New Series Representation for the Trivariate Non-Central Chi-Squared Distribution

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Abstract—This paper derives a new infinite series representation for the trivariate Non-central chi-squared distribution when the underlying correlated Gaussian variables have a tridiagonal form of an inverse covariance matrix. The joint probability density function is derived using Miller's approach and Dougall's identity. Moreover, the trivariate cumulative distribution function (cdf) and characteristic function (chf) are also derived. Finally, the bivariate non-central chi-squared distribution and some known forms are shown to be special cases of the more general distribution. However, the derivation of non-central chi-squared distribution for an arbitrary covariance matrix seems intractable via Miller's approach. Two applications of the newly derived results are provided for performance analysis of multiple input multiple output (MIMO) systems with transmit antenna selection over a correlated Rician fading environment. Some numerical results are also presented to verify the accuracy of the analytical expressions.

Index Terms—Antenna selection, characteristic function (chf), MIMO systems, non-central chi-squared density, Rician fading, trivariate density.

I. INTRODUCTION

THE χ^2 and non-central χ^2 distributions play a major role in the performance analysis of communication systems [1]- [5]. The generalized chi-squared distribution is analyzed in detail in [6]- [9]. Khaled and Williams [10] derive a relationship between non-central χ^2 distribution and the distribution of a generalized Hermite quadratic form. The diagonal elements of a Wishart matrix are well known to have chi-squared distribution [11]. The joint probability density function (pdf) of the diagonal elements of a real central Wishart matrix (i.e., multivariate central χ^2 distribution) is analyzed in [12]- [14]. The multivariate generalized Rayleigh pdf studied in [15] is also another form of multivariate central χ^2 distribution. Nevertheless, the authors in [15], [16] assume a tridiagonal form of inverse correlation matrix for the underlying Gaussian variables to derive a closed-form solution for the generalized

Rayleigh pdf. The trivariate generalized Rayleigh pdf for an arbitrary correlation matrix is given in [16]. In [17], Mallik derives multivariate Rayleigh and exponential distributions from correlated Gaussian random variables. Hagedorn et al. [11] derive a trivariate central chi square distribution from the diagonal elements of a complex Wishart matrix.

Miller's assumption of the tridiagonal form of inverse correlation matrix is significant since it leads to a closed-form solution for the multivariate Rayleigh pdf. Karagiannidis et al. [18], [19] extend Miller's result to n variate Nakagami-m distribution, which is also a form of a multivariate central χ^2 pdf. We further derive quadrivariate Nakagami-m distribution in [20] using Miller's method with the most general correlation matrix.

The Rice pdf is obviously closely coupled with non-central χ^2 distribution [21]. The multivariate non-central χ^2 distribution can be thought of as a generalization of the multivariate Rician distribution. The bivariate Rician pdf is given in [22], [23] and [24]. In [25] the authors propose an infinite series representation involving modified Bessel functions of the first kind for the trivariate Rician distribution when the underlying Gaussian components have a tridiagonal form of inverse covariance matrix. The non-central Wishart distribution derived in [26] can be thought of as a more generalized version of the non-central distribution. Miller in [16] proposes an infinite series representation involving a modified Bessel function of the first kind for the pdf of the bivariate generalized Rician distribution. A search of the previous work related to multivariate distributions reveals that no joint pdf exists for trivariate non-central χ^2 distribution.

For this reason, we propose a novel expression for the non-central chi-squared pdf when the underlying Gaussian components have tridiagonal form of inverse covariance matrix. Our main derivation is inspired by Miller's approach [16] and by Dougall's theorem for a product of two ultraspherical polynomials [27]. Dougall's theorem which linearizes a product of two ultraspherical polynomials plays a major role in the density derivation. However, the derivation of non-central trivariate pdf for an arbitrary covariance matrix seems intractable via Miller's approach. Finally, as applications of the newly derived pdf, we analyze the performance of multiple input multiple output (MIMO) systems with transmit antenna selection (TAS) by using orthogonal space-time block codes (OSTBC) in a correlated Rician fading environment.

This paper is organized as follows. Section II derives the generalized Rician distribution and trivariate non-central chi-

squared distribution. Several simplifications related to previously known results are also discussed there. Section III deals with the derivation of cdf and chf with some simplifications. Applications of the pdf for MIMO systems with TAS are discussed in section IV. Section V concludes the paper.

II. DERIVATION OF TRIVARIATE NON-CENTRAL χ^2 DISTRIBUTION

Let $\{\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3\}$ be three nonzero mean Gaussian vectors with $E(\mathbf{X}_i) = \mathbf{a}$ and $\mathbf{X}_i = (x_{1i}\, x_{2i} \ldots \ldots x_{ni})^T$ for all $1 \leq i \leq 3$. Here, $\mathbf{a} = (a_1\, a_2 \ldots a_n)^T$, $E(\cdot)$ represents the mathematical expectation, and $(\cdot)^T$ denotes the transpose of a matrix. Let $\mathbf{V}_j = (x_{j1}\, x_{j2}\, x_{j3})$, $1 \leq j \leq n$ be independent four-dimensional nonzero mean Gaussian vectors composed of the jth components of $\mathbf{X}_i$. In this display, the columns are the n-dimensional Gaussian vectors

$$
\begin{array}{c|ccc}
 & \mathbf{X}_1 & \mathbf{X}_2 & \mathbf{X}_3 \\
\hline
\mathbf{V}_1 & x_{11} & x_{12} & x_{13} \\
\mathbf{V}_2 & x_{21} & x_{22} & x_{23} \\
 & \ldots & \ldots & \ldots \\
\mathbf{V}_n & x_{n,1} & x_{n,2} & x_{n,3}
\end{array} \tag{1}
$$

and the rows $\mathbf{V}_j$ are independent from each other and have an identical covariance matrix $\mathbf{M}_3$. The inverse covariance matrix of $\mathbf{V}_j$ is

$$
\mathbf{W}_3 = \mathbf{M}_3^{-1} = \begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{12} & w_{22} & w_{23} \\ w_{13} & w_{23} & w_{33} \end{pmatrix}. \tag{2}
$$

The derivation of the joint pdf is analytically resolvable with Miller's approach if one or more off-diagonal elements of $\mathbf{W}_3$ are zero. The most general such realization is the **tridiagonal form of a matrix or in other words, $\mathbf{w_{13}} = 0$**. The exponential type covariance matrix is well known to give a tridiagonal form of inverse matrix [28]. This assumption is common for all the multivariate derivations given in [15], [18], [19].

The amplitudes $s_i = |\mathbf{X}_i|$ ($1 \leq i \leq 3$), being the square root of the sum of the squares of n nonzero mean independent Gaussian random variables, are generalized Rician random variables. Here, $|\cdot|$ denotes the norm of a column vector. The joint pdf of $\{\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3\}$ is clearly given by

$$
\begin{aligned}
f(\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3) &= \prod_{j=1}^{n} f(\mathbf{V_j}) \\
&= \frac{W_3^{\frac{n}{2}}}{(2\pi)^{\frac{3n}{2}}} \exp\left\{ -\frac{1}{2} \sum_{j=1}^{n} (\mathbf{V_j} - a_j \mathbf{1}_3) \mathbf{W}_3 (\mathbf{V_j} - a_j \mathbf{1}_3)^T \right\}
\end{aligned} \tag{3}
$$

where $\mathbf{1}_3 = (1\,1\,1)$ and W_3 denotes the determinant of square matrix $\mathbf{W}$. Expanding the quadratic form in (3) and replacing the $\mathbf{V}_j$'s by $\mathbf{X}_i$ (see the display in (1)), we find that

$$
\begin{aligned}
f(\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3) &= \frac{W_3^{\frac{n}{2}}}{(2\pi)^{\frac{3n}{2}}} \exp\left\{ -\frac{1}{2}\left(\sum_{i=0}^{3} w_{ii}s_i^2 + w_4 a^2 \right) \right\} \\
&\times \exp\left\{ \mathbf{X}_1^T (w_1 \mathbf{a} - w_{12}\mathbf{X}_2) \right\} \exp\left(w_2 \mathbf{X}_2^T \mathbf{a} \right) \\
&\times \exp\left\{ \mathbf{X}_3^T (w_3 \mathbf{a} - w_{23}\mathbf{X}_2) \right\}
\end{aligned} \tag{4}
$$

where $a^2 = |\mathbf{a}|^2, w_1 = w_{11} + w_{12}, w_2 = w_{12} + w_{23} + w_{22}, w_3 = w_{23} + w_{33}$, and $w_4 = w_1 + w_2 + w_3$. From this pdf (4), we need to integrate out $\mathbf{X}_i$, $1 \leq i \leq 3$, subject to the constraints $s_i = |\mathbf{X}_i|$, which will yield the joint pdf of correlated generalized Rician variables $\{s_1, s_2, s_3\}$ [16]. Now, the joint pdf can be written as

