Unified Performance Analysis of Two Hop Amplify and Forward Relaying

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***Abstract*—Wireless relay networks have been studied extensively in the recent literature. Amplify and forward (AF) is one of the most widely used type of relaying. Even though special cases such as channel-noise-assisted, channel-assisted and blind relay modes have been analyzed, a unified performance analysis seems to be not available.**

In this paper, we present unified performance analysis results for two-hop AF relaying over Nakagami-m fading nonidentical source-to-relay (S→R) and relay-to-destination (R→D) links. A general model for the received signal-to-noise ratio, which covers channel-noise-assisted, channel-assisted and blind relay configurations as special cases, is developed. Closed-form expressions are then derived for the cumulative distribution function (cdf), probability density function (pdf), and moment generating function (mgf). Exact results are derived for symbol error rate of special cases. All results are verified through Monte Carlo simulation.

I. Introduction

Wireless relay networks [1], [2] whose nodes cooperate with each other to enhance the overall quality of service by relaying signals meant for other nodes, have been studied extensively in the literature [3]–[6]. Wider coverage, transmit power saving and reduced interference are some of the benefits of cooperation. Moreover, cooperative relay networks may operate as distributed space-time coded multiple-input multiple-output (MIMO) systems [7]. With technology becoming mature enough to support the increased system complexity required for cooperation, user cooperation is receiving wide acceptance. Emerging wireless communication standards (e.g. IEEE 802.16j [8]), ad-hoc sensor networks, broadband wireless networks, and others utilize cooperative relaying. Higher flexibility offered by such cooperation comes hand-in-hand with design challenges, motivating further research in this area.

Cooperation schemes behave and perform quite differently depending on their protocols. Nevertheless they can be broadly categorized [9] as *Decode and Forward* (DF) (or *regenerative* systems) and *Amplify and Forward* (AF) (or *non-regenerative* systems) based on whether or not signal decoding takes place at the relays. DF relays decode and regenerate the incoming signal prior to relaying, whereas AF relays merely amplify and forward it. Thus AF appears simpler and cheaper to implement than DF.

AF relaying may be subdivided based on how source-to-relay Channel State Information (CSI) and noise statistics are used in determining the relay gains.

- In *Blind/ fixed gain* relaying [10], [11] the relay gain is chosen a priori, arbitrarily or sometimes semi-blindly (i.e. based on channel statistics) [12].
- *Channel Assisted* (CA) or *CSI Assisted* relaying [13] adapts the relay gains based on CSI to cancel out the effect of fading.
- *Channel Noise Assisted* (CNA) [4], [14] relaying adapts the relay gains based on CSI to compensate fading, however within the bounds based on noise statistics.

Performance of such a system can be characterized fully if the statistics of the received signal-to-noise ratio (SNR) can be expressed in terms of its cumulative distribution function (cdf), probability density function (pdf), or moment generating function (mgf). Outage probability, bit/ symbol-error-rate, ergodic capacity are commonly used performance measures. Closed-form analysis of these measures provides insights useful for comparative evaluation and optimization of parameters and protocols.

With only source-to-relay (S→R) and relay-to-destination (R→D) links, dual-hop transmission is the simplest and the most widely analyzed. The pdf, mgf, and cdf for CA relaying over non-identical Rayleigh fading links is derived in closed-form [13]. The SNR for CA relaying is used here as a tight bound for that of CNA relaying. These results have been extended in [15] for identical Nakagami-m fading links. Reference [16] analyzes a MIMO system with Rayleigh fading links, a single-antenna relay and no direct S→D paths. But this analysis is equivalent to that of a dual-hope AF system using CA/ CNA relaying over non-identical Nakagami-m fading links. Our approach is inspired by [16] and we generalize [16]. The BER and outage analysis of blind relaying over non-identical Nakagami-m fading links is found respectively in [10] and [17]. Moreover, [18] presents the exact outage results for non-identical Nakagami-m fading links for both CNA and blind relaying. Certain exact results for more complicated systems with: dual-hop with direct path [12], [19]–[21] and multi-hop [11], [22]–[24] are also available. Despite these valuable contributions, complete, exact performance analysis of even the dual-hop case is not available.

