analyzed over an equal gain combiner with i.i.d. Nakagami-*m* fading branches. In [7], detection over square-law and switch diversity combining detectors are considered.

In this paper, we construct the decision variable of an energy detector which employs a selection combiner. By applying an alternative representation of Marcum-Q function, we analyze the detector performance. This approach transforms the related integrals of Marcum-Q function into integrals of special functions that are relatively tractable.

This paper is organized as follows. In Section 2, the system model is provided. The performance of the energy detector over the Nakagami-*m* fading channel is discussed in Section 3. Section 4 extends the decision variable formulation to a selection combiner and derives detection probabilities over Nakagami-*m* fading channels. In Section 5, numerical and simulation results are presented for showing the detector performance over different fading and diversity parameters. The concluding remarks are given in Section 6.

2. SYSTEM MODEL

To be consistent, notations similar to [6] are used as listed below.

h	: Channel coefficient amplitude
E_s	: Received energy over the observation interval T
W	: One sided bandwidth
$u = TW$	: Time bandwidth product
N_{01}	: One sided noise power spectral density
λ	: Energy threshold of the receiver

The detection problem of an unknown deterministic signal is a binary hypothesis test [1]. The probability of detection (P_d) and probability of false alarm (P_f) can be written as in

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(1) and (2) respectively,

$$P_d = Q_u(\sqrt{2\gamma}, \sqrt{\lambda}) \tag{1}$$

$$P_f = \frac{\Gamma\left(u, \frac{\lambda}{2}\right)}{\Gamma(u)} \tag{2}$$

where signal-to-noise-ratio (SNR) is defined by $\gamma = \frac{h^2 E_s}{N_{01}}$ [4]. We refer the reader to [1, 2, 4] for detailed derivations. $Q_u(.,.)$ is the generalized (u^{th} order) Marcum Q-function and $\Gamma(.,.)$ is upper incomplete gamma function. Since γ does not appear in (2), average false alarm probability over any fading channel is same as (2).

3. DETECTION OVER NAKAGAMI-*M* FADING

Using the alternative representation of Marcum-Q function given in [8, eq. (4.74), pp. 104],(1) can be written as,

$$Q_u(\sqrt{2\gamma}, \sqrt{\lambda}) = \sum_{n=0}^{\infty} \frac{\gamma^n e^{-\gamma}}{n!} \sum_{k=0}^{n+u-1} \frac{e^{-\frac{\lambda}{2}}}{k!} \left(\frac{\lambda}{2}\right)^k. \tag{3}$$

The probability density function (PDF) of SNR over Nakagami-*m* channel is,

$$f_\gamma(\gamma) = \frac{1}{\Gamma(m)} \left(\frac{m}{\overline{\gamma}}\right)^m \gamma^{m-1} e^{-\frac{m\gamma}{\overline{\gamma}}},\ \gamma > 0. \tag{4}$$

By integrating (3) over the (4), the average detection probability over Nakagami-*m* fading ($\overline{P}_{d,Nak}$) can be written as,

$$\overline{P}_{d,Nak} = \frac{e^{-\frac{\lambda}{2}}}{\Gamma(m)} \left(\frac{m}{\overline{\gamma}}\right)^m \sum_{n=0}^{\infty} \frac{1}{n!} \int_0^\infty y^{n+m-1} e^{-\left(1+\frac{m}{\overline{\gamma}}\right)y}\, dy \times \sum_{k=0}^{n+u-1} \frac{1}{k!} \left(\frac{\lambda}{2}\right)^k. \tag{5}$$

Using [9, eq. (3.35-3), pp. 340], the integral in (5) can be evaluated for integer values of m as,

$$\overline{P}_{d,Nak} = \frac{e^{-\frac{\lambda}{2}}}{\Gamma(m)} \left(\frac{m}{\overline{\gamma}+m}\right)^m \sum_{n=0}^{\infty} \left(\frac{\overline{\gamma}}{\overline{\gamma}+m}\right)^n \frac{(n+m-1)!}{n!} \times \sum_{k=0}^{n+u-1} \frac{1}{k!} \left(\frac{\lambda}{2}\right)^k. \tag{6}$$

The result in (6) is an alternative derivation of average detection probability over Nakagami-*m* fading channel and is numerically equivalent to the results in [4] and [6].

