Optimal Pilots for Frequency Offset and Channel Estimation in OFDMA Uplink

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***Abstract*—Optimal pilots design and placement for the frequency offset and channel estimation in orthogonal frequency-division multiplexing access (OFDMA) uplink systems are proposed. The received pilots of multiple users can always be demodulated, even if they are totally overlapped due to the large frequency offsets. With the knowledge of channel state information (CSI) at the receiver, the performance of the proposed frequency offset estimation is robust to the channel estimation errors. The frequency offset and CSI can be jointly estimated by employing the proposed pilots.**

I. Introduction

In an OFDMA system, several users simultaneously transmit their own data by modulating an exclusive set of orthogonal sub-carriers. The orthogonality among subcarriers guarantees intrinsic protection against multiple access interference (MAI) [1]. OFDMA therefore is adopted in part of the IEEE 802.16 standard for wireless metropolitan area networks (WMANs) and is considered as a promising candidate for next generation broadband wireless networks.

However, the design of OFDMA faces several technical challenges. First, OFDMA systems are highly sensitive to the frequency offset, which results in the loss of orthogonality among subcarriers. Second, channel should be estimated to coherently detect transmitted data. Frequency offset and channel estimation are particularly challenging in the uplink of OFDMA since the base station (BS) has to estimate frequency offset and multiple transmission channels for each user.

Several methods of frequency offset and/or channel estimation for OFDMA uplink have been presented in [2]–[8]. Reference [2] exploits redundancy offered by the cyclic-prefix (CP). In [3], frequency offset is estimated by looking for the position of null subcarriers within the signal bandwidth. By using the inherent signal structure, a high-resolution blind frequency offset estimator is presented in [5] for interleaved OFDMA systems. The maximum likelihood (ML) algorithm for both frequency offset and channel estimation is studied in [6], and the complexity is reduced by employing an alternating-projection method. The iterative space-alternation generalized expectation-maximization (SAGE) algorithm for frequency offset and time estimation is presented in [7] to overcome MAI. Two iterative joint frequency offset and channel estimation algorithms based on a cyclically equal-spaced, equal-energy interleaved pilot are proposed in [8]. Some good pilots designed for channel estimation in multiple-input multiple-output (MIMO) orthogonal frequency-division multiplexing (OFDM) systems, e.g., [9]–[11], can also be used for OFDMA uplink channel estimation. In many existing algorithms, especially those based on periodically modulated pilots, the pilots for different users are demodulated by using a frequency-domain filter. However, these frequency-domain-filter-based algorithms may fail to identify the pilots for each user if the pilots of different users are overlapped due to the large frequency offsets.

In this paper, the optimal pilots design and placement for both the frequency offset and channel estimation in an OFDMA uplink are treated. Unlike many frequency-domain filter-based algorithms, the pilots for each user can always be identified, regardless the frequency offset is large or not. Thus, the constraint in many algorithms that the abosolute value of all the frequency offsets is less than one-half of subcarriers seperation can be removed. Subcarrier allocation for one user needs not be contiguous in the frequency-domain, and this feature helps to exploit the frequency diversity. Without loss of generality, the frequency offset of each user is assumed to be an independent and identically distributed (i.i.d.) random variable (RV). A joint frequency offset and channel estimation scheme is presented based on the proposed pilots.

The remainder of this paper is organized as follows. The OFDMA uplink signal model is discribed in Section II. Section III presents the optimal pilot design for frequency offset estimation. Section IV discusses the optimal pilots design for the LS channel estimation, and joint frequency offset and channel estimation scheme is proposed in Section V. Simulation results are discussed in Section VI, followed by conclusions drawn in Section VII.

Notation: $(\cdot)^T$ and $(\cdot)^H$ and $(\cdot)^*$ are the transpose, complex conjugate transpose and complex conjugate. $\mathbf{x}[i]$ is the i-th element of vector $\mathbf{x}$, and $\|\mathbf{x}\|_2^2 = \sum_i |\mathbf{x}[i]|^2$ is the sum of the square of each entry in $\mathbf{x}$. $[\mathbf{B}]_{mn}$ is the mn-th entry of matrix $\mathbf{B}$. $\mathbf{I}_N$ and $\mathbf{O}_N$ are the $N \times N$ identity matrix and all-zero matrix, respectively. $\mathbf{0}_N$ is an $N \times 1$ all-zero vector. A circularly symmetric complex Gaussian variable z with mean m and variance σ_z^2 is denoted by $z \sim \mathcal{CN}(m, \sigma_z^2)$. $\mathbb{E}\{x\}$ and Var$\{x\}$ are the mean and variance of x, respectively.

