Spectrum Sharing in Wireless Networks via QoS-Aware Secondary Multicast Beamforming

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***Abstract*—Secondary spectrum usage has the potential to considerably increase spectrum utilization. In this paper, quality-of-service (QoS)-aware spectrum underlay of a secondary multicast network is considered. A multiantenna secondary access point (AP) is used for multicast (common information) transmission to a number of secondary single-antenna receivers. The idea is that beamforming can be used to steer power towards the secondary receivers while limiting sidelobes that cause interference to primary receivers. Various optimal formulations of beamforming are proposed, motivated by different "cohabitation" scenarios, including robust designs that are applicable with inaccurate or limited channel state information at the secondary AP. These formulations are NP-hard computational problems; yet it is shown how convex approximation-based multicast beamforming tools (originally developed without regard to primary interference constraints) can be adapted to work in a spectrum underlay context. Extensive simulation results demonstrate the effectiveness of the proposed approaches and provide insights on the tradeoffs between different design criteria.**

***Index Terms*—Beamforming, multicasting, secondary spectrum usage, semidefinite programming (SDP).**

I. Introduction

RECENTLY, there has been rapid growth in spectrum demand, especially due to the deployment of a variety of wireless devices and emerging wireless services. However, almost all usable frequencies have already been licensed. At the same time, extensive measurements [1] indicate that many frequency bands remain unused for as much as 85% of time due to nonuniform spectral occupation. This mismatch between spectrum licensing and utilization has triggered significant research activity in searching for better spectrum access strategies for improved efficiency. One of the approaches allowing for improved bandwidth efficiency is the introduction of secondary spectrum licensing, where nonlicensed users may obtain provisional usage of the spectrum. Naturally, secondary spectrum usage is possible only if the secondary network causes an acceptably small performance degradation to the primary users [2]. Therefore, the challenge is to construct spectrum sharing schemes that protect primary users from excessive interference, while ensuring a meaningful level of service to the secondary system(s).

Existing works on spectrum sharing/access so far mainly exploit temporal and spatial spectrum opportunities. For example, an approach for maximizing the throughput of a secondary network is proposed in [3] based on a partially observable Markov decision process framework. An *ad hoc* secondary network configuration where the secondary users operate over the spectrum resources unoccupied by the primary system is proposed in [4]. In these approaches, the secondary users first listen to the environment, and then decide to transmit if some channels are not currently used by primary users—interference to the primary users can only be caused by sensing errors due to shadowing, propagation delays, etc. These strategies fall under the *spectrum overlay* category [2].

Our work investigates a special case of the spectrum sharing problem from the *spectrum underlay* perspective [2]. The concept of "*interference temperature*" has been introduced in [5], and it indicates the allowable interference level at the primary receivers. While most of the current literature on secondary spectrum access relies on channel sensing and medium access control (MAC) schemes, an alternative is to exploit the benefits of using multiple antennas [6]–[11]. Through the use of beamforming and power control techniques, the interference to the primary network can be effectively controlled. Therefore, even when the primary users are operating, the network of secondary users is able to exchange information continuously. This alleviates spectrum sensing demands, which are stringent in overlay systems. Whereas spectrum underlay requires channel estimates, spectrum overlay requires *activity detection* at a much faster time scale. Similar to classical random access protocols such as carrier-sense multiple access, activity detection is compounded by the *hidden terminal* problem [12], which is common in wireless systems. In overlay systems, if a hidden (to the secondary AP) primary node transmits to a visible (to the secondary AP) primary node, simultaneous transmission by the

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secondary AP will jam the receiver of the visible primary node. In underlay systems, the secondary AP limits interference to the visible primary node, so even if there is a hidden terminal transmitting to it, the damage is limited.

Beamforming and power control techniques are well-known in the context of cellular systems [13]. In [13], an iterative algorithm is proposed to jointly compute a set of feasible transmit beamforming weight vectors and power allocations for several co-channel unicast transmissions such that the signal-to-interference-plus-noise ratio (SINR) at each mobile receiver is greater than a target value. Whereas most of the literature on beamforming and power control is concerned with cellular systems (without regard to primary interference constraints), several recent papers have adapted and extended existing techniques from the cellular to the cognitive network context [6]–[10]. The downlink case is considered in [6], where two iterative algorithms are proposed for optimal beamforming and power control. The uplink case is considered in [7]. Joint beamforming and user selection has been studied in [8], and capacity-achieving transmission schemes for a single secondary link and multiple primary receivers under different system configurations have been developed in [9]. Robust beamforming in a cognitive network context has been considered in [10], with the objective of maximizing the rate of the secondary user while keeping the interference to the primary user below a certain threshold with high probability.

The main difference between our paper and all aforementioned approaches is that we consider the case of a secondary multicast network (as opposed to multiple cochannel unicast secondary transmissions). Specifically, we consider a secondary access point (AP) equipped with an antenna array. The objective is to transmit a *common* data stream to all the secondary users. The AP uses transmit beamforming to direct signal power towards secondary users while limiting interference to primary users. In this scenario, the design of the transmit beamformer is formulated as an optimization problem. The following specific optimization problems are considered in the context of secondary multicast network underlay:

- minimization of the total transmission power subject to constraints on the quality-of-service (QoS) of each receiver;
- minimization of interference to primary users subject to constraints on the signal-to-noise ratio (SNR) of secondary users and on the total power transmitted by the secondary AP;
- maximization of the smallest receiver SNR over the secondary users subject to constraints on the transmit power and interference caused to the primary users;
- a weighted tradeoff formulation that balances interference caused to the primary system versus the minimum SNR in the secondary system.

The Evolved Multimedia Broadcast/Multicast Service (E-MB/MS) in the context of 3GPP[1]/UMTS-LTE[2] includes explicit provisions for point-to-multipoint physical layer multicasting [14]. While this is a next-generation cellular draft standard, similar developments are underway for fixed wireless networks, and multicast beamforming is also appealing in a cognitive network context for dissemination of digital TV/radio/newsfeed programming and location-dependent content (e.g., maps, traffic alerts). Multicasting is especially appealing in situations where users are geographically clustered, in which case transmitting common information to all users can be far more efficient (in terms of total bit volume transmitted per second and Hertz) than transmitting independent information to each one. To see this, consider a single tone: if two terminals are very close to each other (or, their channel vectors are closely aligned in terms of direction), we have to time-share the tone to efficiently transmit in unicast mode, otherwise SINR will be less than 0 dB; but if they are interested in exactly the same content, we can serve both simultaneously.

We begin by assuming perfect channel knowledge at the design center, but also provide robust extensions that are applicable when the channel vectors are only known to within a certain tolerance. The proposed optimization problems are nonconvex and NP-hard; yet we extend the semidefinite programming (SDP) relaxation approach in [15] to our present context and show that high-quality approximate solutions can be obtained at a modest computational cost. SDP relaxation has recently been used to tackle a variety of NP-hard problems that are important in engineering practice; e.g., see [16]–[19].

The main novelty of our paper is in proposing and exploring various formulations of the multicast beamforming problem in a cognitive underlay context, including the practically important case of imperfect channel state information (CSI). The main difference from the cellular context is the need to protect the primary users. This requirement yields design *problems* that are quite different, and the differences are important. For example, the introduction of primary interference constraints opens the door for infeasibility. The associated (approximate) *solutions* are similar in structure to the corresponding ones for the cellular case.

The paper is organized as follows. In Section II, the system model and assumptions are presented. Various formulations of the spectrum sharing problem via multicast beamforming are developed in Section III for the case when perfect CSI is available. In Section IV, the corresponding formulations are extended to the practically important case of imperfect CSI. A probability-constrained design approach is developed in Section V for the case when the channel vectors can be assumed to follow an independent identically distributed (i.i.d.) Rayleigh fading model, and only the channel variance is known at the design center. In Section VI, semidefinite relaxation and customized randomization approaches for the specific problems in Sections III and IV are developed. Numerical results for simulated and real data that demonstrate the effectiveness of our solutions are presented in Section VII, followed by conclusions in Section VIII.

II. System Model

A network that consists of several secondary users in the presence of multiple primary transmitter-receiver links as shown in Fig. 1 is considered. An example of such network can be the temporary deployment of a secondary wireless local area network (WLAN) in the area of an existing primary WLAN

[1]Third Generation Partnership Project.

[2]Universal Mobile Telecommunications System—Long Term Evolution.

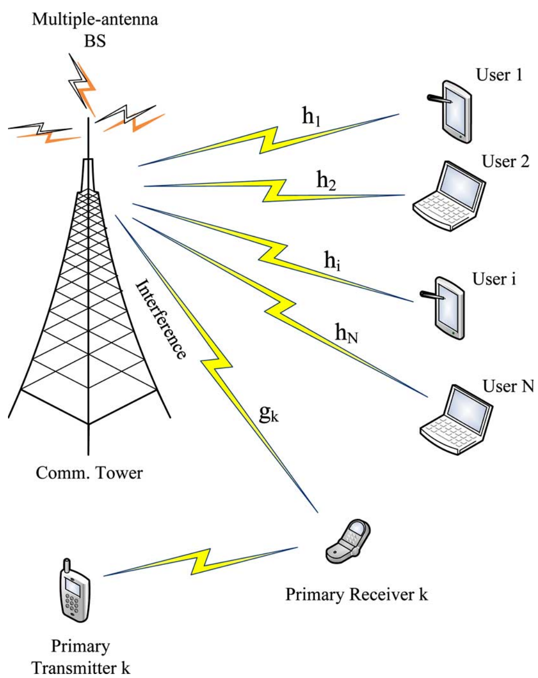

Fig. 1. Secondary cell with N users and a single primary link.

