On the Capacity of Rayleigh Fading Cooperative Systems under Adaptive Transmission

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***Abstract*—In this letter, the use of adaptive source transmission with amplify-and-forward relaying is proposed. Three different adaptive techniques are considered: (i) optimal simultaneous power and rate adaptation; (ii) constant power with optimal rate adaptation; (iii) channel inversion with fixed rate. The capacity upper bounds of these adaptive protocols are derived for the amplify-and-forward cooperative system over both independent and identically distributed (i.i.d.) Rayleigh fading and non-i.i.d. Rayleigh fading environments. The capacity analysis is based on an upper bound on the effective received signal-to-noise ratio (SNR). The tightness of the upper bound is validated by the use of a lower bound and by Monte Carlo simulation. It is shown that at high SNR the optimal simultaneous power and rate adaptation and the optimal rate adaptation with constant power provide roughly the same capacity. Channel inversion is shown to suffer from a deterioration in capacity relative to the other adaptive techniques.**

***Index Terms*—Cooperative diversity, adaptive transmission, rayleigh fading, channel capacity.**

I. Introduction

An efficient way of providing diversity in wireless fading networks is through the use of node cooperation for information relaying [1], [2]. In cooperative communications, the source transmits information to the destination not only through a direct-link but also through the use of relays. The performance of cooperative communication systems has been analyzed for various system and channel models. The average symbol error rate (SER) of a two-hop cooperative system is analyzed in [3]–[5] for the Rayleigh and Nakagami-m fading channels. The outage performance of the cooperative system with Rayleigh fading which operates on a half-duplex mode in the low SNR regime is provided in [6]. In [7] and [8] the authors derive the closed-form expressions for the outage probability for Rayleigh and Nakagami-m channels, respectively, with decode-and-forward relays. The performance of an analytical model for automatic repeat request (ARQ) cooperative diversity in multi-hop wireless networks is presented in [9]. Furthermore, the enhancement of spatial-diversity by applying space-time coding is investigated in [10] and [11] for non-regenerative and distributed regenerative relaying, respectively.

All the aforementioned papers only consider fixed rate and fixed power transmission. However, adaptive transmission techniques for the wireless channel are shown to be effective and popular [12], [13]. Particularly, the transmitter adapts the transmit power level, symbol/bit rate, constellation size, coding rate/scheme or any combination of these parameters in response to the changing channel conditions [12], [13]. Therefore, by transmitting faster and/or higher power under good channel conditions and slower and/or smaller power under poor channel conditions, a higher spectral efficiency without sacrificing performance can be achieved.

More recently, resource allocation in terms of power and bandwidth is investigated for the basic three-node relay network in [14]–[17] and for the m-node relay network in [18], [19]. The majority of the aforementioned work considers power allocation problems in the context of cooperative networks. This problem is formulated by placing a fixed power constraint among the relays, and one seeks to allocate the power to different nodes to optimize some objective. This requires channels state information (CSI) of all the links and fixed source rates, and as a consequence, it is distinctly different than the adaptive policies of [12], [13]. In general, the power allocation problem has a high overhead when the number of nodes in the network is large. This is due to the requirement of having the CSI for all of the links. Furthermore, rate adaptation at the transmitter is not considered in all the aforementioned work.

Motivated by these observations, we propose the use of fixed non-regenerative cooperative systems with adaptive transmission techniques. That is, only the source adapts its rate and/or power level according to the changing channel conditions, while the m relays simply amplify and forward the signals. For the proposed source-adaptation scheme only partial CSI is required at the source. Thus, feedback of the effective SNR is only required to be available at the source, not the m relays. In this letter, we derive the upper bound expressions for the capacity and outage probability of such source-adaptive cooperative networks in both independent and identically distributed (i.i.d.) and non-i.i.d. Rayleigh fading

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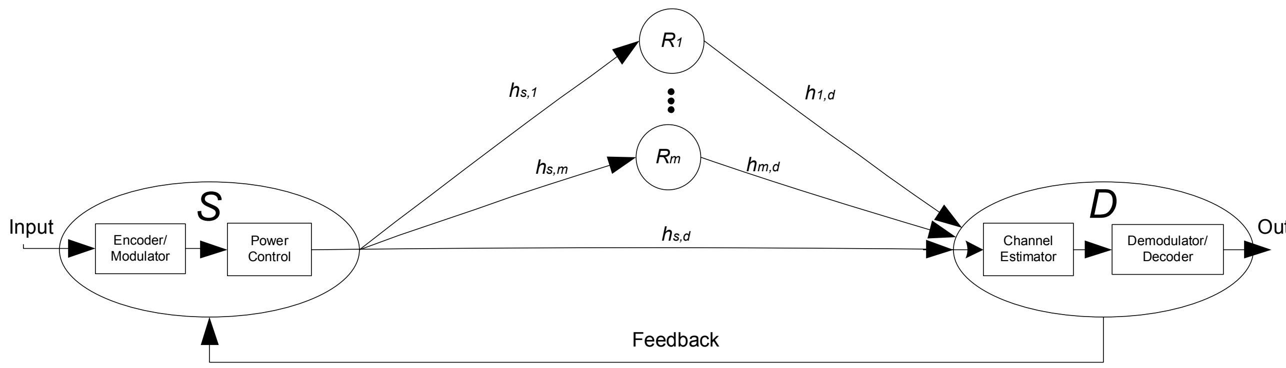

