

On the Trivariate Rician Distribution

Prathapasinghe Dharmawansa, *Member, IEEE*, Nandana Rajatheva, *Senior Member, IEEE*,
and Chinthananda Tellambura, *Senior Member, IEEE*

Abstract—An exact expression for the joint density of three correlated Rician variables is not available in the open literature. In this letter, we derive new infinite series representations for the trivariate Rician probability density function (pdf) and the joint cumulative distribution function (cdf). Our results are limited to the case where the inverse covariance matrix is tridiagonal. This case seems the most general one that is tractable with Miller's approach and cannot be extended to more than three Rician variables. The outage probability of triple branch selective combining (SC) receiver over correlated Rician channels is presented as an application of the density function.

Index Terms—Cumulative distribution function (cdf), exponential correlation, Rician fading, selection combining, trivariate Rician distribution.

I. INTRODUCTION

IN PROPAGATION environments such as satellite or microcellular mobile radio channels with line-of-sight propagation, the received signal consists of a direct component and a number of multipath (reflected) components. Since the received signal in this ubiquitous case has the Rician distribution [1], it is widely used by wireless communications systems researchers for a myriad of analysis and design problems involving diversity reception, multicarrier systems, multiple-input multiple output (MIMO) systems, wireless channel modeling and others.

Consequently, bivariate and multivariate Rice distributions play a pivotal role in the performance analysis of practical wireless communication systems and the related results date back four decades [2]- [10]. Bello and Boardman [3] derive the bivariate Rician density as a single finite-range integral. Middleton [10] has derived the bivariate Rician density as an infinite series of modified Bessel functions. However, his result has gone unnoticed before. Abu-Dayya and Beaulieu [4] use the integral in [3] for performance analysis of switched dual diversity systems over correlated Rician fading. In [9], [11], an infinite series for the bivariate Rician density is derived by expanding the Bessel function in the integrand [3] and is employed for performance analysis of dual diversity SC.

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P. Dharmawansa was with the Telecommunications Field of Study, School of Engineering and Technology, Asian Institute of Technology, Thailand. He is now with the Department of Electronic and Computer Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong (e-mail: eesinghe@ust.hk).

N. Rajatheva is with the Telecommunications Field of Study, School of Engineering and Technology, Asian Institute of Technology, P. O. Box 4, Klong Luang, Pathumthani 12120, Thailand (e-mail: rajath@ait.ac.th).

C. Tellambura is with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6G 2V4, Canada (e-mail: chinth@ece.ualberta.ca).

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Miller [12] derives the pdf of norm of two correlated n -dimensional non zero-mean Gaussian vectors. This result has been used by Simon [13] to derive the pdf and cdf of the bivariate Rician distribution.

The joint density of generalized p Rayleigh variables is derived in [14] when the inverse of the correlation matrix is tridiagonal. The trivariate case is considered there for an arbitrary correlation matrix. Multivariate Rayleigh and Nakagami- m distributions, which have many applications, are analyzed in detail in [15]- [19]. The authors in [7], [8] analyze the performance of L branch SC and equal gain combining (EGC) over equi-correlated Rician environments. The authors in [20] and [21] analyze the performance of several diversity schemes over correlated Rician channels employing the chf based approach. Furthermore, the performance of maximal ratio combining (MRC) in correlated Rician channels are analyzed in [22], [23]. Despite the extensive literature, no exact analytical joint pdf and cdf are available in the open literature for the trivariate (or higher order) Rician distribution.

In this letter, we derive a new series representation of the trivariate Rician density using Miller [12] when the inverse covariance matrix is tridiagonal. This condition holds for exponentially correlated channels. Moreover, pdf and cdf of bivariate and trivariate Rice distributions are also derived. However, the trivariate analysis for an arbitrary correlation matrix seems intractable. We consider only one application - namely the performance analysis of triple branch SC over correlated Rician fading environment.