$$
\begin{aligned}
f(s_1, s_2, s_3) &= \frac{W_3^{\frac{n}{2}}}{(2\pi)^{\frac{3n}{2}}} \exp\left\{ -\frac{1}{2}\left(\sum_{i=1}^{3} w_{ii}s_i^2 + w_4 a^2 \right) \right\} \\
&\times \int_{|\mathbf{X}_1|=s_1} \exp\left\{ \mathbf{X}_1^T (w_1 \mathbf{a} - w_{12}\mathbf{X}_2) \right\} d\sigma_{x_1} \\
&\times \int_{|\mathbf{X}_3|=s_3} \exp\left\{ \mathbf{X}_3^T (w_3 \mathbf{a} - w_{23}\mathbf{X}_2) \right\} d\sigma_{x_3} \\
&\times \int_{|\mathbf{X}_2|=s_2} \exp\left(w_2 \mathbf{X}_2^T \mathbf{a} \right) d\sigma_{x_2},
\end{aligned} \tag{5}
$$

where $d\sigma_{x_i}, 1 \leq i \leq 3$ are the elements of surface area. The first integral in (5) can be evaluated as [16, eq.2.2.9]

$$
\begin{aligned}
\int_{|\mathbf{X}_1|=s_1} &\exp\left\{ \mathbf{X}_1^T (w_1 \mathbf{a} - w_{12}\mathbf{X}_2) \right\} d\sigma_{x_1} \\
&= (2\pi s_1)^{\frac{n}{2}} |w_1 \mathbf{a} - w_{12}\mathbf{X}_2|^{\frac{2-n}{2}} \\
&\times I_{\frac{n-2}{2}} (s_1 |w_1 \mathbf{a} - w_{12}\mathbf{X}_2|),
\end{aligned} \tag{6}
$$

where $I_n(z)$ is the nth order modified Bessel function of the first kind [29], and the second integral follows the same form. Furthermore, the right side of (6) can be written by using the generalized Neumann addition formula [29] when $n > 2$ as

$$
\begin{aligned}
&(2\pi s_1)^{\frac{n}{2}} |w_1 \mathbf{a} - w_{12}\mathbf{X}_2|^{\frac{2-n}{2}} I_{\frac{n-2}{2}} (s_1 |w_1 \mathbf{a} - w_{12}\mathbf{X}_2|) \\
&= \frac{(2\pi s_1)^{\frac{n}{2}} 2^{\frac{n-2}{2}} \Gamma\left(\frac{n-2}{2}\right) s_1}{(w_1 w_{12} a s_2)^{\frac{n-2}{2}}} \sum_{k=0}^{\infty} (-1)^k \nu_k \\
&\times I_{\nu_k} (a w_1 s_1) I_{\nu_k} (w_{12} s_1 s_2) C_k^{\frac{n-2}{2}} (\cos\theta)
\end{aligned} \tag{7}
$$

where $\nu_k = \frac{n}{2} + k - 1$, $\Gamma(z) = \int_0^\infty t^{z-1} \exp(-t), z > 0$ is the Gamma function [30], $C_n^\lambda(z)$ denotes the ultraspherical polynomials [30] defined as

$$
(1 - 2zr + r^2)^{-\lambda} = \sum_{n=0}^{\infty} C_n^\lambda(z) r^n, \qquad \lambda > -\frac{1}{2}
$$

$$
\begin{aligned}
f(s_1, s_2, s_3) =& \frac{W_3^{\frac{n}{2}} 2^{n-2} \Gamma^2\left(\frac{n-2}{2}\right) s_1 s_3}{(2\pi)^{\frac{n}{2}} (w_1 w_{12} w_{23} w_3)^{\frac{n-2}{2}} (a s_2)^{n-2}} \exp\left\{ -\frac{1}{2}\left(\sum_{i=1}^{3} w_{ii}s_i^2 + w_4 a^2 \right) \right\} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} (-1)^{k+l} \nu_k \nu_l I_{\nu_k} (a w_1 s_1) \\
& I_{\nu_k} (w_{12} s_1 s_2) I_{\nu_l} (a w_3 s_3) I_{\nu_l} (w_{23} s_2 s_3) \int_{|\mathbf{X}_2|=s_2} \exp\left(w_2 \mathbf{X}_2^T \mathbf{a} \right) C_k^{\frac{n-2}{2}} (\cos\theta) C_l^{\frac{n-2}{2}} (\cos\theta) d\sigma_{x_2}.
\end{aligned} \tag{8}
$$

and θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{X}_2$. Following (7), (5) can be written as given at the bottom of the previous page. The product of two ultraspherical polynomials can be written by using the Dougall's identity, given in [27, eq.6.8.4] as

$$C_p^\lambda(z)C_q^\lambda(z) = \sum_{m=0}^{min(p,q)} A(m,p,q)C_{p+q-2n}^\lambda(z), \quad (9)$$

where $A(m,p,q)$ is given by

$$A(m,p,q) = \frac{(\mu_\lambda - m)(\lambda)_m(\lambda)_{p-m}(\lambda)_{q-m}(2\lambda)_{\mu_0}(\mu_0 - m)!}{\mu_\lambda m!(p-m)!(q-m)!(\lambda)_{\mu_0}(2\lambda)_{\mu_0 - m}}$$

with $\mu_k = p + q + k - m$, $(\lambda)_n = \frac{\Gamma(\lambda+n)}{\Gamma(\lambda)}$ denoting the Pochhammer symbol, and $min(p,q)$ selecting the minimum of p, q. We use Dougall's identity, given in (9), to yield (10). The integral in (10) can be solved by using [16, eq.2.2.26] to give

$$\int_{|\mathbf{X}_2|=s_2} \exp\left(w_2\mathbf{X}_2^T\mathbf{a}\right) C_{\eta_{kl}}^{\frac{n-2}{2}}(\cos\theta)\, d\sigma_{x_2}$$
$$= \frac{(2\pi)^{\frac{n}{2}} s_2^{n-1}\binom{n+\eta_{kl}-3}{n-3}}{(aw_2s_2)^{\frac{n-2}{2}}} I_{\nu_{\eta_{kl}}}(aw_2s_2)$$

where $\eta_{kl} = k + l - 2m$ and $\binom{n}{r} = \frac{n!}{r!(n-r)!}$. Substituting the above integral in (10) and after some algebraic manipulations, the joint pdf of the multivariate generalized Rician distribution for $n > 2$ can be written as shown in (11). The case when $n = 2$ is given in [25, eq.3].

In order to derive the trivariate non-central χ^2 pdf, the following variable transformations are introduced in (11), $r_1 = s_1^2, r_2 = s_2^2, r_3 = s_3^2$. Now, it is clear that $\{r_1, r_2, r_3\}$ represent the correlated non-central χ^2 variables. After some algebraic manipulation, the trivariate non-central χ^2 pdf can be written as given in (12). To the best of the authors' knowledge, (12) is a novel result. Even though (12) is not valid for $n = 2$, the degenerated cases of (12) are valid for all $n \geq 2$ as shown below. Moreover, if a given covariance matrix does not match with the criteria mentioned above, we can use Green's matrix

approach, given in [19], to approximately find a matrix with a tridiagonal form of inverse. Next, several simplifications of (12) are given.

A. Independent non-central χ^2 case

$\mathbf{W}_3$ is obviously a diagonal matrix with the elements $\{w_{11}, w_{22}, w_{33}\}$ under this scenario. Since all the off-diagonal elements are zero, the following important limit involving the Bessel functions is obtained:

$$\lim_{w_{ii+1}\to 0} \frac{I_{\frac{n}{2}+k-1}\left(w_{ii+1}\sqrt{r_ir_{i+1}}\right)}{w_{ii+1}^{\frac{n-2}{2}}} = \frac{\left(\frac{\sqrt{r_ir_{i+1}}}{2}\right)^{\frac{n}{2}-1}}{\Gamma\left(\frac{n}{2}\right)} \quad (13)$$

which is valid for $i = 1, 2$ if $k = 0$. Using (13) in (12) and after algebraic rearrangements, the joint pdf is obtained as

$$g(r_1, r_2, r_3) = \prod_{i=1}^3 \frac{w_{ii}}{2}\left(\frac{r_i}{a^2}\right)^{\frac{n-2}{4}} \exp\left\{-\frac{w_{ii}}{2}\left(r_i + a^2\right)\right\}$$
$$\times I_{\frac{n}{2}-1}\left(aw_{ii}\sqrt{r_i}\right). \quad (14)$$

The extension for central χ^2 distribution follows from (14).