Fig. 1. Cooperative diversity wireless network with source adaptive transmission.

environments. A lower bound and Monte Carlo simulations are used to substantiate the tightness of the derived upper bound. Three different adaptive techniques are considered, namely (i) optimal simultaneous power and rate adaptation, (ii) constant power with optimal rate adaptation, (iii) channel inversion with fixed rate.

The remainder of this letter is organized as follows. Section II presents the channel and system model. The capacity analysis for the cooperative system under the different adaptive transmission techniques is derived in Section III. In Section IV the results of the capacity derivations are compared. Conclusions are given in Section V.

II. Channel and System Model

A. System Model

The cooperative wireless network of Fig. 1, a source node S communicates with a destination node D via a direct link and through m amplify-and-forward relays R_i, $i \in \{1, 2, ..., m\}$. In the first phase of cooperation, the source transmits the signal x to the destination and the relays. The received signals at the destination and at the ith relay respectively are

$$r_{s,d} = h_{s,d}x + n_{s,d}, \tag{1}$$

$$r_{s,i} = h_{s,i}x + n_{s,i}, \tag{2}$$

where $h_{s,i}$, $h_{i,d}$, and $h_{s,d}$ denote the Rayleigh fading coefficients between the source and the ith relay, the ith relay and destination, and the source and the destination, respectively. The noise is denoted at the relays as $n_{s,i}$ and at the destination as $n_{s,d}$ and $n_{i,d}$.

The ith relay amplifies the received signal and transmits it to the destination in the second phase of cooperation. During the second phase of cooperation, orthogonal transmission is required to transmit the m symbols at each of the relays. Without lost of generality, this can be accomplished by using time division multiple access (TDMA) [2]. That is, each of the m symbols are transmitted from the relays in a round robin fashion. The received signal at the destination from the ith relay is

$$r_{i,d} = G_i h_{i,d} r_{s,i} + n_{i,d} \tag{3}$$

where G_i is the ith relay amplifier gain, chosen as [2], $G_i^2 = E_s / \left(E_s |h_{s,i}|^2 + N_0\right)$ where E_s is the average symbol energy, and N_0 is the noise variance.

Using maximum ratio combining at the destination node the total SNR is easily found as [2]

$$\gamma_{\text{tot}} = \gamma_{s,d} + \sum_{i=1}^{m} \frac{\gamma_{s,i}\gamma_{i,d}}{\gamma_{s,i} + \gamma_{i,d} + 1} \tag{4}$$

where $\gamma_{s,i} = |h_{s,i}|^2 E_s/N_0$, $\gamma_{i,d} = |h_{i,d}|^2 E_s/N_0$, $\gamma_{s,d} = |h_{s,d}|^2 E_s/N_0$ are the instantaneous SNRs between S and R_i, R_i and D, S and D respectively.

An upper bound of the total SNR at the destination node can be found as [4], [5]

$$\gamma_{\text{tot}} \le \gamma_{s,d} + \sum_{i=1}^{m} \gamma_i = \gamma_{ub} \tag{5}$$

where $\gamma_i = \min(\gamma_{s,i}, \gamma_{i,d})$. Our subsequent analysis exclusively relies on γ_{ub}, as this upper bound has been shown to be quite accurate [4], [5]. A lower bound can be formulated as in [4] where $\gamma_i = 0.5 \min(\gamma_{s,i}, \gamma_{i,d})$. As the lower bound is different from the upper bound only by a factor of half, the following analysis can easily be extended to the lower bound, but is omitted for brevity.

B. Probability Distribution Function Derivation

As γ_i and $\gamma_{s,d}$ are independent the moment generating function (MGF) of γ_{ub} is expressed as

$$M_{\gamma_{ub}}(s) = M_{\gamma_{s,d}}(s) \prod_{i=1}^{m} M_{\gamma_i}(s) \tag{6}$$

where $M_{\gamma_{s,d}}(s)$ and $M_{\gamma_i}(s)$ are the MGF of $\gamma_{s,d}$ and γ_i, respectively, and the MGF is defined as $M_X(s) = \mathbf{E}\{e^{-sX}\}$, $\mathbf{E}\{\cdot\}$ denotes the statistical average over the random variable X.

