OFDMA based Cooperative Relay Networks

Chintha Tellambura, Zhongshan Zhang
Department of Electrical and Computer Engineering
University of Alberta, Edmonton
AB T6G 2V4, Canada
{chintha,zszhang}@ece.ualberta.ca

***Abstract*—We evaluate the performance improvement in Orthogonal Frequency-Division Multiplex Access (OFDMA) achievable with cooperative relaying in the presence of frequency offsets. The relays can operate in either the amplify-and-forward (AF) or decode-and-forward (DF) mode. In each transmission, node S has M relays when transmitting its own signal, and it also helps the other M relays transmit their signals. The total power used to transmit each symbol, including the power consumed in S and the relays, is kept constant. The outage information rates of the proposed cooperative transmission scheme are first derived. The performance improvement in the frequency offset estimation by using cooperative relay is then analyzed. Channel estimation in a cooperative OFDMA network in the presence of frequency offsets is also evaluated, and the pairwise error probability (PEP) of orthogonal space-time coding in cooperative OFDMA due to both the frequency offset and channel estimation errors is derived. Numerical results illustrate the superior performance of the proposed scheme over the conventional transmission with regard to outage information rate, frequency offset estimation error and the PEP.**

I. Introduction

In orthogonal Frequency-Division Multiple Access (OFDMA), each user employs a different set of orthogonal sub-carriers to transmit data, and by adaptively allocating and modulating subcarriers for each user, the frequency diversity gain can be improved [1].

Besides frequency diversity, spatial diversity can also be exploited to improve the diversity gain. One way to realize a spatial diversity gain is to employ a space-time code and multiple co-located transmit antennas [2]. Another way is to use "cooperative diversity". Some recent cooperative relaying schemes have been presented [3], [4]. Three time-division multiple-access-based cooperative schemes are proposed in [4], where the relay can operate in either the amplify-and-forward (AF) or decode-and-forward (DF) mode.

Frequency synchronization is critical for the OFDMA transmission. Synchronization issues for OFDMA have been widely researched [5], [6]. However, almost all these existing algorithms perform frequency offset estimation based on a point-to-point connection. OFDMA frequency offset estimation can be improved by using cooperative relays. By creating a duplicated link between the source node and the destination through a third node, i.e., the cooperative relay, the reliability of the transmission can be improved considerably. Some combining schemes can be used to effectively combine the received copies of the training sequence at the destination.

Channel estimation errors can also significantly impact the system performance. Since relay networks are virtual MIMO systems [4], many existing MIMO channel estimation algorithms, e.g., that proposed in [7], can also be adapted. Using Alamouti-coded pilot symbols, [8] provides synchronization and channel estimation for a cooperative orthogonal Frequency-Division Multiplexing (OFDM) system. However, the optimal channel estimation for cooperative OFDM/OFDMA in the presence of the frequency offset is still an open issue.

In this paper, the performance improvement in an OFDMA transmission via cooperative relaying in the presence of frequency offsets is discussed. For each source node having M relays, the source node also acts as a relay for the other M nodes simultaneously. We impose a constraint on each relay that does not permit to receive and transmit signals simultaneously at the same time and the same frequency sub-band, i.e., half-duplex constraint is assumed. The outage information rates of the proposed cooperative OFDMA transmission are first analyzed, and then the frequency offset estimation in the cooperative transmission is also discussed. The optimal pilot design for channel estimation in cooperative OFDMA in the presence of the frequency offset is also proposed, and the PEP performance of the orthogonal space-time coded cooperative OFDMA system is then evaluated.

The remainder of this paper is organized as follows. Section II introduces the cooperative OFDMA uplink signal model. The outage information rates are derived in Section III. Section IV discusses the frequency offset estimation in the cooperative OFDMA, and Section V proposes an optimal channel estimation algorithm in the presence of the frequency offset. The PEP performance of the orthogonal space-time coded system is evaluated in Section VI. Section VII discusses some numerical results. Finally, Section VIII concludes the paper.