B. Bivariate non-central χ^2 distribution

If $\{r_1, r_2\}$ are independent from r_3, the trivariate pdf can be written as a product of bivariate and univariate pdf's. Equating w_{23} to zero and using the limit (13) with $l = 0$ followed by some manipulations, the bivariate pdf is obtained as

$$g(r_1, r_2) = \frac{W_2^{\frac{n}{2}} 2^{\frac{n}{2}} \Gamma\left(\frac{n-2}{2}\right)}{8\left(w_1w_2w_{12}\right)^{\frac{n-2}{2}}(a)^{n-2}} e^{-\frac{1}{2}\left(\sum_{i=1}^2 w_{ii}r_i+w_ia^2\right)}$$
$$\times \sum_{k=0}^\infty (-1)^k\binom{n+k-3}{n-3}\nu_k I_{\nu_k}\left(aw_1\sqrt{r_1}\right)$$
$$\times I_{\nu_k}\left(w_{12}\sqrt{r_1r_2}\right) I_{\nu_k}\left(aw_2\sqrt{r_2}\right), \quad (15)$$

$$f(s_1, s_2, s_3) = \frac{W_3^{\frac{n}{2}} 2^{n-2}\Gamma^2\left(\frac{n-2}{2}\right) s_1 s_3}{(2\pi)^{\frac{n}{2}}(w_1 w_{12} w_{23} w_3)^{\frac{n-2}{2}}(as_2)^{n-2}} \exp\left\{-\frac{1}{2}\left(\sum_{i=1}^3 w_{ii}s_i^2 + w_4 a^2\right)\right\} \sum_{k=0}^\infty\sum_{l=0}^\infty\sum_{m=0}^{min(k,l)} (-1)^{k+l}\nu_k\nu_l A(m,k,l)$$
$$\times I_{\nu_k}(aw_1 s_1) I_{\nu_k}(w_{12}s_1 s_2) I_{\nu_l}(aw_3 s_3) I_{\nu_l}(w_{23}s_2 s_3) \int_{|\mathbf{X}_2|=s_2} \exp\left(w_2\mathbf{X}_2^T\mathbf{a}\right) C_{k+l-2m}^{\frac{n-2}{2}}(\cos\theta)\, d\sigma_{x_2}. \quad (10)$$

$$f(s_1, s_2, s_3) = \frac{W_3^{\frac{n}{2}} 2^{n-2}\Gamma^2\left(\frac{n-2}{2}\right) s_1 s_2 s_3}{(w_1 w_2 w_3 w_{12} w_{23} as_2)^{\frac{n-2}{2}}(a)^{n-2}} \exp\left\{-\frac{1}{2}\left(\sum_{i=1}^3 w_{ii}s_i^2 + w_4 a^2\right)\right\} \sum_{k=0}^\infty\sum_{l=0}^\infty\sum_{m=0}^{min(k,l)} (-1)^{k+l}\nu_k\nu_l\binom{n+\eta_{kl}-3}{n-3}$$
$$\times A(m,k,l) I_{\nu_k}(aw_1 s_1) I_{\nu_k}(w_{12}s_1 s_2) I_{\nu_l}(aw_3 s_3) I_{\nu_l}(w_{23}s_2 s_3) I_{\nu_{\eta_{kl}}}(aw_2 s_2). \quad (11)$$

$$g(r_1, r_2, r_3) = \frac{W_3^{\frac{n}{2}} 2^n \Gamma^2\left(\frac{n-2}{2}\right)}{32\left(w_1 w_2 w_3 w_{12} w_{23}\right)^{\frac{n-2}{2}}(a)^{\frac{3n}{2}-3}\left(\sqrt{r_2}\right)^{\frac{n-2}{2}}} \exp\left\{-\frac{1}{2}\left(\sum_{i=1}^3 w_{ii}r_i + w_4 a^2\right)\right\} \sum_{k=0}^\infty\sum_{l=0}^\infty\sum_{m=0}^{min(k,l)} (-1)^{k+l}\nu_k\nu_l$$
$$\times A(m,k,l)\binom{n+\eta_{kl}-3}{n-3} I_{\nu_k}(aw_1\sqrt{r_1}) I_{\nu_k}(w_{12}\sqrt{r_1 r_2}) I_{\nu_l}(aw_3\sqrt{r_3}) I_{\nu_l}(w_{23}\sqrt{r_2 r_3}) I_{\nu_{\eta_{kl}}}(aw_2\sqrt{r_2}). \quad (12)$$

a	ρ	$r_1=1.5, r_2=2.3, r_3=3.4$	$r_1=3.7, r_2=4.2, r_3=3.8$	$r_1=2.8, r_2=5.1, r_3=4.6$
1	0.1	$K=3, L=3$	$K=3, L=3$	$K=3, L=3$
3		$K=4, L=4$	$K=5, L=4$	$K=5, L=5$
1	0.4	$K=4, L=3$	$K=4, L=4$	$K=4, L=4$
3		$K=4, L=4$	$K=5, L=5$	$K=6, L=6$
1	0.8	$K=4, L=4$	$K=4, L=4$	$K=4, L=4$
3		$K=5, L=5$	$K=6, L=6$	$K=6, L=6$

a	ρ	$r=1$	$r=2$	$r=5$
1	0.1	$K=2, L=2$	$K=3, L=3$	$K=3, L=3$
3		$K=3, L=3$	$K=4, L=4$	$K=4, L=4$
1	0.4	$K=3, L=3$	$K=4, L=4$	$K=4, L=4$
3		$K=3, L=3$	$K=5, L=5$	$K=6, L=6$
1	0.8	$K=3, L=3$	$K=4, L=4$	$K=5, L=5$
3		$K=4, L=4$	$K=6, L=6$	$K=6, L=6$

where W_2 denotes the determinant of 2×2 inverse covariance matrix $\mathbf{W}_2$. Equation (15) is equivalent to Miller's previously published result [16, eq.2.2.18].

Now let us consider the truncation of the series (12) because for all practical numerical calculations the infinite series are truncated to limited number of terms. This operation introduces a truncation error. Suppose that the series (12) is limited to K and L terms in the variables k and l respectively. Hence the truncation error can be expressed as

$$E_{TR(pdf)} = \sum_{k=0}^{K-1} \sum_{l=L}^{\infty} \sum_{m=0}^{min(k,l)} R_\epsilon + \sum_{k=K}^{\infty} \sum_{l=0}^{\infty} \sum_{m=0}^{min(k,l)} R_\epsilon$$

where

$$R_\epsilon = (-1)^{k+l} \frac{W_3^{\frac{n}{2}} 2^n \Gamma^2\left(\frac{n-2}{2}\right) e^{-\frac{1}{2}\left(\sum_{i=1}^{3} w_{ii} r_i + w_4 a^2\right)}}{32 \left(w_1 w_2 w_3 w_{12} w_{23}\right)^{\frac{n-2}{2}} a^{\frac{3n}{2}-3} \left(\sqrt{r_2}\right)^{\frac{n-2}{2}}}$$
$$\times A(m,k,l) \binom{n+\eta_{kl}-3}{n-2} \nu_k \nu_l I_{\nu_k}\left(a w_1 \sqrt{r_1}\right)$$
$$\times I_{\nu_k}\left(w_{12}\sqrt{r_1 r_2}\right) I_{\nu_l}\left(a w_3 \sqrt{r_3}\right) I_{\nu_l}\left(w_{23}\sqrt{r_2 r_3}\right)$$
$$\times I_{\nu_{\eta_{kl}}}\left(a w_2 \sqrt{r_2}\right).$$

Furthermore, to shed some light on the convergence of (12), the number of terms needed in (12) to achieve five significant figure accuracy is given in Table 1 and Table II. The exponential covariance model proposed by Aalo [28] is used in the calculations.

III. CDF AND CHF OF THE TRIVARIATE DISTRIBUTION

A. Cumulative Distribution Function

The trivariate cdf is, by definition [31],

$$G(r_1, r_2, r_3) = \int_0^{r_1} \int_0^{r_2} \int_0^{r_3} g(y_1, y_2, y_3)\, dy_1 dy_2 dy_3. \tag{16}$$

Substituting (12) in (16) and expanding each Bessel function term with its infinite series representation, followed by term-by-term integration, the cdf of trivariate non-central χ^2 distribution is obtained as given in (17), where $\lambda_1 = 2i_1 + k$, $\lambda_2 = 2i_5 + k + l - 2m$, $\lambda_3 = 2i_3 + l$, $\lambda_4 = 2i_2 + k$, $\lambda_5 = 2i_4 + l$, $\delta_1 = i_1 + i_2 + k + \frac{n}{2}$, $\delta_2 = i_2 + i_4 + i_5 + k + \frac{n}{2} - m$, $\delta_3 = i_3 + i_4 + l + \frac{n}{2}$, and $\gamma(a,z) = \int_0^z t^{a-1} \exp(-t) dt$ is the incomplete gamma function.

Moreover, if $n > 2$ is an even integer, the alternative closed form expression given in [21] can be used instead of the incomplete gamma function. The simplifications for special cases are straightforward with (17).