[6]–[10]. The particular scenario considered here is one in which the secondary WLAN AP transmits common information to all secondary users. The secondary AP is equipped with M antennas while each of N secondary and K primary users has a single antenna. Since the primary and secondary networks coexist, the operation of the latter must not cause excessive interference to the former. This can be accomplished in two ways. One is to severely limit the total transmission power of the secondary AP, which will limit the interference to any primary receiver irrespective of the associated coupling channel (the channel between the secondary AP and primary receiver) vector direction, by virtue of the Cauchy–Schwartz inequality. Knowing the maximal coupling channel norm (the largest norm of a channel between the secondary AP and primary receiver) is then sufficient to bound interference power. The drawback of this approach is that it will typically overconstrain the transmission power and thus the spectral efficiency of the secondary network. A more appealing alternative for the secondary AP is to estimate the channel vectors between its antenna array and the primary (and secondary) receivers and use beamforming techniques.

Channel estimation for the secondary system can be accomplished via training and feedback. The AP can send pilots, the receivers can estimate their respective channels, and feed the estimates back to the AP. An important issue here is that this hand-shaking should not disturb the operation of the primary system. If hand-shaking and CSI exchanges are infrequent (low mobility), they can be designed not to interfere with the primary system. In particular, one can use long spreading, frequency hopping, or ultra-wide-band (UWB) transmission and coherent combining to ensure low spectral density (under the noise floor) across the primary system's band. Notice that these techniques cannot be employed for the multicast downlink, because video feeds, for example, are high-rate and require guaranteed QoS for timely delivery.

Estimating the channel vector between the secondary AP and a primary receiver is more challenging, because primary users are unlikely to cooperate with secondary AP. If the primary system operates in a time-division duplex (TDD) mode, however, and assuming reciprocity, this information can be acquired at the secondary AP by listening to the primary receiver when it takes its turn to transmit. Note that this approach is possible only if the same frequency is used for duplexing. Otherwise, blind beamforming techniques could be employed (e.g., [20] and references therein). Finally, the primary system could cooperate (under a "sublet" agreement) with the secondary system to pass along channel estimates (see also [6]–[10] and references therein)—albeit this is far less appealing from a practical standpoint.

Although perfect CSI will not be available in the considered scenario, accurate CSI can be obtained in certain (e.g., fixed wireless or low-mobility) cases. Either way, (approximate or partial/statistical) knowledge of the primary channel vectors enables (approximate) spatial nulling to protect the primary receivers while directing higher power towards the secondary receivers—thereby increasing the transmission rate for the secondary system.

Let $\mathbf{h}_i$, $\mathbf{g}_k$ denote the $M \times 1$ complex vectors which model the channel gains from M transmit antennas to the secondary user i, $i = 1, \ldots, N$ and to the receiver of the primary link k, $k = 1, \ldots, K$, respectively. Also let $\mathbf{w}$ denote the beamforming weight vector applied to the transmit antenna elements. If the transmitted signal is white zero-mean with unit variance, and the noise at the ith receiver is white zero-mean with variance σ_i^2, then the received SNR of the ith user can be expressed as

$$\text{SNR}_i = \frac{|\mathbf{w}^H \mathbf{h}_i|^2}{\sigma_i^2}. \tag{1}$$

Note that, from the viewpoint of secondary receivers, interference caused by primary users can be lumped together with the additive noise term. The interference power to the receiver of the primary link k is given by $|\mathbf{w}^H \mathbf{g}_k|^2, \; k = 1, \ldots, K$.

III. Beamforming for Secondary Multicasting in Wireless Networks With Perfect CSI

The case of perfect CSI is first considered. This will serve as a stepping stone towards developing more realistic robust beamforming designs for the case of inaccurate CSI, as we will see in the sequel.

A. *Transmit Power Minimization Based Beamforming*

Given lower bound constraints on the received SNR of each secondary user and upper bound constraints on the interference to the primary users, the problem of designing the beamformer which minimizes the transmit power can be mathematically posed as

$$\text{minimize}_{\mathbf{w}} \; \|\mathbf{w}\|_2^2 \tag{2a}$$

$$\text{subject to} \; \frac{|\mathbf{w}^H \mathbf{h}_i|^2}{\sigma_i^2} \geq \text{SNR}_i^{\min}, \; i = 1, \ldots, N \tag{2b}$$

$$|\mathbf{w}^H \mathbf{g}_k|^2 \leq \eta_0, \; k = 1, \ldots, K \tag{2c}$$

where $\mathrm{SNR}_i^{\min}$ is the prescribed minimum received SNR for the $i\mathrm{th}$ user, η_0 is the allowable interference threshold value, and $\|\cdot\|_2$ stands for the Euclidean norm of a vector. The constraints (2b) require the SNR for each secondary user be greater than a target minimum SNR denoted as $\mathrm{SNR}_i^{\min}$. The constraints (2c) state that the interference level to any primary receiver must be less than the threshold value η_0.

It can be seen that the problem (2a)–(2c) belongs to the class of quadratically constrained quadratic programming (QCQP) problems. The constraints (2b) are concave homogeneous quadratic constraints. Note that the problem (2a)–(2c) contains as a special case the problem considered in [15] (see also [18]) and is therefore NP-hard. Fortunately, efficient approximate solution to this problem can be computed by appropriately modifying the techniques developed in [15], as we will see in the sequel. Note that the transmit power is not constrained in the above formulation, but an explicit power constraint can be added without changing the nature of the problem.

Observation 1: At optimality, at least one of the constraints (2b) must be met with equality. Otherwise, the beamformer can be scaled down by an appropriate coefficient such that all the constraints are still met, and at the same time the objective function is decreased.

It is also worth noting that the beamforming problem (2a)–(2c) is not always feasible. Geometrically, the feasible region of (2a)–(2c) is the intersection of the exteriors of N co-centered ellipsoids and the interiors of K co-centered ellipsoids [18]. This intersection may turn out to be empty. Checking (in)feasibility is also NP-hard, however, thus one must rely on approximation methods to assess it (see also [21]). We will return to this issue after we present appropriate approximation algorithms for the various problem formulations.

B. Interference Minimization Based Beamforming

Due to the broadcast nature of wireless transmission, the operation of the secondary network inevitably degrades the reception quality of the primary links by creating interference at the primary receivers. Therefore, a possible problem formulation is to minimize the interference level while each secondary user has its SNR above some threshold. This formulation corresponds to scenarios when the secondary network leases the spectrum of the primary network, thus QoS requirements for secondary users must be guaranteed. In practice, the QoS requirements are specified by the agreement with the primary network. Then, mathematically, the beamforming problem can be formulated as

$$\text{minimize}_{\mathbf{w}} \ \sum_{k=1}^{K} |\mathbf{w}^H \mathbf{g}_k|^2 + \xi \|\mathbf{w}\|^2 \tag{3a}$$
$$\text{subject to } \|\mathbf{w}\|_2^2 \le P \tag{3b}$$
$$\frac{|\mathbf{w}^H \mathbf{h}_i|^2}{\sigma_i^2} \ge \mathrm{SNR}_i^{\min}, \ i = 1, \ldots, N \tag{3c}$$

where ξ is a small regularization parameter and P is the total available power at the secondary AP. Constraint (3b) captures regulatory and power supply/amplification considerations. Notice that when $K < M$ and no $\mathbf{h}_i, \forall i$ belongs to the orthogonal complement of the nullspace of the matrix $\mathbf{G} = [\mathbf{g}_1, \mathbf{g}_2, \ldots, \mathbf{g}_K]^H$, it is possible to find a vector $\mathbf{w}_0$ that solves the system $|\mathbf{w}^H \mathbf{g}_k|^2 = 0, \ \forall k = 1, \ldots, K$, and is not orthogonal to any of the $\mathbf{h}_i$'s. The penalty term $\xi \|\mathbf{w}\|^2$ in (3a) then steers the solution towards minimum power.

Similar to the problem (2a)–(2c), it can be shown that the problem (3a)–(3c) is nonconvex [due to the constraints (3c)] and NP-hard.

Observation 2: Since the objective function (3a) is decreasing with respect to (w.r.t.) $\|\mathbf{w}\|_2$, at least one of the constraints (3c) must be met with equality at the optimum. Otherwise, the beamformer can be scaled down such that all the constraints are still met and the objective function is decreased.

Observation 3: If the elements of $\mathbf{h}_i, \ \forall i$ and $\mathbf{g}_k, \ \forall k$ are jointly drawn from a continuous distribution (with respect to the Lebesgue measure in $\mathcal{C}^{MNK}$), the condition that no $\mathbf{h}_i, \ \forall i$ belongs to the orthogonal complement of the nullspace of the matrix $\mathbf{G}$ holds almost surely [22], and thus the power constraint is necessary. Note that the almost surely argument does not exclude the case of having inner products close to zero, which happens with nonzero probability; the power constraint is needed to handle these cases as well.

