A general numerical method for computing the Probability of Outage

D. Senaratne, *Student Member, IEEE* and C. Tellambura, *Senior Member, IEEE*
Department of Electrical and Computer Engineering,
University of Alberta, Edmonton, AB, Canada
{damith, chintha}@ece.ualberta.ca

***Abstract*—The outage probability is a fundamental performance metric, which has been widely investigated in the literature. In this paper, we develop a general method to compute the outage given the moment generating function (MGF). When computing the outage using the MGF, integration along the standard Bromwich contour suffers from a loss of accuracy due to oscillatory nature of the integrand. One can address this difficulty by using the Cauchy's theorem to replace the Bromwich contour by an appropriate equivalent contour. For highly-accurate numerical results, the steepest descent contour is the most suitable replacement. Unfortunately, this optimal contour can not be in general expressed in closed-form. The class of Talbot contours characterized by three parameters provides an alternative. However, it is not clear how these parameters need to be tuned for best results. We propose the use of linear regression to set the parameter values so as to minimize the mismatch between corresponding Talbot contour and the steepest-descent contour. The resulting integral has a smooth and rapidly decaying integrand, making it possible for the outage probability to be evaluated with high accuracy by using a simple numerical integration method. This approach is general in a sense that it works for any system where the MGF is known. Thus, it can handle a wide range of fading distributions and a variety of communication systems.**

***Index Terms*—Probability of Outage, Contour Integration**

I. Introduction

In the recent years, wireless networks have undergone tremendous growth globally. Comparative evaluation, optimization and design of various physical layer and upper layer protocols for wireless networks are thus essential. Since the outage probability is a fundamental quality of service (QoS) parameter that is often used for such purposes, the development of mathematical tools for outage analysis has received much attention. In its simplest version, the outage is the probability that the instantaneous signal-to-noise ratio (SNR) γ falls below a given threshold γ_0 that depends on the system design requirements. Early papers such as [1]–[8] have exhaustively treated the case of outage probability analysis for conventional cellular systems, where the performance is often limited by co-channel interference rather than noise; and small-scale multipath fading models such as Rayleigh, Rice, Nakagami-m, Nakagami-Hoyt and Weibull as well as large-scale path loss effects and shadowing need to be analyzed [9].

More-recent papers focus on outage analysis for multiple-input multiple output (MIMO) systems, cooperative networks, network coding, ultrawide band (UWB) systems and others. [10] [11]. Outage probability for decode-and-forward and amplify-and-relay cooperative diversity systems are analyzed in [12] and [13]. In [14], the performance of several decode-and-forward-based cooperative transmission protocols with and without additional network coding is investigated. Such a wide array of applications and the fact that closed-form solutions are not always available call for the development of efficient outage computation algorithms.

The basic outage probability $P_o(\gamma_o)$ in a fading communication channel is given by the cumulative distribution function (CDF) of instantaneous received SNR γ. Given the moment generating function (MGF) $M_\gamma(s) = \mathbb{E}\left[e^{-s\gamma}\right]$, the outage probability is given by

$$P_o(\gamma_o) = P\{\gamma < \gamma_o\} = \frac{1}{2\pi\jmath}\int_{c-\jmath\infty}^{c+\jmath\infty} \frac{e^{s\gamma_o}}{s} M_\gamma(s)\mathrm{d}s \qquad (1)$$

where $P\{.\}$ is the probability assignment function; c is a constant that lies in the domain of convergence of the MGF; and the optimal choice of c is to be discussed later. The integration in (1) is carried out along the vertical contour (called the Bromwich contour) given by $\Re[s] = c$. Notice that $e^{s\gamma_o}$ term oscillates as $\Im[s] \to \pm\infty$. This fact presents a difficulty in highly accurate evaluation of (1) via simple direct numerical integration techniques. In [6]–[8], the use of Gauss-Chebysev quadrature technique for the computation of (1) is suggested. In [15], the trapezoidal rule and the Euler summation are suggested for the numerical inversion of Laplace transforms. This method has been applied for outage calculation in [16].