B. Characteristic Function

The joint chf is defined as [31]

$$\psi(v_1, v_2, v_3) = E\left\{\exp\left(v_1 r_1 j + v_2 r_2 j + v_3 r_3 j\right)\right\}, \tag{18}$$

where $j = \sqrt{-1}$. As with the cdf derivation, we encounter integrals of the form

$$\int_0^\infty z^{\nu-1} \exp\left(-[p+jq]z\right) dz,$$

which can be solved by using [31, eq.3.381.5] to yield the chf as shown in (19).

The bivariate generalization is straightforward with (19). If all $\{r_1, r_2, r_3\}$ are independent (i.e., $w_{12} = w_{23} = 0, i_2 = i_4 = k = l = 0$ in (19)), then the joint chf can be written as a product of the individual chfs. Thus, we arrive at a product

$$G(r_1, r_2, r_3) = W_3^{\frac{n}{2}} \Gamma^2\left(\frac{n-2}{2}\right) \exp\left(-\frac{a^2 w_4}{2}\right) \sum_{k,l=0}^{\infty} \sum_{m=0}^{min(k,l)} (-1)^{k+l} A(m,k,l) \nu_k \nu_l \binom{n+\eta_{kl}-3}{n-3}$$
$$\times \sum_{i_1, i_2, i_3, i_4, i_5 = 0}^{\infty} \frac{\gamma\left(\delta_1, \frac{w_{11}}{2} r_1\right) \gamma\left(\delta_2, \frac{w_{22}}{2} r_2\right) \gamma\left(\delta_3, \frac{w_{33}}{2} r_3\right)}{2^{\frac{1}{2}(\lambda_1 + \lambda_2 + \lambda_3)} i_1! i_2! i_3! i_4! i_5! \Gamma\left(i_1 + \frac{n}{2} + k\right) \Gamma\left(i_2 + \frac{n}{2} + k\right) \Gamma\left(i_3 + \frac{n}{2} + l\right)} \tag{17}$$
$$\times \frac{a^{\lambda_1 + \lambda_2 + \lambda_3} w_1^{\lambda_1} w_2^{\lambda_2} w_3^{\lambda_3} w_{12}^{\lambda_4} w_{23}^{\lambda_5}}{\Gamma\left(i_4 + \frac{n}{2} + l\right) \Gamma\left(i_5 + \frac{n}{2} + k + l - 2m\right) w_{11}^{\delta_1} w_{22}^{\delta_2} w_{33}^{\delta_3}}$$

TABLE III
NUMBER OF TERMS NEEDED IN EACH INDEX K, L, I_1, I_2, I_3, I_4 AND I_5 IN (17) TO ACHIEVE THREE SIGNIFICANT FIGURE ACCURACY WHEN $r_1 = r_2 = r_3 = r$.

a	ρ	$r = 1$	$r = 2$	$r = 5$
1	0.1	$2, 2, 2, 2, 2, 2, 3$	$2, 2, 3, 3, 3, 3, 3$	$2, 2, 4, 4, 4, 4, 4$
3		$3, 2, 4, 4, 4, 4, 4$	$3, 3, 5, 5, 5, 5, 5$	$4, 4, 6, 6, 6, 6, 6$
1	0.3	$2, 2, 3, 3, 3, 3, 3$	$2, 2, 3, 3, 3, 3, 3$	$3, 3, 4, 4, 4, 4, 4$
3		$3, 3, 4, 4, 4, 4, 4$	$4, 4, 7, 7, 7, 7, 7$	$4, 4, 7, 7, 7, 7, 7$
1	0.6	$4, 4, 4, 4, 4, 4, 4$	$5, 5, 6, 6, 6, 6, 6$	$6, 6, 11, 11, 11, 11, 11$
3		$5, 5, 6, 6, 6, 6, 6$	$9, 9, 7, 7, 7, 7, 7$	$11, 11, 13, 13, 13, 13, 13$

of three infinite series of the form

$$\psi(v_i) = \sum_{k_i=0}^{\infty} \frac{w_{ii}^{2k_i+\frac{n}{2}} a^{2k_i}}{2^{2k_i+\frac{n}{2}} k_i! \left(\frac{w_{ii}^2}{4} + v_i^2\right)^{k_i+\frac{n}{2}}}$$
$$\times \exp\left\{j\left(k_i + \frac{n}{2}\right) \arctan\left(\frac{2v_i}{w_{ii}}\right)\right\}, \quad i = 1, 2, 3.$$

However, the closed-form solution for the chf is not immediately obvious. Nevertheless, it is easy to see that the individual infinite summation is in the form of an exponential series, and with some algebraic manipulation, one can show that the individual chf series reduces to [21, eq.2.1.117].

Before discussing the applications, the truncation error upper bound of (17) is briefly introduced. Let us assume that the infinite series (17) is limited to K, L, I_1, I_2, I_3, I_4 and I_5 in the variables k, l, i_1, i_2, i_3, i_4 and i_5 respectively. Then the rest of the terms denote the truncation error which can easily

be upper bounded using the fact that $\gamma(a, z) < \Gamma(z)$ as shown at the bottom. Further simplification of the upper bound is an arduous task. In fact the former gamma function bound for the incomplete gamma function is not that tight for the most practical purposes. To this end another form of tighter upper bound can be derived following the approach given in Tan and Beaulieu [32] at the expense of more mathematical rigor. Next we shed some light to convergence of the cdf series using numerical figures. For brevity, the number of terms needed in each index in (17) is calculated to achieve three significant figure accuracy, as can be seen from Table III.

IV. APPLICATIONS

The new results developed in Section III enable the performance analysis of MIMO systems with antenna selection by using space-time block codes in a correlated Rician fading environment. This section presents a bit error rate (BER)

$$\psi(v_1, v_2, v_3) = W_3^{\frac{n}{2}} \Gamma^2\left(\frac{n-2}{2}\right) \exp\left(-\frac{a^2 w_4}{2}\right) \sum_{k,l=0}^{\infty} \sum_{m=0}^{min(k,l)} (-1)^{k+l} A(m,k,l) \nu_k \nu_l \binom{n+\eta_{kl}-3}{n-3}$$

$$\times \sum_{i_1,i_2,i_3,i_4,i_5=0}^{\infty} \frac{\exp\left\{j\delta_1 \arctan\left(\frac{2v_1}{w_{11}}\right) + j\delta_2 \arctan\left(\frac{2v_2}{w_{22}}\right) + j\delta_3 \arctan\left(\frac{2v_3}{w_{33}}\right)\right\}}{2^{\sum_{j=1}^{5} \lambda_j - \frac{3n}{2}} i_1! i_2! i_3! i_4! i_5! \Gamma\left(i_1 + \frac{n}{2} + k\right) \Gamma\left(i_2 + \frac{n}{2} + k\right) \Gamma\left(i_3 + \frac{n}{2} + l\right)} \quad (19)$$

$$\times \frac{a^{\sum_{j=1}^{3} \lambda_j} w_1^{\lambda_1} w_2^{\lambda_2} w_3^{\lambda_3} w_{12}^{\lambda_4} w_{23}^{\lambda_5} \Gamma(\delta_1) \Gamma(\delta_2) \Gamma(\delta_3)}{\Gamma\left(i_4 + \frac{n}{2} + l\right) \Gamma\left(i_5 + \frac{n}{2} + k + l - 2m\right) \left(\frac{w_{11}^2}{4} + v_1^2\right)^{\frac{\delta_1}{2}} \left(\frac{w_{22}^2}{4} + v_2^2\right)^{\frac{\delta_2}{2}} \left(\frac{w_{33}^2}{4} + v_3^2\right)^{\frac{\delta_3}{2}}}$$

$$E_{TR(cdf)} < \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} \sum_{m=0}^{min(k,l)} \sum_{i_1=0}^{I_1-1} \sum_{i_2=0}^{I_2-1} \sum_{i_3=0}^{I_3-1} \sum_{i_4=0}^{I_4-1} \sum_{i_5=I_5}^{\infty} S_\epsilon + \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} \sum_{m=0}^{min(k,l)} \sum_{i_1=0}^{I_1-1} \sum_{i_2=0}^{I_2-1} \sum_{i_3=0}^{I_3-1} \sum_{i_4=I_4}^{\infty} \sum_{i_5=0}^{\infty} S_\epsilon$$

$$+ \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} \sum_{m=0}^{min(k,l)} \sum_{i_1=0}^{I_1-1} \sum_{i_2=0}^{I_2-1} \sum_{i_3=I_3}^{\infty} \sum_{i_4=0}^{\infty} \sum_{i_5=0}^{\infty} S_\epsilon + \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} \sum_{m=0}^{min(k,l)} \sum_{i_1=0}^{I_1-1} \sum_{i_2=I_2}^{\infty} \sum_{i_3=0}^{\infty} \sum_{i_4=0}^{\infty} \sum_{i_5=0}^{\infty} S_\epsilon$$