For Rayleigh fading, $\gamma_{s,d}$ is exponentially distributed, thus $M_{\gamma_{s,d}}(s) = (1 + \bar{\gamma}_{s,d}s)^{-1}$, where $\bar{\gamma}_{s,d} = \mathbf{E}\{|h_{s,d}|^2\}E_s/N_0$. The MGF of γ_i is derived via the use of the cumulative distribution function (CDF) of γ_i

$$F_{\gamma_i}(\gamma) = 1 - P(\gamma_{s,i} > \gamma)P(\gamma_{i,d} > \gamma). \tag{7}$$

To proceed further, we consider two different cases of fading channels in the following.

1) I.I.D. Fading Channels: In this case, the statistic of all links is identical and the average SNR on each link is given by $\bar{\gamma} = \mathbf{E}\{|h_{s,i}|^2\}E_s/N_0 = \mathbf{E}\{|h_{i,d}|^2\}E_s/N_0$. Differentiating (7) the probability density function (pdf) of γ_i is easily shown to be $p_{\gamma_i}(\gamma) = (2/\bar{\gamma})e^{-2\gamma/\bar{\gamma}}$. Then, the MGF can be written as $M_{\gamma_i}(s) = (1 + 0.5\bar{\gamma}s)^{-1}$. Applying these results into (6) the expression for the MGF of γ_{ub} is

$$M_{\gamma_{ub}}(s) = (1 + \bar{\gamma}_{s,d}s)^{-1}(1 + 0.5\bar{\gamma}s)^{-m}. \quad (8)$$

Using partial fractions, (8) can be rewritten as

$$M_{\gamma_{ub}}(s) = \beta_0(1 + \bar{\gamma}_{s,d}s)^{-1} + \sum_{i=1}^{m} \beta_i(1 + 0.5\bar{\gamma}s)^{-i} \quad (9)$$

where

$$\beta_0 = \left(1 - \frac{\bar{\gamma}}{2\bar{\gamma}_{s,d}}\right)^{-m} \quad (10)$$

and

$$\beta_i = \frac{(0.5\bar{\gamma})^{(i-m)}}{(m-i)!}\frac{\partial^{m-i}}{\partial s^{m-i}}\left[(1 + \bar{\gamma}_{s,d}s)^{-1}\right]_{s=-\frac{1}{(0.5\bar{\gamma})}}. \quad (11)$$

Taking the inverse Laplace transform of $M_{\gamma_{ub}}(s)$ in (9), and using the fact that $\mathcal{L}^{-1}\{(1+as)^{-k}\} = \frac{1}{(k-1)!a^k}x^{k-1}e^{-\frac{x}{a}}$, the pdf of γ_{ub} is as follows:

$$p_{\gamma_{ub}}(\gamma) = \frac{\beta_0}{\bar{\gamma}_{s,d}}e^{-\frac{\gamma}{\bar{\gamma}_{s,d}}} + \sum_{i=1}^{m} \frac{\beta_i(0.5\bar{\gamma})^{-i}}{(i-1)!}\gamma^{i-1}e^{-\frac{\gamma}{(0.5\bar{\gamma})}}. \quad (12)$$

2) Non-I.I.D. Fading Channels: In practice, the relays are often not symmetrically placed which causes different fading statistics among the relay-destination links. Thus, we consider independent but not identically distributed channels. Similar to the case of i.i.d. fading, differentiating (7) we obtain the pdf of γ_i as $p_{\gamma_i}(\gamma) = (1/\tau_i)e^{-\gamma/\tau_i}$, where $\tau_i = \frac{\bar{\gamma}_{s,i}\bar{\gamma}_{i,d}}{\bar{\gamma}_{s,i}+\bar{\gamma}_{i,d}}$, $\bar{\gamma}_{s,i} = \mathbf{E}\{|h_{s,i}|^2\}E_s/N_0$ and $\bar{\gamma}_{i,d} = \mathbf{E}\{|h_{i,d}|^2\}E_s/N_0$. The MGF can then be written as $M_{\gamma_i}(s) = (1 + \tau_i s)^{-1}$. Applying these results into (6) the MGF of γ_{ub} is

$$M_{\gamma_{ub}}(s) = (1 + \bar{\gamma}_{s,d}s)^{-1}\prod_{i=1}^{m}(1 + \tau_i s)^{-1}. \quad (13)$$