However, since the integrand is an analytic function, via Cauchy's theorem, the vertical contour can be replaced by an equivalent contour, say, $\mathcal{L}$ starting and ending in $\Re[s] \to -\infty$. Along this new contour the integrand decays rapidly while the impact of the oscillations are minimized. How does one find such a contour? In [17], Talbot suggests the replacement contour $\mathcal{L}: s(\theta) = \sigma + \mu(\theta\cot\theta + \nu\jmath\theta), -\pi \le \theta \le \pi$, where σ, μ and ν are three controlling parameters that determine the shape of the contour. However, it is not clear how to choose the optimal set of parameters for a given integrand. In [18], the original Talbot contour is simplified to $\mathcal{L}: s(\theta) = r\theta(\cot\theta + \jmath), -\pi \le \theta \le \pi$, where $r > 0$ is a parameter that controls the shape of the contour. This method has already been used by [10]. The idea of deforming the Bromwich contour also appears in Helstrom [19], where the following contour is suggested $\mathcal{L}: s = \sigma + \mu y^2 + \jmath y, -\infty < y < \infty$.

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However, the resulting integral in this case has infinite range and requires some truncation.

It is clear that in all these cases, the optimal contour depends on one or more parameters and it is not immediately clear how to find the best set of the Talbot parameters, which is the problem addressed in this paper. Our solution is to use linear regression to select the values of the parameters in order to minimize the mismatch between the Talbot contour and the steepest-descent contour. About five points on the latter is used for the parameter estimation. The resulting integral has a smooth and rapidly decaying integrand that is limited to $[0, \pi]$; and thus can be evaluated with high accuracy using a simple numerical integration method. This method can be applied to a wide range of fading distributions (including Rice, Nakagami-m, Nakagami-Hoyt and Weibull stochastic channel models) and to a variety of communication systems.

This paper is organized as follows. Section II formulates the approach highlighting why integrating along the steepest-descent contour (which happens to be a constant-phase contour) provides high accuracy. It also points out the drawbacks of seeking for a constant-phase contour, which are overcome by the Talbot-contour approximation proposed in Section III. Section IV demonstrates the approach by numerically computing and comparing against the analytical results the outage probability of L Rayleigh fading diversity branches. Possible use of the approach elsewhere is exemplified in Section V where it is used to compute the information outage of STBCs.

Mathematical Notations: $\log(x)$ is the natural logarithm of x while $\dot{f}(x)$ is the derivative of a function $f(x)$ with respect to x. $\jmath = \sqrt{-1}$. $\Re$ denotes the set of real numbers; $\Re[z]$ and $\Im[z]$ denote respectively the real and imaginary components of a complex number z.

II. OPTIMAL CONSTANT-PHASE CONTOUR

A. Contour Integration

We briefly justify how the Bromwich contour in (1) can be deformed. A complex function $f(z) = u(x,y) + \jmath v(x,y)$ of complex variable $z = x + \jmath y$ where $x, y, u(x,y), v(x,y) \in \mathbb{R}$. If $f(z)$ is differentiable, we have

$$f'(z) = \frac{\partial u(x,y)}{\partial x} + \jmath \frac{\partial v(x,y)}{\partial x} = \frac{\partial v(x,y)}{\partial y} - \jmath \frac{\partial u(x,y)}{\partial y}. \quad (2)$$

As a result, if the Cauchy-Riemann conditions

$$\frac{\partial u(x,y)}{\partial x} = \frac{\partial v(x,y)}{\partial y}, \quad \frac{\partial v(x,y)}{\partial x} = -\frac{\partial u(x,y)}{\partial y} \quad (3)$$

are satisfied, then $f(z)$ is **analytic**. These conditions imply that the gradient of $u(x,y)$ and $v(x,y)$ are orthogonal; thus, the contour lines of constant $u(x,y)$ are everywhere orthogonal to the contour lines of constant $v(x,y)$. If a function $f(z)$ is analytic, $f'(z_0) = 0$ then $z_0 = x_0 + \jmath y_0$ is a called a saddle point of $f(z)$. This name arises because contours of both $u(x,y)$ and $v(x,y)$ have valleys and hills around this point. Any contour (path) that passes through this point and that satisfies $v(x,y) = v(x_0,y_0)$ is path of **steepest descent** for $u(x,y)$.