Notation: $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and conjugate transpose of a matrix, respectively. $(\cdot)^{-1}$ represents the inverse of a matrix. The imaginary unit is $\jmath = \sqrt{-1}$. A circularly symmetric complex Gaussian variable with mean a and variance σ^2 is denoted by $z \sim \mathcal{CN}(a, \sigma^2)$. A real Gaussian variable with mean m and variance σ_x^2 is denoted by $x \sim \mathcal{N}(m, \sigma_x^2)$. $\mathbf{x}[i]$ represents the i-th element of vector $\mathbf{x}$. $[\mathbf{A}]_{ij}$ represents the ij-th element of matrix $\mathbf{A}$. The nn-th element of diagonal matrix diag$\{\mathbf{x}\}$ equals to $\mathbf{x}[n]$. The mean and the variance are represented as $\mathbb{E}\{\bullet\}$ and Var$\{bullet\}$. $\theta \in \mathcal{A}$ means θ is an element of set $\mathcal{A}$.

II. Cooperative OFDMA Uplink Signal Model

In OFDMA, the total number of subcarriers is assumed to be N, and $\mathcal{M}$ users are accessing the base station with each user being allocated N_u unique subcarriers. The input data bits are first mapped to complex symbols drawn from a typical signal constellation such as phase-shift keying (PSK) or quadrature amplitude modulation (QAM). An $N \times 1$ vector $\tilde{\mathbf{X}}_S = \tilde{\mathbf{X}}_S^d + \tilde{\mathbf{X}}_S^p = [X_S[0], X_S[1], \cdots, X_S[N-1]]$ is used to represent these symbols sent by the node S, where $\tilde{\mathbf{X}}_S^p$ and $\tilde{\mathbf{X}}_S^d$ are the pilot and data vectors, respectively. Discrete samples are generated by taking the Inverse Discrete Fourier Transform (IDFT) of $\tilde{\mathbf{X}}_S$. A length-L_{CP} cyclic prefix (CP) is appended before transmission to mitigate the inter-symbol interference (ISI).

In cooperative OFDMA, each cooperative transmission is performed in two time slots. In the first time slot, each mobile node transmits its first symbol to all its relays and the base station (node D) and simultaneously receives the first symbol of the other nodes. In the second time slot, each node will send its second symbol and forward symbols for the other nodes simultaneously. Without loss of generality, node $R_k, k \in \{1, \cdots, M\}$ represents the k-th relay of S, where M is the total number of relays of S.

A. Channel Model

In this paper, the time-invariant composite channel impulse response between node a and node b is modelled as

$$h_{a,b}(\tau) = \sum_{l=0}^{L-1} h_{a,b}[l]\delta(\tau - lT_s), \tag{1}$$

where $h_{a,b}[l]$ is the channel gain between nodes a and b at the l-th tap, and $\delta(x)$ is the unit impulse function. L is the maximum channel order for any pair of nodes, and $T_s = 1/B$ with B representing the total bandwidth. Accordingly, the discrete T_s-spaced channel impulse response between nodes a and b is $\tilde{\mathbf{h}}_{a,b} = [h_{a,b}(0), h_{a,b}(1), \cdots, h_{a,b}(L-1)]^T$. The frequency-domain channel coefficient matrix is $\mathbf{H}_{a,b} = \text{diag}\{H_{a,b}[0], \cdots, H_{a,b}[N-1]\}$, where $H_{a,b}[n] = \sum_d h_{a,b}(d)e^{-\frac{j2\pi nd}{N}}$ is the channel frequency response on the n-th subcarrier. Without loss of generality, we assume that the $S \rightarrow R_k$ channel suffers only the small-scale fading, but both the $S \rightarrow D$ and $R_k \rightarrow D$ channels suffer the large-scale fading with $0 < \mathcal{L}_u < 1$ representing large-scale fading coefficient.

In cooperative transmission, the relays of S can be chosen as follows: we first set a threshold ν where $\nu > 0$, and R_k will act as a relay of S if $\hbar_k = \text{trace}\left\{\mathbf{H}_{S,R_k}\mathbf{H}_{S,R_k}^H\right\} \geq \nu N_u$, $1 \leq k \leq \mathcal{M}-1$. From [9, page 41], we have $P_\nu = \text{P}_\text{r}\{\hbar_k \geq \nu N_u\} = e^{-N_u\nu}\sum_{m=0}^{N_u-1}\frac{\nu^m N_u^m}{m!}$. The probability that S has M relays is $P_{\nu,M} = \binom{\mathcal{M}-1}{M} P_\nu^M (1-P_\nu)^{\mathcal{M}-M-1}$.