Again, using partial fractions, (13) can be rewritten

$$M_{\gamma_{ub}}(s) = \widehat{\beta}_0(1 + \gamma_{s,d}s)^{-1} + \sum_{i=1}^{m} \widehat{\beta}_i(1 + \tau_i s)^{-1} \quad (14)$$

where

$$\widehat{\beta}_0 = \prod_{i=1}^{m}\left(1 - \frac{\tau_i}{\bar{\gamma}_{s,d}}\right)^{-1} \quad (15)$$

and

$$\widehat{\beta}_i = \left(1 - \frac{\bar{\gamma}_{s,d}}{\tau_i}\right)^{-1}\prod_{k=1,k\neq i}^{m}\left(1 - \frac{\tau_k}{\tau_i}\right)^{-1}, \quad i = 1, \ldots, m. \quad (16)$$

Taking the inverse Laplace transform of $M_{\gamma_{ub}}(s)$ in (14) gives the pdf of γ_{ub} as:

$$p_{\gamma_{ub}}(\gamma) = \frac{\widehat{\beta}_0}{\bar{\gamma}_{s,d}}e^{-\frac{\gamma}{\bar{\gamma}_{s,d}}} + \sum_{i=1}^{m}\frac{\widehat{\beta}_i}{\tau_i}e^{-\frac{\gamma}{\tau_i}}. \quad (17)$$

The next section investigates the capacity of the cooperative system under adaptive transmission. In the proposed system, only the source performs adaptation, i.e., the source will vary its rate and/or power while the relays simply amplify and forward their received signal. In order to implement adaptive transmission, it is assumed that the received SNR γ_{tot} is perfectly tracked at the destination and is then fed back error free to the source. The channel is assumed to be slow fading and feedback delay is negligible, thus allowing the source to change the power and/or rate. These are more or less standard assumptions in [12-13].

III. Capacity Analysis Under Adaptive Transmission

It is well-known that the Shannon capacity of the fading channel defines the theoretical upper bound on the rate for reliable data transmission. One way to achieve this bound is to employ adaptive transmission, i.e., the transmitter at the source adapts its power, rate, and/or coding scheme to the channel variation. For fixed amplify-and-forward relaying where only the source performs adaptation, the capacity of different adaptive schemes are as follows.

A. Optimal Simultaneous Power and Rate Adaptation

1) I.I.D. Fading Channels: The channel capacity C_{opra} (in bits/second) given the pdf of the received SNR $p_{\gamma_{ub}}(\gamma)$, under the condition of optimal simultaneous power and rate adaptation is given by [12], [13]:

$$C_{\text{opra}} = \frac{B}{(m+1)\ln 2}\int_{\gamma_0}^{\infty}\ln\left(\frac{\gamma}{\gamma_0}\right)p_{\gamma_{ub}}(\gamma)d\gamma \quad (18)$$

where B (in hertz) is the bandwidth of the channel and γ_0 is the optimal cutoff SNR below which the transmission is stopped. The factor $1/(m+1)$ accounts for the fact that the transmission process takes place in $(m+1)$ orthogonal channels or time-slots.

The optimal cutoff SNR below which the transmission is halted satisfies

$$\int_{\gamma_0}^{\infty}\left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right)p_{\gamma_{ub}}(\gamma)d\gamma = 1. \quad (19)$$

As the transmission is halted when $\gamma_{ub} < \gamma_0$, there is a probability that the SNR falls below the optimal threshold γ_0. This outage probability is given by:

$$\begin{aligned} P_{\text{out}} &= P[\gamma_{ub} < \gamma_0] = \textstyle\int_0^{\gamma_0} p_{\gamma_{ub}}(\gamma)d\gamma \\ &= 1 - \textstyle\int_{\gamma_0}^{\infty} p_{\gamma_{ub}}(\gamma)d\gamma. \end{aligned} \quad (20)$$

The capacity C_{opra} is achieved when the source adapts its power and rate simultaneously according to the perfect channel state information (CSI) at the transmitter. Substituting (12) into (18), and making use of the integral $\mathcal{J}_n(\mu) = \int_1^{\infty} t^{n-1}\ln(t)e^{-\mu t}dt$, $\mu > 0$; $n = 1, 2, \ldots$, which is evaluated in closed-form [13, eq. (70)], the closed-form expression for C_{opra} is

$$\begin{aligned} C_{\text{opra}} &= \frac{B}{(m+1)\ln 2}\left[\frac{\beta_0\gamma_0}{\bar{\gamma}_{s,d}}\mathcal{J}_1\left(\frac{\gamma_0}{\bar{\gamma}_{s,d}}\right)\right. \\ &+ \left.\sum_{i=1}^{m}\frac{\beta_i(0.5\bar{\gamma})^{-i}}{(i-1)!\gamma_0^{-i}}\mathcal{J}_i\left(\frac{2\gamma_0}{\bar{\gamma}}\right)\right] \end{aligned} \quad (21)$$