This paper provides a unified performance analysis of dual-hop AF for independent but non-identical Nakagami-m fading links. We consider a received SNR model that encompasses CA, CNA, and blind relaying as special cases. Closed-form, exact expressions are then derived for the pdf, cdf and mgf.

Further, the exact symbol error rate is derived for specific cases. Our analysis also gives an insight into how AF schemes other than CA, CNA and blind relaying may behave.

The paper is organized as follows. Section II presents the system model assumed in our analysis. Closed-form expressions derived for the pdf, cdf, mgf and symbol error rate are presented in section III. Section IV verifies the results through simulation. Section V concludes the paper highlighting possible extensions to this analysis.

Notation: $K_\nu(.)$ is the *Modified Bessel function of the second kind* [25, 9.6] of order ν. $\mathcal{W}_{\mu,\nu}(.)$ denotes the *Whittaker W function* [25, 13.1]. ${}_2\mathcal{F}_1(\alpha,\beta;\gamma;.)$ represents the *Gauss Hypergeometric function* [25, 15.1] while $\mathcal{Q}(.)$ denotes the *Q-function* [25, 26.2.3]. $\mathcal{E}_\Lambda\{.\}$ is the *expectation operator* with respect to random variable Λ. $f_\Lambda(.)\,, F_\Lambda(.)\,, \bar{F}_\Lambda(.)$ and $M_\Lambda(.)$ denote respectively the pdf, cdf, ccdf and mgf of continuous random variable Λ. MGF is defined as $M_\Lambda(s) = \mathcal{E}_\Lambda\left\{e^{-s\Lambda}\right\}$.

II. System and Channel Model

Our analysis considers a two-hop relayed path (Fig. 1) in a cooperative wireless network. Node S acts as the data source AF relaying data through another node R to a third node D, which is the destination. The direct path and other relayed paths that may lead from S to D are not considered in this analysis.

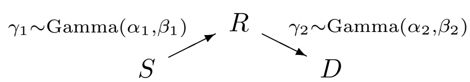

Fig. 1. System Model

The S→R and R→D links undergo independent Nakagami-m fading. Thus received SNR γ_i for each link $i \in \{1,2\}$ is Gamma distributed with parameters α_i and β_i.

We consider following unified model for the received SNR

$$\Lambda = \frac{\gamma_1\gamma_2}{a\gamma_1 + \gamma_2 + b}, \tag{1}$$

where $\gamma_1 \sim \text{Gamma}(\alpha_1,\beta_1)$ and $\gamma_2 \sim \text{Gamma}(\alpha_2,\beta_2)$. The parameters a and b are real and non-negative; and chosen to reflect the relay configuration. For instance blind, CA and CNA relay configurations may be represented respectively with $(a,b) \in \{(0,C),(1,0),(1,1)\}$, where C is a constant. Other values (e.g. $a = 0.5, b = 1$) represent intermediate configurations. Our analysis is therefore valid for all but the trivial and unrealistic case $a = b = 0$.

III. Mathematical Formulation

Given Λ in (1), the random variable denoting the received SNR at D (Fig. 1), we derive exact expressions for the cdf, pdf, mgf and symbol error rate results assuming independent fading over S→R and R→D links.

A. Cumulative distribution function of Λ

We begin by deriving the cdf (or equivalently ccdf) because its differentiation is guaranteed to yield the pdf in closed-form.

Theorem 1: complementary cdf of Λ

Let γ_1 and γ_2 be distributed $\text{Gamma}(\alpha_1,\beta_1)$ and $\text{Gamma}(\alpha_2,\beta_2)$, respectively. The complementary cdf $\bar{F}_\Lambda(x)$ of Λ in (1) is given for $x \geq 0$ by

$$\bar{F}_\Lambda(x) = 2e^{-\left(\frac{1}{\beta_1}+\frac{a}{\beta_2}\right)x} \sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{m=0}^{k} C_1(n,k,m)\, K_{n-m+1}\left(2\sqrt{\frac{x(ax+b)}{\beta_1\beta_2}}\right)\left(a+\frac{b}{x}\right)^{\frac{n+m+1}{2}} x^{\alpha_1+k} \tag{2}$$

where $C_1(n,k,m) = \dfrac{a^{k-m}\beta_1^{\frac{n-m+1-2\alpha_1}{2}}\beta_2^{\frac{m-n-1-2k}{2}}}{m!\,(k-m)!\,n!\,(\alpha_1-n-1)!}$.