B. First Time Slot

The received samples at node D and relay R_k are

$$\mathbf{y}_{z,1} = \sqrt{\alpha\bar{P}}\mathbf{E}_{z,S}\mathbf{F}\mathbf{H}_{z,S}\tilde{\mathbf{X}}_S + \mathbf{w}_{z,1}, \tag{2}$$

where $z \in \{D, R_k\}$, $0 < \alpha < 1$ represents the power allocation ratio between S and its relays, $\bar{P}$ stands for the average power of each subcarrier, $\mathbf{w}_{z,1}$ is an additive white Gaussian noise (AWGN) vector with $w_{z,1}(n) \sim \mathcal{CN}(0, \sigma_w^2)$, and $[\mathbf{F}]_{nk} = e^{\frac{j2\pi nk}{N}}/\sqrt{N}$ for $0 \leq n, k \leq N-1$. The matrix $\mathbf{E}_{a,b}$ is defined by $\mathbf{E}_{a,b} = \text{diag}\left\{1, e^{\frac{j2\pi\varepsilon_{a,b}}{N}}, \cdots, e^{\frac{j2\pi\varepsilon_{a,b}(N-1)}{N}}\right\}$, where $\varepsilon_{a,b}$ is the normalized frequency offset between nodes a and b with $\varepsilon_{z,S} \sim \mathcal{N}(0, \sigma_\epsilon^2)$.

C. Second Time Slot

In the second time slot, the relays will forward the signal for S by using the total power $(1-\alpha)N\bar{P}$, and in this paper, an identical power is allocated to each relay.

1) AF Mode: The received symbol at D is given by

$$\mathbf{y}_{D,2}^{\text{AF}} = \sqrt{\frac{(1-\alpha)\bar{P}}{M\left(\alpha\bar{P}+\sigma_w^2\right)}}\sum_{k=1}^{M}\mathbf{E}_{D,R_k}\mathbf{F}\mathbf{H}_{D,R_k}\mathbf{y}_{R_k,1} + \mathbf{w}_{D,2}. \tag{3}$$

2) DF Mode: In the DF mode, each relay first demodulates and decodes the received symbol, but only the correct-decoding relays will perform retransmission. We assume that m out of M relays can decode correctly, where $0 \leq m \leq M$, and the received symbol at node D is

$$\mathbf{y}_{D,2}^{\text{DF}} = \sqrt{\frac{(1-\alpha)\bar{P}}{m}}\sum_{k=1}^{m}\mathbf{E}_{D,R_k}\mathbf{F}\mathbf{H}_{D,R_k}\tilde{\mathbf{X}}_{R_k} + \mathbf{w}_{D,2}. \tag{4}$$

III. Outage Information Rate Improvement by Using Cooperative Transmission

In this section, we will derive a closed-form representation of the outage information rate in a cooperative OFDMA transmission in the presence of frequency offsets.

A. Outage Information Rate in the AF Mode

The averaged p $(0 \leq p \leq 1)$ outage information rate of the proposed AF cooperative transmission is given by

$$\overline{\mathcal{I}_{\text{out}}^{\text{AF}}} = \sum_{M=1}^{\mathcal{M}-1} P_{\nu,M}\mathcal{I}_{p,\alpha,M}^{\text{AF}}, \tag{5}$$

where $\mathcal{I}_{p,\alpha,M}^{\text{AF}}$, $\mathcal{Z}_{\alpha,M}^{AF}$, $\sigma^2_{\boldsymbol{\xi}_{S,1}^{\text{AF}}}$ and $\sigma^2_{\boldsymbol{\xi}_{S,2}^{\text{AF}}}$ are defined in (6), $\beta_1 = 1 - \frac{\pi^2\sigma_\epsilon^2}{3} + \frac{\pi^4\sigma_\epsilon^4}{20}$, $\theta_\nu = 1 + \left(N_u!\sum_{m=0}^{N_u-1}\frac{\nu^{m-N_u}N_u^{m-N_u}}{m!}\right)^{-1}$, and $\gamma = \bar{P}/\sigma_w^2$ represents the average Signal-to-Noise Ratio (SNR).