The optimal cutoff SNR γ_0 is found by solving for γ_0 in (19), which can be rewritten as

$$\frac{1}{\gamma_0}\int_{\gamma_0}^{\infty} p_{\gamma_{ub}}(\gamma)d\gamma - \int_{\gamma_0}^{\infty}\frac{1}{\gamma}p_{\gamma_{ub}}(\gamma)d\gamma = 1. \quad (22)$$

First,

$$\int_{\gamma_0}^{\infty} p_{\gamma_{ub}}(\gamma)d\gamma = \beta_0 e^{-\frac{\gamma_0}{\bar{\gamma}_{s,d}}} + e^{-\frac{2\gamma_0}{\bar{\gamma}}}\sum_{i=1}^{m}\beta_i\sum_{k=0}^{i-1}\frac{1}{k!}\left(\frac{2\gamma_0}{\bar{\gamma}}\right)^k \quad (23)$$

is established by using [20, eq. (3.351.2)]. Second,

$$\int_{\gamma_0}^{\infty}\frac{p_{\gamma_{ub}}(\gamma)d\gamma}{\gamma} = \frac{\beta_0}{\bar{\gamma}_{s,d}}E_1\left(\frac{\gamma_0}{\bar{\gamma}_{s,d}}\right) + \frac{2\beta_1}{\bar{\gamma}}E_1\left(\frac{2\gamma_0}{\bar{\gamma}}\right) + e^{-\frac{2\gamma_0}{\bar{\gamma}}}\sum_{i=2}^{m}\frac{\beta_i}{(i-1)}\sum_{k=0}^{i-2}\frac{\gamma_0^k}{k!(0.5\bar{\gamma})^{k+1}} \quad (24)$$

is found similarly using [20, eq. (3.351.2)] where $E_n(x)$ is the exponential integral of order n, defined by [13] $E_n(x) = \int_1^{\infty} t^{-n}e^{-xt}dt,\ x \geq 0$. Substituting (23) and (24) into (22) the optimal cutoff SNR γ_0 can be obtained numerically. Asymptotically as $\bar{\gamma} = \bar{\gamma}_{s,d} \to \infty$, $\gamma_0 \to 1$. Numerical results also indicate, but not shown in this paper, that γ_0 lies in the interval of $[0, 1]$.

The probability of outage is found by substituting (23) into (20)

$$P_{\text{out}} = 1 - \left[\beta_0 e^{-\frac{\gamma_0}{\bar{\gamma}_{s,d}}} + e^{-\frac{2\gamma_0}{\bar{\gamma}}}\sum_{i=1}^{m}\beta_i\sum_{k=0}^{i-1}\frac{1}{k!}\left(\frac{2\gamma_0}{\bar{\gamma}}\right)^k\right]. \quad (25)$$

2) Non-I.I.D. Fading Channels: The closed-form expression for C_{opra} is

$$C_{\text{opra}} = \frac{B}{(m+1)\ln 2}\left[\frac{\widehat{\beta}_0\gamma_0}{\bar{\gamma}_{s,d}}\mathcal{J}_1\left(\frac{\gamma_0}{\bar{\gamma}_{s,d}}\right) + \sum_{i=1}^{m}\frac{\widehat{\beta}_i\gamma_0}{\tau_i}\mathcal{J}_1\left(\frac{\gamma_0}{\tau_i}\right)\right]. \quad (26)$$

The optimal cutoff SNR γ_0 in (26) is found by numerically solving for γ_0 in (22), where

$$\int_{\gamma_0}^{\infty} p_{\gamma_{ub}}(\gamma)d\gamma = \widehat{\beta}_0 e^{-\frac{\gamma_0}{\bar{\gamma}_{s,d}}} + \sum_{i=1}^{m}\widehat{\beta}_i e^{-\frac{\gamma_0}{\tau_i}} \quad (27)$$

and,

$$\int_{\gamma_0}^{\infty}\frac{p_{\gamma_{ub}}(\gamma)d\gamma}{\gamma} = \frac{\widehat{\beta}_0}{\bar{\gamma}_{s,d}}E_1\left(\frac{\gamma_0}{\bar{\gamma}_{s,d}}\right) + \sum_{i=1}^{m}\frac{\widehat{\beta}_i}{\tau_i}E_1\left(\frac{\gamma_0}{\tau_i}\right). \quad (28)$$