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II. OFDMA Uplink Signal Model

M users are assumed to share N subcarriers and simultaneously communicate with the BS. Let $N \times 1$ vector $\mathbf{x}_k$ denote the frequency-domain symbols sent by the k-th user, the corresponding time-domain vector is generated by taking the *Inverse Discrete Fourier Transform* (IDFT) of $\mathbf{x}_k$, where the $N \times N$ IDFT matrix is defined by $[\mathbf{F}]_{nk} = \frac{1}{\sqrt{N}} e^{\frac{j2\pi nk}{N}}$ with $0 \leq n, k \leq N-1$. $\mathbf{x}_k$ can be expressed as $\mathbf{x}_k = \mathbf{x}_k^p + \mathbf{x}_k^d$, where $\mathbf{x}_k^p$ and $\mathbf{x}_k^d$ represent the pilot and data vector, respectively. The number of subcarriers and the number of pilots allocated to each user are assumed to be $\mathcal{N}$ and $\mathcal{N}_p$, respectively. An identical power is assumed to be allocated to each user, i.e., $\|\mathbf{x}_k^p\|_2^2 = E_p/M$ and $\mathcal{E}\{\|\mathbf{x}_k^d\|_2^2\} = E_d/M$, where E_p and E_d denote the total pilot power and data power of M users.

By using $h_k(n)$ to represent the discrete-time impulse response of the n-th tap channel of the k-th user, the related channel response vector can be represented as $\mathbf{h}_k = [\tilde{\mathbf{h}}_k^T, \mathbf{0}_{N-L_{max}}^T]^T = [h_k(0), h_k(1), \cdots, h_k(L_{\max} - 1), \mathbf{0}_{N-L_{max}}^T]^T$ with L_{max} representing the maximum length of all channels. The corresponding frequency-domain channel attenuation matrix is given by $\mathbf{H}_k = \text{diag}\left\{H_k^{(0)}, H_k^{(1)}, \cdots, H_k^{(N-1)}\right\}$ with $H_k^{(n)} = \sum_{d=0}^{L_{max}-1} h_k(d) e^{-\frac{j2\pi nd}{N}}$ representing the channel attenuation at the n-th subcarrier of the k-th user.

With ε_k denoted as normalized frequency offset of the k-th user, the received signal at the BS can be represented as

$$\begin{aligned}\mathbf{y} &= \sum_{k=1}^{M} \left(\mathbf{E}_k \mathbf{F} \underbrace{\mathbf{H}_k \mathbf{x}_k^p}_{\tilde{\mathbf{x}}_k^p} + \mathbf{E}_k \mathbf{F} \underbrace{\mathbf{H}_k \mathbf{x}_k^d}_{\tilde{\mathbf{x}}_k^d} \right) + \mathbf{w} \\ &= \sum_{k=1}^{M} (\boldsymbol{\Xi}_k + \boldsymbol{\Omega}_k) \boldsymbol{\nu}_k + \mathbf{w},\end{aligned} \tag{1}$$

where $\boldsymbol{\Xi}_k = \text{diag}\left\{\mathbf{f}_0^T \tilde{\mathbf{x}}_k^p, \cdots, \mathbf{f}_{N-1}^T \tilde{\mathbf{x}}_k^p\right\}$ and $\boldsymbol{\Omega}_k = \text{diag}\left\{\mathbf{f}_0^T \tilde{\mathbf{x}}_k^d, \cdots, \mathbf{f}_{N-1}^T \tilde{\mathbf{x}}_k^d\right\}$ with $\mathbf{f}_i$ representing the i-th column of $\mathbf{F}$, $\mathbf{E}_k = \text{diag}\left\{1, e^{\frac{j2\pi\varepsilon_k}{N}}, \cdots, e^{\frac{j2\pi\varepsilon_k(N-1)}{N}}\right\}$, $\boldsymbol{\nu}_k = \left[1, e^{\frac{j2\pi\varepsilon_k}{N}}, \cdots, e^{\frac{j2\pi\varepsilon_k(N-1)}{N}}\right]^T$, and $\mathbf{w}$ in (1) is a vector of additive white Gaussian noise (AWGN) with $\mathbf{w}[i] \sim \mathcal{CN}(0, \sigma_w^2)$.