For the case $a = 0$, (2) reduces to

$$\bar{F}_\Lambda(x) = 2e^{-\frac{x}{\beta_1}} \sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1} C_2(n,k)\, K_{n-k+1}\left(2\sqrt{\frac{bx}{\beta_1\beta_2}}\right) x^{\frac{2\alpha_1+k-n-1}{2}} \tag{3}$$

where $C_2(n,k) = \dfrac{\beta_1^{\frac{n-k+1-2\alpha_1}{2}}\left(\frac{b}{\beta_2}\right)^{\frac{n+k+1}{2}}}{k!\,n!\,(\alpha_1-n-1)!}$.

The cdf for each case is given by

$$F_\Lambda(x) = 1 - \bar{F}_\Lambda(x)\,. \tag{4}$$

Proof: Given in [26]. ∎

B. Probability density function of Λ

The pdf $f_\Lambda(x)$ relates to the ccdf by

$$f_\Lambda(x) = -\frac{\mathrm{d}}{\mathrm{d}x}\bar{F}_\Lambda(x)\,, \tag{5}$$

which allows the pdf to be computed straightaway using the product rule of differentiation.

Theorem 2: pdf of Λ

Let γ_1 and γ_2 be distributed $\text{Gamma}(\alpha_1,\beta_1)$ and $\text{Gamma}(\alpha_2,\beta_2)$, respectively. The pdf $f_\Lambda(x)$ of Λ in (1) is given for $x \geq 0$ by

$$f_\Lambda(x) = 2e^{-\left(\frac{1}{\beta_1}+\frac{a}{\beta_2}\right)x} \sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{m=0}^{k} C_1(n,k,m)\, \mathbb{I}_1(n,k,m)\left(a+\frac{b}{x}\right)^{\frac{n+m-1}{2}} x^{\alpha_1+k-2} \tag{6}$$

where
$$\mathbb{I}_1(n,k,m) = \left((ax+b)\left(\left(\frac{1}{\beta_1}+\frac{a}{\beta_2}\right)x - (\alpha_1+k-n-1)\right) - amx\right) K_{n-m+1}\left(2\sqrt{\frac{x(ax+b)}{\beta_1\beta_2}}\right) + (2ax+b)\sqrt{\frac{x(ax+b)}{\beta_1\beta_2}}\, K_{n-m}\left(2\sqrt{\frac{x(ax+b)}{\beta_1\beta_2}}\right).$$

For the case $a = 0$, (6) reduces to

$$f_\Lambda(x) = 2e^{-\frac{x}{\beta_1}} \sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1} C_2(n,k)\, \mathbb{I}_2(n,k)\left(\frac{b}{x}\right)^{\frac{n+k-1}{2}} x^{\alpha_1+k-2} \quad (7)$$

where

$$\mathbb{I}_2(n,k) = b\left(\frac{x}{\beta_1} - (\alpha_1+k-n-1)\right) K_{n-k+1}\left(2\sqrt{\frac{bx}{\beta_1\beta_2}}\right) + b\sqrt{\frac{bx}{\beta_1\beta_2}} K_{n-k}\left(2\sqrt{\frac{bx}{\beta_1\beta_2}}\right).$$

Proof: Given in [26]. ∎

C. Moment generating function of Λ

Given the ccdf $\bar{F}_\Lambda(x)$, the mgf can be computed using

$$M_\Lambda(s) = \int_0^\infty e^{-sx} f_\Lambda(x)\,\mathrm{d}x = 1 - s\int_0^\infty e^{-sx}\bar{F}_\Lambda(x)\,\mathrm{d}x. \quad (8)$$

Cases $a = 0$ and $a \neq 0$ are treated separately to make the problem mathematically tractable. Unlike with previous results, the result for case $a = 0$ is not a direct reduction of that for the case $a \neq 0$.

