Super-imposed Pilot-Aided Channel Estimation and Power Allocation for Relay Systems

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***Abstract*—Super-imposed pilots can be used as an alternative to traditional pilots that are used for channel estimation. Superimposed pilots improve bandwidth efficiency. We apply it to the Amplify and Forward (AF) relay systems. In this paper we give the channel estimation (CE) results, analyze the system performance, study the power allocation methods and extend our result to multi-hop relay systems. Our main contribution is that we suggest and prove the existence of minimum bit error rate (BER) as a function of pilot signal power, find the best power allocation ratio value that can reach the minimum BER, analyze parameters' influence on the ratio value, and extend our result to multi-hop systems.**

I. Introduction

The idea of super-imposed pilots first appeared in [1] for analog systems. Reference [1] describes a method of superimposing a pseudorandom channel-sounding signal upon a conventional frequency modulation information-bearing signal. Super-imposed pilots for digital systems were first proposed by Makrakis and Feher [2] [3]. The general idea is to linearly add a known pilot sequence to the data sequence in order to improve bandwidth efficiency. This technique can be used for phase synchronization, frame synchronization, time and frequency synchronization and channel estimation [4]–[10]. This techniques has also been used in orthogonal frequency division multiplexing (OFDM) systems [4] [9] [10].

The advantages of the super-imposed pilot technique lie in two aspects: first, it can improve spectral efficiency because pilots are transmitted together with data and do not require additional frequency or time slots; second, it can perform better in frequency selective fading channels compared to traditional pilot techniques. Reference [11] gives a detailed comparison of the Cramer-Rao bound (CRB) and throughput performance between super-imposed pilots and conventional pilots.

There are two types of CE methods using super-imposed pilot. The first type explores the first-order statistics. Reference [7] proposes a simple such CE scheme on the condition that data signal and the noise have zero mean. Reference [8] suggests a first-order statistics based super-imposed CE. The second type uses precoding techniques. [12] proposes an affine precoding method that combines the super-imposed pilot to get the channel information. By exploiting the periodicity of super-imposed pilots in OFDM systems, [9] derives a least square estimator and provides optimal pilot symbols that minimize the mean square error; [10] derives maximum-likelihood (ML) estimator for both timing and frequency offsets. Reference [13] uses the super-imposed pilot for both synchronization and CE. It also gives the performance analysis and power allocation between pilot and data signals.

To the best of our knowledge, CE using super-imposed pilots for relay systems has not been developed. This motivates our present work. In this paper, we apply super-imposed pilots for one-hop AF relay systems, give the CE results, propose the method to find the system performance, explore the power allocation methods, and extend our results to multi-hop relay systems.

The remainder of this paper is organized as follows. Section II introduces the one-hop and multi-hop relay system model using super-imposed pilot. Section III gives the methods of CE and variance of CE error. Section IV proposes the methods for system performance analysis, including the influence of CE error on system performance. Section V suggests and proves two conclusions about power allocation, gives the power allocation ratio value that can nearly get the minimum BER, and extend it to multi-hop models. Section VI gives simulation results. Finally, section VII summarizes the main results of the paper.

II. Relay System Model

Relays can be of type Amplify and Forward(AF) or Decode and Forward (DF). DF relays decode the information received from the source, re-encode and retransmit to the destination. Both traditional methods and super-imposed pilot estimation method can work in such a scenario. However, in the AF mode, the relay simply amplifies and transmits the received signal. Channel estimation is performed at the destination, but not at the relay [14]. Moreover, since AF relays are simple and can outperform DF relays under certain conditions [15] [16], the AF mode is studied in this paper.

