Performance Analysis of Decode-and-Forward Relay Network Under Adaptive M-QAM

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Abstract—In this paper, the performance of a cooperative Decode-and-Forward (DF) relay network with adaptive M-ary Quadrature Amplitude Modulation (M-QAM) is analyzed. A five mode adaptive M-QAM scheme is employed, and the system performance is investigated for both fixed switching levels and for optimized switching levels with a target bit error rate (BER) constraint. Expressions for average BER, spectral efficiency, and outage probability are derived to investigate the performance of the cooperative system under independently and identically distributed (i.i.d.) Nakagami-m fading wireless environments. Adaptive M-QAM with optimized switching levels maintains the target BER constraint while fixed switching levels achieve a conservative BER, which is lower than the target.

I. INTRODUCTION

The global wireless industry has been growing and continue to grow rapidly; beyond third generation (3G) and fourth generation (4G) standards are being developed. These aim at providing 100 Mbps and 1 Gbps speeds in both indoor and outdoor wireless channels. The most fundamental characteristic of wireless channels is fading (random variations of signal strength). Since the current view on multi-path fading recognizes that random fading increases the number of degrees of freedom available for communications [1]. These degrees of freedom need to be exploited in the emerging standards. The use of multiple antennas at the transmitter and receiver is aimed at this exploitation, but incurs increased hand-set complexity. Therefore, the cooperative network concept has received much attention as a realization of distributed (virtual) multiple antenna systems [2], [3]. The transmission between the source and destination is thus assisted by one or more of cooperative relays. Spatial separation between individual relays ensures spatial diversity. Performance analysis of various relay networks could be found in, among many others, [4]–[8].

Adaptive transmission techniques can also exploit the number of degrees available for communications by adapting the transmission parameters according to channel conditions. During favorable conditions, the transmitter can use higher power, larger symbol constellations and reduced coding and error correction schemes. When the channel is under deep fade and conditions are unfavorable, the transmitter switches to low power, smaller symbol constellations and include improved coding and error correction schemes. Adaptive transmission techniques have been widely studied under conventional multiple-antenna and single antenna systems (i.e., non-cooperative communication systems) [9]–[12].

Cooperative networks using adaptive transmission techniques has received attention recently. In [13], Hasna studied the performance of adaptive modulation in a single relay network considering both amplify-and-forward (AF) and decode-and-forward (DF) relays and obtained capacity expressions for three different adaptive transmission schemes. Nechiporenko et al. [14] have generalized the work of [13] to \( N \geq 1 \) AF cooperative network deriving expressions for capacity for AF relaying over both independent and identically distributed (i.i.d) and non-identical and independently distributed (non-i.i.d.) Rayleigh fading environments. In [15] Nechiporenko et al. extend [14] by considering optimum switching thresholds for adaptive modulation.

This paper provides a performance analysis of a DF cooperative network under five-mode adaptive M-QAM transmission with constant power in i.i.d. Nakagami-\( m \) fading wireless channels. We extend our previous work [15], where only AF relays were considered under i.i.d and non-i.i.d. Rayleigh fading. Theoretical expressions for the average bit error rate (BER), the outage probability and the achievable spectral efficiency are derived, and Monte Carlo simulation is used for verification of the analysis.

The remainder of this paper is organized as follows. In Section II the system and channel model is presented. Section III presents the performance parameters used for the evaluation of the system. Section IV provides information on switching levels. The simulation results are provided in Section V. Conclusions are made in Section VI.

II. SYSTEM MODEL

A. DF Cooperative Network

The DF cooperative network is illustrated in Fig. 1. Here, \( N \) number of DF relays cooperate to convey the message of the source (\( S \)) to a particular destination (\( D \)). Two phases are involved in the cooperative communication. During the first phase, the source transmits its message \( x \) which is received by the destination and all the \( N \) relays due to the broadcast nature of the wireless channel. In the second phase, each of the relays transmits the message received from the source through an orthogonal channel to the destination.
It is assumed that all channels experience i.i.d. Nakagami-$m$ fading. $h_{s,i}$, $h_{i,d}$, and $h_{s,d}$ in Fig. 1 denote the fading coefficients from $S$ to $i$th relay ($R_i$), $R_i$ to $D$, and from $S$ to $D$, respectively. Nakagami fading parameters of all channels are equal, and is taken as $m$ where $m \in \mathbb{Z}^+$. Additive White and Gaussian noise (AWGN) is assumed to have zero mean and variance of $N_0$. The noise term is denoted as $n_{s,i}$ at the relays and $n_{s,d}$ and $n_{i,d}$ at the destination. Instantaneous signal to noise ratios (SNR) at $R_i$ is $\gamma_{s,i} = |h_{s,i}|^2 \tilde{\gamma}$ and at $D$ is $\gamma_{i,d} = |h_{i,d}|^2 \tilde{\gamma}$ from $R_i$, and $\gamma_{s,d} = |h_{s,d}|^2 \tilde{\gamma}$ from $S$, where $\tilde{\gamma} = E_s/N_0$ is the transmit SNR at the source with average signal energy $E_s$.