It can be seen that the interference minimization based beamforming problem (3a)–(3c) is not always feasible. The feasibility of the problem (3a)–(3c) depends on many factors: the bound on transmit power P, the number of transmit antennas M, the number of receivers N, the channel realizations $\mathbf{h}_i, \ i = 1, \ldots, N$, and the SNR constraints for the secondary users. A practical implication of infeasibility is that it may not be possible to serve all the secondary subscribers at their desired QoS from a single power-limited AP. Moreover, since the objective function in the problem (3a)–(3c) is a sum of interferences to all primary receivers, there may be excessive interference to some particular primary receiver at the optimum. If protecting individual receivers is more important than systemwide optimization, then the following formulation is more appropriate:

$$\text{minimize}_{\mathbf{w}} \ \max_{k=1,\ldots,K} \left\{ |\mathbf{w}^H \mathbf{g}_k|^2 \right\} \tag{4a}$$
$$\text{subject to constraints (3b), (3c).} \tag{4b}$$

Our methodology can be adapted to cover the above formulation as well; we skip this due to space limitations.

C. Max–Min Fairness Based Beamforming

In addition to providing preferential treatment to high priority connections, a meaningful level of service should also be provided to low priority users. This suggests the following formulation:

$$\text{maximize}_{\mathbf{w}} \ \min_{i=1,\ldots,N} \left\{ \frac{|\mathbf{w}^H \mathbf{h}_i|^2}{\sigma_i^2} \right\} \tag{5a}$$
$$\text{subject to } \|\mathbf{w}\|_2^2 \le P \tag{5b}$$
$$|\mathbf{w}^H \mathbf{g}_k|^2 \le \eta_0, \ k = 1, \ldots, K. \tag{5c}$$

Observation 4: At optimality, at least one of the constraints in (5b) or (5c) will be met with equality. Otherwise, it would have been possible to scale up the beamformer and thereby improve the objective without violating any constraint.

Note that other forms of fairness, for example, weighted fairness can be considered. While a weighted sum-rate utility function is often preferable in the case of transmitting independent information to different users, the situation is different for multicasting. In a multicast context, it is the worst-user SNR that determines the *common* (multicast) rate, in the information-theoretic sense. This is because a common information signal is transmitted to all the users, and the Shannon rate is determined by the weakest link. In other words, the capacity of the multicast channel is determined by the weakest link—see [15] and references therein.

Introducing a new variable t, the problem (5a)–(5c) can be equivalently rewritten as the following optimization problem:

$$\text{minimize}_{\mathbf{w},\, t} \quad -t \tag{6a}$$
$$\text{subject to} \quad \frac{|\mathbf{w}^H\mathbf{h}_i|^2}{\sigma_i^2} \geq t,\ i=1,\ldots,N \tag{6b}$$
$$\|\mathbf{w}\|_2^2 \leq P,\ t \geq 0 \tag{6c}$$
$$|\mathbf{w}^H\mathbf{g}_k|^2 \leq \eta_0,\ k=1,\ldots,K. \tag{6d}$$

It is easy to check that the constraints (6c)–(6d) are convex on $\mathbf{w}$. However, the constraints (6b) are nonlinear and *nonconvex* on $\mathbf{w}$ and t. Moreover, this nonconvex problem contains the one in [15] as a special case; it is therefore NP-hard.

D. Worst User SNR-Interference Tradeoff Analysis

The interference to the primary users is minimized in the problem (3a)–(3c), while the minimum received SNR over all secondary receivers is maximized in the problem (5a)–(5c). Obviously, simultaneous maximization of the users' received SNRs and minimization of the interference caused to the primary users are desirable. However, there is a tradeoff between these two objectives. Given the available transmit power, a mathematical model for the tradeoff analysis between these two objectives can be posed as

$$\text{maximize}_{\mathbf{w}} \quad p_2 \min_{i=1,\ldots,N} \left\{ \frac{|\mathbf{w}^H\mathbf{h}_i|^2}{\sigma_i^2} \right\} - p_1 \max_{k=1,\ldots,K} \left\{ |\mathbf{w}^H\mathbf{g}_k|^2 \right\} \tag{7a}$$
$$\text{subject to} \quad \|\mathbf{w}\|_2^2 \leq P \tag{7b}$$

where p_1 and p_2 are the importance parameters.

The optimization problem (7a)–(7b) can be shown to be a nonconvex QCQP problem that is also NP-hard. The arbitrary importance parameters p_1 and p_2 quantify the desire to make the largest interference level small and the SNR of the worst user large, respectively. Moreover, the ratio of p_1 and p_2, i.e., p_1/p_2, can be seen as a relative importance of the interference to the primary users and the performance of the secondary users. In particular, for a fixed value of p_2, a larger value of p_1 results in smaller interference at the cost of performance degradation for the worst user in the network. Without loss of generality, we can set $p_2 = 1$ and by varying $p_1 > 0$, obtain the Pareto optimal points by solving (7a)–(7b). We further notice that the objectives are *competing* since in order to decrease one objective, the other must be increased.

The problem (7a)–(7b) has another interesting interpretation. The parameters p_1, p_2 can be seen as the prices per unit interference level and SNR gain. Therefore, as for the secondary network operator, $p_2 \min_{i=1,\ldots,N} \ |\mathbf{w}^H\mathbf{h}_i|^2/\sigma_i^2$ can be viewed as the total revenue obtained for serving the secondary network. Similarly, $p_1 \max_{k=1,\ldots,K} |\mathbf{w}^H\mathbf{g}_k|^2$ can be seen as the total cost spent for causing interference to the primary network. Therefore, the optimization problem (7a)–(7b) is to determine the appropriate transmit strategy to maximize "profit."

IV. Beamforming for Secondary Multicasting in Wireless Networks With Imperfect CSI

The previously considered assumption of perfect CSI is not always practical due to the time-varying nature of wireless propagation channels and the mobility of the users. Therefore, we propose an approach to robust beamforming design in the case of erroneous CSI which uses the concept of worst-case design (see, e.g., [23] and references therein). Specifically, we assume that all channel vectors are known with certain errors $\boldsymbol{\delta}$ and that these errors are all norm-bounded, that is, $\|\boldsymbol{\delta}\|_2 \leq \varepsilon$ where the parameter ε is assumed to be known. Due to space limitations, we consider in detail only the transmit power minimization based beamforming with imperfect CSI, and comment on how the same technique can be applied to the max–min fairness based beamforming. Note that other beamforming formulations can be similarly extended to the case of imperfect CSI.

A. Transmit Power Minimization Based Beamforming

The robust modification of the beamforming problem (2a)–(2c) can be written as

$$\text{minimize}_{\mathbf{w}} \quad \|\mathbf{w}\|_2^2 \tag{8a}$$
$$\text{subject to} \quad \min_{\|\boldsymbol{\delta}_i\|_2 \leq \varepsilon} \frac{\left|\mathbf{w}^H(\mathbf{h}_i + \boldsymbol{\delta}_i)\right|^2}{\sigma_i^2} \geq \text{SNR}_i^{\min},\ \forall i \tag{8b}$$
$$\max_{\|\boldsymbol{\delta}_k\|_2 \leq \varepsilon} \left|\mathbf{w}^H(\mathbf{g}_k + \boldsymbol{\delta}_k)\right|^2 \leq \eta_0,\ \forall k. \tag{8c}$$

It can be seen that both the SNR and the interference constraints are satisfied for all realizations of the channel error vectors $\boldsymbol{\delta}$. Therefore, statistical information about the channel error vectors is not required in this approach, and the rough knowledge of the upper-bound of channel error vector norms is sufficient.[3] To simplify the problem (8a)–(8c), we modify the inequality constraints (8b) and (8c) using an approach similar to the one developed in [23], [26], and [27]. From the triangle inequality, it follows

$$\left|\mathbf{w}^H(\mathbf{g}_k + \boldsymbol{\delta}_k)\right| \leq \left|\mathbf{w}^H\mathbf{g}_k\right| + \left|\mathbf{w}^H\boldsymbol{\delta}_k\right|. \tag{9}$$

Applying the Cauchy–Schwarz inequality, we have

$$|\mathbf{w}^H\boldsymbol{\delta}_k| \leq \|\mathbf{w}\|_2 \|\boldsymbol{\delta}_k\|_2 \leq \varepsilon\|\mathbf{w}\|_2 \tag{10}$$

[3]However, we should note that if the statistical information regarding $\boldsymbol{\delta}$ is available, a more efficient approach may be possible [24], [25].

where the fact that $\|\boldsymbol{\delta}_k\|_2 \leq \varepsilon$ has been also used. Hence, we find that

$$\max_{\|\boldsymbol{\delta}_k\|_2\leq\varepsilon} |\mathbf{w}^H\boldsymbol{\delta}_k| = \varepsilon\|\mathbf{w}\|_2. \tag{11}$$