A. Dual-hop relay system

The source node transmits the data signal $s[n]$ plus pilot signal $p[n]$. Let h_1 and h_2 be the Rayleigh fading complex channels between source and relay, between relay and destination, and $h_1 \sim CN(0, \sigma_{h_1}^2)$ and $h_2 \sim CN(0, \sigma_{h_2}^2)$. Then $|h_1|^2$ and $|h_2|^2$ are exponential distributed with the means $\sigma_{h_1}^2$ and $\sigma_{h_2}^2$. The noise samples are $w_1[n] \sim CN(0, \sigma_{n_1}^2)$ and $w_2[n] \sim CN(0, \sigma_{n_2}^2)$. The relay amplifier factor is A_1. Let E_s be the power of data signal $s[n]$, E_p the power of pilot signal $p[n]$, and E be the total transmission power of the source node, E_{r_1} the transmission power of the relay node.

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The received signals for an AF system are

$$\begin{aligned} r_1[n] &= h_1(s[n]+p[n])+w_1[n] \\ r_2[n] &= A_1 h_2(r_1[n])+w_2[n] \\ &= A_1h_2h_1s[n]+A_1h_2h_1p[n]+A_1h_2w_1[n]+w_2[n], \end{aligned}$$

where $A_1^2 = \frac{E_{r_1}}{\sigma_{h_1}^2 E+\sigma_{n_1}^2}$.

B. Multi-hop relay systems

Suppose there are m ($m \geq 2$) hops between the source node and destination node. The signal received by the third relay node is:

$$\begin{aligned} r_3[n] &= h_3A_2r_2[n]+w_3[n] \\ &= h_3A_2h_2A_1h_1(s[n]+p[n]) \\ &\quad +h_3A_2h_2A_1w_1[n]+h_3A_2w_2[n]+w_3[n] \quad (1) \end{aligned}$$

The signal received by the destination is:

$$\begin{aligned} r_{m+1}[n] &= h_{m+1}[n]r_m[n]+w_{m+1}[n] \\ &= h_{m+1}\{(s[n]+p[n])\prod_{i=1}^{m} A_ih_i + \\ &\quad \sum_{i=1}^{m} w_i[n]\prod_{k=i}^{m} A_kh_{k+1}\}+w_{m+1}[n] \quad (2) \end{aligned}$$

where $k+1 \leq m$, $A_i^2 = \frac{E_{r_i}}{\sigma_{h_i}^2 E_{i-1}+\sigma_{n_i}^2}$, and E_{r_i} the transmission power of the i^{th} relay node.

One limitation of both one-hop and multi-hop AF relay systems is that E, the power of the source node, is limited:

$$E = E_s + E_p \quad (3)$$

Let

$$\beta = E_p/E, 0 \leq \beta \leq 1 \quad (4)$$

then

$$E_s = (1-\beta)E \quad (5)$$

Our main problem is to find good β that can give a good performance.

III. Channel Estimation

First-order statistics are often used for channel estimation. For simplicity and easy understanding, we use the dual-hop model to introduce the CE method. For brevity, the results for the multi-hop case are just presented without the details. Suppose h_1 and h_2 remain static in a frame. Let P be the period of super-imposed pilot $p[n]$. Let $0 \leq j \leq R-1, 0 \leq l \leq P-1$. RP is the number of bits per frame. The average of $r_2[n]$ in one frame is:

$$\begin{aligned} &\frac{1}{R}\sum_{j=0}^{R-1} r_2(jP+l) \\ &= A_1h_1h_2p(jP+l)+\frac{1}{R}\sum_{j=0}^{R-1}u[n], \quad (6) \end{aligned}$$

where $u[n] = A_1h_1h_2s(jP+l)+A_1h_2w_1(jP+l)+w_2(jP+l)$. $u(n)$ and $u(k)$ are uncorrelated if $n \neq k$. $A_1h_1h_2$ is the channel information.