Denote by $g(z)$ the integrand of (1). We can readily shown that $g(z)$ is analytic everywhere, except for several isolated singular points in the complex plane. Cauchy's theorem shows that deforming the contour does not change the value of the integral, provided the contour stays in the region of analyticity. Let $f(z) = \log(g(z))$. Since $u(x,y)$ determines the magnitude of the integrand, and $v(x,y)$ its phase, the oscillations can be totally eliminated by choosing a contour along which $v(x,y)$ remains constant. The contributions along such a contour are all in phase (they add coherently). Moreover, this contour can be chosen to pass through the saddle point z_0. This method is often used to derive asymptotic expansions [20]. The idea of using steepest-descent contours for numerical evaluation, as opposed to asymptotic expansions, is not new. References [21]–[23] use this idea for numerical computations.

In sum, the steepest-descent contour is given by the constant-phase condition

$$\mathcal{L} = \{z | z \in \mathbb{Z},\ \Im[f(z)] = v(x,y) = \Im[f(z_0)] = v(x_0,y_0)\}. \quad (4)$$

Then the integral like (1) can be expressed as

$$I = \frac{e^{\jmath v(x_0,y_0)}}{2\pi\jmath} \int_{\mathcal{L}} e^{u(x,y)}\, dz \quad (5)$$

with the remaining integrand no longer oscillating at all. Moreover, this contour ensures that the integrand has the fastest rate of decay and hence facilitates the use of simple numerical integration methods with high accuracy.

However, the drawback is that the steepest-descent contour can not be found in closed-form. Thus, we need to find in closed-form a contour that approximates the steepest-descent contour.

B. Outage

In this section, we identify the most optimal (i.e. most efficient for numerical integration) contour. However, evaluating an integral along this contour is challenging because an explicit closed-form equation is not generally available for it.

Eqn. (1) can be rewritten as

$$P_o(\gamma_o) = \frac{1}{2\pi\jmath} \int_{c-\jmath\infty}^{c+\jmath\infty} e^{\varphi(s)} \mathrm{d}s = \frac{1}{2\pi\jmath} \int_{\mathcal{L}} e^{\varphi(s)} \mathrm{d}s \quad (6)$$

where $\varphi(s) = s\gamma_o - \log(s) + \log(\mathrm{M}(s))$ and $\mathcal{L}$ is suitable deformation of the Bromwich contour.

The convergence rate can be greatly improved by selecting the contour $\mathcal{L}$ to make the integrand 'constant-phase'. The optimal contour $\mathcal{L}$ is thus given by

$$\mathcal{L}: \quad \Im[\varphi(s)] = \Im[\varphi(z_0)] \quad (7)$$

where the point $z_0 \in \mathcal{L}$ is a parameter that uniquely specifies $\mathcal{L}$. It is customary to select c of (1) to be the saddle point of the integrand $e^{\varphi(s)}$; given as a solution to $\dot{\varphi}(z) = 0$. Since $\mathrm{M}(z)$ and $\dot{\mathrm{M}}(z)$ are real-valued for real z one would always get a real root for

$$\dot{\varphi}(z) = \gamma_o - \frac{1}{z} + \frac{\dot{\mathrm{M}}(z)}{\mathrm{M}(z)} = 0. \quad (8)$$

We propose selecting $z_0 = c$ to be a *positive* root of (8). Realistic integrands are known not to have essential singularities in positive real half of the s-plane. Hence the essential singularities would remain to the left of $\mathcal{L}$, justifying our approach. Since z_0 is a real number, $\mathcal{L}$ is a 'zero-phase contour' for this application, letting (6) to be simplified as

$$P_o(\gamma_o) = \frac{1}{2\pi\jmath} \int_{\mathcal{L}} e^{\Re[\varphi(s)]} \mathrm{d}s. \tag{9}$$

Substituting $s = re^{\jmath\theta}$ in (9) considering the fact that $P_o(\gamma_o) \in \Re$ yields

$$P_o(\gamma_o) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{\varphi\left(r(\theta)e^{\jmath\theta}\right)} \left(r(\theta)\cos(\theta) + \dot{r}(\theta)\sin(\theta)\right) \mathrm{d}\theta \tag{10}$$

where $r = r(\theta),\ \forall s = re^{\jmath\theta} \in \mathcal{L}$. The integrand of (10) is typically an even function of θ, based on which the range of integration can be reduced to $[0, \pi]$.

$$P_o(\gamma_o) = \frac{1}{\pi} \int_{0}^{\pi} e^{\varphi\left(r(\theta)e^{\jmath\theta}\right)} \left(r(\theta)\cos(\theta) + \dot{r}(\theta)\sin(\theta)\right) \mathrm{d}\theta. \tag{11}$$

Below we try to provide an insight into how (10) can be implemented in a computational environment. This needs computation of z_0, $r(\theta)$ and $\dot{r}(\theta)$.