Received message signals during the first phase of communication at the destination and $i$th relay are $r_{s,i} = h_{s,i} \sqrt{E_s} x + n_{s,i}$, and $r_{s,i} = h_{s,i} \sqrt{E_s} x + n_{s,i}$ where $x$ is the transmit symbol with unit energy. The received signal at the destination from the $i$th relay during the second phase is $r_{i,d} = h_{i,d} \sqrt{E_s} \hat{x} + n_{i,d}$ where $\hat{x}$ is the decoded and re-encoded symbol of $x$. Here an adaptive DF relaying scheme is assumed as in [3, 16, 17] where the relay is able to accurately decode the received signal from source, without outage, only if the instantaneous received SNR is above a particular threshold value $\gamma_T$, i.e., $\gamma_{s,i} > \gamma_T$ with $\gamma_T = 2R(N+1) - 1$ where $R$ is the target information rate below which outage occurs. For the simplicity of BER analysis, we neglect the bit errors at the relays and assume that decoding is performed perfectly. Under this adaptive DF scheme, the number of relays forwarding source message to destination is a binomial random variable $X$, where $X = 0, 1, \ldots, N$. Hence the probability of $X$ relays transmitting to the destination is given as

$$P_X(X = x) = \binom{N}{x} \left[ \frac{\Gamma(m, \frac{x}{\gamma_T})}{\Gamma(m)} \right]^x \left[ \frac{\gamma_{s,i}(m, \frac{x}{\gamma_T})}{\Gamma(m)} \right]^{N-x},$$

where $\Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt$, $\gamma_{f,n}(a, x) = \int_0^x t^{a-1} e^{-t} dt$, and $\Gamma(a) = (a-1)!$, $a \in \mathbb{Z}^+$. We assume that the destination employs maximum ratio combining (MRC), which results in the total received SNR, $\gamma_{tot}$, to be $\gamma_{tot} = \sum_{j=1}^{x} \gamma_j$, where $\gamma_j$ are Gamma random variables with shape parameter $m$ and scale parameter $\frac{1}{m}$. The moment generating function (MGF) of $\gamma_{tot}$ is derived as

$$M_{\gamma_{tot}}(s|x) = \prod_{i=1}^{x+1} M_{\gamma_i}(s) = \left(1 + \frac{x}{m} s\right)^{-m(x+1)}. \hspace{1cm} (2)$$

Laplace inversion of (2) gives the probability density function (PDF) of $\gamma_{tot}$ conditional on $X$ as

$$f_{\gamma_{tot}}(\gamma|x) = \left(\frac{m}{\gamma}\right)^{m(x+1)} \frac{\gamma^{m(x+1)-1}}{(m(x+1)-1)!} e^{-\frac{mx}{\gamma}}. \hspace{1cm} (3)$$

Using the theorem of total probability and equations (1) and (3), the unconditional PDF of total received SNR $\gamma_{tot}$ can be written as

$$f_{\gamma_{tot}}(\gamma) = \sum_{i=0}^{N} P_X(X = i) \left(\frac{m}{\gamma}\right)^{m(i+1)} \frac{\gamma^{m(i+1)-1}}{(m(i+1)-1)!} e^{-\frac{mx}{\gamma}}. \hspace{1cm} (4)$$