Substituting (11) into (9), we obtain

$$\max_{\|\boldsymbol{\delta}_k\|_2\leq\varepsilon} \left|\mathbf{w}^H(\mathbf{g}_k+\boldsymbol{\delta}_k)\right|^2 \leq (|\mathbf{w}^H\mathbf{g}_k|+\varepsilon\|\mathbf{w}\|_2)^2. \tag{12}$$

Expanding the right-hand side of (12), we have

$$\begin{aligned}\Big(|\mathbf{w}^H\mathbf{g}_k|+\varepsilon\|\mathbf{w}\|_2\Big)^2 &= \left|\mathbf{w}^H\mathbf{g}_k\right|^2+\varepsilon^2\|\mathbf{w}\|_2^2+2\varepsilon\|\mathbf{w}\|_2|\mathbf{w}^H\mathbf{g}_k| \\ &\leq \left|\mathbf{w}^H\mathbf{g}_k\right|^2+\varepsilon^2\|\mathbf{w}\|_2^2+2\varepsilon\|\mathbf{w}\|_2^2\|\mathbf{g}_k\|_2 \\ &= \left|\mathbf{w}^H\mathbf{g}_k\right|^2+\varepsilon(\varepsilon+2\|\mathbf{g}_k\|_2)\|\mathbf{w}\|_2^2 \\ &= \mathbf{w}^H\tilde{\mathbf{G}}_k\mathbf{w}\end{aligned} \tag{13}$$

where the Cauchy–Schwarz inequality has been used again in the second line and the matrix $\tilde{\mathbf{G}}_k$ is computed as

$$\tilde{\mathbf{G}}_k = \mathbf{g}_k\mathbf{g}_k^H+\varepsilon\left(\varepsilon+2\sqrt{\mathbf{g}_k^H\mathbf{g}_k}\right)\mathbf{I}. \tag{14}$$

Following similar lines of argument, the left-hand side of the constraint (8b) can be modified as follows:

$$\begin{aligned}|\mathbf{w}^H(\mathbf{h}_i+\boldsymbol{\delta}_i)| &\geq |\mathbf{w}^H\mathbf{h}_i| - |\mathbf{w}^H\boldsymbol{\delta}_i| && \text{(15a)}\\ &\geq |\mathbf{w}^H\mathbf{h}_i| - \max_{\|\boldsymbol{\delta}_i\|\leq\epsilon}|\mathbf{w}^H\boldsymbol{\delta}_i| && \text{(15b)}\\ &= |\mathbf{w}^H\mathbf{h}_i| - \epsilon\|\mathbf{w}\|_2 \geq 0. && \text{(15c)}\end{aligned}$$

Note that it has been assumed that $|\mathbf{w}^H\mathbf{h}_i| \geq |\mathbf{w}^H\boldsymbol{\delta}_i|$ in (15a), and $|\mathbf{w}^H\mathbf{h}_i| \geq \epsilon\|\mathbf{w}\|$ in (15c). This assumption essentially means that the errors $\boldsymbol{\delta}_i, \forall i$ are sufficiently small or equivalently ϵ is sufficiently small. This is a practical assumption since large channel estimation errors can cause large beamforming errors and no robustness can be guaranteed in such case.

Using (15a)–(15c), the left-hand side of the constraint (8b) can be lower bounded as follows:

$$\min_{\|\boldsymbol{\delta}_i\|_2\leq\varepsilon} |\mathbf{w}^H(\mathbf{h}_i+\boldsymbol{\delta}_i)|^2 \geq \mathbf{w}^H\tilde{\mathbf{H}}_i\mathbf{w} \tag{16}$$

where the matrix $\tilde{\mathbf{H}}_i = \mathbf{h}_i\mathbf{h}_i^H + \varepsilon(\varepsilon - 2\sqrt{\mathbf{h}_i^H\mathbf{h}_i})\mathbf{I}$.

Using the above results, the original problem (8a)–(8c) can be modified (strengthened) as

$$\begin{aligned}\text{minimize}_{\mathbf{w}}\quad & \|\mathbf{w}\|_2^2 && \text{(17a)}\\ \text{subject to}\quad & \mathbf{w}^H\tilde{\mathbf{H}}_i\mathbf{w}\geq\sigma_i^2\mathrm{SNR}_i^{\min},\ i=1,\ldots,N && \text{(17b)}\\ & \mathbf{w}^H\tilde{\mathbf{G}}_k\mathbf{w}\leq\eta_0,\ k=1,\ldots,K. && \text{(17c)}\end{aligned}$$

Note that for the optimization problem (17a)–(17c), the matrices $\tilde{\mathbf{G}}_k,\ k=1,\ldots,K$ are always positive definite, while the positive definiteness of the matrices $\tilde{\mathbf{H}}_i,\ i=1,\ldots,N$ depends on the bound value for the channel estimation error ε.

B. Max–Min Fairness Based Beamforming

Similarly, the robust modification of the beamforming problem (5a)–(5c) can be written as

$$\begin{aligned}\text{maximize}_{\mathbf{w}}\quad & \min_{i=1,\ldots,N}\left\{\min_{\|\boldsymbol{\delta}_i\|_2\leq\varepsilon}\frac{\left|\mathbf{w}^H(\mathbf{h}_i+\boldsymbol{\delta}_i)\right|^2}{\sigma_i^2}\right\} && \text{(18a)}\\ \text{subject to}\quad & \max_{\|\boldsymbol{\delta}_k\|_2\leq\varepsilon}\left|\mathbf{w}^H(\mathbf{g}_k+\boldsymbol{\delta}_k)\right|^2 \\ & \leq\eta_0,\ k=1,\ldots,K && \text{(18b)}\\ & \|\mathbf{w}\|_2^2\leq P && \text{(18c)}\end{aligned}$$

or, equivalently, as

$$\begin{aligned}\text{minimize}_{\mathbf{w},\,t}\quad & -t && \text{(19a)}\\ \text{subject to}\quad & \mathbf{w}^H\tilde{\mathbf{H}}_i\mathbf{w}\geq\sigma_i^2 t, i=1,\ldots,N && \text{(19b)}\\ & \mathbf{w}^H\tilde{\mathbf{G}}_k\mathbf{w}\leq\eta_0, k=1,\ldots,K && \text{(19c)}\\ & \|\mathbf{w}\|_2^2\leq P,\quad t\geq 0. && \text{(19d)}\end{aligned}$$

It can be easily seen that the optimization problems (17a)–(17c) and (19a)–(19d) are also nonconvex QCQP problems. In Section VI, we show how the proposed formulations can be approximately solved using SDP relaxation.

V. Beamforming for Secondary Multicasting in Wireless Networks With Channel Statistics Only

In the absence of any instantaneous CSI information, instantaneous QoS levels cannot be guaranteed. However, if at least the channel correlation matrices are known, then *ensemble-average* QoS can be ensured. The change is particularly simple to implement in the context of the semidefinite relaxation algorithms to be discussed in Section VI, for all it takes is replacing the rank-one channel outer products by the given correlation matrices—everything else carries over verbatim.

If the channel correlation matrices are also unavailable, one can take recourse to a simple but parsimonious channel model and aim for probabilistic service guarantees. This is exemplified in the sequel for the commonly adopted i.i.d. Rayleigh fading model.

We consider the following probabilistic constraint

$$\Pr\left\{\min_{i=1,\ldots,N}\ \frac{\theta_i^2}{\sigma_i^2}\left|\mathbf{w}^H\mathbf{h}_i\right|^2\geq\eta\right\}\geq\rho \tag{20}$$

where $\Pr\{\cdot\}$ denotes probability, θ_i accounts for signal attenuation, and $\mathbf{h}_i,\ i=1,\ldots,N$ consist of independent zero-mean unit-variance Gaussian random variables. Introducing the notation $\hat{\eta}_i \triangleq \sigma_i^2\eta/\theta_i^2$, it can be shown that

$$\Pr\left\{\min_{i=1,\ldots,N}\left|\mathbf{w}^H\mathbf{h}_i\right|^2\geq\hat{\eta}_i\right\} = \Pr\left\{\min_{i=1,\ldots,N}\ \|\mathbf{w}\|_2^2\lambda_i\geq\hat{\eta}_i\right\} \tag{21}$$

where $\lambda_i,\ i=1,\ldots,N$ are i.i.d. exponentially distributed random variables with unit mean. Let us assume for brevity that $\hat{\eta}_i=\hat{\eta},\ i=1,\ldots,N$. The random variables $\gamma_i \triangleq \|\mathbf{w}\|_2^2\lambda_i$, are i.i.d. exponentially distributed with mean $\bar{\gamma}_i = \bar{\gamma} = \|\mathbf{w}\|_2^2$. Using order statistics, it can be shown that (20) is equivalent to

$$\theta^2 \log(\rho)\|\mathbf{w}\|_2^2 \geq -N\sigma^2\eta. \tag{22}$$

Similarly, the probabilistic constraint on the interference to the primary users can be written as

$$\Pr\left\{\max_{k=1,\ldots,K} \theta_i^2 \left|\mathbf{w}^H \mathbf{g}_k\right|^2 \geq \zeta\right\} \leq \rho \tag{23}$$

and, for $\theta_i^2 = 1, \ \forall i$, it reduces to

$$\log\left(1-(1-\rho)^{1/K}\right)\|\mathbf{w}\|_2^2 \leq -\zeta. \tag{24}$$

We can see that the problem boils down to a simple power budget calculation. It is worth mentioning that there is no benefit from multiple transmit antennas in this scenario, because the assumed (oversimplified) channel model exhibits no "directionality".