B. Outage Information Rate in the DF Mode

The channel information rate of the DF mode is different from that of the AF mode, because only the correct-decoding relays can perform re-transmission in the former. The averaged p outage information rate for the DF mode is given by

$$\overline{\mathcal{I}_{\text{out}}^{\text{DF}}} = \sum_{M=1}^{\mathcal{M}-1} P_{\nu,M}\mathcal{I}_{p,\alpha,M}^{\text{DF}}, \tag{7}$$

where $\mathcal{I}_{p,\alpha,M}^{\text{DF}}$, β_2, $\mathcal{Z}_{\alpha,M,\text{r}}^{DF}$ and $\mathcal{Z}_{\alpha,M}^{DF}$ are defined in (8), and $\sigma^2_{\boldsymbol{\xi}_S^{\text{DF}}} = \frac{\mathcal{L}_u M\pi^2\sigma_\epsilon^2\bar{P}}{3\mathcal{M}} + \sigma_w^2$.

$$\mathcal{I}^{\mathrm{AF}}_{p,\alpha,M}=\frac{N_u}{2}\log_2\left(\mathcal{Z}^{AF}_{\alpha,M}+\frac{\mathcal{L}_u\cdot\theta_\nu(1-\alpha)\bar{P}^2\beta_1\left(p\cdot M!\right)^{\frac{1}{M}}}{M\sigma^2_{\boldsymbol{\xi}^{\mathrm{AF}}_{S,2}}(\theta_\nu\bar{P}+\sigma^2_w)}\right),\quad \mathcal{Z}^{AF}_{\alpha,M}=\left(1+\frac{\alpha\cdot\mathcal{L}_u\cdot\bar{P}\cdot\beta_1\cdot\ln\frac{1}{1-p}}{\sigma^2_{\boldsymbol{\xi}^{\mathrm{AF}}_{S,2}}}\right)\left(1+\frac{\mathcal{L}_u\cdot\bar{P}\cdot\beta_1\cdot\ln\frac{1}{1-p}}{\sigma^2_{\boldsymbol{\xi}^{\mathrm{AF}}_{S,1}}}\right),$$
$$\sigma^2_{\boldsymbol{\xi}^{\mathrm{AF}}_{S,1}}=\frac{\mathcal{L}_uM\pi^2\sigma^2_\epsilon\bar{P}}{3\mathcal{M}}+\sigma^2_w,\quad \sigma^2_{\boldsymbol{\xi}^{\mathrm{AF}}_{S,2}}=\frac{2\mathcal{L}_u\theta_\nu(M^2-M+1)(1-\alpha)\pi^2\sigma^2_\epsilon\bar{P}^2+\mathcal{L}_u\alpha M\pi^2\sigma^2_\epsilon\bar{P}}{3\mathcal{M}}+\left(\mathcal{L}_u(M+1)(1-\alpha)\bar{P}+1\right)\sigma^2_w. \tag{6}$$

$$\mathcal{I}^{\mathrm{DF}}_{p,\alpha,M}=\frac{N_u}{2}\times\log_2\min\left\{\mathcal{Z}^{DF}_{\alpha,M,\mathrm{r}},\ \mathcal{Z}^{DF}_{\alpha,M}+\frac{\mathcal{L}_u(1-\alpha)\bar{\gamma}\beta_1\left(p\cdot M!\right)^{\frac{1}{M}}}{\frac{\mathcal{L}_uM^2\pi^2\sigma^2_\epsilon\bar{\gamma}}{3\mathcal{M}}+M}\right\},\quad \beta_2=1-\frac{2\pi^2\sigma^2_\epsilon}{3}+\frac{\pi^4\sigma^4_\epsilon}{5},$$
$$\mathcal{Z}^{DF}_{\alpha,M,\mathrm{r}}=\left(1+\frac{\alpha\cdot\mathcal{L}_u\cdot\bar{\gamma}\cdot\beta_1\cdot\ln\frac{1}{1-p}}{\frac{\mathcal{L}_uM\pi^2\sigma^2_\epsilon\bar{\gamma}}{3\mathcal{M}}+1}\right)\left(1+\frac{\theta_\nu\cdot\bar{\gamma}\cdot\beta_2\cdot\ln\frac{1}{1-p}}{\frac{2(M-1)\pi^2\sigma^2_\epsilon\bar{\gamma}}{3\mathcal{M}}+1}\right),\quad \mathcal{Z}^{DF}_{\alpha,M}=\left(1+\frac{\alpha\cdot\mathcal{L}_u\cdot\bar{\gamma}\cdot\beta_1\cdot\ln\frac{1}{1-p}}{\frac{\mathcal{L}_uM\pi^2\sigma^2_\epsilon\bar{\gamma}}{3\mathcal{M}}+1}\right)\left(1+\frac{\mathcal{L}_u\cdot\bar{\gamma}\cdot\beta_1\cdot\ln\frac{1}{1-p}}{\frac{\mathcal{L}_uM\pi^2\sigma^2_\epsilon\bar{\gamma}}{3\mathcal{M}}+1}\right). \tag{8}$$