As in the case of i.i.d. fading, the probability of outage is found by substituting (27) into (20)

$$P_{\text{out}} = 1 - \left[\widehat{\beta}_0 e^{-\frac{\gamma_0}{\bar{\gamma}_{s,d}}} + \sum_{i=1}^{m}\widehat{\beta}_i e^{-\frac{\gamma_0}{\tau_i}}\right]. \quad (29)$$

B. Optimal Rate Adaptation with Constant Transmit Power

1) I.I.D. Fading Channels: For optimal rate adaptation with constant transmit power, the channel capacity C_{ora} is given by [12], [13]:

$$C_{\text{ora}} = \frac{B}{(m+1)\ln 2}\int_0^{\infty}\ln(1+\gamma)p_{\gamma_{ub}}(\gamma)d\gamma. \quad (30)$$

In fact, as discussed in [12], [13], C_{ora} (30) is the capacity of a flat-fading $(m+1)$ orthogonal wireless channel, without adaptation. In other words, C_{ora} (30) is the channel capacity with receiver side information (i.e., CSI is known only at the receiver).

Substituting (12) into (30), and making use of the integral $\mathcal{I}_n(\mu) = \int_0^{\infty} t^{n-1}\ln(1+t)e^{-\mu t}dt,\ \mu > 0;\ n = 1, 2, ...,$ which can be evaluated in a closed-form as in [13, eq. (78)], the closed-form expression for the capacity C_{ora} is

$$C_{\text{ora}} = \frac{B}{(m+1)\ln 2}\left[\frac{\beta_0}{\bar{\gamma}_{s,d}}\mathcal{I}_1\left(\frac{1}{\bar{\gamma}_{s,d}}\right) + \sum_{i=1}^{m}\frac{\beta_i(0.5\bar{\gamma})^{-i}}{(i-1)!}\mathcal{I}_i\left(\frac{2}{\bar{\gamma}}\right)\right]. \quad (31)$$

2) Non-I.I.D. Fading Channels: Likewise, the closed-form expression for the capacity C_{ora} is

$$C_{\text{ora}} = \frac{B}{(m+1)\ln 2}\left[\frac{\widehat{\beta}_0}{\bar{\gamma}_{s,d}}\mathcal{I}_1\left(\frac{1}{\bar{\gamma}_{s,d}}\right) + \sum_{i=1}^{m}\frac{\widehat{\beta}_i}{\tau_i}\mathcal{I}_1\left(\frac{1}{\tau_i}\right)\right]. \quad (32)$$

As the transmitter adapts only its rate, instead of both rate and power, to the changing channel conditions, the scheme of optimal rate adaptation with constant transmit power can be implemented at reduced complexity, and thus, is more practical than that of optimal simultaneous power and rate adaptation. Furthermore, it is worth mentioning that in [12], it is stated that optimal rate adaptation with constant transmit power is arguably more complex than the system based on receiver side information alone.

C. Channel Inversion with Fixed Rate

1) I.I.D. Fading Channels: Truncated channel inversion with fixed rate is the least complex adaptive technique as the transmitter only adjusts the power level to provide a constant SNR at the destination. Truncated channel inversion is possible as long as the received SNR is above a cutoff γ_0. This scheme achieves the outage capacity of the system. The channel capacity C_{tifr} is given by [12], [13]:

$$C_{\text{tifr}} = \frac{B}{(m+1)\ln 2}\ln\left(1 + \left[\int_{\gamma_0}^{\infty}\frac{p_{\gamma_{ub}}(\gamma)d\gamma}{\gamma}\right]^{-1}\right)(1 - P_{\text{out}}). \quad (33)$$

The truncated channel inversion capacity with fixed rate is simply found by substituting (24) and (25) into (33) but omitted here for brevity.

2) Non-I.I.D. Fading Channels: The truncated channel inversion capacity with fixed rate is similarly found by substituting (28) and (29) into (33) but is omitted for brevity.

The capacity of this scheme is shown to be always less than the previous two schemes as it does not performs rate adaptation relative to the channel conditions. It should be noted that when channel inversion is done without truncation i.e., $\gamma_0 = 0$, the capacity is zero for i.i.d Rayleigh fading. However, some applications require constant rate even with the loss in achievable capacity.