The error result in truncating infinite series in (6) by N terms ($\mid E_{Nak} \mid$) can be bounded as in (7) by upper bounding the summation over index k in (6) by $e^{\frac{\lambda}{2}}$ and constructing the Hypergeometric series of the form ${}_1F_0(a;;x)$ where generalized Hypergeometric function is given by (8). Using this bound, the number of terms required to obtain a given figure accuracy can be found.

$$\mid E_{Nak} \mid = \left(\frac{m}{\overline{\gamma}+m}\right)^m \left[{}_1F_0\left(m;;\frac{\overline{\gamma}}{\overline{\gamma}+m}\right) - \sum_{n=0}^{N} \left(\frac{\overline{\gamma}}{\overline{\gamma}+m}\right)^n \frac{(m)_n}{n!} \right] \tag{7}$$

$${}_pF_q(a_1,...,a_p;b_1,...,b_q;x) = \sum_{n=0}^{\infty} \frac{(a_1)_n...(a_p)_n}{(b_1)_n...(b_p)_n} \frac{x^n}{n!} \tag{8}$$

The notation $(.)_n$ denotes the Pochhammer symbol and $(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}$ and for integer values of a, $\Gamma(a) = (a-1)!$.

4. DETECTION WITH SELECTION COMBINER (SC)

In the selection combiner, the branch with the maximum SNR is selected and hence the PDF of output SNR of SC (γ_{sc}) can be written as in (9) by setting i.i.d. branch statistics in equation (6) of [10],

$$f_{\gamma_{sc}}(y) = \frac{L}{\Gamma^L(m)} \left(\frac{m}{\overline{\gamma}}\right)^m y^{m-1} e^{-\left(\frac{my}{\overline{\gamma}}\right)} \left[G\left(m, \frac{my}{\overline{\gamma}}\right)\right]^{L-1} \tag{9}$$

where $G(.,.)$ is the lower incomplete gamma function defined by the integral form $G(a,x) = \int_o^x t^{a-1}e^{-t}dt$. Thus, the average detection probability of L branch SC receiver ($\overline{P}_{d,sc,L}$), can be calculated by averaging (1) over (9) as shown in (10).

$$\overline{P}_{d,sc,L} = \frac{L}{\Gamma^L(m)} \left(\frac{m}{\overline{\gamma}}\right)^m \int_0^\infty y^{m-1}\, e^{-\left(\frac{my}{\overline{\gamma}}\right)} \times \left[G\left(m, \frac{my}{\overline{\gamma}}\right)\right]^{L-1} Q_u(\sqrt{2y}, \sqrt{\lambda})\, dy \tag{10}$$

4.1. Dual Diversity Combiner ($L = 2$)

The special function $G(.,.)$ can be written as in (11) with the aid of [9, eq. (8.351-2), pp. 899].

$$G(a,x) = \frac{x^a}{a} e^{-x}\ {}_1F_1(1; 1+a; x) \tag{11}$$

The Marcum-Q function in (1) can be expressed in terms of an infinite series as in (12) [8, (4.63)].

$$Q_u\left(\sqrt{2\gamma}, \sqrt{\lambda}\right) = 1 - e^{-\left(\gamma+\frac{\lambda}{2}\right)} \sum_{n=u}^{\infty} \left(\frac{\lambda}{2\gamma}\right)^{\frac{n}{2}} I_n(\sqrt{2\lambda\gamma}) \tag{12}$$

Substituting (11) and (12) in (10) and replacing ${}_1F_1(1; 1+a; x)$ from the respective series using (8), $\overline{P}_{d,sc,2}$ (i.e. L=2) can be expressed as shown in (13).