IV. Frequency Offset Estimation in Cooperative Transmission

In this section, a cooperative scheme is proposed to improve the performance of any conventional training/pilot-based OFDMA frequency offset estimation, but the exact training/pilot design is beyond the scope of this paper. Without loss of generality, perfect time synchronization is assumed here. The relays in this section can operate in either the AF or the decode-and-compensation-and-forward (DcF) mode, and only the relay with the best $S\rightarrow R$ channel will perform retransmission.[1]

In the first time slot, ε_{SD} is estimated as $\hat{\varepsilon}_{SD,1}=\varepsilon_{SD}+e_{SD,1}$, where $e_{SD,1}$ represents the estimation error. The frequency offset estimation in the second time slot can be represented as $\hat{\varepsilon}_{SR_sD,2}=\varepsilon_{SD}+e_{SR_sD,2}$. By combining the results in two time slots, the minimum variance of e_{SD} in a two-time-slot period can be achieved as

$$\mathrm{Var}\left\{e_{SD}\middle|\mathrm{AF}\right\}\geq\frac{1}{\mathcal{A}_T\left(\gamma_{SD,1}+\gamma_2^{\mathrm{AF}}\right)}, \tag{9a}$$

$$\mathrm{Var}\left\{e_{SD}\middle|\mathrm{DcF}\right\}\geq\frac{1}{\mathcal{A}_T\left(\gamma_{SD,1}+\frac{\gamma_{SR_s,1}\gamma_2^{\mathrm{DcF}}}{\gamma_{SR_s,1}+\gamma_2^{\mathrm{DcF}}}\right)}, \tag{9b}$$

where $\mathcal{A}_T$ is a positive coefficient specified by the structure of the training sequence $\mathbf{X}_S$.

If the mobile nodes does not receive the CSI feedback from the base station, the optimal α should be obtained by minimizing the expected variance error in (9), with the expectation being performed in terms of $(\nu_{SR_s},\nu_{R_sD},\nu_{R_sD})$. With CSI feedback from the base station, for a given (ν_{SD}, ν_{SR_s}, ν_{R_sD}), the adaptively optimized α can be obtained by directly minimizing (9).

[1]The DcF in this paper is slightly different from the DF proposed in [4]: in the DcF mode, the relay should first estimate the frequency offset between the source node and itself, and then demodulate and decode the received training sequence. After doing so, the relay will use the estimation result to pre-compensate for the frequency offset in the decoded training sequence and re-transmits this re-generated training sequence to D.

V. Cooperative OFDMA Channel Estimation with Frequency Offsets

This section investigates the optimal channel estimation for a cooperative OFDMA systems in the presence of frequency offsets. Different from the first time slot, the destination receives the same signal from multiple relays in the second time slot and, therefore, MAI occurs. Optimal channel estimation requires that the MAI be totally eliminated at the destination.

A. Optimal Pilot to Eliminate MAI in The Second Time Slot

Since an identical pilot is received at each relay in the first time slot, the same pilot will be received at the destination D if the relays simply retransmit the received signal without modifying it. In this case, node D will not be able to identify the channel related to different relays. With multiple active relays, the MAI must be eliminated to minimize the Mean Square Error (MSE). Without loss of generality, we assume that the retransmitted pilot for the relay R_k is $\mathbf{X}^p_{R_k}$.[2]

1) AF Mode: In the AF mode, the truncated channel $S\rightarrow R_k\rightarrow D$ is given by $\tilde{\mathbf{h}}_{D,R_k,S}=\left(\tilde{\mathbf{h}}^T_{R_k,S}\otimes\tilde{\mathbf{h}}^T_{D,R_k}\right)^T$, where $\otimes$ represents a convolutionary product operation. A modified AF relaying mode is applied in each relay to eliminate the MAI. With this modified AF mode, the received signal in the k-th relay R_k is multiplied by a pre-modulation matrix $\mathbf{\Pi}_k$ to modify the received pilot into its own pilot.