$$+ \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} \sum_{m=0}^{min(k,l)} \sum_{i_1=I_1}^{\infty} \sum_{i_2=0}^{\infty} \sum_{i_3=0}^{\infty} \sum_{i_4=0}^{\infty} \sum_{i_5=0}^{\infty} S_\epsilon + \sum_{k=0}^{K-1} \sum_{l=L}^{\infty} \sum_{m=0}^{min(k,l)} \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \sum_{i_3=0}^{\infty} \sum_{i_4=0}^{\infty} \sum_{i_5=0}^{\infty} S_\epsilon$$

$$+ \sum_{k=K}^{\infty} \sum_{l=0}^{\infty} \sum_{m=0}^{min(k,l)} \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \sum_{i_3=0}^{\infty} \sum_{i_4=0}^{\infty} \sum_{i_5=0}^{\infty} S_\epsilon$$

$$S_\epsilon = \frac{W_3^{\frac{n}{2}} \Gamma^2\left(\frac{n-2}{2}\right) \exp\left(-\frac{a^2 w_4}{2}\right) A(m,k,l) \nu_k \nu_l \binom{n+\eta_{kl}-3}{n-2} \nu_k \nu_l a^{\sum_{j=1}^{3} \lambda_j}}{2^{\frac{\sum_{j=1}^{3} \lambda_j}{2}} i_1! i_2! i_3! i_4! i_5! \Gamma\left(i_1 + \frac{n}{2} + k\right) \Gamma\left(i_2 + \frac{n}{2} + k\right) \Gamma\left(i_3 + \frac{n}{2} + l\right) \Gamma\left(i_4 + \frac{n}{2} + l\right)}$$

$$\times \frac{w_1^{\lambda_1} w_2^{\lambda_2} w_3^{\lambda_3} |w_{12}|^{\lambda_4} |w_{23}|^{\lambda_5} \Gamma(\delta_1) \Gamma(\delta_2) \Gamma(\delta_3)}{\Gamma\left(i_5 + \frac{n}{2} + k + l - 2m\right) w_{11}^{\delta_1} w_{22}^{\delta_2} w_{33}^{\delta_3}}.$$

expression for a Gray coded M-ary amplitude modulation scheme with OSTBC and transmit antenna selection. A capacity expression is also derived for the same scenario.

The MIMO technology is well known to mitigate the adversary effects of fading and to enhance the capacity of wireless links. The full-diversity order promised by MIMO technology can be exploited by using OSTBC, which utilizes low-complexity maximum-likelihood detection [5]. Nevertheless, full-rate OSTBC do not exist for all transmit antenna configurations. The most celebrated and the only full-rate complex designed OSTBC scheme is known as the Alamouti scheme [33], which employs two transmit antennas. Transmit antenna selection (TAS) [3] is another scheme where a subset of transmit antennas is selected at a particular time. This scheme reduces the amount of hardware, yet retains the diversity benefit. The performance of TAS with OSTBC is analyzed in many papers [34]- [37]. However, no fading correlation is considered in them. In the following, the performance of TAS with OSTBC in the presence of transmit correlation is analyzed.

We consider a MIMO system in a correlated Rician fading environment with 3 transmit and 2 receive antennas. Two transmit antennas out of 3 are selected and activated for the transmission of the Alamouti code, while the remaining transmit antenna is inactive. Let $\widetilde{\mathbf{H}} \in \mathbb{C}^{2 \times 2}$ be a submatrix of the channel matrix $\mathbf{H} \in \mathbb{C}^{2 \times 3}$. The elements of $\mathbf{H} = [h_{ij}]$, which represent the channel gain between jth transmit and ith receive antenna can be modeled as complex Gaussian variables each having a mean $b/\sqrt{2}$ and variance $\sigma^2/2$ per dimension. Moreover, without loss of generality, $b^2 + \sigma^2 = 1$ for normalizing the average channel gains. Now, it is clear that $\widetilde{\mathbf{H}}$ consists of the channel gains for the selected two transmit and two receive antennas. Suppose that $\mathbf{h}_j (j = 1, 2, 3)$ are the columns of the channel matrix $\mathbf{H}$. The columns are sorted according to their norms such that $||\mathbf{h}_3|| \geq ||\mathbf{h}_2|| \geq ||\mathbf{h}_1||$ [34]. Since only transmit correlation is considered, the covariance matrix of $vec(\mathbf{H})$, $\mathbf{C_H}$ can be written as

$$\mathbf{C_H} = \sigma^2 \begin{pmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{pmatrix} \otimes \mathbf{I}_{2 \times 2} = \mathbf{R}_T \otimes \mathbf{I}_{2 \times 2}, \quad (20)$$

where $\rho^{|j-k|} = \frac{E(h_{ij}h_{ik}^*)-b^2}{\sigma^2}(i = 1, 2, 3)$, $\mathbf{R}_T$ denotes the transmit correlation matrix, $\mathbf{I}_{2 \times 2}$ is the 2×2 identity matrix, $vec(.)$ is the vectorization operation, $\otimes$ denotes the Kronecker product, and $*$ denotes the complex conjugate operation. When the Alamouti code is used, the MIMO system can be replaced with equivalent single input single output (SISO) systems

defined as in [5], [34], [35]:

$$\tilde{s}_q = \sqrt{\frac{E_s}{2}} \left(||\widetilde{\mathbf{H}}||_F^2 \right) s_q + \tilde{\nu}_q, \quad q = 1, 2, \quad (21)$$

where $\sqrt{\frac{E_s}{2}}$ ensures that the total transmitted power in each channel use is E_s and is independent of the number of transmit antennas, $\tilde{s}_q, s_q$ denote the received and transmitted symbols, $||\cdot||_F$ is the Frobenius norm, and $\tilde{\nu}_q$ is the circular symmetric additive complex Gaussian noise having a zero mean and variance $\frac{||\widetilde{\mathbf{H}}||_F^2 N_0}{2}$ per dimension.

A. BER Analysis of M-ary PAM Scheme

The achievable SNR per bit in a M-ary PAM constellation with TAS and the Alamouti scheme can be written as [34]

$$\gamma_b = \frac{\gamma}{2 \log_2 M} ||\widetilde{\mathbf{H}}||_F^2 = c\gamma ||\widetilde{\mathbf{H}}||_F^2, \quad (22)$$

where $\gamma = \frac{E_s}{N_0}$ and $c = \frac{1}{2 \log_2 M}$. Now, it is clear that the antenna-selection criterion mentioned above maximizes the instantaneous SNR.

Let $\gamma_k = c\gamma ||\mathbf{h}_k||^2, k = 1, 2, 3$ be the scaled norms of the columns of $\mathbf{H}$. Alternatively, if the lth element of $\mathbf{h}_k$ is rewritten as $(\Re[h_{lk}] \Im[h_{lk}])^T$, then the original $\mathbf{H}$ can be written as a 4×3 matrix, $\mathbf{H}_{eq}$ defined as

$$\mathbf{H}_{eq} = \begin{pmatrix} \Re[h_{11}] & \Re[h_{12}] & \Re[h_{13}] \\ \Im[h_{11}] & \Im[h_{12}] & \Im[h_{13}] \\ \Re[h_{21}] & \Re[h_{22}] & \Re[h_{23}] \\ \Im[h_{21}] & \Im[h_{22}] & \Im[h_{23}] \end{pmatrix},$$

where $\Re(.)$ and $\Im(.)$ denote the real and imaginary part of a complex number. Since only the transmit correlation is considered, the rows of $\mathbf{H}_{eq}$ will be independent and the columns will have the covariance of $\frac{1}{2}\mathbf{R}_T$. Then, it is clear that γ_k are correlated non-central chi-squared random variables having underlying Gaussian variables with a tridiagonal form of inverse covariance matrix. Then the achievable SNR (22) can be written as

$$\gamma_b = \gamma_2 + \gamma_3. \quad (23)$$

Since this SNR is a sum of order statistics, we may follow the approach given in [38] to obtain the joint pdf of γ_2, γ_3 as (see Appendix A) shown at the bottom, where $y \geq x > 0$, $d_k = \frac{w_{kk}}{2c\gamma}$, A_Σ is given at the top of the next page, with $2b^2 = a^2, W_3 = \frac{8}{\sigma^6(1-\rho^2)^2}, w_{11} = w_{33} = \frac{2}{\sigma^2(1-\rho^2)}, w_{22} = \frac{2(1+\rho^2)}{\sigma^2(1-\rho^2)}$, and $w_{12} = w_{23} = \frac{-2\rho}{\sigma^2(1-\rho^2)}$. Using the definition for the Laplace transform, the moment generating function (mgf) of