VI. Approximate Solutions Via Semidefinite Programming (SDP)

A. Transmit Power Minimization Based Beamforming

Although the optimization problem (2a)–(2c) is NP-hard (thus an arbitrary instance of it cannot be solved in polynomial time), it can be relaxed to a convex problem using SDP relaxation, and the relaxed problem can be efficiently solved. Specifically, using the fact that $\mathbf{h}_i^H \mathbf{w}\mathbf{w}^H \mathbf{h}_i = \mathrm{trace}(\mathbf{w}\mathbf{w}^H \mathbf{h}_i \mathbf{h}_i^H)$ where $\mathrm{trace}(\cdot)$ denotes the trace of a matrix, we can recast (2a)–(2c) as follows:

$$\begin{aligned} \text{minimize}_{\mathbf{w}} \ & \mathrm{trace}\ (\mathbf{w}\mathbf{w}^H) && \text{(25a)}\\ \text{subject to} \ & \mathrm{trace}\ (\mathbf{w}\mathbf{w}^H \mathbf{H}_i) \\ & \quad \geq \mathrm{SNR}_i^{\min}, \ i=1,\ldots,N && \text{(25b)}\\ & \mathrm{trace}\ (\mathbf{w}\mathbf{w}^H \mathbf{G}_k) \leq \eta_0, \ k=1,\ldots,K && \text{(25c)} \end{aligned}$$

where $\mathbf{H}_i \triangleq \mathbf{h}_i\mathbf{h}_i^H/\sigma_i^2, \ i=1,\ldots,N$ and $\mathbf{G}_k \triangleq \mathbf{g}_k\mathbf{g}_k^H, \ k = 1,\ldots,K$.

Introducing a new variable $\mathbf{X} \triangleq \mathbf{w}\mathbf{w}^H$ with $\mathbf{X}$ being symmetric positive semi-definite matrix, i.e., $\mathbf{X} \succcurlyeq \mathbf{0}$, the problem (25a)–(25c) can be equivalently rewritten as

$$\begin{aligned} \text{minimize}_{\mathbf{X}} \ & \mathrm{trace}\ (\mathbf{X}) && \text{(26a)}\\ \text{subject to} \ & \mathrm{trace}(\mathbf{X}\mathbf{H}_i) \geq \mathrm{SNR}_i^{\min}, \ i=1,\ldots,N && \text{(26b)}\\ & \mathrm{trace}\ (\mathbf{X}\mathbf{G}_k) \leq \eta_0, \ k=1,\ldots,K && \text{(26c)}\\ & \mathbf{X} \succcurlyeq \mathbf{0}, \ \mathrm{rank}(\mathbf{X}) = 1 && \text{(26d)} \end{aligned}$$

where $\mathrm{rank}(\cdot)$ denotes the rank of a matrix. The objective function and the trace constraints are linear in $\mathbf{X}$, while the set of symmetric positive semidefinite matrices is convex. However, the rank constraint is nonconvex. Dropping the rank constraint, we obtain the so-called SDP relaxation, that is,

$$\begin{aligned} \text{minimize}_{\mathbf{X}} \ & \mathrm{trace}\ (\mathbf{X}) && \text{(27a)}\\ \text{subject to} \ & \mathrm{trace}(\mathbf{X}\mathbf{H}_i) \geq \mathrm{SNR}_i^{\min}, \ i=1,\ldots,N && \text{(27b)}\\ & \mathrm{trace}\ (\mathbf{X}\mathbf{G}_k) \leq \eta_0, \ k=1,\ldots,K && \text{(27c)}\\ & \mathbf{X} \succcurlyeq \mathbf{0} && \text{(27d)} \end{aligned}$$

which is an SDP problem. This SDP problem is convex and can be efficiently solved using interior point methods, at a complexity cost that is at most $\mathcal{O}((N+K+M^2)^{3.5})$ [28]. SeDuMi [29]—a Matlab toolbox that implements modern interior point methods for SDP—can be used for solving problem (27a)–(27d) efficiently.

Dropping the rank-one constraint may seem completely *ad-hoc*, and, at any rate, solving the relaxed SDP problem *does not* solve the original NP-hard problem. However, it has been shown in [30] that *rank relaxation* of a general QCQP problem yields the Lagrange bi-dual problem, which is the closest convex problem to the original NP-hard problem, in a certain sense. Furthermore, researchers in the optimization community have long recognized the value of rank relaxation for obtaining approximate solutions to hard nonconvex problems, and have developed suitable procedures for converting the solution of the relaxed problem to an approximate solution of the original problem. In many cases, this is accomplished via *randomization* techniques, whose complexity is small relative to that of solving the relaxed SDP problem.

B. Randomization Algorithm

Let $\mathbf{X}_{\mathrm{opt}}$ denote the optimal solution to the problem (27a)–(27d). If the matrix $\mathbf{X}_{\mathrm{opt}}$ is rank-one, then the optimal weight vector can be straightforwardly recovered from it by finding the principal eigenvector corresponding to the only nonzero eigenvalue. However, because of the SDP relaxation step, i.e., relaxation of the rank-one constraint, the matrix $\mathbf{X}_{\mathrm{opt}}$ may not be rank-one in general. Similar to [15], once the relaxed SDP problem (27a)–(27d) is solved, a *randomization* approach can be used to obtain an approximate solution to the original problem from the solution to its relaxed version. Various randomization techniques have been developed so far (see [31], [32] and references therein). A common idea of these techniques is to generate a set of candidate vectors $\{\tilde{\mathbf{w}}_{\mathrm{cand},l}\}_{l=1}^{L}$ using $\mathbf{X}_{\mathrm{opt}}$ and choose the best solution from these candidate vectors. Here, L is the number of randomizations used.

In application to our problem, the randomization technique can be modified as follows. First, to obtain the candidate vectors, the eigendecomposition of $\mathbf{X}_{\mathrm{opt}}$ is calculated in the form

$$\mathbf{X}_{\mathrm{opt}} = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^H \tag{28}$$

and the candidate beamforming vector

$$\tilde{\mathbf{w}}_{\mathrm{cand},l} = \mathbf{U}\mathbf{\Sigma}^{1/2}\mathbf{v}_l \tag{29}$$

is selected as a candidate vector, where $\mathbf{U}$ is an unitary matrix of eigenvectors, $\mathbf{\Sigma}$ is a diagonal matrix of eigenvalues, and $\mathbf{v}_l$ is a random vector whose elements are independent random variables uniformly distributed on the unit circle in the complex plane. This ensures that $\tilde{\mathbf{w}}_{\text{cand},l}^H \tilde{\mathbf{w}}_{\text{cand},l} = \mathbf{v}_l^H (\mathbf{\Sigma}^{1/2})^H \mathbf{U}^H \mathbf{U} \mathbf{\Sigma}^{1/2} \mathbf{v}_l = \text{trace } (\mathbf{\Sigma} \mathbf{v}_l \mathbf{v}_l^H) = \text{trace } (\mathbf{\Sigma}) = \text{trace } (\mathbf{X}_{\text{opt}})$ for any realization of $\mathbf{v}_l$, where we have used the fact that $\mathbf{\Sigma}$ is diagonal, and the elements of $\mathbf{v}_l$ are drawn from the unit circle in the complex plane. When randomization is needed (i.e., $\text{rank}(\mathbf{X}_{\text{opt}}) > 1$), then it follows that $\tilde{\mathbf{w}}_{\text{cand},l}$ must violate at least one of the constraints in (2b) or (2c)—otherwise a contradiction emerges. This is because $\tilde{\mathbf{w}}_{\text{cand},l}$ generates a rank-one covariance, has the same cost as $\mathbf{X}_{\text{opt}}$, and if it satisfies all constraints then this means that a rank-two covariance is not needed. When at least one of each type of constraints is violated, then $\tilde{\mathbf{w}}_{\text{cand},l}$ is discarded, and a new randomization round begins. When $\tilde{\mathbf{w}}_{\text{cand},l}$ only violates one or more constraints in (2b), then it must be scaled up by a coefficient $\sqrt{\alpha} > 1$, which can be determined as

$$\alpha = \max_{i=1,\ldots,N} \frac{\sigma_i^2 \text{SNR}_i^{\min}}{|\tilde{\mathbf{w}}_{\text{cand},l}^H \mathbf{h}_i|^2} > 1. \tag{30}$$

Thus, a new candidate vector $\tilde{\mathbf{w}}_{\text{cand},l} = \sqrt{\alpha} \tilde{\mathbf{w}}_{\text{cand},l}$ can be found. This scaled candidate vector is guaranteed to satisfy all the QoS constraints (2b). However, it is also necessary to check whether the constraints (2c) are satisfied using this new scaled candidate vector. If (2c) is violated, then the particular candidate is discarded, and a new randomization round begins. Likewise, when $\tilde{\mathbf{w}}_{\text{cand},l}$ only violates one or more constraints in (2c), then it must be scaled down, and the resulting vector must be checked for admissibility with respect to the constraints in (2b). Finally, among the feasible candidates $\tilde{\mathbf{w}}_{\text{cand},l}$ (if any), the one with smallest norm is chosen as the sup-optimal beamformer vector.