Theorem 3: mgf of Λ

Let γ_1 and γ_2 be distributed $\mathrm{Gamma}(\alpha_1, \beta_1)$ and $\mathrm{Gamma}(\alpha_2, \beta_2)$, respectively. The mgf $M_\Lambda(s)$ of Λ in (1) is given by

Case $a \neq 0$:

$$M_\Lambda(s) = 1 - 2s\sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{m=0}^{k} C_1(n,k,m)\, a^{\frac{n+m+1}{2}} \sum_{q=0}^{n+m+2}\binom{n+m+2}{q}\left(\frac{b}{a}\right)^q \mathbb{J}_1(n,k,m,q) \quad (9)$$

where

$$\mathbb{J}_1(n,k,m,q) = \frac{\sqrt{a\beta_1\beta_2}\,\Gamma(n+2)\,\Gamma(m+1)}{2b\,(-1)^{\alpha_1+k-q+1}} \frac{\mathrm{d}^{\alpha_1+k-q+1}}{\mathrm{d}t^{\alpha_1+k-q+1}}\left\{ e^{\frac{bt}{2a}} \mathcal{W}_{-\frac{n+m+2}{2},\frac{n-m+1}{2}}\left(\frac{b\left(t-\sqrt{t^2-\frac{4a}{\beta_1\beta_2}}\right)}{2a}\right) \mathcal{W}_{-\frac{n+m+2}{2},\frac{n-m+1}{2}}\left(\frac{b\left(t+\sqrt{t^2-\frac{4a}{\beta_1\beta_2}}\right)}{2a}\right)\right\}\Bigg|_{t\doteq s+\frac{1}{\beta_1}+\frac{a}{\beta_2}}.$$

The mgf can be obtained in a relatively compact form as follows whenever $(a \neq 0, b = 0)$.

$$M_\Lambda(s) = 1 - 2s\sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{m=0}^{k} C_1(n,k,m)\, \mathbb{J}_3(n,k,m)\, a^{\frac{n+m+1}{2}} \quad (10)$$

where $\mathbb{J}_3(n,k,m) =$

$$\frac{\sqrt{\pi}\,\Gamma(\alpha_1+k+n-m+2)\,\Gamma(\alpha_1+k-n+m)}{\Gamma\left(\alpha_1+k+\frac{3}{2}\right)}\left(\frac{16a}{\beta_1\beta_2}\right)^{\frac{n-m+1}{2}} \frac{{}_2\mathcal{F}_1\left(\alpha_1+k+n-m+2, n-m+\frac{3}{2}; \alpha_1+k+\frac{3}{2}; \bar{s}\right)}{\left(s+\left(\frac{1}{\sqrt{\beta_1}}+\sqrt{\frac{a}{\beta_2}}\right)^2\right)^{\alpha_1+k+n-m+2}},$$

such that $\bar{s} = \frac{s+\left(\frac{1}{\sqrt{\beta_1}}-\sqrt{\frac{a}{\beta_2}}\right)^2}{s+\left(\frac{1}{\sqrt{\beta_1}}+\sqrt{\frac{a}{\beta_2}}\right)^2}$.

Case $a = 0$:

$$M_\Lambda(s) = 1 - 2s\sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1} C_2(n,k)\,\mathbb{J}_2(n,k) \quad (11)$$

where

$$\mathbb{J}_2(n,k) = \frac{\Gamma(\alpha_1+1)\,\Gamma(\alpha_1+k-n)}{2\sqrt{\frac{b}{\beta_1\beta_2}}}\left(s+\frac{1}{\beta_1}\right)^{\frac{n-k-2\alpha_1}{2}} e^{\frac{b}{2\beta_2(1+\beta_1 s)}}\mathcal{W}_{\frac{n-2\alpha_1-k}{2},\frac{n-k+1}{2}}\left(\frac{b}{\beta_2(1+\beta_1 s)}\right).$$

Proof: Given in [26]. ∎

Although (9) appears complicated it may be expanded and implemented without much difficulty for given $\alpha_1, \beta_1, \alpha_2, \beta_2, a$ and b, using *Generalized Leibniz rule* [27] for higher derivatives. Moreover, symbolic evaluation with common mathematical software such as MAPLE, MATHEMATICA and MATLAB is fairly easy. In fact, numerical results for this paper have been produced through symbolic evaluation of (9) with MATHEMATICA.