II. TRIVARIATE DISTRIBUTION

A. Probability Density Function

Let $\{\mathbf{X}, \mathbf{Y}, \mathbf{Z}\}$ be two dimensional Gaussian vectors with means given by $E(\mathbf{X}) = E(\mathbf{Y}) = E(\mathbf{Z}) = \mathbf{a}$, where $\mathbf{X} = (x_1 \ x_2)^T$, $\mathbf{Y} = (y_1 \ y_2)^T$, $\mathbf{Z} = (z_1 \ z_2)^T$ and $\mathbf{a} = (a_1 \ a_2)^T$. Here $(\cdot)^T$, $E(\cdot)$ denote the transpose of a matrix and the mathematical expectation, respectively. Let $\mathbf{V}_i = (x_i \ y_i \ z_i)$, $1 \leq i \leq 2$, be independent three dimensional Gaussian vectors composed of the i th components of \mathbf{X} , \mathbf{Y} and \mathbf{Z} with positive definite covariance matrix of order 3×3 , \mathbf{M}_3 . In the following display, the columns are the 2-dimensional Gaussian vectors

$$\begin{array}{c|ccc} & \mathbf{X} & \mathbf{Y} & \mathbf{Z} \\ \hline \mathbf{V}_1 & x_1 & y_1 & z_1 \\ \mathbf{V}_2 & x_2 & y_2 & z_2 \end{array} \quad (1)$$

and the rows \mathbf{V}_j are independent from each other and with identical covariance matrix \mathbf{M}_3 . The inverse covariance matrix

of \mathbf{V}_j is

$$\mathbf{W}_3 = \mathbf{M}_3^{-1} = \begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{12} & w_{22} & w_{23} \\ w_{13} & w_{23} & w_{33} \end{pmatrix}. \quad (2)$$

As mentioned above, we assume that \mathbf{W}_3 is a **tridiagonal matrix** (i.e., $w_{13} = 0$). The amplitudes $r_1 = |\mathbf{X}|$, $r_2 = |\mathbf{Y}|$ and $r_3 = |\mathbf{Z}|$, being the square root of sum of squares of 2 nonzero-mean Gaussian variates, are Rician variates [1]. Here $|\cdot|$ denotes the Euclidean norm of a column vector. The joint density of \mathbf{X} , \mathbf{Y} and \mathbf{Z} can then be written as [12]

$$f(\mathbf{X}, \mathbf{Y}, \mathbf{Z}) = \frac{1}{8\pi^3 M_3} \exp\left\{-\frac{1}{2}\left(\sum_{i=1}^3 w_{ii} r_i^2 + a^2 w_4\right)\right\} \\ \times \exp\left\{\mathbf{X}^T (w_1 \mathbf{a} - w_{12} \mathbf{Y}) + w_2 \mathbf{Y}^T \mathbf{a}\right\} \\ \times \exp\left\{\mathbf{Z}^T (w_3 \mathbf{a} - w_{23} \mathbf{Y})\right\} \quad (3)$$

where $w_1 = w_{11} + w_{12}$, $w_2 = w_{22} + w_{23} + w_{12}$, $w_3 = w_{33} + w_{23}$, $w_4 = w_1 + w_2 + w_3$, $a = |\mathbf{a}|$ and M_3 denotes the determinant of square matrix \mathbf{M}_3 . The power correlation coefficient of the Rician variates can be written as $\rho_{ij} = \text{Cov}(r_i^2, r_j^2) / \sqrt{\text{var}(r_i^2) \text{var}(r_j^2)}$ for all $1 \leq i, j \leq 3$.