1) Computing saddle point z_0 : z_0 can be easily computed using the Bisection algorithm on (8). Accuracy of z_0 is not critical since (10) holds true for any z_0 larger than the real part of essential singularities. The largest root may be picked where multiple solutions exist for (8).

2) Computing the contour $r(\theta)$: Substituting $\varphi(s) = s\gamma_o - \log(s) + \log\{\mathrm{M}(s)\}$ and $s = re^{\jmath\theta}$ in $\mathcal{L} : \Im[\varphi(s)] = 0$ one gets

$$\gamma_o r\sin(\theta) - \theta + \Im\left[\log\left(\mathrm{M}\left(re^{\jmath\theta}\right)\right)\right] = 0, \tag{12}$$

$$\Rightarrow \mathcal{L} : \gamma_o r\sin(\theta) - \theta + \angle\mathrm{M}\left(re^{\jmath\theta}\right) = 0. \tag{13}$$

The optimal contour is symmetric about the real axis; thus, it is sufficient to compute it for $\theta \in [0, \pi)$. $\angle\mathrm{M}$ need not be the principle argument of M; its range may be shifted by multiples of 2π as long as continuity of the computed curve is retained.

Value of $r(\theta)$ that satisfies (13) can be found for given θ using the Bisection algorithm. One may also make use of the fact that $r = r(\theta)$ is an even function of θ for all probability distributions.

3) Computing the derivative $\dot{r}(\theta)$: Derivative $\dot{r}(\theta)$ can be computed by differentiating (12) with respect to θ, after applying the identity $\Im[z] = \frac{z-z^*}{2\jmath}$. Conjugate symmetry exhibited by $\mathrm{M}\left(re^{\jmath\theta}\right)$ and $\dot{\mathrm{M}}\left(re^{\jmath\theta}\right)$ allows the result to be expressed as

$$\dot{r}(\theta) = -\frac{\gamma_o r(\theta)\cos(\theta) - 1 + r(\theta)\Re\left[\frac{\dot{\mathrm{M}}\left(r(\theta)e^{\jmath\theta}\right)e^{\jmath\theta}}{\mathrm{M}\left(r(\theta)e^{\jmath\theta}\right)}\right]}{\gamma_o\sin(\theta) + \Im\left[\frac{\dot{\mathrm{M}}\left(r(\theta)e^{\jmath\theta}\right)e^{\jmath\theta}}{\mathrm{M}\left(r(\theta)e^{\jmath\theta}\right)}\right]}. \tag{14}$$

Saddle point $s = z_0$ corresponds to $\theta = 0$, at which both numerator and denominator of $\dot{r}(\theta)$ tends to 0. L'Hopital's Rule may be used to evaluate the limit; using $\mathrm{M}(.), \dot{\mathrm{M}}(.)$ and second derivative of the MGF $\ddot{\mathrm{M}}(.)$. However $\theta = 0$ can either be avoided or separately handled if evaluating (11) instead of (10).

The above steps indicate that integrating along the optimal zero-phase contour $\mathcal{L}$ is fairly difficult. The main drawbacks are: computing $r(\theta)$ though numerical search, evaluating $\dot{r}(\theta)$ and dealing with the removable singularity. The solution developed in the next section is determining and using the Talbot contour that is closest to the optimal contour.

III. Talbot Contour

The Talbot contour $\mathcal{L}_\mathcal{T}$, specified by three parameters, is given by [17]

$$\mathcal{L}_\mathcal{T} : s(\theta) = \sigma + \mu(\theta\cot(\theta) + \nu\jmath\theta) \tag{15}$$

where $\sigma, \mu, \nu \in \Re$ are parameters to be estimated.

Our key idea is to use *regression analysis* to find the contour parameters that ensure that $\mathcal{L}_\mathcal{T}$ approximates the constant-phase contour $\mathcal{L}$. In regression analysis, a dependent variable is modeled as a function F of independent variables specified by some parameters so that F 'best fits' the available data. Least squares method is typically used for estimation. It can be easily implemented in computational environments such as MATLAB.