D. Symbol error rate

We consider here only the modulation schemes $\mathcal{M}$ whose symbol error rate P_s expression is given by

$$P_s = \mathcal{E}_\Lambda\left\{\mu\mathcal{Q}\left(\sqrt{2\nu\Lambda}\right)\right\}, \quad (12)$$

where μ, ν are constants dependent on the modulation scheme $\mathcal{M}$. For instance $(\mu = 1, \nu = 1)$ gives the exact symbol error rate for BPSK; while that for many other schemes can be approximated with (12) using union bound arguments.

The symbol error rate can be simplified to the integral representation (13), considering the fact that $f_\Lambda(x) = 0, x \leq 0$.

$$P_s = \frac{\mu}{2} - \frac{\mu}{2}\sqrt{\frac{\nu}{\pi}}\int_0^\infty \frac{e^{-\nu x}\bar{F}_\Lambda(x)}{\sqrt{x}}\mathrm{d}x \quad (13)$$

Substituting the cdf results, it can be simplified to obtain closed-form expressions for the cases: $a = 0, b \neq 0$ (15) and $a \neq 0, b = 0$ (14).

Theorem 4: symbol error rate

Let γ_1 and γ_2 be distributed $\mathrm{Gamma}(\alpha_1, \beta_1)$ and $\mathrm{Gamma}(\alpha_2, \beta_2)$, respectively. For a modulation scheme $\mathcal{M}$ upon which the parameters μ and ν of (12) are determined, the symbol error rate P_s is given by

Case $a \neq 0, b = 0$

$$P_s = \frac{\mu}{2} - \frac{\mu}{\sqrt{\frac{\pi}{\nu}}} \sum_{n=0}^{\alpha_1-1} \sum_{k=0}^{\alpha_2-1} \sum_{m=0}^{k} C_1(n,k,m)\mathbb{I}_3(n,k,m) a^{\frac{n+m+1}{2}} \tag{14}$$

where $\mathbb{I}_3(n,k,m)$

$$= \frac{\sqrt{\pi}\Gamma\left(\alpha_1+k+n-m+\frac{3}{2}\right)\Gamma\left(\alpha_1+k-n+m-\frac{1}{2}\right)}{\Gamma(\alpha_1+k+1)} \frac{{}_2\mathcal{F}_1\left(\alpha_1+k+n-m+\frac{3}{2}, n-m+\frac{3}{2}; \alpha_1+k+1; \bar{\nu}\right)}{\left(\nu+\left(\frac{1}{\sqrt{\beta_1}}+\sqrt{\frac{a}{\beta_2}}\right)^2\right)^{\alpha_1+k+n-m+\frac{3}{2}} \left(\frac{16a}{\beta_1\beta_2}\right)^{-\frac{n-m+1}{2}}},$$

such that $\bar{\nu} = \frac{\nu+\left(\frac{1}{\sqrt{\beta_1}}-\sqrt{\frac{a}{\beta_2}}\right)^2}{\nu+\left(\frac{1}{\sqrt{\beta_1}}+\sqrt{\frac{a}{\beta_2}}\right)^2}$.

Case $a = 0$

$$P_s = \frac{\mu}{2} - \frac{\mu\Gamma\left(\alpha_1+\frac{1}{2}\right)}{2}\sqrt{\frac{\nu\beta_1\beta_2}{b\pi}} e^{\frac{b}{2\beta_2(1+\beta_1\nu)}} \sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1} C_2(n,k)\Gamma\left(\alpha_1+k-n-\frac{1}{2}\right)\left(\nu+\frac{1}{\beta_1}\right)^{-\frac{2\alpha_1+k-n-1}{2}} \mathcal{W}_{-\frac{2\alpha_1+k-n-1}{2},\frac{n-k+1}{2}}\left(\frac{b}{\beta_2(1+\beta_1\nu)}\right). \tag{15}$$

Proof: Omitted. ■

P_s for case $(a \neq 0, b \neq 0)$ has not been derived yet.