One simple and effective choice [7] of $p[n]$ is

$$p[n] = \sum_{j=0}^{R-1} a\delta(n-jP) \quad (7)$$

a is the amplitude of the pilot signal. If $P=1$, then $E_p = a^2$. The channel estimates are then given by

$$\begin{aligned} \widehat{h} &= \triangle\, h+\phi \\ &= \triangle\, A_1h_1h_2+\phi \quad (8) \end{aligned}$$

where ϕ is the error of CE given by

$$\phi = \frac{1}{aR}\sum_{j=0}^{R-1} u[jP+l]. \quad (9)$$

Because the mean value of $u[n]$ is zero, this is unbiased estimation and $\triangle = 1$ and $mean(\phi) = 0$. If we treat h_1 and h_2 as unchanged, then the variance of CE error ϕ is:

$$var(\phi) = \frac{1}{E_pR}(A_1^2h_2^2h_1^2E_s + A_1^2h_2^2\sigma_{n_1}^2 + \sigma_{n_2}^2) \quad (10)$$

If we treat h_1 and h_2 as time-varying, then the variance of ϕ is

$$var(\phi) = \frac{1}{E_pR}(A_1^2\sigma_{h_2}^2\sigma_{h_1}^2E_s + A_1^2\sigma_{h_2}^2\sigma_{n_1}^2 + \sigma_{n_2}^2) \quad (11)$$

Similarly, for m-hop models, we find

$$\begin{aligned} \widehat{h} &= h+\phi \\ &= h_{m+1}\prod_{i=1}^{m} A_ih_i+\phi \quad (12) \\ var(\phi) &= \frac{1}{E_pR}(\sigma_h^2E_s + L + \sigma_{n_{m+1}}^2) \quad (13) \\ \sigma_h^2 &= \sigma_{h_{m+1}^2}\prod_{i=1}^{m} A_i^2\sigma_{h_i}^2 \quad (14) \\ L &= \sigma_{h_{m+1}}^2\sum_{i=1}^{m}\sigma_{n_i}[n]\prod_{k=i}^{m} A_k^2\sigma_{h_{k+1}}^2 \quad (15) \end{aligned}$$

Simulation results show that ϕ is a zero-mean complex Gaussian.

IV. Performance analysis

The influence of CE on dual-hop relay system performance is analyzed. First, the ideal CE case is analyzed. Second, the case with CE errors is analyzed.

A. Accurate channel estimation

If $\widehat{h} = A_1h_1h_2$, $\phi = 0$

$$r_2[n] - \widehat{h}P = A_1h_1h_2s[n] + A_1h_2w_1[n] + w_2[n]$$

Then, the instantaneous SNR is

$$\gamma = \frac{A_1^2|h_2|^2|h_1|^2E_s}{A_1^2|h_2|^2\sigma_{n_1}^2+\sigma_{n_2}^2} = \frac{|h_1|^2E_s|h_2|^2E_{r_1}}{E|h_2|^2\sigma_{n_1}^2+\sigma_{n_2}^2\sigma_{h_1}^2E+\sigma_{n_1}^2} = \frac{\gamma_1\gamma_2}{\gamma_2+C} \quad (16)$$

where $\gamma_1 = \frac{E_s|h_1|^2}{\sigma_{n_1}^2}$, $\gamma_2 = \frac{E_{r_1}|h_2|^2}{\sigma_{n_2}^2}$, and $C = 1+\frac{\sigma_{h_1}^2E}{\sigma_{n_1}^2}$.

The outage probability, pdf of SNR, moment generating function (MGF) of SNR, and BER for two-hop relay networks with fixed relay gain and Rayleigh fading has been studied in [17]. The pdf of SNR is:

$$p_\gamma(\gamma) = \frac{2}{\overline{\gamma_1}}e^{(-\gamma/\overline{\gamma}_1)}[\sqrt{\frac{C\gamma}{\overline{\gamma_1}\cdot\overline{\gamma_2}}}K_1(2\frac{C\gamma}{\overline{\gamma_1}\cdot\overline{\gamma_2}})+\frac{C}{\overline{\gamma_2}}K_0(2\sqrt{\frac{C\gamma}{\overline{\gamma_1}\cdot\overline{\gamma_2}}})]$$

where $\overline{\gamma_1} = E_s\sigma_{h_1}^2/\sigma_{n_1}^2$ and $\overline{\gamma_2} = E_{r_1}\sigma_{h_2}^2/\sigma_{n_2}^2$.