$$\overline{P}_{d,sc,2} = 1 - \frac{2\,e^{-\frac{\lambda}{2}}}{m\,\Gamma^2(m)} \sum_{n=u}^{\infty}\sum_{k=0}^{\infty} \left(\frac{m}{\overline{\gamma}}\right)^{k+2m} \left(\frac{\lambda}{2}\right)^{\frac{n}{2}} \times \frac{1}{(m+1)_k} \int_0^{\infty} y^{\left(k+2m-\frac{n}{2}-1\right)} e^{-\left(\frac{2m}{\overline{\gamma}}+1\right)y} I_n(\sqrt{2\gamma y})dy \tag{13}$$

Using [9, eq. (6.643-2), pp.709] and the relation of Whittaker function to the degenerate Hypergeometric function given in (14), $\overline{P}_{d,sc,2}$ can be derived as given in (15).

$$M_{\lambda,\mu}(z) = z^{\mu+\frac{1}{2}}\, e^{-\frac{z}{2}}\, {}_1F_1\left(\mu-\lambda+\frac{1}{2}; 2\mu+1; z\right) \tag{14}$$

$$\overline{P}_{d,sc,2} = 1 - \frac{2e^{-\frac{\lambda}{2}}}{m\,\Gamma^2(m)} \sum_{n=u}^{\infty}\sum_{k=0}^{\infty} \left(\frac{\lambda}{2}\right)^{n} \left(\frac{m}{\overline{\gamma}\beta_2}\right)^{k+2m} \times \frac{\Gamma(m+1)\Gamma(2m+k)}{\Gamma(m+k+1)\Gamma(n+1)}\, {}_1F_1\left(2m+k; n+1; \frac{\lambda}{2\beta_2}\right) \tag{15}$$

Expanding ${}_1F_1(.;.;.)$ by using (8) and constructing the Hypergeometric function of two variables of the form given in (16) [11], $\overline{P}_{d,sc,2}$ can be expressed as in (17).

$$\Psi_1(\alpha,\beta;\gamma,\acute{\gamma};x,y) = \sum_{m,n=0}^{\infty} \frac{(\alpha)_{m+n}(\beta)_m}{(\gamma)_m(\acute{\gamma})_n}\frac{x^m}{m!}\frac{y^n}{n!}, |x|<1 \tag{16}$$

$$\overline{P}_{d,sc,2} = 1 - \frac{2e^{-\frac{\lambda}{2}}}{m}\frac{\Gamma(2m)}{\Gamma^2(m)}\left(\frac{m}{\overline{\gamma}+2m}\right)^{2m} \sum_{n=u}^{\infty}\frac{1}{n!}\left(\frac{\lambda}{2}\right)^{n} \times \Psi_1\left(2m,1;m+1,n+1;\frac{m}{\overline{\gamma}+2m},\frac{\lambda\overline{\gamma}}{2(\overline{\gamma}+2m)}\right) \tag{17}$$

For fixed values of m, λ and $\overline{\gamma}$, the term of $\Psi_1\,(.,.;.,.;.,.)$ monotonically decreases as n increases. Hence, the error result in truncating the infinite series in (17) by N terms ($| E_{sc,2} |$) can be bounded as in (18).

$$| E_{sc,2} | \leq \frac{2\,e^{-\frac{\lambda}{2}}}{m}\frac{\Gamma(2m)}{\Gamma^2(m)}\left(\frac{m}{\overline{\gamma}+2m}\right)^{2m} \times \left(e^{\frac{\lambda}{2}} - \sum_{n=0}^{N}\frac{1}{n!}\left(\frac{\lambda}{2}\right)^{n}\right) \times \Psi_1\left(2m,1;m+1,N+1;\frac{m}{\overline{\gamma}+2m},\frac{\lambda\overline{\gamma}}{2(\overline{\gamma}+2m)}\right) \tag{18}$$

Using the bound given in (18), number of terms ($\tilde{N}$) required to compute $\overline{P}_{d,eg,2}$ to a given figure accuracy can be found.