From the pdf (3), we must integrate out \mathbf{X} , \mathbf{Y} , \mathbf{Z} , subject to the constraints $r_1 = |\mathbf{X}|$, $r_2 = |\mathbf{Y}|$, $r_3 = |\mathbf{Z}|$, which will yield the joint pdf of correlated Rician variables $\{r_1, r_2, r_3\}$. Consequently the trivariate Rician density of $r_1 = |\mathbf{X}|$, $r_2 = |\mathbf{Y}|$ and $r_3 = |\mathbf{Z}|$ can be written as

$$g(r_1, r_2, r_3) = \frac{1}{8\pi^3 M_3} \exp\left\{-\frac{1}{2}\left(\sum_{i=1}^3 w_{ii} r_i^2 + a^2 w_4\right)\right\} \\ \times \int_{|\mathbf{X}|=r_1} \exp\left\{\mathbf{X}^T (w_1 \mathbf{a} - w_{12} \mathbf{Y})\right\} d\sigma_x \\ \times \int_{|\mathbf{Z}|=r_3} \exp\left\{\mathbf{Z}^T (w_3 \mathbf{a} - w_{23} \mathbf{Y})\right\} d\sigma_z \\ \times \int_{|\mathbf{Y}|=r_2} \exp(w_2 \mathbf{Y}^T \mathbf{a}) d\sigma_y \quad (4)$$

where $d\sigma_x$, $d\sigma_y$ and $d\sigma_z$ are the elements of surface area. The first integral can be evaluated as [12, eq.2.2.9]

$$\int_{|\mathbf{X}|=r_1} \exp\left\{\mathbf{X}^T (w_1 \mathbf{a} - w_{12} \mathbf{Y})\right\} d\sigma_x \\ = 2\pi r_1 I_0(r_1 |w_1 \mathbf{a} - w_{12} \mathbf{Y}|)$$

where $I_0(z)$ is the modified Bessel function of the first kind and zeroth order and the second integral follows the same form. Using the Neumann addition formula for Bessel functions [24: pp.365], followed by change of variables in the second integral to polar coordinates and choosing \mathbf{a} as the polar axis [12], we arrive at (5), where ε_k is the Neumann factor ($\varepsilon_0 = 1, \varepsilon_k = 2$ for $k = 1, 2, \dots$), θ is the angle between \mathbf{a} and \mathbf{Y} and $I_n(z)$ is the modified Bessel function of the first kind and order n . The integral in (5) can be evaluated by using the identity $\cos k\theta \cos p\theta \equiv \frac{1}{2} [\cos(k+p)\theta + \cos(k-p)\theta]$ and [25: eq.7.34], to give the trivariate Rician distribution as shown at the bottom.

To the best of our knowledge, the joint pdf (6) is a new result. Note however that (6) is not valid unless $w_{13} = 0$. Nevertheless, if a given covariance matrix has no tridiagonal inverse, then we can follow the Green's matrix approach suggested in [17] to produce the best approximate covariance matrix with a tridiagonal inverse. Next we illustrate some simplifications of (6).

1) *Bivariate Density*: Here we obtain the bivariate Rician distribution from the Trivariate distribution. Although it is natural to integrate out r_3 in (6) to get the joint density of r_1, r_2 , we use an alternative method by taking $w_{23} = 0$, which implies $k = 0$ and (6) then can be written as a product of the bivariate density and a univariate Rician density. Thus we get the bivariate Rician density as

$$g_2(r_1, r_2) = \frac{r_1 r_2}{M_2} \exp\left\{-\frac{1}{2}\left(\sum_{i=1}^2 w_{ii} r_i^2 + w_i a^2\right)\right\} \\ \times \sum_{p=0}^{\infty} \varepsilon_p (-1)^p I_p(w_1 a r_1) I_p(w_{12} r_1 r_2) \\ \times I_p(w_2 a r_2). \quad (7)$$

where M_2 denotes the determinant of 2×2 covariance matrix of the underlying Gaussian variables corresponding to the Rice variables r_1, r_2 . This agrees with the previous result [10: eq.9.56].