From (15) we get

$$\begin{aligned} F_1 &: \Re[s(\theta)] = \sigma + (\theta\cot(\theta))\,\mu, \\ F_2 &: \Im[s(\theta)] = \theta\ \mu\nu. \end{aligned} \tag{16}$$

Note that F_1, which is the real part of the contour, is linear in σ an μ. Likewise, F_2, the imaginary part of the contour, is linear in $\mu\nu$. Because of symmetry it is sufficient to estimate the parameters for some $\theta_k \in [0, \pi), k \in \{0, \ldots, N-1\}$ values. Regression on F_1 yields σ and μ; while regression on F_2 produces an estimate of $\mu\nu$ product. Once the parameters are known, numerical evaluation of the integral along $\mathcal{L}_\mathcal{T}$ is possible as follows.

The variable of integration is first changed to θ using

$$\frac{\mathrm{d}}{\mathrm{d}\theta}s = \mu\left(\cot(\theta) - \theta\csc^2(\theta)\right) + \mu\nu\jmath,$$

which is obtained from (15) using the fact that $s(\theta)$ has conjugate symmetry. The outage integral is now given by

$$\begin{aligned} P_o(\gamma_o) &= \frac{1}{2\pi\jmath}\int_{-\pi}^{\pi} e^{\varphi(s(\theta))}\left(\mu\left(\cot(\theta) - \theta\csc^2(\theta)\right) + \mu\nu\jmath\right)\mathrm{d}\theta \\ &= \frac{1}{2\pi\jmath}\int_{0}^{\pi} e^{\varphi(s(\theta))}\left(\mu\left(\cot(\theta) - \theta\csc^2(\theta)\right) + \mu\nu\jmath\right)\mathrm{d}\theta \\ &+ \frac{1}{2\pi\jmath}\int_{0}^{\pi} e^{\varphi(s^*(\theta))}\left(-\mu\left(\cot(\theta) - \theta\csc^2(\theta)\right) + \mu\nu\jmath\right)\mathrm{d}\theta. \end{aligned} \tag{17}$$

Moreover, $\varphi(s)$ too can be shown to exhibit conjugate symmetry, based on which the integral simplifies as

$$\begin{aligned} P_o(\gamma_o) = \frac{\mu}{\pi}\int_{0}^{\pi} e^{\Re[\varphi(s)]}\,&\left[\left(\cot(\theta) - \theta\csc^2(\theta)\right)\sin\left(\Im[\varphi(s)]\right)\right. \\ &\left. + \nu\cos\left(\Im[\varphi(s)]\right)\right]\mathrm{d}\theta. \end{aligned} \tag{18}$$

In practice, as few as five points on $\mathcal{L}$ are sufficient to obtain the approximate contour. An exact match of the constant-phase contour is not essential because (18) does not assume zero phase in the integrand. As could be observed with numerical examples, this approximate contour retains the benefits of a steepest descent contour.

The outline of complete algorithm is given in **Algorithm 1**.

input : outage threshold γ_o,
MGF $\mathrm{M}(.)$
number of points N used for estimation
desired precision ϵ
output: outage probability $P_o(\gamma_o)$

compute saddle point z_0 that satisfies (8);
for $k = 0, \dots, N-1$ **do**
  $\theta_k = k\frac{\pi}{N} + \frac{\pi}{2N}$;
  compute $r(\theta_k)$ which satisfies (12);
  $s(\theta_k) = r(\theta_k)e^{\jmath\theta_k}$;
end
use linear regression for (16) and $s(\theta_k)$ to estimate σ, μ and ν;
use a simple numerical integration technique to evaluate (18) at precision ϵ;
return $P_o(\gamma_o)$;

Algorithm 1: Numerical Evaluation of Outage

IV. Example 1: Maximal-ratio combined Rayleigh fading L diversity branches

Consider a system with L diversity branches undergoing independent and identically-distributed Rayleigh fading with $\sigma = 1$.