4.2. Integer m

By means of series form expansion of $G(.,.)$ [9, eq. (8.352-6), pp. 900] and the well known binomial expansion, $\left[G\left(m,\frac{my}{\overline{\gamma}}\right)\right]^{L-1}$ can be written as in (19) for integer values of m.

$$\left[G\left(m,\frac{my}{\overline{\gamma}}\right)\right]^{L-1} = \Gamma^{L-1}(m)\sum_{k=0}^{L-1}\binom{L-1}{k}(-1)^k \times \left[e^{-\frac{my}{\overline{\gamma}}}\sum_{i=0}^{m-1}\left(\frac{m}{\overline{\gamma}}\right)^i\frac{y^i}{i!}\right]^k \tag{19}$$

Using (12) and applying multinomial expansion in (19), (10) can be written as in (20),

$$\overline{P}_{d,sc,L} = 1 - \frac{L}{\Gamma(m)}\left(\frac{m}{\overline{\gamma}}\right)^m e^{-\frac{\lambda}{2}}\sum_{n=u}^{\infty}\sum_{k=0}^{L-1}\binom{L-1}{k}(-1)^k \times \left(\frac{\lambda}{2}\right)^{\frac{n}{2}}\sum_{i=0}^{k(m-1)}\zeta_i\left(m,k,\overline{\gamma}\right) \times \int_0^{\infty} y^{\left(i+m-\frac{n}{2}-1\right)}\, e^{-\beta_L\, y}\, I_n\left(\sqrt{2\lambda y}\right)\, dy \tag{20}$$

where $\zeta_i\left(m,k,\overline{\gamma}\right)$ is the coefficient of multinomial expansion of $\left[\sum_{i=0}^{m-1}\left(\frac{m}{\overline{\gamma}}\right)^i\frac{y^i}{i!}\right]^k$ and $\beta_L = 1+\frac{m(k+1)}{\overline{\gamma}}$. Hence, using [9, eq. (6.643-2), pp.709] and with simplifications using (14), $\overline{P}_{d,sc,L}$ can be derived as in (21). Pd,L integer m for SC (Final Answer)

$$\overline{P}_{d,sc,L} = 1 - Le^{-\frac{\lambda}{2}}\left(\frac{m}{\overline{\gamma}}\right)^m\sum_{n=u}^{\infty}\sum_{k=0}^{L-1}\binom{L-1}{k}(-1)^k\frac{1}{n!} \times \left(\frac{\lambda}{2}\right)^{n}\sum_{i=0}^{k(m-1)}\frac{\zeta_i\left(m,k,\overline{\gamma}\right)(m)_i}{\beta_L^{(i+m)}} \times {}_1F_1\left(i+m;n+1;\frac{\lambda}{2\beta_L}\right) \tag{21}$$

The multinomial expansion in (21) reduces to 1 for $m=1$ and to binomial expansion for $m=2$. Under the constraint of $m=1$ (equivalently the Rayleigh fading channel), results in (15), (17) and (21) are numerically equivalent to the results in [5] and [3] with the correction given in [4, pp. 22].

For fixed values of m, λ and $\overline{\gamma}$, ${}_1F_1\left(i+m;n+1;\frac{\lambda}{2\beta_L}\right)$ monotonically decreases as n increases [12]. Hence, the error result in truncating the infinite series in (21) by N terms (|

Table 1. Number of Terms Required to Obtain a Five Figure Accuracy ($\tilde{N}$)

$\mid E_{Nak} \mid$	$SNR = 10\,dB$ $m = 1$ $\tilde{N} = 120$	$SNR = 10\,dB$ $m = 4$ $\tilde{N} = 54$	$SNR = 5\,dB$ $m = 2$ $\tilde{N} = 28$	$SNR = 10\,dB$ $m = 2$ $\tilde{N} = 77$	$SNR = 15\,dB$ $m = 2$ $\tilde{N} = 231$
$\mid E_{sc,2} \mid$	$u = 1$ $P_f = 0.01$ $\overline{\gamma} = 10$ $m = 2$ $\tilde{N} = 12$	$u = 5$ $P_f = 0.01$ $\overline{\gamma} = 10$ $m = 2$ $\tilde{N} = 29$	$u = 1$ $P_f = 0.01$ $\overline{\gamma} = 20$ $m = 2$ $\tilde{N} = 6$	$u = 1$ $P_f = 0.01$ $\overline{\gamma} = 10$ $m = 4$ $\tilde{N} = 5$	$u = 1$ $P_f = 0.0001$ $\overline{\gamma} = 10$ $m = 2$ $\tilde{N} = 24$
$\mid E_{sc,L} \mid$	$u = 2$ $P_f = 0.01$ $S = 10\,dB$ $m = 2$ $L = 2$ $\tilde{N} = 19$	$u = 2$ $P_f = 0.01$ $S = 10\,dB$ $m = 2$ $L = 3$ $\tilde{N} = 20$	$u = 2$ $P_f = 0.01$ $S = 10\,dB$ $m = 2$ $L = 4$ $\tilde{N} = 21$	$u = 8$ $P_f = 0.0001$ $S = 10\,dB$ $m = 2$ $L = 2$ $\tilde{N} = 47$	$u = 8$ $P_f = 0.0001$ $S = 10\,dB$ $m = 2$ $L = 3$ $\tilde{N} = 47$