B. Channel estimation with errors

If $\widehat{h} = A_1h_1h_2+\phi$, $\phi \neq 0$, then

$$r_2[n]-\widehat{h}P = A_1h_1h_2s[n]+A_1h_2w_1[n]+w_2[n]-\phi p[n] \quad (17)$$

Then,

$$\gamma = \frac{a^2|h_2|^2|h_1|^2E}{a^2|h_2|^2\sigma_{n_1}^2+\sigma_{n_2}^2+|\phi|^2E_p} = \frac{\gamma_1\gamma_2}{\gamma_2+C+\gamma_3D} \quad (18)$$

where $\gamma_3 = |\phi|^2$ and $D = \frac{E_p(E\sigma_{h_1}^2+\sigma_{n_1}^2)}{\sigma_{n_1}^2\sigma_{n_2}^2}$.

γ_1, γ_2 have independent exponential distribution. γ_3 can also be assumed as a variable with exponential distribution. We can assume the three variables are independent since γ_3 has weak dependence on γ_1 (or γ_2).

The outage probability is

$$\begin{aligned} P_{out} &= P(\gamma < \gamma_{th}) \\ &= P(\frac{\gamma_1\gamma_2}{\gamma_2+C+\gamma_3D} < \gamma_{th}|\gamma_2\gamma_3)P(\gamma_2\gamma_3) \\ &= \int_0^\infty\int_0^\infty P(\frac{\gamma_1\gamma_2}{\gamma_2+C+\gamma_3D} < \gamma_{th}|\gamma_2\gamma_3)p(\gamma_2)p(\gamma_3)d\gamma_2d\gamma_3 \\ &= \int_0^\infty\int_0^\infty \frac{1}{\overline{\gamma}_1}(1-\exp(-\frac{\gamma_{th}(\gamma_2+C+D\gamma_3)}{\gamma_2\overline{\gamma}_1})) \\ &\quad p(\gamma_2)p(\gamma_3)d\gamma_2d\gamma_3 \end{aligned} \quad (19)$$

Thus, from (19) we can get outage probability. Then the pdf of γ is

$$p_\gamma(\gamma) = \frac{d(P_{out})}{d\gamma_{th}} \quad (20)$$

Using the same way in [17], the moment generating function (MGF) of γ can be found. Finally, we can get the BER function using MGF.

Since the BER function is a function of E_s and E_p, we can find the best pilot power of E_p after taking the derivative of BER function. But this method is very complicated. Moreover, it is too complicated to get the solution when there are several hops. In the following section, we give an easy way to find the power allocation ratio between pilot and data signals.

V. Power Allocation

In this section, we consider the problem of power allocation between pilot and data signals. We give two conclusions and find the value of power allocation ratio that can nealy reach minimum BER, and extend our method to multi-hop models.

A. A special case of power allocation

Generally speaking, the channel information h can be written as $h = |h|e^{j\theta_h}$, where θ_h means the phase error in synchronization. If there is perfect synchornization, then $\theta_h = 0$ and $h = |h| > 0$, which means the channel only has fading. In this situation, we have the following conclusion for BPSK signals without ISI.

theorem 1: if the relay networks use BPSK and if there is perfect synchronization ($h = |h| > 0$), then $E_p = 0, E_s = E$ is the best power allocation.

Proof: Firstly, we only consider the fading channel with AWGN and no ISI.

$$r[n] = hs[n]+w[n] \quad (21)$$

$s[n]$ is +1 or -1. h means channel fading, $0 < h \leq 1$. $s[n]$ is the noise signal. The decision rule of ML or MMSE is that:

$$\begin{cases} s[n] = +1, & \text{if } |r[n]-\widehat{h}(+1)| > |r[n]-\widehat{h}(-1)| \\ s[n] = -1, & \text{if } |r[n]-\widehat{h}(+1)| < |r[n]-\widehat{h}(-1)| \end{cases} \quad (22)$$

which is equal to that :

$$\begin{cases} s[n] = +1, & \text{if } r[n] > 0 \\ s[n] = -1, & \text{if } r[n] < 0 \end{cases} \quad (23)$$

The distance rule (22) $\Leftrightarrow$ the pass-zero rule (23) can be shown by Fig. 1:

We can see that if $r[n]$ is close to -1, then it is the same as $r[n] < 0$, and vice versa. Therefore, (22) $\Leftrightarrow$ (23).