B. Five-mode Adaptive M-QAM

In this adaptive modulation scheme, the transmission mode is selected based on the received SNR at the destination. The range of received SNR is partitioned into $K$ regions using a set of switching levels $S = \{\gamma_0, n = 0, 1, \ldots, K\}$ with $\gamma_0 = 0$ and $\gamma_K = \infty$ where $K = 5$ gives five-mode adaptive M-QAM. The transmitter selects mode $n$ when the received SNR falls in the $n$th fading region; i.e., $\gamma_n \leq \gamma_{tot} < \gamma_{n+1}$. Information of which transmission mode to select, $n$, is communicated to the source through a reliable, low delay feedback link as shown in Fig. 1. Error-free feedback from the destination to source is assumed. The transmitter may have to send side information to the relays, or potentially the relays may employ automatic modulation classification techniques to identify the incoming modulation mode without side information. Table I gives the parameters of the five-mode adaptive M-QAM scheme, where $b_n$ is the number of bits per a transmitted symbol, $\gamma$ is the instantaneous received SNR and $M_n$ is the constellation size of the modulation scheme used.

III. SYSTEM PERFORMANCE PARAMETERS

This section presents the parameters that will be used to analyze the performance of the system. We define the transmission mode selection probability (MSP) [18] as

$$\delta_n = P(\gamma_n \leq \gamma < \gamma_{n+1}) = \int_{\gamma_n}^{\gamma_{n+1}} f_{\gamma_{tot}}(\gamma) d\gamma. \hspace{1cm} (5)$$

Here $\delta_n$ is the probability that the received SNR lies in the $n$th region and transmission is done using transmission mode $n$. By using the PDF of $\gamma_{tot}$ given in (4) the MSP can be derived as

$$\delta_n = \sum_{i=0}^{N} P_X(X = i) \left[ \frac{\Gamma(m(i+1), \frac{\gamma_{n+1}}{m}) - \Gamma(m(i+1), \frac{\gamma_{n}}{m})}{\Gamma(m(i+1))} \right]. \hspace{1cm} (6)$$

A. Outage Probability

Clearly no transmission takes place when the received SNR falls in region 0 according to the parameters given in Table I.
\[ R = \sum_{i=0}^{N} P_X(X = i) \sum_{n=1}^{K-1} b_n \left[ \frac{\Gamma(m(i+1), \frac{m-a_n}{r})}{\Gamma(m(i+1))} \right]^{\gamma_n+1}. \]  

\[ I_i(a_l, \beta, \gamma_n, \gamma_{n+1}) = Q(\sqrt{a_l\gamma_n})\beta^{-i}(i-1)! \left[ 1 - e^{-\beta r} \sum_{r=0}^{\infty} \frac{(\beta r)^r}{r!} \right] \frac{\gamma_{n+1}}{\gamma_n} - \sqrt{\frac{r}{\pi}} \sum_{r=0}^{\infty} \frac{2^r}{r!} (a_l/2 + \beta)^{-r-0.5}\Gamma(r + 0.5, (a_l/2 + \beta)\gamma) \frac{\gamma_{n+1}}{\gamma_n}. \]  

The probability of such an outage of transmission is given by
\[ P_{out} = P(\gamma_0 \leq \gamma < \gamma_1) = \delta_0. \]  

By using (6) and (9) the outage probability can be found as
\[ P_{out} = 1 - \sum_{i=0}^{N} P_X(X = i) \left[ \frac{\Gamma(m(i+1), \frac{m-a_n}{r})}{\Gamma(m(i+1))} \right]^{\gamma_n+1}. \]  

B. Achievable Spectral Efficiency

The achievable spectral efficiency is the average throughput achieved by the system, which can be expressed as the sum of the data rates \( b_n \) in each region weighted by the MSP of each region [15, 18]

\[ R = \frac{B}{N+1} \sum_{n=1}^{K-1} b_n \delta_n, \]  

where \( b_n = \log_2(M_n) \) is the data rate of the \( n \)th region, \( B \) is the bandwidth and \( N \) is the number of DF relay nodes forwarding the source signal to the destination. The division by \( (N+1) \) is to cater for the fact that transmission takes place using \( (N+1) \) orthogonal channels. Using (6) and (11) the average throughput can be expressed as in (7).