IV. Numerical Results and Comparisons

This section presents, the channel capacity and the outage probability for cooperative systems with adaptive transmissions. For i.i.d. and non-i.i.d Rayleigh fading, the system with

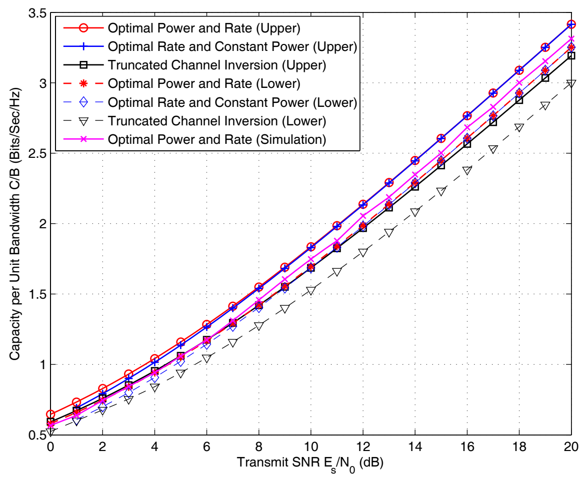

Fig. 2. Channel capacity comparison of adaptive schemes with i.i.d. Rayleigh fading channels. Upper bound, lower bound, and simulation results plotted for $m = 1$ relay.

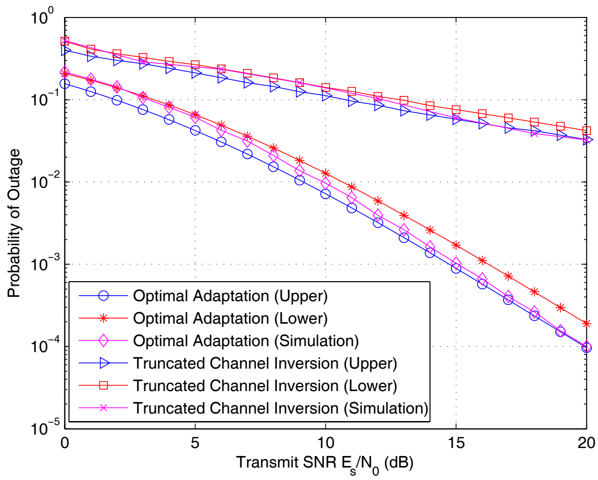

Fig. 3. Outage probability of adaptive schemes with i.i.d. Rayleigh fading channels. Upper bound, lower bound, and simulation results plotted for $m = 1$ relay.

one relay, $m = 1$ and two relays, $m = 2$ is considered, respectively. However, the results of Section III can be used for any number of relays. Moreover, the average SNR of the links are chosen to represent a practical model of a cooperative communication system.

In Fig. 2 the closed-form channel capacity derived in (21), (31), and (33) are plotted for i.i.d. Rayleigh fading, with $\bar{\gamma}_{s,d} = \bar{\gamma}$. Note that the approximate lower bound of [4] where $\gamma_i = 0.5\min(\gamma_{s,i}, \gamma_{i,d})$ is also plotted along with the Monte Carlo simulation results for the case of simultaneous power and rate adaptation. There is a distinct gap between the upper and lower bounds, which agrees with the results in [4]. The capacity of the optimal simultaneous power and rate adaptation and the capacity of the optimal rate adaptation with constant transmit power scheme are basically indistinguishable at high SNR. It is noticeable at low SNR that the Monte Carlo simulation is tight to the lower bound. Similar results for the

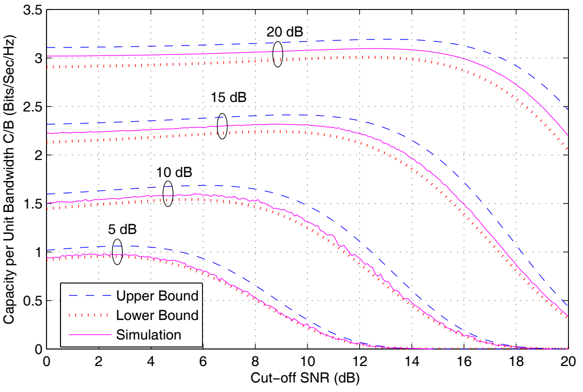

Fig. 4. Channel capacity C_{tifr} versus cut-off SNR γ_0^* for $\bar{\gamma}_{s,d} = \bar{\gamma}$. Upper bound, lower bound, and simulation results plotted for $m = 1$ relay with i.i.d. Rayleigh fading.

Monte Carlo simulation result were obtained for the other adaptive policies, but are omitted for clarity. Furthermore, the capacity performance of the truncated channel inversion with fixed rate is clearly suboptimal to the other schemes, as previously discussed.