(22) is always the same as (23) no matter if $\widehat{h} = h$ or not. (23) do not need any information of h or $\widehat{h}$. Therefore, the knowledge of channel estimation does not help in BER performance. Then it is a waste if $E_p \neq 0$ because E_p do not help in this situation and on the contrary it will reduce the effective SNR.

Since we assume no ISI for relay systems, so the conclusion can also hold true for AF relay systems. ∎

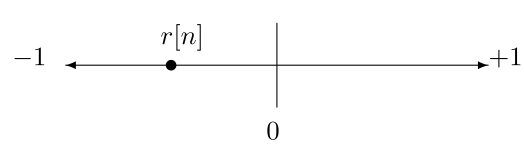

Fig. 1. MMSE vs Compare Zero Rule

B. The existence of minimum BER

If there is no perfect synchronization, we have the following conclusion:

theorem 2: There exists a minimum value for the BER of relay systems using super-imposed pilot, a function of power of super-imposed pilot, if the sum of pilot power and data power is a contant.

Proof: The BER is monotonic non-increasing function of SNR. So the change of SNR can reflect the change of BER. We only need to prove that SNR is a concave function of β.

$$\begin{aligned} & r_2[n] - \widehat{h}P \\ =& hs[n] + A_1 h_2 w_1[n] + w_2[n] - \phi p[n] \\ =& (h+\phi)s[n] + A_1 h_2 w_1[n] + w_2[n] - \phi(p[n] + s[n]) \end{aligned} \quad (24)$$

Then,

$$SNR = \frac{(A_1^2 \sigma_{h_2}^2 \sigma_{h_1}^2 + \sigma_\phi^2) E_s}{\sigma_\phi^2 E + {A_1}^2 \sigma_{h_2}^2 \sigma_{n_1}^2 + \sigma_{n_2}^2} \quad (25)$$

Using (11)(4)(5), we can write (25) as:

$$SNR = \frac{[M\beta R + M(1-\beta) + N](1-\beta)}{M(1-\beta) + N + \beta RN} \quad (26)$$

where $M = EA_1^2 \sigma_{h_2}^2 \sigma_{h_1}^2$, $\quad N = A_1^2 \sigma_{h_2}^2 \sigma_{n_1}^2 + \sigma_{n_2}^2$.

$$\frac{d^2(SNR)}{d\beta^2} = \frac{2RN(M-N)(M+N)(R+1)}{(-M + M\beta - N - \beta NR)^3} \quad (27)$$

Since $0 \le \beta \le 1$ and usually $M - N > 0$, $-M + M\beta < 0$. So, (27) < 0. Thus, SNR is a concave function of β and thus also a concave function of pilot power. ■

C. The optimal power allocation value

When E_p is low (near zero), there are big errors in CE and BER is high. When E_p is high (near E), the signal power E_s is low and BER is still high no matter how accurate the CE is. So we need to find the best power allocation for pilot signal. Here we use the method of maximizing SNR that is used in [13].

Our goal is to maximize (26) under the condition that $E = E_s + E_p$.