C. Average Bit Error Rate

When the performance of the system is investigated in terms of the BER, since different modulation schemes are being employed under signal fading (Table I), an average BER value is needed which averages the effects of both fading and the use of different modulation schemes. The average BER for five-mode adaptive M-QAM scheme is given as [15, 18]

\[ BER_{avg} = \frac{\sum_{n=1}^{K-1} b_n P_n}{\sum_{n=1}^{K-1} b_n \delta_n}, \]  

where \( P_n \) is the BER of the \( n \)th transmission mode which is calculated as

\[ P_n = \int_{\gamma_n}^{\gamma_{n+1}} P_{M_n, QAM}(\gamma) f_{\theta_n}(\gamma) d\gamma. \]  

\( P_{M_n, QAM} \) is the BER of square M-QAM in an AWGN channel, with coherent detection and Gray coding given as [18]

\[ P_{M_n, QAM} = \sum_{l=1}^{M} A_l Q(\sqrt{a_l\gamma}), \]  

where \( Q(\cdot) \) is the standard Gaussian Q-function defined as
\[ Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-z^2/2} dz, \]  

\( \gamma \) is the received SNR, and \( A_l \) and \( a_l \) are constants particular to the QAM constellation used, given in [18]. Using (4), (13) and (14) expression for \( P_n \) can be derived as

\[ P_n = \sum_{i=0}^{N} P_X(X = i) \left( \frac{\pi}{(m+1)\gamma} \right)^{m(i+1)-1} e^{-\frac{m-a_n}{\gamma}} \]  

\[ \times \sum_{l=1}^{M} A_l \int_{\gamma_n}^{\gamma_{n+1}} Q(\sqrt{a_l\gamma}) \gamma^{m(i+1)-1} e^{-\frac{m-a_n}{\gamma}} d\gamma. \]  

Let \( I_i(a_l, \beta, \gamma_n, \gamma_{n+1}) = \int_{\gamma_n}^{\gamma_{n+1}} Q(\sqrt{a_l\gamma}) \gamma^{m(i+1)-1} e^{-\frac{m-a_n}{\gamma}} d\gamma. \) From our previous work in [15], \( I_i(a_l, \beta, \gamma_n, \gamma_{n+1}) \) can be written in closed form as given in (8). Hence, by substituting (8) into (15) a closed form expression for \( P_n \) can be written as

\[ P_n = \sum_{i=0}^{N} P_X(X = i) \left( \frac{\pi}{(m+1)\gamma} \right)^{m(i+1)} \]  

\[ \times \sum_{l=1}^{M} A_l I_{m(i+1)}(a_l, m/\gamma, \gamma_n, \gamma_{n+1}). \]  

Substituting for \( P_n \) and \( \delta_n \) in (12) using (6) and (16) the average BER can be obtained.

IV. SWITCHING LEVELS

The goal of the above adaptive modulation system is to communicate using as high a number of bits per symbol while maintaining the required quality of service (QoS). Here a target BER level is considered as the level of QoS required. Hence, the switching levels, which determine the transmission mode to be used, are chosen such that they would maximize the average throughput given in (11) while satisfying the required target BER level. We discuss two types of switching level assignment methods for the five-mode adaptive M-QAM scheme used in the cooperative network, (i) fixed switching levels and (ii) optimum switching levels.

A. Fixed Switching Levels

In fixed switching, assignment of the levels or region boundaries is done so that the SNR level at the boundary satisfies the BER target with the modulation scheme used in an AWGN channel. We have adopted the criteria used in [12] for determining the fixed switching levels:

\[ \gamma_0 = 0, \] 

\[ \gamma_1 = [\text{erfc}^{-1}(2\text{BER_0})]^2, \] 

\[ \gamma_n = \frac{2}{3} K_0(M_n - 1); \quad n = 2, 3, \ldots, K - 1, \] 

\[ \gamma_K = +\infty, \]
TABLE I
FIVE-MODE ADAPTIVE M-QAM PARAMETERS

<table>
<thead>
<tr>
<th>SNR</th>
<th>$n$</th>
<th>$M_n$</th>
<th>$b_n$</th>
<th>mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_0 \leq \gamma &lt; \gamma_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>No Tx</td>
</tr>
<tr>
<td>$\gamma_1 \leq \gamma &lt; \gamma_2$</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>BPSK</td>
</tr>
<tr>
<td>$\gamma_2 \leq \gamma &lt; \gamma_3$</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>QPSK</td>
</tr>
<tr>
<td>$\gamma_3 \leq \gamma &lt; \gamma_4$</td>
<td>3</td>
<td>16</td>
<td>4</td>
<td>16-QAM</td>
</tr>
<tr>
<td>$\gamma_4 \leq \gamma &lt; \gamma_5$</td>
<td>4</td>
<td>64</td>
<td>6</td>
<td>64-QAM</td>
</tr>
</tbody>
</table>

where BER_0 is the target BER level, efrc^{-1}(\cdot) is the inverse complementary error function and $K_0 = -\ln(5\text{BER}_0)$.