MGF is given by $M(s) = \frac{1}{(s+1)^L}$; making

$$\varphi(s) = s\gamma_o - \log(s) - L\log(s+1),$$
$$z_0 = \frac{1}{2}\left\{\left(\frac{L+1}{\gamma_o} - 1\right) + \sqrt{\left(\frac{L+1}{\gamma_o} - 1\right)^2 + \frac{4}{\gamma_o}}\right\}.$$

Let $s = re^{\jmath\theta}$ where (r, θ) gives the polar coordinates of a point s in the s-plane. Optimal contour $\mathcal{L}$ for this case would be given by

$$\mathcal{L}:\ \gamma_o r\sin(\theta) - \theta - L\tan^{-1}\left(\frac{r\sin(\theta)}{1 + r\cos(\theta)}\right) = 0. \qquad (19)$$

Unfortunately, there is no simple way to get $r = r(\theta)$ in closed-form which leaves no alternatives but to numerically find $r(\theta)$ for each θ of concern.

As highlighted in section III complexity can be reduced greatly by evaluating along a Talbot-contour approximating $\mathcal{L}$. In practice it is sufficient to compute $5-25$ points on $\mathcal{L}$ to estimate the parameters σ, μ and ν needed to characterize $\mathcal{L}_{\mathcal{T}}$. For instance $(\sigma = -0.120444, \mu = 4.99419, \nu = 0.985651)$ parameterizes $\mathcal{L}_{\mathcal{T}}$ for the case: $g_o = 0.4, L = 1$, Fig. 1 shows the constant-phase contours for outage thresholds $g_o \in \{0.4, 1\}$ and $L \in \{1, 2, 5, 10\}$; Talbot-contour approximations obtained with regression on 25 points are marked with dots. Due to the large number of points (i.e., 25) used in this case, the Talbot contour matches extremely well with the steepest-descent contour (which can be calculated by the Bisection algorithm). This level of extreme agreement is not necessary for the accuracy of the method and is shown here for the purpose of illustrations only. Extremely high accuracy is achieved by using, say, five points to get the Talbot parameters.

Since $|s|$ increases rapidly as θ reaches π, the contours tend to match better for θ_k close to 0; notably it is from θ close to 0 that the integral gets most of the contribution. The approximation not only shows performance almost comparable with the constant-phase contour, but also provides high precision for all g_o values for realistic L values.

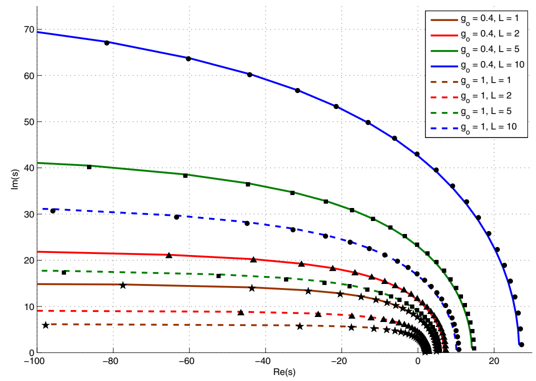

Fig. 1. Optimal Contours for Maximal-ratio combining of Rayleigh fading L diversity branches.

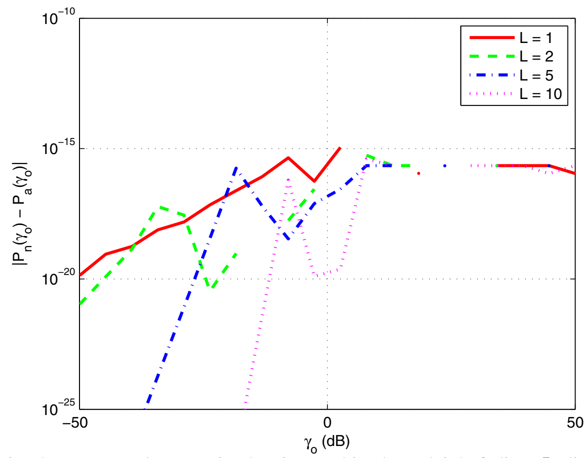

Fig. 2. Error Plot: Maximal-ratio combined Rayleigh fading L diversity branches, where $M(s) = (1+s)^{-L}$.

Fig. 2 shows how values evaluated numerically (with MATLAB routine `quadl` at internal precision of 10^{-15}) using (18) compares with the exact analytical result given by

$$P_{o\,(\text{analytical})}(\gamma_o) = 1 - \mathrm{e}^{-\gamma_o}\sum_{k=0}^{L-1}\frac{\gamma_o^k}{k!}. \qquad (20)$$

Apparent discontinuities of the curves are due to points being treated as 0 (i.e. $10^{-\infty}$) which cannot be plot on a log axis. Precision of Fig. 2 is the best achievable with MATLAB (which has limited precision). Much smaller errors can be expected if our algorithm is ported to an arbitrary precision environment such as Maple or Mathematica.