2) *Independent Rice Envelopes*: If r_1, r_2 and r_3 are independent, then $w_{12} = w_{23} = 0$ and the two infinite summations in (6) can be simplified to

$$g(r_1, r_2, r_3) = \prod_{i=1}^3 w_{ii} r_i \exp\left\{-\frac{1}{2} w_{ii} (r_i^2 + a^2)\right\} I_0(w_{ii} a r_i).$$

Therefore we get a product of three Rician density functions.

$$g(r_1, r_2, r_3) = \frac{r_1 r_2 r_3}{2\pi M_3} \exp\left\{-\frac{1}{2}\left(\sum_{i=1}^3 w_{ii} r_i^2 + a^2 w_4\right)\right\} \sum_{k=0}^{\infty} \sum_{p=-\infty}^{\infty} \varepsilon_k (-1)^{k+p} I_k(w_3 a r_3) I_k(w_{23} r_2 r_3) \\ \times I_p(w_1 a r_1) I_p(w_{12} r_1 r_2) \int_0^{2\pi} \cos k\theta \cos p\theta \exp(w_2 a r_2 \cos \theta) d\theta \quad (5)$$

$$g(r_1, r_2, r_3) = \frac{r_1 r_2 r_3}{M_3} \exp\left\{-\frac{1}{2}\left(\sum_{i=1}^3 w_{ii} r_i^2 + a^2 w_4\right)\right\} \sum_{k=0}^{\infty} \sum_{p=-\infty}^{\infty} \varepsilon_k (-1)^{k+p} I_k(w_3 a r_3) I_k(w_{23} r_2 r_3) \\ \times I_p(w_1 a r_1) I_p(w_{12} r_1 r_2) I_{k+p}(w_2 a r_2). \quad (6)$$

3) *Trivariate Rayleigh Density*: When $a = 0$, the trivariate Rayleigh distribution can be obtained from (6) by considering the degenerated infinite summation when $k = p = 0$ as

$$g(r_1, r_2, r_3) = \frac{r_1 r_2 r_3}{M_3} \exp\left\{-\frac{1}{2} \sum_{i=1}^3 w_{ii} r_i^2\right\} \\ \times I_0(|w_{12}|r_1 r_2) I_0(|w_{23}|r_2 r_3).$$

This exactly coincides with the density function given in [17: eq.2] when $n = 2$.

The exponential correlation model gives a tri-diagonal form of inverse [19]. If we assume $\sigma_i^2, \sigma_j^2, \rho^{|i-j|}$ as the variances and correlation coefficient of the underlying Gaussian variables corresponding to the Rice variables r_i and r_j respectively, we can write $\rho_\eta = \text{Cov}(r_i^2, r_j^2) = \frac{\rho^{|i-j|}(a^2 + \rho^{|i-j|}\sigma_i\sigma_j)}{\sqrt{(a^2 + \sigma_i^2)(a^2 + \sigma_j^2)}}$. For Rayleigh random variables, it reduces to $\rho_\eta = \rho^2|^i-j|$ [19].

B. Cumulative Density Function

The trivariate cumulative density function is given by

$$G(r_1, r_2, r_3) = \int_0^{r_1} \int_0^{r_2} \int_0^{r_3} g(x, y, z) dx dy dz. \quad (8)$$

Since $I_n(z) = I_{-n}(z)$ for any integer n , we can expand $I_n(z)$ as

$$I_n(z) = \sum_{k=0}^{\infty} \frac{z^{2k+|n|}}{2^{2k+|n|} k! \Gamma(k+|n|+1)} \quad (9)$$

where $\Gamma(z)$ is the gamma function. By replacing the Bessel function terms in (6) with (9) and integrating term-by-term because of the uniform convergence of the series, we derive the trivariate joint cdf as shown in (10), where $\kappa_1 = i_3 + i_4 + |p| + 1$, $\kappa_{21} = i_1 + i_3 + i_5 + \frac{k+|p|+|k+p|}{2}$, $\kappa_{22} = i_2 + i_4 + i_5 + 1 + \frac{k+|p|+|k+p|}{2}$, $\kappa_3 = i_1 + i_2 + k + 1$, $\gamma(\alpha, z) = \int_0^z t^{\alpha-1} e^{-t} dt$ is the incomplete gamma function [25]. When $a = 0$, (10) can easily be simplified to the trivariate Rayleigh cdf given in [17, eq.6] when $n = 3$.