The aforementioned *randomization* process is different from the existing techniques such as, for example, the randomization technique used in [15]. This is because the beamforming problem (2a)–(2c) incorporates both convex and concave constraints. Therefore, it is essential to check that the candidate beamformer satisfies both types of constraints.

C. Interference Minimization Based Beamforming

Following the approach developed for the case of transmit power minimization based beamforming, the SDP relaxation of the problem (3a)–(3c) can be written as

$$\text{minimize}_{\mathbf{X}} \sum_{k=1}^{K} \text{trace}(\mathbf{X}\mathbf{G}_k) \tag{31a}$$
$$\text{subject to } \text{trace } (\mathbf{X}) \le P \tag{31b}$$
$$\text{trace}(\mathbf{X}\mathbf{H}_i) \ge \text{SNR}_i^{\min},\ i=1,\ldots,N \tag{31c}$$
$$\mathbf{X} \succcurlyeq \mathbf{0}. \tag{31d}$$

In the *randomization* step (needed when $\text{rank}(\mathbf{X}_{\text{opt}}) > 1$), the initial candidate vector $\tilde{\mathbf{w}}_{\text{cand},l}$ can be obtained from $\mathbf{X}_{\text{opt}}$ and $\|\tilde{\mathbf{w}}_{\text{cand},l}\|_2^2 = \text{trace } (\mathbf{X}_{\text{opt}}) \le P$. At least one of the constraints (3c) is violated for the candidate vector $\tilde{\mathbf{w}}_{\text{cand},l}$, and $\sum_{k=1}^{K} |\tilde{\mathbf{w}}_{\text{cand},l}^H \mathbf{g}_k|^2$ is only a lower bound on the optimal value, i.e., $\sum_{k=1}^{K} |\tilde{\mathbf{w}}_{\text{cand},l}^H \mathbf{g}_k|^2 \le \sum_{k=1}^{K} \text{trace}(\mathbf{X}_{\text{opt}} \mathbf{G}_k)$ for any feasible candidate vector. Therefore, $\tilde{\mathbf{w}}_{\text{cand},l}$ needs to be scaled up as $\tilde{\mathbf{w}}_{\text{cand},l} = \sqrt{\alpha} \tilde{\mathbf{w}}_{\text{cand},l}$ where α can be chosen according to (31c).

Moreover, since the initial candidate vector was scaled by a coefficient $\sqrt{\alpha} > 1$, it is also necessary to check whether the scaled vector satisfies the constraint $\|\tilde{\mathbf{w}}_{\text{cand},l}\|_2^2 \le P$. If this holds, then this is a feasible candidate for the suboptimal beamformer. Finally, among the feasible candidate vectors, the vector $\tilde{\mathbf{w}}_{\text{cand},l}$, for which $\sum_{k=1}^{K} |\tilde{\mathbf{w}}_{\text{cand},l}^H \mathbf{g}_k|^2$ is smallest, is chosen as the sup-optimal beamformer vector.

D. Max–Min Fairness Based Beamforming

In the same manner as before, the SDP relaxation of the optimization problem (6a)–(6d) can be written as

$$\text{minimize}_{\mathbf{X},\, t}\ -t \tag{32a}$$
$$\text{subject to } \text{trace } (\mathbf{X}\mathbf{H}_i) \ge t,\ i=1,\ldots,N \tag{32b}$$
$$\text{trace } (\mathbf{X}) \le P \tag{32c}$$
$$\text{trace } (\mathbf{X}\mathbf{G}_k) \le \eta_0,\ k=1,\ldots,K \tag{32d}$$
$$\mathbf{X} \succcurlyeq \mathbf{0},\ \ t \ge 0. \tag{32e}$$

The objective function and the trace constraints in (32a)–(32e) are linear and, hence, convex on $\mathbf{X}$ and t. Therefore, the optimization problem (32a)–(32e) is an SDP problem.

The *randomization* step can also be developed as before with some appropriate modifications. First, the initial candidate vector $\tilde{\mathbf{w}}_{\text{cand},l}$ is obtained using $\mathbf{X}_{\text{opt}}$ and $\|\tilde{\mathbf{w}}_{\text{cand},l}\|_2^2 = \text{trace}(\mathbf{X}_{\text{opt}}) \le P$. It is also necessary to check if the interference constraints (5c) are satisfied. If all K interference constraints are satisfied as inequalities, the objective (5a) can be increased by scaling the candidate beamforming vector $\tilde{\mathbf{w}}_{\text{cand},l}$ up by $\sqrt{\alpha}$

$$\alpha = \min \left\{ \frac{P}{\|\tilde{\mathbf{w}}_{\text{cand},l}\|^2}; \left. \frac{\eta_0}{|\tilde{\mathbf{w}}_{\text{cand},l}^H \mathbf{g}_k|^2} \right|_{k=1,\ldots,K} \right\} \ge 1. \tag{33}$$

If at least one of K interference constraints is not satisfied, the candidate beamforming vector $\tilde{\mathbf{w}}_{\text{cand},l}$ must be scaled down by $\sqrt{\beta}$

$$\beta = \min_{k=1,\ldots,K} \left\{ \frac{\eta_0}{|\tilde{\mathbf{w}}_{\text{cand},l}^H \mathbf{g}_k|^2} \right\} \le 1. \tag{34}$$

The so-obtained new scaled candidate beamforming vector always satisfies both the power constraint (5b) and the interference constraints (5c). Therefore, the suboptimal beamforming vector is the new scaled candidate vector which yields the largest $\min_{i=1,\ldots,N} \left\{ |\tilde{\mathbf{w}}_{\text{cand},l}^H \mathbf{h}_i|^2 / \sigma_i^2 \right\}$ and, therefore, provides the maximum to the objective (5a).

E. Worst User SNR-Interference Tradeoff Analysis

Introducing new variables t and $\tilde{t}$ and using SDP relaxation, we obtain the following relaxed version of the optimization problem (7a)–(7b)

$$\text{maximize}_{\mathbf{X},\, t,\, \tilde{t}}\ t - p_1 \tilde{t} \tag{35a}$$
$$\text{subject to } \text{trace } (\mathbf{X}) \le P \tag{35b}$$
$$\text{trace } (\mathbf{X}\mathbf{G}_k) \le \tilde{t},\ k=1,\ldots,K \tag{35c}$$
$$\text{trace } (\mathbf{X}\mathbf{H}_i) \ge t,\ i=1,\ldots,N \tag{35d}$$
$$\mathbf{X} \succcurlyeq \mathbf{0},\ t \ge 0,\ \tilde{t} \ge 0. \tag{35e}$$

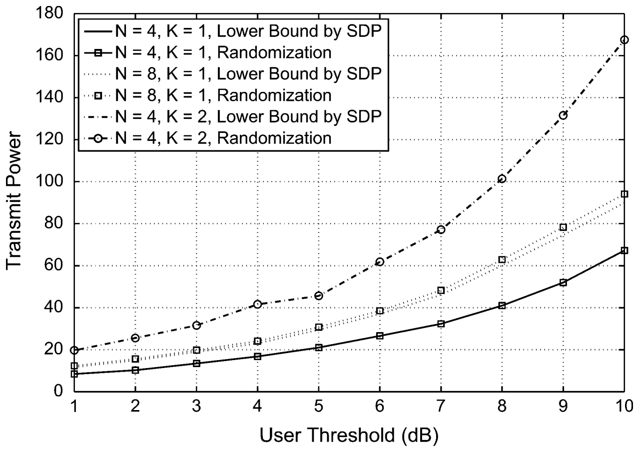

Fig. 2. Transmit power minimization based beamforming: transmit power versus users' SNR thresholds.

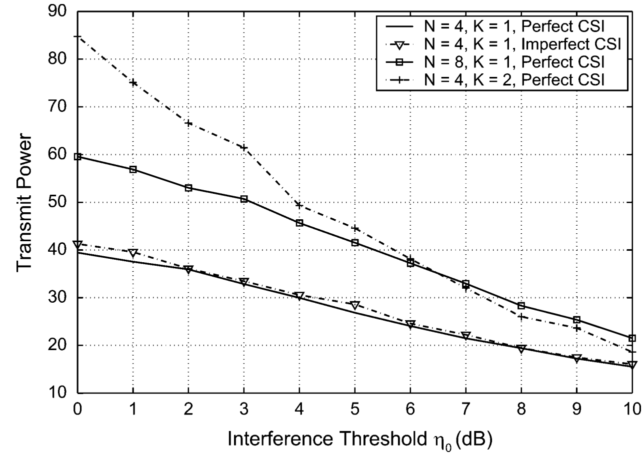

Fig. 3. Transmit power minimization based beamforming: transmit power versus interference threshold.