B. Optimum Switching Levels

Here the switching levels are optimized. The optimization is performed so that average throughput given in (11) is maximized under the constraint of $BER_{avg} = BER_0$, where both the average throughput and the average bit error rate $BER_{avg}$ are functions of transmit SNR. We have used the results of the Lagrangian optimization problem discussed in [18] together with equations for MSP (6) and mode specific BER (16) to arrive at the optimized switching levels. From the work of [18] it is clear that any set of optimized switching levels $s \in S$ is a function of the optimized switching level $\gamma_1$ and is independent of the underlying propagation environment. The relationship between $\gamma_1$ and the rest of the switching levels $\gamma_2, \gamma_3$ and $\gamma_4 \ (\gamma_0 = 0$ and $\gamma_5 = \infty$ are fixed) is given in [18, Fig. 4]. The set of optimized switching levels for a particular transmit SNR $\bar{\gamma}$ and target BER level BER_0 is found by solving the constraint function given by [18]

$$Y(\bar{\gamma}; s(\gamma_1)) = \sum_{n=1}^{N} b_n P_n - BER_0 \sum_{n=1}^{N} b_n \delta_n = 0,$$

where $s(\gamma_1)$ is the set of switching levels as a function of $\gamma_1$. Fig. 2 compares the optimum and fixed switching levels for a target BER of $10^{-3}$. A set of optimized switching levels exists only when there is a solution to $Y(\bar{\gamma}; s(\gamma_1)) = 0$.

V. RESULTS

This section presents a comparison of the analytical and simulation results of the performance parameters discussed in Section III. The results for the outage probability, average BER, and achievable spectral efficiency are presented under fixed and optimum switching level assignments. A two-relay cooperative network is considered with fading parameter $m = 2$ and a decoding threshold $\gamma_T$ of 5 dB.

Fig. 2 shows the optimized and fixed switching levels. For small transmit SNRs, the difference between the fixed and the optimized switching levels is fairly small, but the difference increases as the transmit SNR increases. At a particular SNR value known as the avalanche SNR [18], all the switching levels abruptly reach zero for optimized switching levels. This event occurs when the BER of the highest order modulation scheme equals the target BER, after which adaptation is abandoned and modulation is performed using the highest order modulation scheme.

The achievable spectral efficiency is plotted in Fig. 3, where the target BER is set to be $10^{-3}$. Optimum switching levels yield higher throughput for equal transmit SNR. A performance gain of 2 - 3 dB is achieved when optimum switching is used as opposed to fixed switching.

Fig. 4 provides the average bit error rate performance analysis. Clearly the system under optimized switching levels maintains the target BER requirement until the avalanche SNR is reached where the adaptation stops and the highest modulation scheme is used for transmission. Under fixed switching levels, the average BER of five-mode adaptive M-QAM is always well below the target BER and hence satisfies the QoS requirement; however this QoS conformance comes at the cost of sacrificing system performance in the form of average throughput, which is clearly observed in the plots of achievable spectral efficiency in Fig. 2.

Outage probability of the system is depicted in Fig. 5. The system under optimized switching levels experience less outage as compared to the system under fixed switching levels.

VI. CONCLUSION

The performance of a DF cooperative network under optimum and fixed switching adaptive M-QAM in i.i.d. Nakagami-$m$ fading channels has been investigated. Closed-form expressions for the outage probability, achievable spectral efficiency and the BER were derived and validated using Monte Carlo simulations. Results show that optimization of switching levels yields 2 - 3 dB performance gain over fixed switching. This work can be extended in several ways. For example, performance analysis could be extended to the case where the assumption of error-free DF operation is removed. Performance analysis can also be extended to other fading models such as Rician fading. We are currently investigating these and similar topics.
**Fig. 3.** Achievable spectral efficiency for i.i.d. Nakagami-$m$ fading ($m = 2$) for a two relay network.

**Fig. 4.** Average bit error rate for i.i.d. Nakagami-$m$ fading ($m = 2$) for a two relay network.

**Fig. 5.** Outage probability for i.i.d. Nakagami-$m$ fading ($m = 2$) for a two relay network.

REFERENCES