The bivariate joint cumulative distribution function can be written using (10) as shown in (11). It can easily be simplified to [13, eq.11] once suitable parameters are assumed. Now ρ_η simplifies to $\rho_\eta = \text{Cov}(r_1^2, r_2^2) = \frac{\rho(a^2 + \rho\sigma_1\sigma_2)}{\sqrt{(a^2 + \sigma_1^2)(a^2 + \sigma_2^2)}}$.

III. TRUNCATION ERROR

Assume that the trivariate cdf series (10) is limited to $K, 2P - 1, I_1, I_2, I_3, I_4$ and I_5 terms in the variables k, p, i_1, i_2, i_3, i_4 and i_5 respectively. It should be noted that the second summation is truncated symmetrically around the zeroth term. Then the rest of the terms represent the truncation error. Following the fact that $\gamma(a, z) \leq \Gamma(a)$ [18], we can write the truncation error upper bound as

$$|E_{TR}| \leq \sum_{k=0}^{K-1} \sum_{p=-P+1}^{P-1} \sum_{i_1, i_2, i_3, i_4=0}^{I_1-1, I_2-1, I_3-1, I_4-1} \sum_{i_5=I_5}^{\infty} E_p \\ + \sum_{k=0}^{K-1} \sum_{p=-P+1}^{P-1} \sum_{i_1, i_2, i_3=0}^{I_1-1, I_2-1, I_3-1} \sum_{i_4=I_4}^{\infty} \sum_{i_5=0}^{\infty} E_p \\ + \sum_{k=0}^{K-1} \sum_{p=-P+1}^{P-1} \sum_{i_1, i_2=0}^{I_1-1, I_2-1} \sum_{i_3=I_3}^{\infty} \sum_{i_4, i_5=0}^{\infty} E_p \\ + \sum_{k=0}^{K-1} \sum_{p=-P+1}^{P-1} \sum_{i_1=0}^{I_1-1} \sum_{i_2=I_2}^{\infty} \sum_{i_3, i_4, i_5=0}^{\infty} E_p \\ + \sum_{k=0}^{K-1} \sum_{p=-P+1}^{P-1} \sum_{i_1=I_1}^{\infty} \sum_{i_2, i_3, i_4, i_5=0}^{\infty} E_p \\ + \sum_{k=0}^{K-1} \sum_{p=P}^{\infty} \sum_{i_1, i_2, i_3, i_4, i_5=0}^{\infty} [E_p + E_{-p}] \\ + \sum_{k=K}^{\infty} \sum_{p=-\infty}^{\infty} \sum_{i_1, i_2, i_3, i_4, i_5=0}^{\infty} E_p \quad (12)$$

where E_p is defined at the bottom of the previous page with all the symbols having their usual meaning. Further simplification

$$G(r_1, r_2, r_3) = \frac{1}{M_3} \exp\left\{-\frac{a^2 w_4}{2}\right\} \sum_{k=0}^{\infty} \sum_{p=-\infty}^{\infty} \varepsilon_k (-1)^{k+p} \sum_{i_1, i_2, i_3, i_4, i_5=0}^{\infty} \frac{a^{2\kappa_{21}} w_1^{2i_3+|p|} w_2^{2i_5+|k+p|} w_3^{2i_1+k} w_{23}^{2i_2+k} w_{12}^{2i_4+|p|}}{2^{\kappa_{21}} w_{11}^{\kappa_{11}} w_{22}^{\kappa_{22}} w_{33}^{\kappa_3} i_1! i_2! i_3! i_4! i_5!} \\ \times \frac{\gamma\left(\kappa_1, \frac{w_{11} r_1^2}{2}\right) \gamma\left(\kappa_{22}, \frac{w_{22} r_2^2}{2}\right) \gamma\left(\kappa_3, \frac{w_{33} r_3^2}{2}\right)}{(i_1+k)! (i_2+k)! (i_3+|p|)! (i_4+|p|)! (i_5+|k+p|)!} \quad (10)$$