IV. Numerical results

Theorem 1, 2 and 3 are verified through Monte Carlo simulation. We consider here a non-identical fading case: $(\alpha_1 = 3, \beta_1 = 2, \alpha_2 = 4, \beta_1 = 1)$ for illustration. The performance of four AF relaying schemes: CNA $(a = 1, b = 1)$, CA $(a = 1, b = 0)$, blind $(a = 0, b = 1)$ and non-standard $(a = 0.5, b = 1)$ are considered. Any scheme that selects relay gain as the reciprocal of a linear combination of γ_1 (based on CSI) and noise statistics may be characterized by (1). The non-standard scheme we have considered here is a scheme chosen to perform in between CA and blind relaying.

MGF for the case $(a \neq 0, b \neq 0)$ was implemented in MATHEMATICA, while all other computations were carried out using MATLAB. All Monte Carlo simulations have been carried out by averaging over 10^7 sample points.

Figure 2 depicts the pdf of the received SNR. The pdfs of CA and CNA schemes agree closely. This justifies approximation of the latter by the former [15]. Blind relaying shows the highest dispersion.

Given noise outage threshold γ_o and transmission rate φ, noise P_{no} (16) and information P_{io} (17) outage probabilities can be expressed in terms of the cdf as

$$P_{no}(\gamma_o) = F_\Lambda(\gamma_o) \tag{16}$$

$$P_{io}(\gamma_o) = F_\Lambda(2^\varphi - 1) \tag{17}$$

This makes the ccdf of received SNR (Fig. 3) a good criteria for characterizing an AF relaying strategy. CNA, which is more conservative with respect to transmit power, has the highest probability of outage; while blind relaying suffers the least outage.

The mgf results we derive hold true for complex arguments.

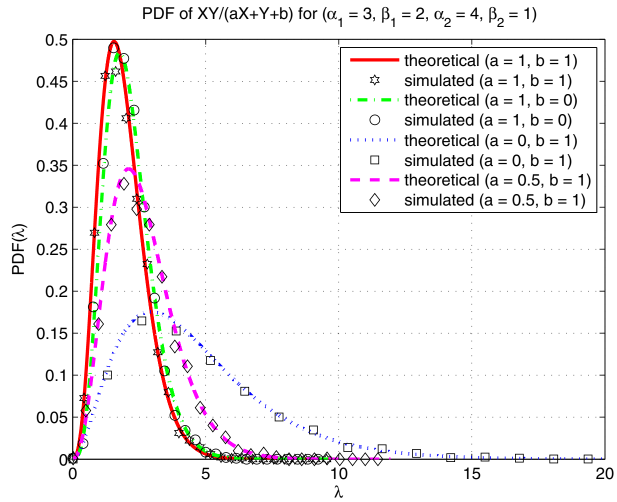

Fig. 2. verification of the pdf $(\alpha_1 = 3, \beta_1 = 2, \alpha_2 = 4, \beta_2 = 1)$

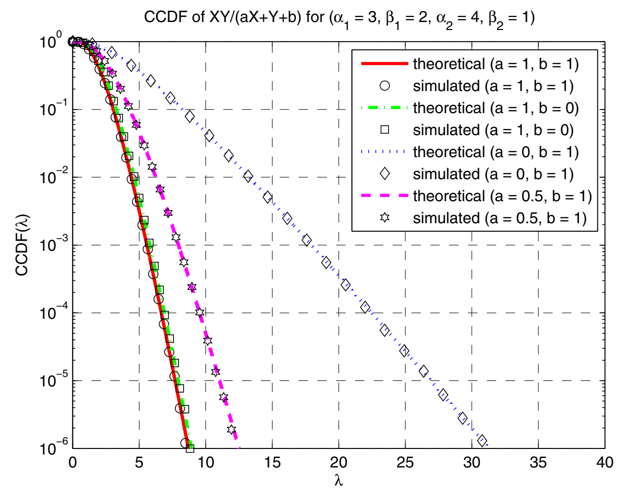

Fig. 3. verification of the ccdf $(\alpha_1 = 3, \beta_1 = 2, \alpha_2 = 4, \beta_2 = 1)$