$$f_{\gamma_2,\gamma_3}(x,y) = A_\Sigma \sum_{\substack{u,v,w=1 \\ u \neq v \neq w}}^{3} d_u^{\delta_u} d_v^{\delta_v} x^{\delta_u-1} y^{\delta_v-1} \exp\left\{-(d_u x + d_v y)\right\} \left[(\delta_w-1)! - \exp(-d_w x) \sum_{q=0}^{\delta_w-1} \frac{(\delta_w-1)!}{q!} d_w^q x^q \right], \quad (24)$$

$$\begin{array}{llll} \delta_1 = i_1 + i_2 + k + 2 & \delta_2 = i_2 + i_4 + i_5 + k + 2 - m & \delta_3 = i_3 + i_4 + l + 2 & \lambda_1 = 2i_1 + k \\ \lambda_2 = 2i_5 + k + l - 2m & \lambda_3 = 2i_3 + l & \lambda_4 = 2i_2 + k & \lambda_5 = 2i_4 + l. \end{array}$$

$$A_\Sigma = W_3^2 \exp\left(-b^2 w_4\right) \sum_{k,l=0}^{\infty} \sum_{m=0}^{min(k,l)} \sum_{i_1,i_2,i_3,i_4,i_5=0}^{\infty} (-1)^{k+l} A(m,k,l) \binom{k+l+1-2m}{1}$$
$$\times \frac{(k+1)(l+1)b^{\lambda_1+\lambda_2+\lambda_3} w_1^{\lambda_1} w_2^{\lambda_2} w_3^{\lambda_3} w_{12}^{\lambda_4} w_{23}^{\lambda_5}}{w_{11}^{\delta_1} w_{22}^{\delta_2} w_{33}^{\delta_3} i_1! i_2! i_3! i_4! i_5 \, (i_1+k+1)! \, (i_2+k+1)! \, (i_3+l+1)! \, (i_4+l+1)! \, (i_5+k+l+1-2m)!}$$

γ_b can be written as shown in (25). The average BER of an OSTBC with M-PAM is given by [34]

$$P_M(\gamma) = \frac{1}{\log_2 M} \sum_{n=1}^{\log_2 M} P_M(n;\gamma), \qquad (26)$$

where

$$P_M(n;\gamma) = \frac{2}{M} \sum_{i=0}^{k_n} B_i(n) \frac{1}{\pi} \int_0^\infty \frac{\Phi_{\gamma_b}\left(\frac{D_i^2(1+t^2)}{2}\right)}{1+t^2} dt \quad (27)$$

with

$$k_n = \left(1 - \frac{1}{2^n}\right) M - 1$$
$$B_i(n) = (-1)^{\lfloor \frac{i2^{n-1}}{M} \rfloor} \left(2^{n-1} - \lfloor \frac{i2^{n-1}}{M} + \frac{1}{2} \rfloor\right)$$
$$D_i = (2i+1)\sqrt{\frac{6\log_2 M}{M^2 - 1}}.$$

Substituting (25) in (27) and subsequent integration yields the integrals of the form

$$I(\alpha_1, \alpha_2, m, n) = \int_0^\infty \underbrace{\frac{1}{(1+z^2)\left(\alpha_1^2 + z^2\right)^m \left(\alpha_2^2 + z^2\right)^n}}_{f(z)} dz.$$

These integrals can be solved by evaluating the residues of $f(z)$ in the complex plane as

$$I(\alpha, \beta, m, n) = \pi j \left[\sum_{i=1}^{2} Res_{z=j\alpha_i} f(z) + Res_{z=j} f(z) \right],$$

where $Res_{z=z_0} f(z)$ is the residue of function $f(z)$ evaluated at $z = z_0$. Then the BER can be written as shown in (28) at the bottom.

Since (28) has seven fold infinite summations, the numerical evaluation of it would definitely be an arduous task. Hence it is natural to truncate the infinite summations, thereby introducing truncation error in the numerical calculations. Let us assume that the index values k, l, i_1, i_2, i_3, i_4 and i_5 are limited to K, L, I_1, I_2, I_3, I_4 and I_5 terms respectively. Then the truncation error can be written as shown at the bottom. Furthermore, the number of terms needed in (28) to achieve three significant figure accuracy is given in Table IV. It should be noted that the Rician factor K and b are related as $b = \sqrt{\frac{K}{K+1}}$.

B. Capacity Analysis

Since equal power allocation is assumed, the average capacity in nats/s/Hz is given by [35], [39]:

$$C = E\{\log(1+\gamma_s)\} = \int_0^\infty \log(1+\lambda) f_{\gamma_s}(\lambda) d\lambda, \quad (29)$$

$$\Phi_{\gamma_b}(s) = \int_0^\infty \int_x^\infty \exp\{-s(x+y)\} f_{\gamma_2 \gamma_3}(x,y) dy dx$$
$$= A_\Sigma \sum_{\substack{u,v,w=1 \\ u \neq v \neq w}}^{3} d_u^{\delta_u} d_v^{\delta_v} (\delta_w - 1)! (\delta_v - 1)! \left\{ \sum_{p=0}^{\delta_v - 1} \frac{(p + \delta_u - 1)!}{p! (d_v + s)^{p+\delta_v} (d_u + d_v + 2s)^{p+\delta_u}} \right.$$
$$\left. - \sum_{p=0}^{\delta_v - 1} \sum_{q=0}^{\delta_w - 1} \frac{d_w^q (p + q + \delta_u - 1)!}{p! q! (d_v + s)^{p+\delta_v} (d_u + d_v + d_w + 2s)^{p+q+\delta_u}} \right\}. \qquad (25)$$

$$P(\gamma) = \frac{2}{M\pi \log_2 M} \sum_{n=1}^{\log_2 M} \sum_{i=0}^{k_n} A_\Sigma B_i(n) \sum_{\substack{u,v,w=1 \\ u \neq v \neq w}}^{3} 2^{\delta_v} d_u^{\delta_u} d_v^{\delta_v} (\delta_w - 1)! (\delta_v - 1)!$$
$$\times \left\{ \sum_{p=0}^{\delta_v - 1} \frac{2^p (p + \delta_u - 1)!}{p! D_i^{2(\delta_u + \delta_v + 2p)}} I\left(\sqrt{\frac{2d_v}{D_i^2} + 1}, \sqrt{\frac{d_u + d_v}{D_i^2} + 1}, \delta_v + p, \delta_u + p \right) \right.$$
$$\left. - \sum_{p=0}^{\delta_v - 1} \sum_{q=0}^{\delta_w - 1} \frac{2^p d_w^q (p + q + \delta_u - 1)!}{p! q! D_i^{2(\delta_u + \delta_v + 2p + q)}} I\left(\sqrt{\frac{2d_v}{D_i^2} + 1}, \sqrt{\frac{d_u + d_v + d_w}{D_i^2} + 1}, \delta_v + p, \delta_u + p + q \right) \right\}. \qquad (28)$$

where the achievable SNR per symbol for an M-ary constellation, γ_s, is defined as

$$\gamma_s = c\gamma||\widetilde{\mathbf{H}}||_F^2 \tag{30}$$

with $c = 1/2$. Following analogous steps as in (22)-(25) and inverting $\Phi_{\gamma_s}(\gamma_s)$ (see Appendix B) and subsequent integration of (29) with $\widehat{C}_{m-1}(\theta)$ yields (31). Note that $\widehat{C}_{m-1}(\theta)$ is

defined as [39, App. B] (for $m = 1, 2...$ and $\theta > 0$)

$$\widehat{C}_{m-1}(\theta) = \int_0^\infty \log(1+\lambda)\lambda^{m-1}\exp(-\theta\lambda)d\lambda,$$
$$= (m-1)!\exp(\theta)\sum_{k=1}^m \frac{\Gamma(-m+k,\theta)}{\theta^k}, \tag{32}$$

$$E_{TR(P(\gamma))} = \frac{2}{M\pi\log_2 M}\sum_{n=1}^{\log_2 M}\sum_{i=0}^{k_n}\sum_{k=0}^{K-1}\sum_{l=0}^{L-1}\sum_{m=0}^{min(k,l)}\sum_{i_1=0}^{I_1-1}\sum_{i_2=0}^{I_2-1}\sum_{i_3=0}^{I_3-1}\sum_{i_4=0}^{I_4-1}\sum_{i_5=I_5}^{\infty} T_\epsilon B_i(n)$$

$$+ \frac{2}{M\pi\log_2 M}\sum_{n=1}^{\log_2 M}\sum_{i=0}^{k_n}\sum_{k=0}^{K-1}\sum_{l=0}^{L-1}\sum_{m=0}^{min(k,l)}\sum_{i_1=0}^{I_1-1}\sum_{i_2=0}^{I_2-1}\sum_{i_3=0}^{I_3-1}\sum_{i_4=I_4}^{\infty}\sum_{i_5=0}^{\infty} T_\epsilon B_i(n)$$