$$G_2(r_1, r_2) = \frac{1}{M_2} \exp\left\{-\frac{a^2}{2}(w_1 + w_2)\right\} \sum_{p=0}^{\infty} \varepsilon_p (-1)^p \sum_{l, m, n=0}^{\infty} \frac{a^{2(l+n+p)} w_1^{2l+p} w_2^{2n+p} w_{12}^{2m+p}}{2^{l+n+p} w_{11}^{l+m+p+1} w_{22}^{m+n+p+1} l! m! n! (l+p)! (m+p)! (n+p)!} \\ \times \gamma\left(l+m+p+1, \frac{w_{11} r_1^2}{2}\right) \gamma\left(m+n+p+1, \frac{w_{22} r_2^2}{2}\right). \quad (11)$$

$$E_p = \frac{\varepsilon_k \exp\left\{-\frac{a^2 w_4}{2}\right\} a^{2\kappa_{21}} \Gamma(\kappa_1) \Gamma(\kappa_{22}) \Gamma(\kappa_3) w_1^{2i_3+|p|} w_2^{2i_5+|k+p|} w_3^{2i_1+k} |w_{23}|^{2i_2+k} |w_{12}|^{2i_4+|p|}}{M_3 2^{\kappa_{21}} w_{11}^{\kappa_{11}} w_{22}^{\kappa_{22}} w_{33}^{\kappa_3} i_1! i_2! i_3! i_4! i_5! (i_1+k)! (i_2+k)! (i_3+|p|)! (i_4+|p|)! (i_5+|k+p|)!}$$

of (12) is a difficult task. However, the upper bound may be loose for some small values of r_i 's since we bounded the incomplete gamma function with the gamma function.

A more tighter bound can be obtained for the truncation error of the bivariate Rician cumulative density using the arguments given in [15] as shown in (13) below, where ${}_1F_1(\lambda; \mu; z)$ is the confluent hypergeometric function [25]. The number of terms needed in (10), (11) to achieve four and five significant figure accuracy are tabulated in Table I and Table II respectively. In the case of (10), we have to deal with seven truncated summations and it reduces the calculation speed significantly. We have used the exponential correlation model and Mathematica software to obtain those numerical values.

IV. APPLICATION: OUTAGE PROBABILITY OF TRIPLE BRANCH SC

In this section, the new density function and related statistics are used to analyze the performance of triple branch SC over correlated Rician environment. Outage probability being the probability that the output SNR, γ , falls below a specified level γ_{th} is an important system performance measure. We assume that the noise components at different diversity branches are additive white Gaussian noise (AWGN) with identical power spectral density. Let γ_k and $\bar{\gamma}_k$ denote the instantaneous and the average SNR at the k -th branch ($k = 1, 2, 3$). In SC, the branch with the largest instantaneous SNR is selected as the output, $\gamma_{sc} = \max(\gamma_1, \gamma_2, \gamma_3)$. Using the relation $\gamma_k = \frac{\bar{\gamma}_k}{E(r_k^2)} r_k^2 = \frac{\bar{\gamma}_k}{(2m_{kk} + a^2)} r_k^2$, where r_k is the amplitude of the received signal at the k -th branch, we may obtain the outage probability as given in (14), where $G(\lambda_1, \lambda_2, \lambda_3)$ is the joint cdf of the branch amplitudes (10). Note that the covariance matrix \mathbf{M} specifies the correlation (fading correlation) between three Gaussian samples.