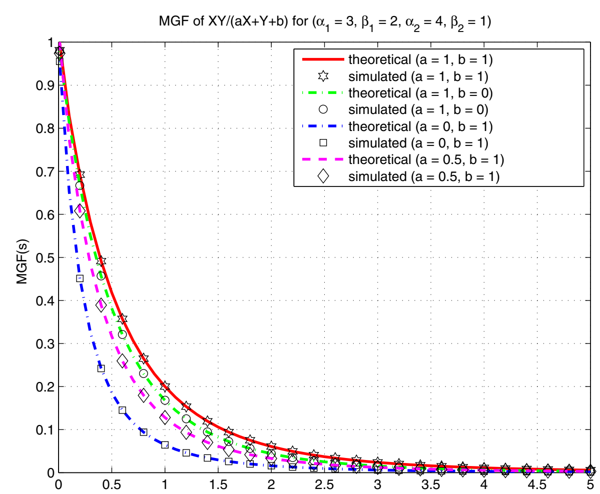

Fig. 4. verification of the mgf $(\alpha_1 = 3, \beta_1 = 2, \alpha_2 = 4, \beta_2 = 1)$

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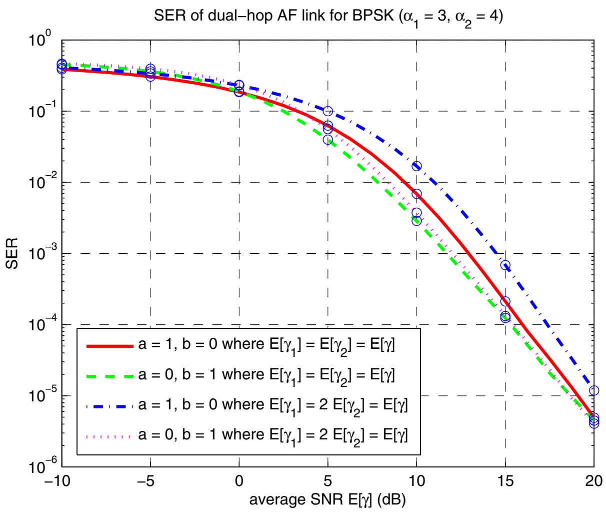

Fig. 5. verification of the symbol error rate ($\alpha_1 = 3, \alpha_2 = 4$)

Figure 4 plots it for real s, for the same systems. Like with the pdf and ccdf results, the curves for CA and CNA agree closely; while that of blind relaying deviates considerably. All figures 2, 3 and 4 show the curve for case $(a = 0.5, b = 1)$ lying in between those of blind and CA relaying.

Theorem 4 is applicable only for blind and CA relaying. Figure 5 depicts how symbol error rate varies with $\bar{\gamma}$ under BPSK modulation $(\mu = 1, \nu = 1)$ for the cases

- $\mathcal{E}_\Lambda \{\gamma_1\} = \mathcal{E}_\Lambda \{\gamma_2\} = \bar{\gamma}$ and $(\alpha_1 = 3, \alpha_2 = 4)$
- $\mathcal{E}_\Lambda \{\gamma_1\} = 2\mathcal{E}_\Lambda \{\gamma_2\} = \bar{\gamma}$ and $(\alpha_1 = 3, \alpha_2 = 4)$

As seen in the figures, analytical results agree well with corresponding Monte Carlo results.

V. Conclusion

Two-hop AF relaying over non-identical Nakagami-m fading links was analyzed. A unified model for the received SNR of two-hop relaying, which covers channel-noise-assisted, channel-assisted, and blind relay configurations as special cases, was considered. The closed-form cdf, pdf, mgf and symbol error rate expressions have been derived. The results are proved analytically and verified through Monte Carlo simulation.

Although this work focuses on the performance of two-hop AF relaying without the direct $S \to D$ path, this analysis may be extended to incorporate the direct $S \to D$ path. Exact analysis of schemes based on relay selection might also be possible. The analysis may also be extended to cover other useful performance measures including ergodic capacity and diversity order.

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