V. Example 2: Information Outage Probability of STBC Systems

Fig. 3 depicts a possible use of the proposed approach in outage analysis for MIMO systems. It replicates [24, Fig.

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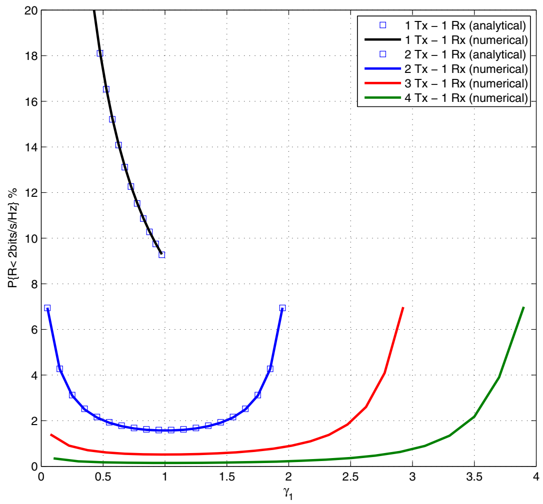

Fig. 3. Information Outage Probability of an STBC system.

3] with the proposed approach, showing as a percentage the information outage probability for four systems:

- SISO
- 2 transmit 1 receive antenna system using Alamouti scheme
- 3/4-rate STBC over 3 transmit-1 receive antenna system
- 3/4-rate STBC over 4 transmit-1 receive antenna system

for 15 dB received SNR and 2 bits/s/Hz information outage threshold. Where the number of transmit antennas $M_t > 1$, first antenna uses $\gamma_1(< M_t)$ power, while the others use equal transmit powers $(M_t - \gamma_1)/(M_t - 1)$. Analytical results have been produced using [24, (8)] for the first two systems; analytical results for the other two cases are not shown because [24, (8)] does not hold true when $\exists i \neq j$ such that $\gamma_i = \gamma_j$. For the first two systems: 6-digit, 12-digit and 15-digit accuracy could be achieved respectively with $15 - 25$, $30 - 50$ and $90 - 135$ points on the approximate contour.

VI. Conclusions

The paper develops a general method to compute the outage given the MGF. When computing the outage using the MGF, integration along the standard Bromwich contour suffers from a loss of accuracy due to oscillatory nature of the integrand. Based on complex analysis, one can replace the Bromwich contours by any of the many equivalent contours. For highly-accurate numerical results, the steepest descent or a 'constant-phase' contour is the most suitable. Unfortunately, this optimal contour can not be generally expressed in closed-form.

We consider a class of contours due to Talbot, which are characterized by three parameters. We use linear regression to select the values of the parameters in order to minimize the mismatch between the Talbot contour and the steepest-descent contour. About five points on the latter is sufficient for the parameter estimation. The resulting integral has a smooth and rapidly decaying integrand; has its range limited to $[0, \pi]$; and thus can be evaluated with high accuracy using a simple numerical integration method. The total error on the order of 10^{-15} or smaller can be easily achieved. Thus this method is extremely accurate, readily programmable; and applicable to a wide range of fading distributions and to a variety of communication systems.

References

[1] G. Stuber and L.-B. Yiin, "Downlink outage predictions for cellular radio systems," *IEEE Trans. Veh. Technol.*, vol. 40, no. 3, pp. 521–531, Aug. 1991.

[2] H. Haas and G. Povey, "Outage probability of cdma-tdd micro cells in a cdma-fdd environment," in *Proc. Ninth IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, vol. 1, 8–11 Sept. 1998, pp. 94–98.

[3] W. Jeong and M. Kavehrad, "Cochannel interference reduction in dynamic-tdd fixed wireless applications, using time slot allocation algorithms," *IEEE Trans. Commun.*, vol. 50, no. 10, pp. 1627–1636, Oct. 2002.

[4] J.-P. Linnartz and I. Davis, J.S., "Outage probability in digital cellular radio networks," in *Conference Record of The Twenty-Sixth Asilomar Conference on Signals, Systems and Computers*, 26–28 Oct. 1992, pp. 88–92.