$$+ \frac{2}{M\pi\log_2 M}\sum_{n=1}^{\log_2 M}\sum_{i=0}^{k_n}\sum_{k=0}^{K-1}\sum_{l=0}^{L-1}\sum_{m=0}^{min(k,l)}\sum_{i_1=0}^{I_1-1}\sum_{i_2=0}^{I_2-1}\sum_{i_3=I_3}^{\infty}\sum_{i_4=0}^{\infty}\sum_{i_5=0}^{\infty} T_\epsilon B_i(n)$$

$$+ \frac{2}{M\pi\log_2 M}\sum_{n=1}^{\log_2 M}\sum_{i=0}^{k_n}\sum_{k=0}^{K-1}\sum_{l=0}^{L-1}\sum_{m=0}^{min(k,l)}\sum_{i_1=0}^{I_1-1}\sum_{i_2=I_2}^{\infty}\sum_{i_3=0}^{\infty}\sum_{i_4=0}^{\infty}\sum_{i_5=0}^{\infty} T_\epsilon B_i(n)$$

$$+ \frac{2}{M\pi\log_2 M}\sum_{n=1}^{\log_2 M}\sum_{i=0}^{k_n}\sum_{k=0}^{K-1}\sum_{l=0}^{L-1}\sum_{m=0}^{min(k,l)}\sum_{i_1=I_1}^{\infty}\sum_{i_2=0}^{\infty}\sum_{i_3=0}^{\infty}\sum_{i_4=0}^{\infty}\sum_{i_5=0}^{\infty} T_\epsilon B_i(n)$$

$$+ \frac{2}{M\pi\log_2 M}\sum_{n=1}^{\log_2 M}\sum_{i=0}^{k_n}\sum_{k=0}^{K-1}\sum_{l=L}^{\infty}\sum_{m=0}^{min(k,l)}\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\sum_{i_3=0}^{\infty}\sum_{i_4=0}^{\infty}\sum_{i_5=0}^{\infty} T_\epsilon B_i(n)$$

$$+ \frac{2}{M\pi\log_2 M}\sum_{n=1}^{\log_2 M}\sum_{i=0}^{k_n}\sum_{k=K}^{\infty}\sum_{l=0}^{\infty}\sum_{m=0}^{min(k,l)}\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\sum_{i_3=0}^{\infty}\sum_{i_4=0}^{\infty}\sum_{i_5=0}^{\infty} T_\epsilon B_i(n)$$

$$T_\epsilon = (-1)^{k+l}W_3^2\exp\left(-b^2 w_4\right)A(m,k,l)\binom{k+l+1-2m}{1}$$

$$\times \frac{(k+1)(l+1)b^{\lambda_1+\lambda_2+\lambda_3}w_1^{\lambda_1}w_2^{\lambda_2}w_3^{\lambda_3}w_{12}^{\lambda_4}w_{23}^{\lambda_5}}{w_{11}^{\delta_1}w_{22}^{\delta_2}w_{33}^{\delta_3}i_1!i_2!i_3!i_4!i_5\,(i_1+k+1)!\,(i_2+k+1)!\,(i_3+l+1)!\,(i_4+l+1)!\,(i_5+k+l+1-2m)!}$$

$$\times \sum_{\substack{u,v,w=1\\u\neq v\neq w}}^3 2^{\delta_v}d_u^{\delta_u}d_v^{\delta_v}(\delta_w-1)!(\delta_v-1)!\left\{\sum_{p=0}^{\delta_v-1}\frac{2^p(p+\delta_u-1)!}{p!D_i^{2(\delta_u+\delta_v+2p)}}I\left(\sqrt{\frac{d_v}{D_i^2}+1},\sqrt{\frac{d_u+d_v}{D_i^2}+1},\delta_v+p,\delta_u+p\right)\right.$$

$$\left. -\sum_{p=0}^{\delta_v-1}\sum_{q=0}^{\delta_w-1}\frac{2^p d_w^q(p+q+\delta_u-1)!}{p!q!D_i^{2(\delta_u+\delta_v+2p+q)}}I\left(\sqrt{\frac{d_v}{D_i^2}+1},\sqrt{\frac{d_u+d_v+d_w}{D_i^2}+1},\delta_v+p,\delta_u+p+q\right)\right\}.$$

$$C = A_\Sigma\sum_{\substack{u,v,w=1\\u\neq v\neq w}}^3 d_u^{\delta_u}d_v^{\delta_v}(\delta_w-1)!(\delta_v-1)!\left\{\sum_{p=0}^{\delta_v-1}\frac{(p+\delta_u-1)!}{p!2^{p+\delta_u}}\left[\sum_{r_1=1}^{\delta_v+p}\frac{A_{r_1}}{(r_1-1)!}\widehat{C}_{r_1-1}(d_v)\right.\right.$$

$$\left.\sum_{r_2=1}^{\delta_u+p}\frac{B_{r_2}}{(r_2-1)!}\widehat{C}_{r_2-1}\left(\frac{d_u+d_v}{2}\right)\right] - \sum_{p=0}^{\delta_v-1}\sum_{q=0}^{\delta_w-1}\frac{d_w^q(p+q+\delta_u-1)!}{p!q!2^{p+q+\delta_u}}\left[\sum_{r_1=1}^{\delta_v+p}\frac{C_{r_1}}{(r_1-)!}\widehat{C}_{r_1-1}(d_v)\right.$$

$$\left.\left. + \sum_{r_2=1}^{\delta_u+p+q}\frac{D_{r_2}}{(r_2-1)!}\widehat{C}_{r_2-1}\left(\frac{d_u+d_v+d_w}{2}\right)\right]\right\}, \tag{31}$$

TABLE IV
NUMBER OF TERMS NEEDED IN EACH INDEX K, L, I_1, I_2, I_3, I_4 AND I_5 IN (28) TO ACHIEVE THREE SIGNIFICANT FIGURE ACCURACY FOR BPSK SIGNALING.

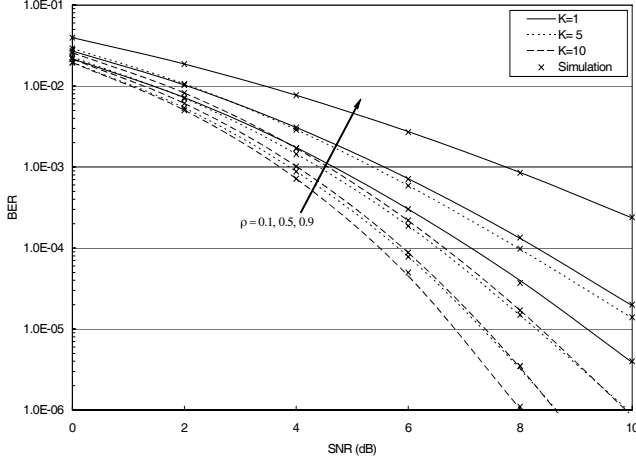| K	ρ	$\gamma = 1(dB)$	$\gamma = 5(dB)$	$\gamma = 8(dB)$
1	0.1	$3, 3, 3, 3, 3, 2, 2$	$4, 4, 4, 4, 5, 5, 4$	$4, 4, 5, 5, 5, 5, 5$
10		$3, 4, 4, 4, 4, 4, 4$	$5, 5, 4, 4, 5, 5, 5$	$4, 4, 6, 6, 6, 6, 6$
1	0.3	$4, 4, 4, 4, 4, 3, 3$	$5, 5, 6, 6, 6, 5, 5$	$6, 6, 7, 7, 7, 6, 6$
10		$4, 4, 5, 5, 5, 4, 4$	$6, 6, 7, 7, 7, 6, 6$	$7, 7, 9, 9, 9, 7, 9$
1	0.6	$4, 4, 6, 6, 5, 6, 5$	$6, 6, 8, 8, 7, 7, 6$	$7, 7, 11, 11, 11, 11, 10$
10		$5, 5, 7, 7, 7, 6, 6$	$9, 9, 11, 11, 11, 7, 10$	$11, 11, 15, 15, 15, 13, 13$

Fig. 1. BER of MIMO with TAS employing Alamouti code and BPSK signaling for different Rician factors (K values).
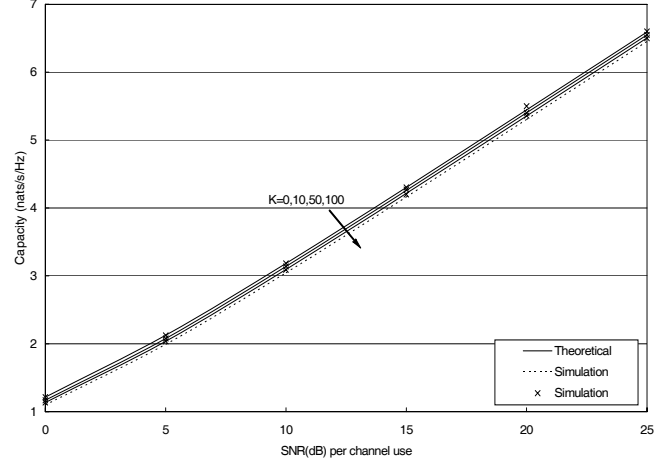

Fig. 2. Capacity of MIMO with TAS employing Alamouti code and BPSK signaling for different Rican factors (K values).