Note that p_2 is set to be equal to 1 for brevity. At least one of the constraints (35c) and at least one of the constraints (35d) must be met with equality at the optimum. Otherwise, $\tilde{t}$ and t can always be decreased and increased, respectively, improving the optimal value and hence contradicting optimality.

The *randomization* step is much simpler for this problem than for the previous problems. In fact, any of the initial candidate vector $\tilde{\mathbf{w}}_{\mathrm{cand},l}$ obtained from $\mathbf{X}_{\mathrm{opt}}$ is a feasible one. This is due to the fact that $\|\tilde{\mathbf{w}}_{\mathrm{cand},l}\|_2^2 = \mathrm{trace}\,(\mathbf{X}_{\mathrm{opt}}) \leq P$, and thus, any $\tilde{\mathbf{w}}_{\mathrm{cand},l}$ satisfies the power constraint, which is the only constraint in the optimization problem (7a)–(7b). Therefore, the final beamforming vector is the candidate vector which provides the smallest objective value.

VII. Simulation Results

A. Beamforming With Simulated Data

Two system configurations are considered. The first configuration has a secondary network with four-antenna AP and four users, while the second configuration has a secondary network with four-antenna AP and eight users. The standard i.i.d. Rayleigh fading channel model is assumed with noise variance $\sigma_i^2 = 1, \ i = 1, \ldots, N$. Either one or two primary links are considered and it is assumed that all secondary users have the same received SNR thresholds. Both cases of perfect and imperfect CSI are evaluated. For the case of imperfect CSI, the error vector $\boldsymbol{\delta}$ is uniformly and randomly generated in a sphere centered at zero with the radius ε.[4] All results are averaged over 1000 simulation runs with $L = 2000$ randomizations.

1) Transmit Power Minimization Based Beamforming: Fig. 2 shows the transmit power[5] versus the SNR requirement of the users for both configurations when there is no interference allowed.[6] It can be seen that the transmit power increases when the SNR threshold increases, or equivalently, more power is needed to improve the users' performance. With the same QoS requirements, more power is required to satisfy eight users than four users. For the four-user network, the transmit power corresponding to the approximate solution obtained via the randomization process is indistinguishable from its lower bound obtained by SDP relaxation. Therefore, the approximate solution is very close to optimal in this case. For the eight-user network, however, there is an appreciable gap between the lower bound obtained by SDP relaxation and the solution obtained via randomization. This gap increases for high QoS requirements. Moreover, as the user SNR demands become stricter, one needs to pay a lot more in terms of excess transmit power to give the users an extra 1 dB in guaranteed SNR. It can also be seen that higher power is required to meet the QoS of all secondary users in the case of two primary links.

Fig. 3 shows the required transmit power versus the interference threshold η_0 for the cases of perfect CSI and imperfect CSI with $\varepsilon = 0.001$. The users' SNR thresholds are fixed at 10 dB. It can be seen that the required transmit power is smaller when the allowable interference level is higher. Mathematically, if a higher level of interference is allowed, the feasible set of the proposed power minimization beamforming problem expands, thus giving an opportunity to further decrease the objective function. Moreover, the decrease of the transmit power is less noticeable in the high interference region. For the same interference and SNR levels, more power is needed in the eight-user network than in the four-user network. One can also see that more power is required to meet the QoS requirements for the network with two primary links than that for one primary link. As expected, when estimation errors are present, more power is needed to satisfy the QoS requirements of the secondary users in the network.

2) Interference Minimization Based Beamforming: In this example, the interference level at the primary receiver is examined when the SNR of secondary users is guaranteed to be larger than a given threshold. The transmit power is constrained to be less than 15 and 20. Fig. 4 shows the interference level versus the users' SNR threshold. Note that this problem is not always feasible. Therefore, the average is taken only over those channel realizations which make the problem feasible. It can be seen that

[4]Note that the radius ε depends on the accuracy of the channel estimation. Larger transmit power may provide better channel estimates, and thus, smaller ε.

[5]Throughout our simulations, $P = 1$ yields SNR $= 0$ dB when all the power is given to one transmit antenna.

[6]In this case, randomization is performed using randomly-generated vectors in the null space of $\mathbf{g}$.

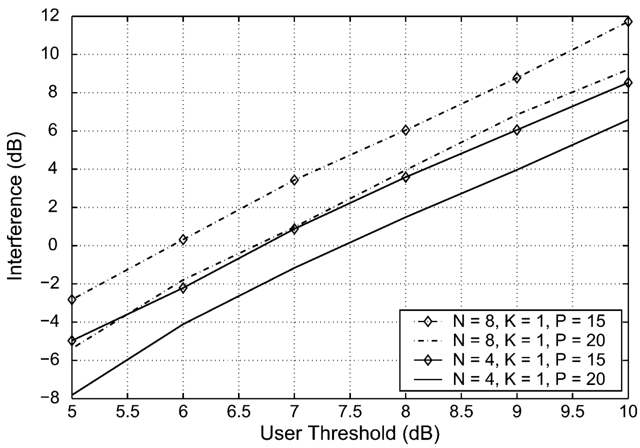

Fig. 4. Interference minimization based beamforming: interference versus users' SNR thresholds.

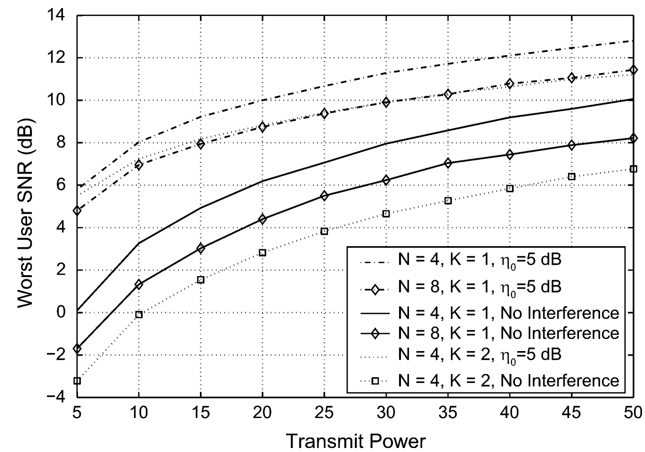

Fig. 6. Max–min fairness based beamforming: worst user SNR versus transmit power with no interference.

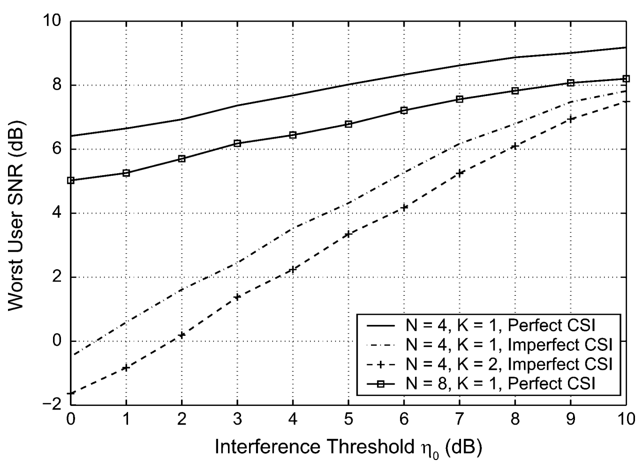

Fig. 5. Max–min fairness based beamforming: worst user SNR versus interference threshold.

for both four-user and eight-user networks the interference level can be reduced when there is more available transmit power. For example, to achieve 10 dB for all users in the eight-user network, the interference must be about 9 and 12 dB for $P = 20$ and $P = 15$, respectively. Therefore, the interference can be reduced by 3 dB if the transmit power is increased by 5. Furthermore, it can be seen that the resulting interference at the primary receiver is low when the secondary users operate at 5–6 dB.

3) Max–Min Fairness Based Beamforming: In this example, two scenarios are considered. The first scenario corresponds to the case of fixed transmit power and varying interference threshold in the interval $[0, 10]$ dB, while the second scenario corresponds to the case of fixed interference threshold and varying transmit power in the interval $[5, 50]$.

Fig. 5 displays the SNR of the worst user versus the interference threshold for several network configurations when the transmit power is equal to 10. Both cases of perfect and imperfect CSI with $\varepsilon = 0.01$ are considered. It can be seen that as the interference threshold increases, the performance of the worst user also increases. Mathematically, the feasible set of the corresponding optimization problem is larger when the allowable interference level is higher. Furthermore, for the same transmit power, the SNR of the worst user in the four-user network is larger than that in the eight-user network. Fig. 5 reveals that the secondary network performance improves when more transmit power is available. As expected, coarse CSI can substantially reduce the attainable beamforming gain, especially in the low η_0 region. In other words, the degradation caused by imperfect CSI becomes less pronounced when η_0 is large.

Fig. 6 shows the performance of the worst user versus the transmit power for two cases of no allowable interference and 5 dB interference. It can be seen that the SNR of the worst user in the secondary network improves significantly when the interference threshold is 5 dB as compared to the case of no allowable interference. For example, for the four-user secondary network, the SNR is equal to 6 dB and 10 dB in the former and latter cases, respectively, if $P = 20$. Secondary network performance improves when more power is available, and the performance of the worst user of the four-user network is always better than that of the eight-user network. In the case of two primary links, the secondary system performance is worse than that in the case of only one primary link.