$E_{sc,L} \mid$) can be bounded as in (22).

$$\mid E_{sc,L} \mid \leq L\, e^{-\frac{\lambda}{2}} \left(\frac{m}{\overline{\gamma}}\right)^{m} \left(e^{\frac{\lambda}{2}} - \sum_{n=0}^{N} \frac{1}{n!} \left(\frac{\lambda}{2}\right)^{n}\right) \times \sum_{k=0}^{L-1} \binom{L-1}{k} \sum_{i=0}^{k(m-1)} \frac{\zeta_i\left(m,k,\overline{\gamma}\right)(m)_i}{\beta_L^{(i+m)}} \times {}_1F_1\left(i+m; N+1; \frac{\lambda}{2\beta_L}\right) \tag{22}$$

Using the bound given in (22), the number of terms ($\tilde{N}$) required to compute $\overline{P}_{d,sc,L}$ to a given figure accuracy can be found.

4.3. False Alarm Probability

Under selection combining, the branch with maximum SNR i.e $max(\gamma_1, \gamma_2, ..., \gamma_L)$, is selected where $\gamma_l = \frac{h_l^2 E_s}{N_{01}}, l = 1, 2, ..., L$. Hence the receiver picks up the branch with maximum h_l. However, the samples under H_0 are coming from noise alone irrespective of the detector branch selection. Consequently the decision statistic is equivalent to fading channel and hence the average false alarm probability over SC ($\overline{P}_{f,sc,L}$) is the same as given by (2). Therefore, the $\overline{P}_{f,sc,L}$ is different from the result given in [5, eq. (27)].

5. NUMERICAL AND SIMULATION RESULTS

Fig.1 shows the truncation error against the number of terms N from (7), (18) and (22). By using the derived error bounds, the Table 1 is constructed. The minimum number of terms required to obtained a five figure accuracy ($\tilde{N}$). The truncation error reduces rapidly as the number of terms increases, especially in the bounds given in (18) and (22). However, the one given in (7) is not tight enough to reflect the actual number of terms required.

The detector performance is evaluated by means of complementary receiver operating characteristic (ROC) curves similar to [4], The decision is taken by comparing Y and corresponding λ values and hence average P_m and average P_f are calculated. Simulation results of average P_f are within the range specified for that particular set up.