Figure 1 depicts the outage performance of triple branch SC over correlated Rician fading environment. For brevity we consider uniform power delay profile with exponential covariance matrix defined by $m_{ij} = \sigma^2 \rho^{|i-j|}$, $0 < \rho < 1$ [19]. Furthermore, we normalize the channel such that $\sigma^2 = \sqrt{\frac{1}{2(1+K)}}$ and $a = \sqrt{\frac{K}{K+1}}$ with K being the Rician factor. We have considered different K and ρ values to show the performance variation. As can be seen from the graph, the correlation among the diversity branches degrades the performance. Since the theoretical outage expression is in the form of an infinite series, we truncate the series for numerical evaluation. The number of terms being used from each index, $k, p, i_1, i_2, i_3, i_4, i_5$ vary in between the minimum of 7, 7, 7, 3, 7, 4, 7 and the maximum of 17, 17, 16, 9, 16, 9, 16. Usually it is tedious to work with nested infinite summations even if they are truncated, since the calculation time and accuracy solely depend on the available computing power.

V. CONCLUSION

In this paper, a new infinite series is derived for the trivariate Rician pdf using Miller's approach when the underlying Gaussian variables have the tridiagonal form of inverse covariance matrix. This assumption is valid if the exponential correlation model holds. However, the trivariate density for an arbitrary correlation matrix seems intractable and remains as an open problem. The trivariate cdf is also derived and the previously available result for the bivariate case is turned out to be a special case of our results. Historically, the bivariate Rician case goes back four decades and the original result of Middleton [10, eq.9.56] has gone unnoticed before.

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$$\begin{aligned}
|E_{TR}| \leq & \frac{{}_1F_1\left(I_3 + 1; I_3 + 2; -\frac{w_{22}r_2^2}{2}\right)}{(I_3 + 1)} \sum_{p=0}^{P-1} \sum_{i_1=0}^{I_1-1} \sum_{i_2=0}^{I_2-1} \sum_{i_3=I_3}^{\infty} \frac{R_{\epsilon 1} F_1\left(p + 1; p + 2; -\frac{w_{11}r_1^2}{2}\right)}{(i_1 + p)!(i_2 + p)!(I_3 + p)!} \\
& + \frac{{}_1F_1\left(I_2 + 1; I_2 + 2; -\frac{w_{11}r_1^2}{2}\right)}{(I_2 + 1)^2} {}_1F_1\left(I_2 + 1; I_2 + 2; -\frac{w_{22}r_2^2}{2}\right) \sum_{p=0}^{P-1} \sum_{i_1=0}^{I_1-1} \sum_{i_2=I_2}^{\infty} \sum_{i_3=0}^{\infty} \frac{R_{\epsilon}}{(i_1 + p)!(I_2 + p)!(i_3 + p)!} \\
& + \frac{{}_1F_1\left(I_1 + 1; I_1 + 2; -\frac{w_{11}r_1^2}{2}\right)}{(I_1 + 1)} \sum_{p=0}^{P-1} \sum_{i_1=I_1}^{\infty} \sum_{i_2=0}^{\infty} \sum_{i_3=0}^{\infty} \frac{R_{\epsilon 1} F_1\left(p + 1; p + 2; -\frac{w_{22}r_2^2}{2}\right)}{(I_1 + p)!(i_2 + p)!(i_3 + p)!} \\
& + \frac{{}_1F_1\left(P + 1; P + 2; -\frac{w_{11}r_1^2}{2}\right) {}_1F_1\left(P + 1; P + 2; -\frac{w_{22}r_2^2}{2}\right)}{(P + 1)^2} \sum_{p=P}^{\infty} \sum_{i_1=I_1}^{\infty} \sum_{i_2=0}^{\infty} \sum_{i_3=0}^{\infty} \frac{R_{\epsilon}}{(i_1 + P)!(i_2 + P)!(i_3 + P)!} \\
R_{\epsilon} = & \frac{\varepsilon_p \exp\left\{-\frac{a^2}{2}(w_1 + w_2)\right\} a^{2(i_1+i_3+p)} w_{12}^{2i_2+p} r_1^{2(i_1+i_2+p+1)} r_2^{2(i_2+i_3+p+1)} w_1^{2i_1+p} w_2^{2i_3+p}}{M_{2i_1!i_2!i_3!2^{2(i_1+i_2+i_3+p+1)+p}}}
\end{aligned} \tag{13}$$

$$P_{out} = \Pr(0 \leq \gamma_{sc} \leq \gamma_{th}) = G\left(\sqrt{\frac{\gamma_{th}(2m_{11} + a^2)}{\bar{\gamma}_1}}, \sqrt{\frac{\gamma_{th}(2m_{22} + a^2)}{\bar{\gamma}_2}}, \sqrt{\frac{\gamma_{th}(2m_{33} + a^2)}{\bar{\gamma}_3}}\right) \tag{14}$$