$\Gamma(a, z) = \int_z^\infty \exp(-t) t^{a-1} dt$ is the incomplete complementary gamma function,

$$A_{r_1} = \frac{1}{(\delta_v + p - r_1)!} \frac{\partial^{\delta_v + p - r_1}}{\partial s^{\delta_v + p - r_1}} [f_1(s)] \mid_{s = -d_v},$$

$$B_{r_2} = \frac{1}{(\delta_u + p - r_2)!} \frac{\partial^{\delta_u + p - r_2}}{\partial s^{\delta_u + p - r_2}} [f_1(s)] \mid_{s = -(d_u + d_v)},$$

$$C_{r_1} = \frac{1}{(\delta_v + p - r_1)!} \frac{\partial^{\delta_v + p - r_1}}{\partial s^{\delta_v + p - r_1}} [f_2(s)] \mid_{s = -d_v},$$

$$D_{r_2} = \frac{\frac{\partial^{\delta_u + p + q - r_2}}{\partial s^{\delta_u + p + q - r_2}} [f_2(s)] \mid_{s = -(d_u + d_v + d_w)}}{(\delta_u + p + q - r_2)!},$$

and

$$f_1(s) = \frac{1}{(d_v + s)^{\delta_v + p} \left(\frac{d_u + d_v}{2} + s\right)^{\delta_u + p}},$$

$$f_2(s) = \frac{1}{(d_v + s)^{\delta_v + p} \left(\frac{d_u + d_v + d_w}{2} + s\right)^{\delta_u + p + q}}.$$

Figure 1 depicts the BER performance of MIMO with TAS by using the Alamouti scheme and binary phase shift keying (BPSK) signaling over correlated Rician fading channels. The BPSK signaling is used to reduce the complexity of the evaluation of (28) numerically for different SNR values. As the graph reveals, the fading correlation among the transmit antennas generally downgrades the performance. However, a strong line of sight path between the transmitter and receiver improves the performance.

Figure 2 shows the capacity of MIMO with TAS by using the Alamouti scheme and BPSK signaling over Rician fading

channels having different Rician factors (K values). The results are analogous to those given in [40 Fig. 3] for Rician channels with a fixed gain. Since the Rician fading scenario is an intermediate state between the Rayleigh fading (i.e. $K = 0$) and the deterministic channel (i.e. $K = \infty$), a fixed channel gain increase in the K value decreases the amount of scattering. This result is highly undesirable in a MIMO system [40]. The effect of the correlation is not shown here because the curves cannot be distinguished in the figure. Nevertheless, a performance degradation can be observed with the increased correlation even in this scenario.

V. CONCLUSION

A new infinite series representation for the trivariate non-central χ^2 pdf has been derived for the case when the underlying Gaussian components have the tridiagonal form of an inverse covariance matrix. An identity for a product of two ultraspherical polynomials, based on Dougall's identity and Miller's approach, is used in the derivation. Moreover, the chf and cdf series are also derived. Some special cases of the joint pdf are also discussed. However, the derivation for an arbitrary covariance matrix seems intractable when using this approach. Furthermore, two applications, namely the analysis of the BER and the capacity of MIMO with TAS and Alamouti coding over correlated Rician fading channels, are considered.

ACKNOWLEDGMENT

APPENDIX A
DERIVATION OF (24)

Since $\gamma_1 \leq \gamma_2 \leq \gamma_3$, following [38] we can write the cdf of γ_2 and γ_3 using the first principles as

$$F_{\gamma_2,\gamma_3}(\alpha,\beta) = \Pr(\gamma_1 \leq \beta, \gamma_2 \leq \alpha, \gamma_3 \leq \alpha)$$
$$+ \Pr(\gamma_1 \leq \alpha, \gamma_2 \leq \beta, \gamma_3 \leq \alpha) \quad (33)$$
$$+ \Pr(\gamma_1 \leq \alpha, \gamma_2 \leq \alpha, \gamma_3 \leq \beta)$$
$$- 2\Pr(\gamma_1 \leq \alpha, \gamma_2 \leq \alpha, \gamma_3 \leq \alpha)$$

where $\beta \geq \alpha > 0$. Now it can easily be rewritten using (17) as

$$F_{\gamma_2,\gamma_3}(\alpha,\beta) = G\left(\frac{\beta}{c\gamma},\frac{\alpha}{c\gamma},\frac{\alpha}{c\gamma}\right) + G\left(\frac{\alpha}{c\gamma},\frac{\beta}{c\gamma},\frac{\alpha}{c\gamma}\right) \quad (34)$$
$$+ G\left(\frac{\alpha}{c\gamma},\frac{\alpha}{c\gamma},\frac{\beta}{c\gamma}\right) - 2G\left(\frac{\alpha}{c\gamma},\frac{\alpha}{c\gamma},\frac{\alpha}{c\gamma}\right).$$

Successive differentiation of (34) with respect to α and β yields the joint pdf as

$$\frac{\partial^2 F_{\gamma_2,\gamma_3}(\alpha,\beta)}{\partial\alpha\partial\beta} = f_{\gamma_2,\gamma_3}(\alpha,\beta) \quad (35)$$
$$= A \sum_{\substack{u,v,w=1\\u\neq v\neq w}}^{3} d_u^{\delta_u} d_v^{\delta_v} \alpha^{\delta_u-1}\beta^{\delta_v-1}$$
$$\times \exp\{-(d_u\alpha + d_v\beta)\}\gamma(\delta_w, d_w\alpha).$$

Since δ_w is an integer, we can express the incomplete gamma function term with the following finite series representation

$$\gamma(\delta_w, d_w\alpha)$$
$$= (\delta_w-1)! - \exp(-d_w\alpha)\sum_{k=0}^{\delta_w-1}\frac{(\delta_w-1)!(d_w\alpha)^k}{k!}. \quad (36)$$

Substitution of (36) in (35) gives (24).

APPENDIX B
DERIVATION OF (31)

Before we start with the derivation of (31) the following Laplace transform pair is worth of mentioning

$$\frac{1}{(s+a)^m(s+b)^n} \longrightarrow \sum_{r_1=1}^{m} A_{r_1}\frac{\exp(-at)t^{r_1-1}}{(r_1-1)!}$$
$$+ \sum_{r_2=1}^{n} B_{r_2}\frac{\exp(-bt)t^{r_2-1}}{(r_2-1)!} \quad (37)$$

where

$$A_{r_1} = \frac{1}{(m-r_1)!}\frac{\partial^{m-r_1}[F(s)]}{\partial s^{m-r_1}}\Big|_{s=-a},$$
$$B_{r_2} = \frac{1}{(n-r_2)!}\frac{\partial^{n-r_2}[F(s)]}{\partial s^{n-r_2}}\Big|_{s=-b}.$$

Now we can make use of (37) in (25) (of course now $c = \frac{1}{2}$) to obtain the inverse Laplace transform to yield the pdf of $f_{\gamma_s}(\lambda)$ as shown in (38). Then we substitute (38) in (29) and subsequent integration using (32) gives (31).

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$$f_{\gamma_s}(\lambda) = A\sum_{\substack{u,v,w=1\\u\neq v\neq w}}^{3} d_u^{\delta_u}d_v^{\delta_v}(\delta_w-1)!(\delta_v-1)!\left\{\sum_{p=0}^{\delta_v-1}\frac{(p+\delta_u-1)!}{p!2^{p+\delta_u}}\left[\sum_{r_1=1}^{\delta_v+p}\frac{A_{r_1}}{(r_1-1)!}\exp(-d_v\lambda)\lambda^{r_1-1}\right.\right.$$
$$\times\sum_{r_2=1}^{\delta_u+p}\frac{B_{r_2}}{(r_2-1)!}\exp\left(-\frac{d_u+d_v}{2}\lambda\right)\lambda^{r_2-1}\right] - \sum_{p=0}^{\delta_v-1}\sum_{q=0}^{\delta_w-1}\frac{d_w^q(p+q+\delta_u-1)!}{p!q!2^{p+q+\delta_u}}$$
$$\times\left[\sum_{r_1=1}^{\delta_v+p}\frac{C_{r_1}}{(r_1-1)!}\exp(-d_v\lambda)\lambda^{r_1-1} + \sum_{r_2=1}^{\delta_u+p+q}\frac{D_{r_2}}{(r_2-1)!}\exp\left(-\frac{d_u+d_v+d_w}{2}\lambda\right)\lambda^{r_2-1}\right]\right\}. \quad (38)$$

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