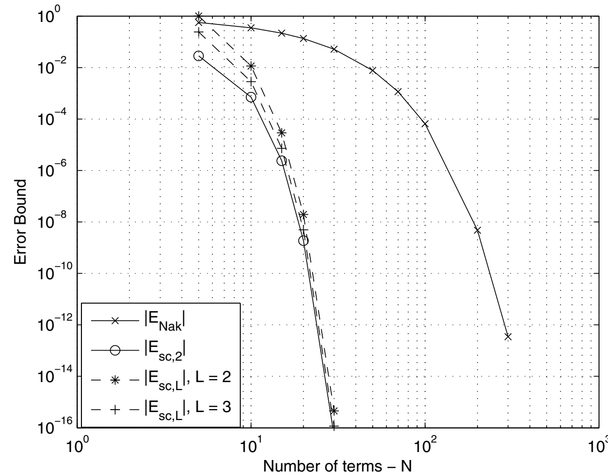

Fig. 1. Error Bounds $u = 1, m = 2, SNR = 10\,dB, L = 2$ and $P_f = 0.01$

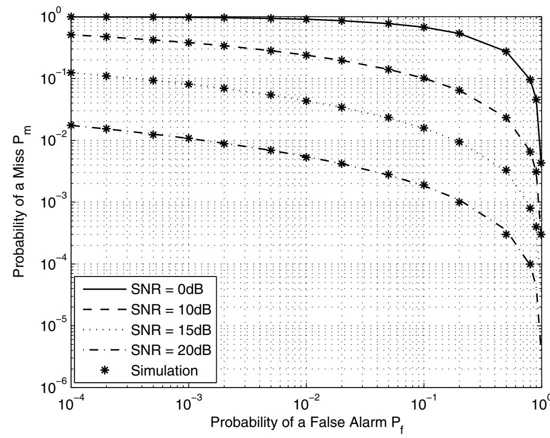

Fig. 2. Complementary ROC curves over Nakagami-*m* channel with $(u = 1, m = 2, SNR = \{0, 10, 15, 20\}\,dB)$

Fig.2 and Fig.3 show the detector performance variation over the average SNR $\overline{\gamma}$ and fading severity index m. As expected higher m and SNR values give better performance. However, for acceptable detector performance, requirement $\overline{\gamma}$ is moderate.

The performance improvement of SC receiver over the number of diversity branches can be observed in Fig.4. Compared to the no diversity case, performance improvement through SC diversity is significant over the considered range of average SNR values. Further, the highest performance gain is observed from no diversity case to the dual branch SC receiver.

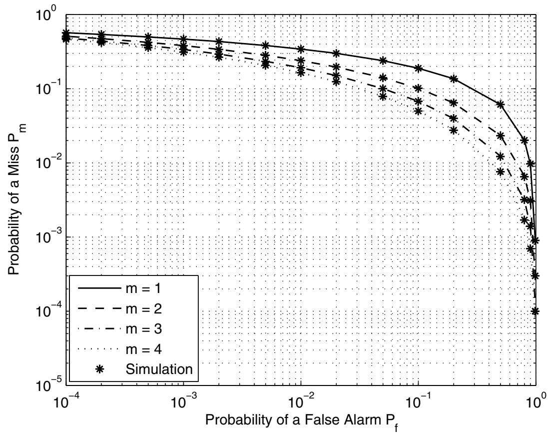

Fig. 3. Complementary ROC curves over Nakagami-*m* channel with $(u = 1, m = \{1, 2, 3, 4\}, SNR = 10\,dB)$

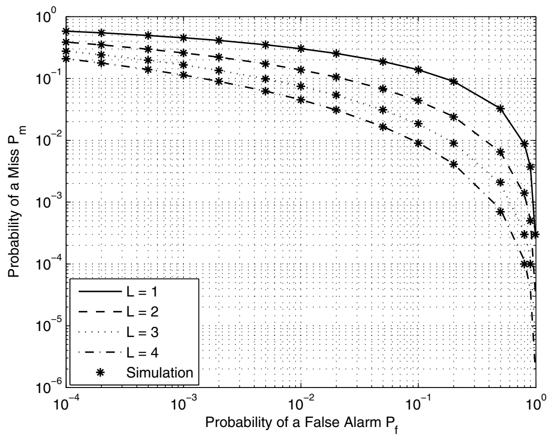

Fig. 4. Complementary ROC curves over SC with $(L = \{1, 2, 3, 4\}, u = 2, m = 2, SNR = 10\,dB)$

6. CONCLUSION

The problem of detecting an unknown deterministic signal through an energy detector was analyzed. An alternative expression for exact detection probability over Nakagami-*m* fading channel was derived. The decision variable of a selection combined energy detector was constructed and the detector performance was explored. To avoid mathematical difficulties of the integrals involving the Marcum-Q function, alternative series representations were used. In order to support the decision variable formulation and to explore the performance of the detector, simulation and numerical results were provided.

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