Using $\frac{d(SNR)}{d\beta} = 0$, we can find the best ratio:

$$\beta_0 = \frac{M(M-N)(R-1) + \sqrt{MN(M^2 - N^2)R(R^2-1)}}{M(RN - M)(R-1)} \quad (28)$$

D. Extension to the m-hop model

Now we show that our best power allocation ratio value β_0 also apply to m-hop relay model.

$$\begin{aligned} & r_{m+1}[n] - \widehat{h}p[n] \\ =& (h+\phi)s[n] - \phi(s[n] + p[n]) \\ & + \sum_{i=1}^{m} w_i[n] \prod_{k=i, k+1 \le m}^{m} A_k h_{k+1} + w_{m+1}[n] \end{aligned} \quad (29)$$

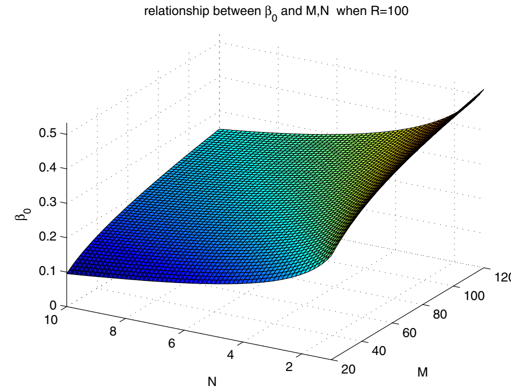

Fig. 2. Relationship between β_0 and M, N

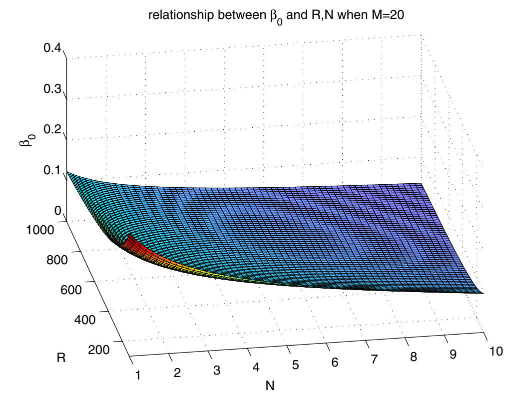

Fig. 3. Relationship between β_0 and R, N

Then we can get:

$$SNR = \frac{(\sigma_h^2 + \sigma_\phi^2) E_s}{\sigma_\phi^2 E + L + \sigma_{m+1}^2} \quad (30)$$

Using (13),(4) and (5), (30) can be written as:

$$SNR = \frac{[M\beta R + N + M(1-\beta)](1-\beta)}{M(1-\beta) + N + \beta RN} \quad (31)$$

where $M = \sigma_h^2 E, N = L + \sigma_{n_{m+1}}^2$, and σ_h^2 is defined in (14), L in (15). Then (31) is the same with (26) except that M, N have different values. Therefore we can still use (28) but with different M, N to find the best power allocation ratio value for m-hop relay systems.

E. Influence of M, N, R on β_0

From (28) we can tell that β_0 is related with M, N, R that is determined by E, E_{r_i}, A_i, R and channel information such as $h_i, \sigma_{h_i}^2, \sigma_{n_i}^2$. Let β_0 is a function of M, N, R and then we can use to graphs to find the influence of M, N, R on β_0 which can be shown in fig.2 and 3.

We can tell that increase of N, increase of R, or decrease of M can result in decrease of β_0. For example, the more hops in the system, the less the value of β_0.

One thing that worth noticing is, as [13] points it out, the method that maximizes the SNR in (24) does not necessarily