4) Worst User SNR- Interference Tradeoff Analysis: In this example, the tradeoff between the performance of the user with the worst SNR and the interference is investigated. Fig. 7 shows the worst SNR and the interference level when the transmit power is varied in the interval [2, 20], and $p_1 = 0.5$ and $p_2 = 1$ in the optimization problem (35a)–(35e). It can be seen that both the SNR and interference curves increase with a constant ratio as the transmit power increases. The eight-user secondary network has smaller SNR for the worst user and smaller interference level as compared to the four-user secondary network. The simulation results clearly show the tradeoffs between the interference and SNR. It can also be seen that improved performance of the secondary network causes more interference to the primary network.

Fig. 8 displays the interference level versus the worst user SNR for different values of the parameter $p_1 \in [0.1, 1]$, while the parameter p_2 is fixed to 1, and the transmit power is fixed at 10 and 20. It should be noted that for smaller p_1, the proposed multiobjective beamforming tries to improve the worst user SNR, while it tries to suppress interference for large p_1. It

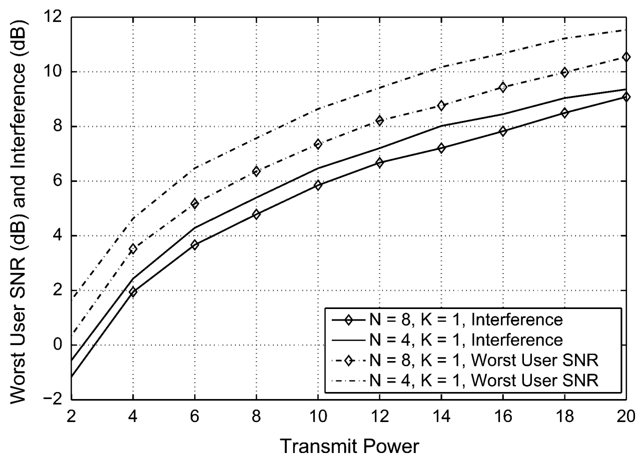

Fig. 7. Multiobjective beamforming: worst-user SNR and interference versus transmit power.

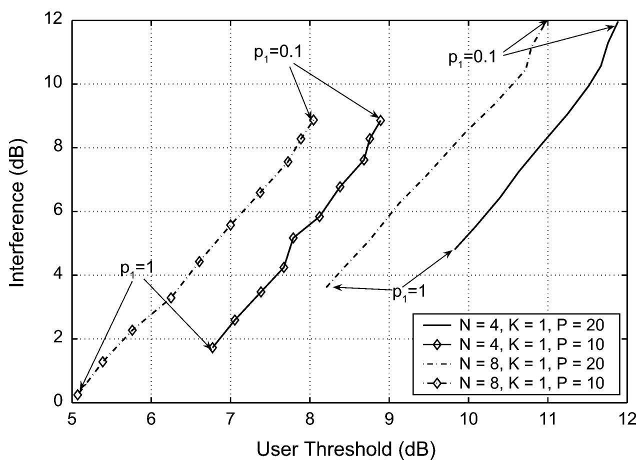

Fig. 8. Multiobjective beamforming with different weight parameters: interference versus worst-user SNR.

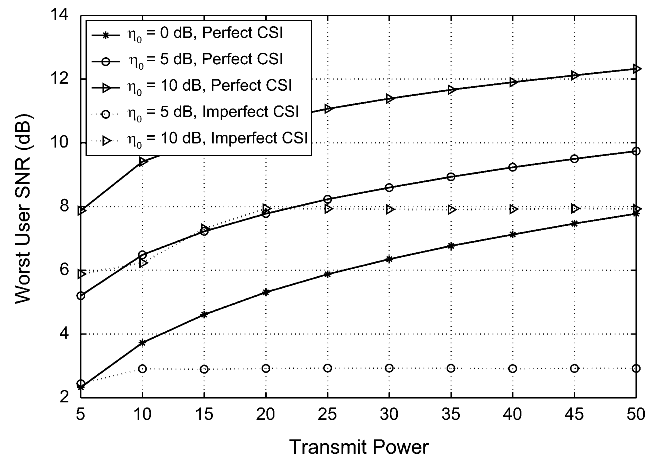

Fig. 9. Worst-user SNR versus transmit power.

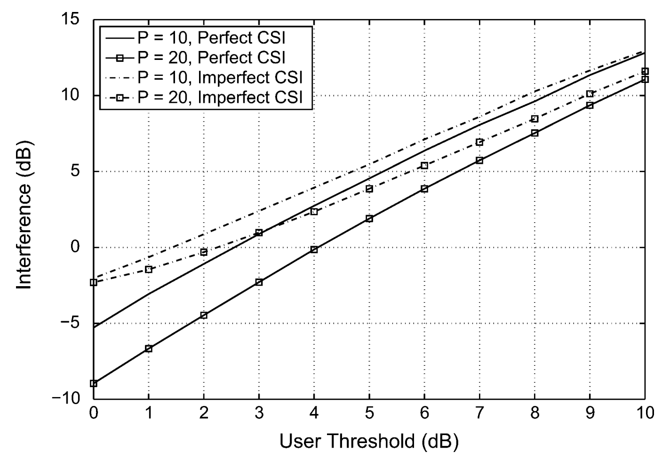

Fig. 10. Interference minimization based beamforming: interference versus users' SNR thresholds.

can be seen in Fig. 8 that for $p_1 = 0.1$, the interference is indeed the largest and the worst user SNR is the largest, while for $p_1 = 1$, the worst user SNR is the smallest and the interference is the smallest. Fig. 8 shows that the interaction between those two metrics depends on the weight factors that we set for each objective.

B. Beamforming With Measured Data

In this example, we consider a secondary network with four-antenna AP serving ten secondary users and interfering with two primary users. Measured channel data (downloaded from the iCORE HCDC Lab. web site in the University of Alberta, http://www.ece.ualberta.ca/~mimo/) are used. Both cases of perfect and imperfect CSI are considered. The data come from the *Quad* measurements, and our results are averaged over 30 temporal channel snapshots measured in the 902–928 MHz (ISM) band. The location area of the experiment is a 150×60-m lawn surrounded by buildings with heights ranging from 15 to 30 m. The transmitter is equipped with four vertically polarized dipoles spaced $\lambda/2$ apart. The channels are frequency-flat and slowly time selective fading due to pedestrian motions and other factors. Further details can be found in the aforementioned website and in [33].

The worst user SNRs versus transmit power for the max–min fairness based beamforming are plotted in Fig. 9. Different values of interference thresholds $\eta_0 \in \{0, 5, 10\}$ dB are considered. For the case of imperfect CSI, ε is taken to be equal to 0.1. It can be seen that better performance can be achieved for more relaxed interference constraints. Furthermore, we can see that the imperfect CSI degrades the performance of the system. These results are consistent with the previous ones for simulated data. Note that *Quad* channels exhibit directionality due to significant line-of-sight components. Secondary multicast beamforming is more effective in such directional scenarios, and in this sense, testing with the *Quad* data is closer to what would likely be encountered in practical deployments.

Fig. 10 shows the interference versus the user threshold $\mathrm{SNR}^{\min}$ for interference minimization based beamforming. Transmit power is taken to be equal to 10 and 20 and $\varepsilon = 0.01$ for the case of imperfect CSI. It can be seen that larger transmit power helps to reduce the interference caused to the primary users. Since ε is quite small in this case, the degradation effects because of imperfect CSI diminish in the high power regime, which shows that the proposed approach based on bounding the channel error norm is quite tight when the channel errors are relatively small.

It is possible that the SDP relaxation version of any of the above problems is feasible, but the original NP-hard problem is not. The opposite cannot happen, for if the original problem is feasible then the relaxed problem will be feasible as well. It is also possible that both the original problem and its SDP relaxation are feasible, but the randomization process fails to discover a feasible point for the original problem. Checking for (in)feasibility of the original problem is also NP-hard, however, which means that there is potential for cases where randomization fails and we cannot tell if the original problem is feasible. These issues have been numerically assessed in [21] (without primary interference constraints), where it was found that such cases are rare. This was also confirmed in our simulations in the present context—for example, with SNR threshold set to 5 dB, it happens once in about 1000 runs.

VIII. Conclusion

The multicast beamforming problem for secondary wireless networks has been considered. It has been shown that using the CSI available at the transmitter, the QoS for both primary and secondary users in a cognitive network can be effectively controlled. A number of practically important design scenarios with different criteria involving the interference level at the primary receivers, the received SNR of the secondary users and the transmit power have been considered. The cases of perfect CSI and imperfect CSI at the transmitter of the secondary network have been studied. Although the proposed designs are nonconvex and NP-hard, a convex relaxation approach coupled with suitable randomization postprocessing provides approximate solutions at a moderate computational cost that is strictly bounded by a low-order polynomial. We also note that our approach can be applicable in conventional cellular systems when broadcasting to a number of receivers and at the same time protecting some specific "directions" from interference.

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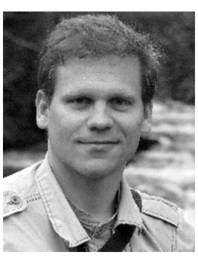

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