Assuming that a total of $\mathcal{N}_p$ pilots are allocated per node, and that the pilot subcarriers for R_k are $\theta_{k,1},\cdots,\theta_{k,\mathcal{N}_p}$. The optimal pilots allocated to R_k are

$$\begin{aligned}&\left[\mathbf{X}^p_{R_k}\right]_{\theta_{k,i}\theta_{k,i}}=e^{\frac{\jmath2\pi\theta_{k,i}(k-1)(2L-1)}{N}},\quad i=1,\cdots,\mathcal{N}_p\\ &\quad s.t.\quad (2L-1)M\leq\mathcal{N}_p\leq N,\quad N/\mathcal{N}_p=\text{integer};\\ &\qquad\quad \theta_{k,i}(k-l)(2L-1)/N\neq\text{integer},\quad k\neq l;\\ &\qquad\quad \theta_{k,2}-\theta_{k,1}=\cdots=\theta_{k,\mathcal{N}_p}-\theta_{k,\mathcal{N}_p-1}=N/\mathcal{N}_p.\end{aligned} \tag{10}$$

To satisfy (10), a matrix $\mathbf{\Pi}_k$ should be used before the re-transmission of R_k to modify the pilots' phase rotation. This

[2]In the following analysis, we assume that a unique pilot is allocated to each relay, and each relay knows its own pilot, but how to perform this pilot-allocation scheme is beyond the scope of this paper.

matrix is given by

$$\mathbf{\Pi}_k = \mathbf{E}^{\text{cir}}_{R_k,S}\mathbf{\Lambda}_k(\mathbf{E}^{\text{cir}}_{R_k,S})^{-1} = \mathbf{F}^H\mathbf{E}_{R_k,S}\mathbf{F}\mathbf{\Lambda}_k\mathbf{F}^H\mathbf{E}^{-1}_{R_k,S}\mathbf{F}. \tag{11}$$

where $\mathbf{\Lambda}_k$ is a diagonal matrix with $[\mathbf{\Lambda}_k]_{\theta_{k,i}\theta_{k,i}} = e^{\frac{\jmath 2\pi\theta_{k,i}(k-1)(2L-1)}{N}}$, and $[\mathbf{\Lambda}_k]_{ll} = 0$ for each $l \neq \theta_{k,i}$. Note that $\mathbf{\Pi}_k$ does nothing to the data subcarriers.

2) DF Mode: In the DF mode, each correct-decoding relay will perform retransmission in the second time slot by modulating the pilot subcarriers with its own pilot but without changing the data subcarriers. The optimal pilot for R_k in the DF mode is

$$\begin{aligned}
&\left[\mathbf{X}^p_{R_k}\right]_{\theta_{k,i}\theta_{k,i}} = e^{\frac{\jmath 2\pi\theta_{k,i}(k-1)L}{N}}, \quad i = 1, \cdots, \mathcal{N}_p \\
&\quad s.t. \quad LM \leq \mathcal{N}_p \leq N, \quad N/\mathcal{N}_p = \text{integer}; \\
&\qquad \theta_{k,i}(k-l)L/N \neq \text{integer}, \quad k \neq l; \\
&\qquad \theta_{k,2} - \theta_{k,1} = \cdots = \theta_{k,\mathcal{N}_p} - \theta_{k,\mathcal{N}_p-1} = N/\mathcal{N}_p.
\end{aligned} \tag{12}$$

VI. Pairwise Error Probability (PEP) Analysis

In this section, the PEP of cooperative OFDMA by considering both the frequency offset and channel estimation errors is derived. An orthogonal space-time signal matrix $\bar{\mathbf{X}}_S = [\tilde{\mathbf{X}}_S(1), \tilde{\mathbf{X}}_S(2), \cdots, \tilde{\mathbf{X}}_S(T)]$ is assumed.

A. PEP for the AF Mode

The PEP is the probability that $\bar{\mathbf{X}}_S$ will be mistaken for another code $\bar{\mathbf{L}}_S$, i.e., $\mathrm{P}^{\text{AF}}_{\text{r}}\left\{\bar{\mathbf{X}}_S \to \bar{\mathbf{L}}_S \middle| 0 < \alpha < 1\right\}$. In a high SINR regime with $\sigma^2_e \to 0$, it can be approximated as in (13), where ℓ_n is the n-th eigenvalue of $\left(\bar{\mathbf{X}}_S - \bar{\mathbf{L}}_S\right)\left(\bar{\mathbf{X}}_S - \bar{\mathbf{L}}_S\right)^H$.

B. PEP for the DF Mode

In the DF mode, by using P_{relay} to represent the average probability of decoding error at each relay, the probability that m out of M relays successfully decode the received signal is a Binomial distribution, i.e., $P_{\text{relay},m} = \binom{M}{m}(1 - P_{\text{relay}})^m P^{M-m}_{\text{relay}}$. We also use $P_{S\to D}$ to represent the probability of the decoding error at D in the first time slot.