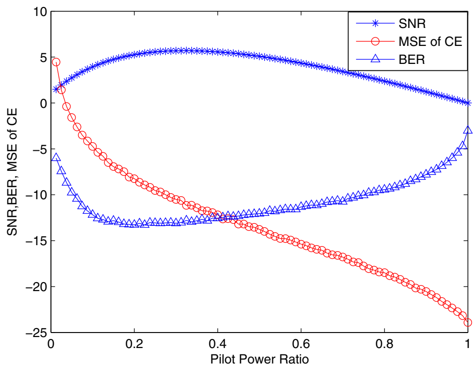

Fig. 4. β vs SNR,BER,CE MSE in One-hop Relay Systems

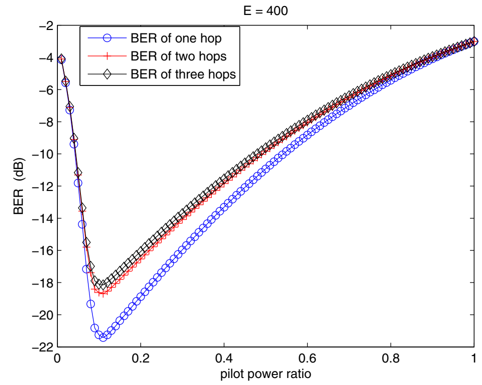

Fig. 6. BER for systems with 1,2,or 3 Hops

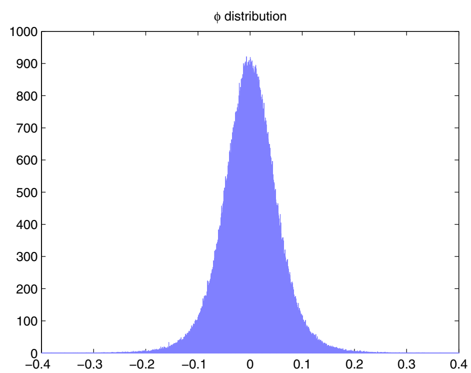

Fig. 5. Distribution of ϕ

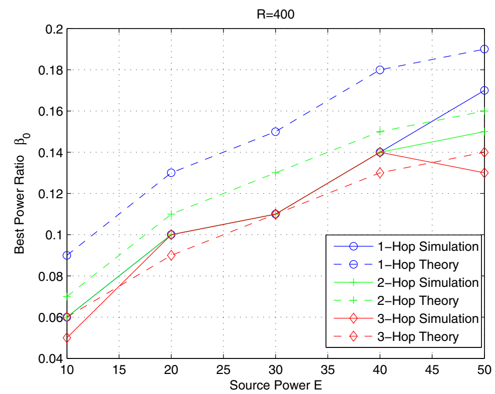

Fig. 7. BER for systems with 1,2,or 3 Hops

optimize the BER performance. However, it is stated for AWGN fading channel, not for relay channel. Simulations show that it can be a good approximation, especially for multi-hop models.

VI. Simulation Results

We first investigate the CE error, BER performance, and optimization of pilot power in one-hop BPSK relay system. Let $h_1, h_2 \sim CN(0,1); w_1[n], w_2[n] \sim CN(0,1)$ and $E = E_{r_1}$. Fig.4 shows that with the increase of pilot power, the CE error will decrease; and the BER will firstly decrease and then increase. There exists a lowest point in the BER curve.

Let $P = 1, E = 100, R = 200$, and $E_p = 10$. Using the CE method introduced in section III, we can find the CE error $\phi = \hat{h} - h$. Fig.5 shows the distribution of real part ϕ. The imaginary part ϕ is alike. Change P, E, R and E_p, we can get similar results. It shows that ϕ can be treated as a Gaussian variable.

Secondly we investigate the relationship between β and BER in 1-hop, 2-hop, and 3-hop relay systems. Let $h_i \sim CN(0,1), w_i[n] \sim CN(0,1), i = 1,2,3,4$ and the power of each relay is equal to the source power $E = 400$. For each power ratio β, we can get a BER. Increase the β from 0 to 1, and we can get fig. 6. It tells that BER, as a function of β, exists a minimum value. Fig. 6 also shows us those β_0 that can reach minimum BER are close in the three systems.

Then we increase E from 10 to 50. For each E, we can get a best power ratio β_0 from simulation. We can also use (28) to get the theoretical values of β_0. Thus we compare the simulation values of β_0 with the theoretical values of β_0 (28) which is shown in Fig.7.

VII. Conclusion

This paper applied super-imposed pilot-aided CE for relay systems. The papers suggests the CE method, presents the method that can analyze the influence of CE errors on system performance, proves two theorems about power allocation and finds the power allocation ratio value that can closely reach minimum BER, and extends the results to multi-hop relay systems. The simulations show that our CE results agree with our theoretical analysis.

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