[5] J. P. M. G. Linnartz, A. J. T. Jong, and R. Prasad, "Effect of coding in digital microcellular personal communication systems with co-channel interference, fading, shadowing, and noise," *IEEE J. Sel. Areas Commun.*, vol. 11, no. 6, pp. 901–910, Aug. 1993.

[6] A. Annamalai, C. Tellambura, and V. Bhargava, "Simple and accurate methods for outage analysis in cellular mobile radio systems-a unified approach," *IEEE Trans. Commun.*, vol. 49, no. 2, pp. 303–316, Feb 2001.

[7] A. Annamalai and C. Tellambura, "A general approach for evaluating the outage probability in microcellular mobile radio systems," in *Proc. IEEE International Conference on Communications ICC '99*, vol. 3, 6–10 June 1999, pp. 1836–1840.

[8] C. Tellambura and A. Annamalai, "An unified numerical approach for computing the outage probability for mobile radio systems," *IEEE Commun. Lett.*, vol. 3, no. 4, pp. 97–99, Apr. 1999.

[9] G. L. Stüber, *Principles of mobile communication (2nd ed.)*. Norwell, MA, USA: Kluwer Academic Publishers, 2001.

[10] R. C. Palat, A. Annamalai, and J. H. Reed, "An efficient method for evaluating information outage probability and ergodic capacity of OSTBC system," *IEEE Commun. Lett.*, vol. 12, no. 3, pp. 191–193, Mar. 2008.

[11] M. D. Renzo, F. Graziosi, and F. Santucci, "A unified framework for performance analysis of UWB receiver architectures in multipath channels," in *Ultra-Wideband, The 2006 IEEE 2006 International Conference on*, Waltham, MA, Sep. 2006, pp. 61–66.

[12] Y. Zhao, R. Adve, and T. J. Lim, "Outage probability at arbitrary SNR with cooperative diversity," *IEEE Commun. Lett.*, vol. 9, no. 8, pp. 700–702, Aug. 2005.

[13] A. Bletsas, H. Shin, and M. Z. Win, "Outage analysis for co-operative communication with multiple amplify-and-forward relays," *Electronics Letters*, vol. 43, pp. 51–52, Mar. 2007.

[14] D. H. Woldegebreal, S. Valentin, and H. Karl, "Outage probability analysis of cooperative transmission protocols without and with network coding: inter-user channels based comparison," in *MSWiM '07: Proceedings of the 10th ACM Symposium on Modeling, analysis, and simulation of wireless and mobile systems*. New York, NY, USA: ACM, 2007, pp. 36–44.

[15] J. Abate and W. Whitt, "Numerical inversion of laplace transforms of probability distributions," *ORSA J. Comput.*, vol. 7, pp. 36–43, 1995.

[16] Y.-C. Ko, M. S. Alouini, and M. K. Simon, "Outage probability of diversity systems over generalized fading channels," *IEEE Trans. Commun.*, vol. 48, no. 11, pp. 1783–1787, Nov. 2000.

[17] A. Talbot, "The accurate numerical inversion of Laplace transforms," *J. Inst. Math. Appl.*, vol. 23, no. 1, pp. 97–120, 1979.

[18] J. Abate and P. P. Valko, "Multi-precision laplace inversion," *Internat. J. Numer. Meth. Engrg.*, vol. 60, pp. 979–993, 2004.

[19] C. W. Helstrom, "Computing the generalized marcum q-function," *IEEE Trans. Inf. Theory*, vol. 38, no. 4, pp. 1422–1428, Jul. 1992.

[20] J. D. Murray, *Asymptotic analysis*, 2nd ed., ser. Applied Mathematical Sciences. New York: Springer-Verlag, 1984, vol. 48.

[21] C. W. Helstrom, "Evaluating the detectability of gaussian stochastic signals by steepest descent integration," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 19, no. 3, pp. 428–437, May 1983.

[22] C. W. Helstrom and J. A. Ritcey, “Evaluating radar detection probabilities by steepest descent integration,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 20, no. 5, pp. 624–634, Sep. 1984.

[23] A. Gil, J. Segura, and N. M. Temme, *Numerical methods for special functions*. Philadelphia, PA: Society for Industrial and Applied Mathematics (SIAM), 2007.

[24] M. Dohler and H. Aghvami, “Information outage probability of distributed STBCs over nakagami fading channels,” *IEEE Commun. Lett.*, vol. 8, no. 7, pp. 437–439, Jul. 2004.