In the second time slot, the m relays with correct decoding will retransmit. The PEP that $\bar{\mathbf{X}}_S$ will be mistaken for another codeword $\bar{\mathbf{L}}_S$ is $\mathrm{P}^{\text{DF}}_{\text{r},m}\left\{\bar{\mathbf{X}}_S \to \bar{\mathbf{L}}_S \middle| 0 < \alpha < 1\right\}$, and the averaged PEP of the DF mode is upper bounded by

$$\overline{\text{PEP}^{\text{DF}}} \leq P_{S\to D}\sum_{m=0}^{M} P_{\text{relay},m}\mathrm{P}^{\text{DF}}_{\text{r},m}\left\{\bar{\mathbf{X}}_S \to \bar{\mathbf{L}}_S \middle| 0 < \alpha < 1\right\}. \tag{14}$$

In the high SINR regime with $\sigma^2_e \to 0$, the PEP can be approximated as

$$\begin{aligned}
&\lim_{\substack{\sigma^2_e \to 0 \\ \text{SNR}\to\infty}} \mathrm{P}^{\text{AF}}_{\text{r}}\left\{\bar{\mathbf{X}}_S \to \bar{\mathbf{L}}_S \middle| 0 < \alpha < 1\right\} \\
&\leq \left(\frac{4}{(1-\alpha)\text{SNR}}\right)^L \times \prod_{n=0}^{L-1} \frac{m}{\left|\sum_{k=1}^{m} \tilde{\mathbf{h}}_{D,R_k}[n]\right|^2 \ell_n}.
\end{aligned} \tag{15}$$

VII. Numerical Results

In our simulation, an OFDMA uplink transmission with $N = (128, 1024)$ is considered. A cyclic-prefix of length 64 is padded to the front of each symbol. Uniform power-delay profiles are used between any pair of nodes.[3]

The outage information rate as a function of α is illustrated in Fig. 1, where we defined that $\overline{\mathcal{I}^{\text{AF}}_{\text{out}}} = \max_{\nu} \mathcal{I}^{\text{AF}}_{\text{out}}$ and $\overline{\mathcal{I}^{\text{DF}}_{\text{out}}} = \max_{\nu} \mathcal{I}^{\text{DF}}_{\text{out}}$, for the AF and DF modes, respectively. For a given SNR and p, the outage information rate of the AF mode is a monotonically decreasing function of α. However, for the DF mode, as α increases, the outage information rate first increases to its maximum value, and then decreases if α continuously increases. For a given σ^2_ϵ, the DF mode outperforms the AF mode only when α is larger than a specified threshold. For example, when $\sigma^2_\epsilon = 10^{-2}$, this threshold is $\alpha = 0.034$. This threshold for $\sigma^2_\epsilon = 10^{-3}$ is $\alpha = 0.037$.

Table I evaluates the frequency offset estimation in the proposed cooperative scheme as a function of M when the base station feedbacks CSI to the mobile nodes. In this simulation, the AF mode still outperforms the DcF mode for each M. We can explain this finding as follows: In the interference-limited cooperative transmission, the interference due to the frequency offset in $S \to R_s$ link is twice that of either the $S \to D$ or $R_s \to D$ link. If the relay operates in the DcF mode, R_s should estimate ε_{SR_s}, and the estimation error will be accumulated and propagated to the final result. However, this error propagation from R_s to D can be mitigated in the AF mode.

The PEP performances as functions of SNR are shown in Fig. 2 for $N = 128$, $L = 4$, $M = 16$, $\sigma^2_e = 10^{-2}$ and 10^{-3}. The PEP performance of the DF mode is about 9 dB better than that of the AF mode at an error rate of 5×10^{-3}. As σ^2_e increases to 10^{-2}, the system becomes interference-limited, and an error floor appears in both the AF and the DF modes. In this case, the performance increases to about 11.3 dB; i.e., the DF mode has a higher interference-mitigation capability than the AF mode.