TABLE I
NUMBER OF TERMS NEEDED IN EACH INDEX $K, P, I_1, I_2, I_3, I_4, I_5$ IN (11) TO ACHIEVE FOUR SIGNIFICANT FIGURE ACCURACY.

a	ρ	$r_1 = r_2 = 1$	$r_1 = r_2 = 2$	$r_1 = r_2 = 5$
1	0.1	3, 3, 3, 2, 3, 2, 3	5, 5, 5, 3, 5, 3, 4	6, 7, 6, 4, 6, 4, 6
3		4, 4, 5, 3, 5, 3, 4	5, 5, 8, 3, 8, 3, 7	9, 9, 11, 5, 11, 5, 11
1	0.3	3, 3, 3, 2, 3, 2, 3	6, 5, 5, 4, 5, 4, 5	11, 11, 8, 6, 8, 6, 8
3		5, 5, 5, 4, 5, 4, 5	6, 6, 8, 6, 8, 6, 8	11, 11, 12, 6, 12, 6, 12

TABLE II
NUMBER OF TERMS NEEDED IN EACH INDEX P, L, N, M IN (12) TO ACHIEVE FIVE SIGNIFICANT FIGURE ACCURACY.

a	ρ	$r_1 = r_2 = 1$	$r_1 = r_2 = 2$	$r_1 = r_2 = 5$
1	0.1	3, 4, 4, 2	5, 5, 5, 3	7, 7, 4, 6
3		5, 6, 6, 2	7, 9, 9, 4	11, 15, 15, 15
1	0.3	3, 4, 4, 3	5, 5, 4, 4	7, 7, 7, 6
3		5, 6, 6, 3	7, 8, 8, 4	15, 13, 13, 9
1	0.6	3, 3, 3, 3	6, 4, 4, 6	9, 6, 6, 14
3		5, 5, 5, 3	9, 7, 7, 7	17, 12, 19, 11
1	0.8	5, 3, 3, 5	7, 4, 4, 10	9, 6, 28, 5
3		7, 5, 5, 5	10, 7, 7, 11	17, 11, 11, 38

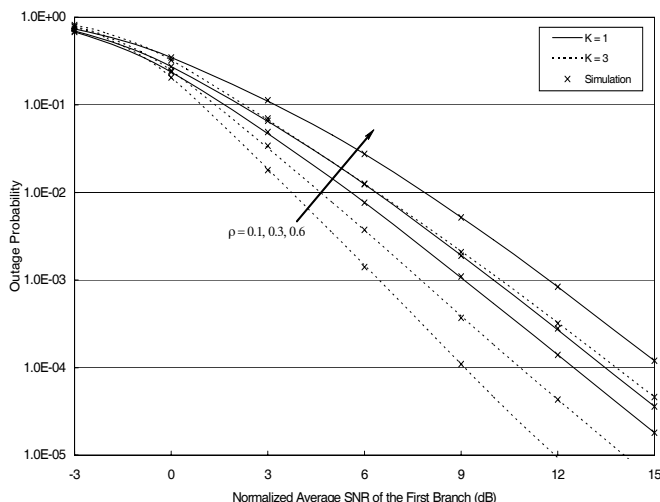


Fig. 1. Outage probability of three branch SC versus normalized average SNR of the first branch $\bar{\gamma}_1/\gamma_{th}$ for several values of K and ρ over correlated Rician fading channel.

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