Fig. 3 shows the probability of outage for the simultaneous optimal rate-power adaptation and for the truncated channel inversion schemes. In the former case, the optimal cut-off SNR was numerically found as in (22). In the latter case, the optimal cut-off SNR is the value that maximizes the capacity (33). However, this maximization occurs at the cost of increased probability of outage. These results are illustrated in Fig. 4, where the capacity per unit bandwidth is plotted as a function of the cut-off SNR γ_0^*, for $\bar{\gamma}_{s,d} = \bar{\gamma} = 5$, 10, 15 and 20 dB. The optimal cut off SNR γ_0^* that maximizes the capacity occurs at $\gamma_0 < \gamma_0^*$, as γ_0 is restricted between the interval [0,1]. This indicates that increased capacity occurs at the cost of some outage probability, as in Fig. 3. Furthermore, it can be seen that the lower bound on the outage probability is tight to the Monte Carlo simulation at low SNR and that the upper bound is tight to the Monte Carlo simulation at high SNR.

For non-i.i.d. Rayleigh fading, the channel capacity with adaptive transmissions are plotted in Fig. 5. The average SNRs on the branches are as follows: $\bar{\gamma}_{s,1} = E_s/N_0$, $\bar{\gamma}_{s,2} = 0.8E_s/N_0$, $\bar{\gamma}_{1,d} = 0.3E_s/N_0$, $\bar{\gamma}_{2,d} = 0.56E_s/N_0$, and $\bar{\gamma}_{s,d} = 0.2E_s/N_0$. The results obtained are similar to the i.i.d. Rayleigh fading case, but, for this two-relay system, the gap between the truncated adaptive scheme and the optimal scheme is decreased. Also it was noticed that as the cooperative diversity increases (i.e., the number of relays increases) the difference in capacity of the adaptive channel inversion technique with respect to the other adaptive techniques depreciates. The outage probability is plotted in Fig. 6.

V. Conclusion

We proposed the use of adaptive source transmission for the cooperative networks with fixed amplify-and-forward relay processing. To the best of our knowledge, this is the first time such source-adaptive relay networks have been analyzed. We derived the Shannon capacity of the non-regenerative

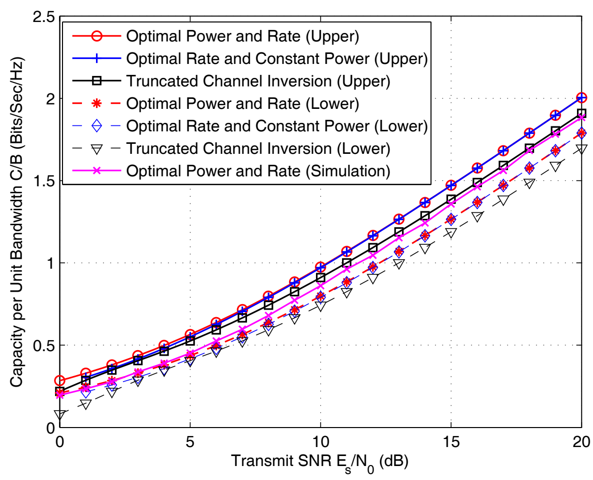

Fig. 5. Channel capacity comparison of adaptive schemes with non-i.i.d. Rayleigh fading channels. Upper bound, lower bound, and simulation results plotted for $m = 2$ relays.

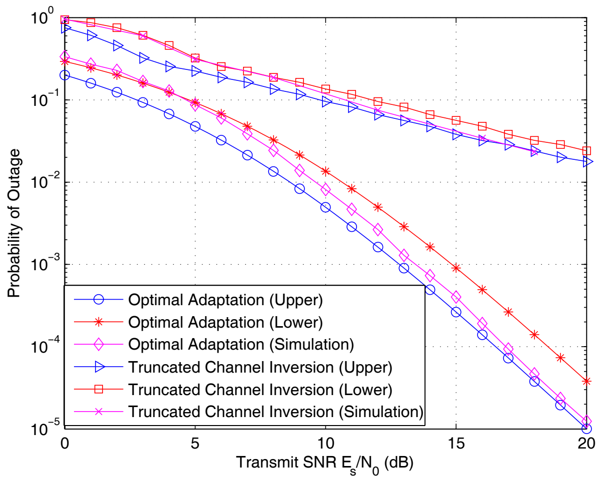

Fig. 6. Outage probability of adaptive schemes with non-i.i.d. Rayleigh fading channels. Upper bound, lower bound, and simulation results plotted for $m = 2$ relays.

cooperative i.i.d. and non-i.i.d. Rayleigh fading channels with adaptive transmission. The closed-form capacity bounds were found using a tight upper bound on the effective received SNR. The three adaptive techniques considered were: (i) optimal simultaneous power and rate adaptation; (ii) constant power with optimal rate adaptation; (iii) channel inversion with fixed rate. These capacity results represent achievable bounds for source-adaptive relay networks. The proposed adaptive relay networks open a large number of research problems such as design of decoding-and-forward relaying, node selection strategies, resource allocation among nodes, and distributed space-time coding for these networks. We are currently investigating some of these topics.

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