VIII. Conclusions

This paper discussed the OFDMA-based cooperative transmission in the presence of frequency offsets. By keeping the total transmit power of each symbol fixed, the outage information rate of each user was improved considerably. Since the accumulation of the frequency offset estimation errors in the relay in the DcF mode degraded the estimation performance, the AF mode's cooperative transmission outperformed the DcF mode with regards to the frequency offset variance error. The optimal pilots are also designed to mitigate the MAI in the cooperative transmission. By considering both the channel and frequency offset estimation errors, the PEP performance of the cooperative OFDMA is also derived, and numerical results

[3] Although our discussion is limited to the uniform profile, the performance analysis is also valid in other profiles.

$$\lim_{\substack{\sigma_e^2\to 0\\ \mathrm{SNR}\to\infty}} \mathrm{P}_{\mathrm{r}}^{\mathrm{AF}}\left\{\bar{\mathbf{X}}_S \to \bar{\mathbf{L}}_S \middle| 0<\alpha<1\right\} \le \left(\frac{4M\left[\frac{\mathcal{L}_u(M+1)(1-\alpha)}{\alpha}+1\right]}{\alpha(1-\alpha)\mathrm{SNR}}\right)^{2L-1}\left(\frac{4}{\alpha\mathrm{SNR}}\right)^L \times \prod_{m=0}^{2L-2}\prod_{n=0}^{L-1}\frac{1}{\left|\sum\limits_{k=1}^{M}\tilde{\mathbf{h}}_{D,R_k,S}[m]\right|^2\cdot\left|\tilde{\mathbf{h}}_{D,S}[n]\right|^2\ell_m\ell_n} \qquad (13)$$

TABLE I
PERFORMANCE IMPROVEMENT IN THE PROPOSED COOPERATIVE SCHEME WITH FEEDBACK AS A FUNCTION OF M

Mode	$N=1024,\ N_u=8,\ \mathcal{L}_u=0.1$, SNR=20 dB, $\sigma_\epsilon=10^{-2}$			
No	M	2	4	16
Relay	Error	5.15×10^{-5}	5.15×10^{-5}	5.15×10^{-5}
AF	M	2	4	16
Mode	Error	4.603×10^{-6}	4.585×10^{-6}	4.562×10^{-6}
DF	M	2	4	16
Mode	Error	1.134×10^{-5}	1.121×10^{-5}	1.1089×10^{-5}

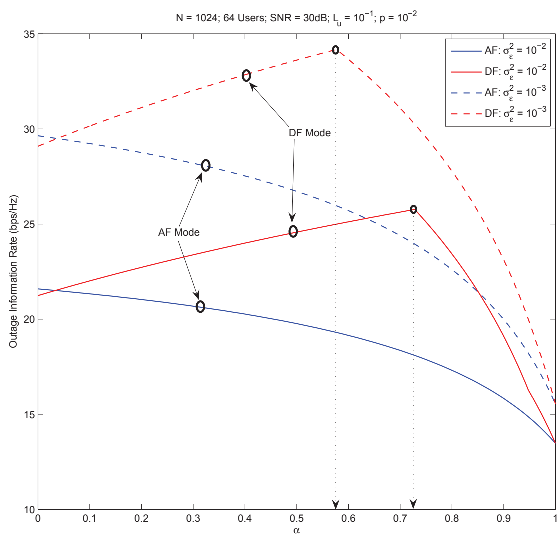

Fig. 1. Outage information rate as a function of α.

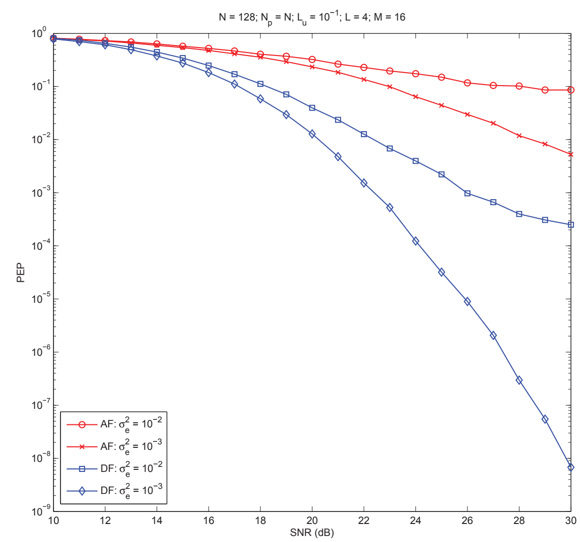

Fig. 2. PEP of the proposed cooperative transmission as a function of SNR with $L=4$, $M=16$ and $\sigma_e^2=10^{-2},10^{-3}$.

prove that the DF mode always outperforms the AF mode in terms of PEP.

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