III. Frequency Offsets Estimation

We assume that ε_k for each user k is an i.i.d. RV with zero mean and variance σ_ϵ^2. Since $\boldsymbol{\nu}_k$ is a sufficient statistic of ε_k, ε_k can be estimated from the estimation of $\boldsymbol{\nu}_k$, i.e. $\hat{\boldsymbol{\nu}}_k$. We design some pilots to make $\hat{\boldsymbol{\nu}}_k$ be orthogonal to each other and, therefore, the MAI can thus be totally eliminated.

A. Estimation of $\boldsymbol{\nu}_k$

Based on (1), an LS estimator of $\boldsymbol{\nu}_k$ can be designed as

$$\hat{\boldsymbol{\nu}}_k = \boldsymbol{\Xi}_k^\dagger \mathbf{y} = \boldsymbol{\nu}_k^\perp + \sum_{l=1, l\neq k}^{M} \boldsymbol{\Xi}_k^\dagger \boldsymbol{\Xi}_l \boldsymbol{\nu}_l + \sum_{m=1}^{M} \boldsymbol{\Xi}_k^\dagger \boldsymbol{\Omega}_m \boldsymbol{\nu}_m + \boldsymbol{\Xi}_k^\dagger \mathbf{w}, \tag{2}$$

where $\boldsymbol{\Xi}_k^\dagger = \left(\boldsymbol{\Xi}_k^H \boldsymbol{\Xi}_k\right)^{-1} \boldsymbol{\Xi}_k^H$, $\boldsymbol{\nu}_k^\perp = \left(\boldsymbol{\Xi}_k^H \boldsymbol{\Xi}_k\right)^{-1} \boldsymbol{\Xi}_k^H \boldsymbol{\Xi}_k \boldsymbol{\nu}_k$, and $\boldsymbol{\Xi}_k^H \boldsymbol{\Xi}_k$ is not necessarily a full-rank matrix. We represent $\boldsymbol{\Xi}_k^H \boldsymbol{\Xi}_k$ as $\boldsymbol{\Xi}_k^H \boldsymbol{\Xi}_k = \text{diag}\left\{0, \cdots, \left|\mathbf{f}_i^T \tilde{\mathbf{x}}_k^p\right|^2, \cdots\right\}$, where $\left|\mathbf{f}_i^T \tilde{\mathbf{x}}_k^p\right|^2 > 0$ and $\sum_i \left|\mathbf{f}_i^T \tilde{\mathbf{x}}_k^p\right|^2 = E_p/M$. Therefore, $\left(\boldsymbol{\Xi}_k^H \boldsymbol{\Xi}_k\right)^{-1}$ can be represented as

$$\left(\boldsymbol{\Xi}_k^H \boldsymbol{\Xi}_k\right)^{-1} = \text{diag}\left\{0, \ldots, |\mathbf{f}_i^T \tilde{\mathbf{x}}_k^p|^{-2}, \ldots\right\}.$$

$\boldsymbol{\nu}_k^\perp = \boldsymbol{\nu}_k$ is satisfied if and only if $\boldsymbol{\Xi}_k^H \boldsymbol{\Xi}_k$ is full-rank; if not, the number of non-zero elements in $\boldsymbol{\nu}_k^\perp$ is less than N. However, the non-zero elements in $\boldsymbol{\nu}_k^\perp$ can also provide sufficient statistics of ε_k. With assuming the entries of $\mathbf{x}_k^d$ are i.i.d., the MSE of $\hat{\boldsymbol{\nu}}_k$ is given by

$$\begin{aligned}&\text{MSE}(\hat{\boldsymbol{\nu}}_k) = \mathbb{E}\left\{\left\|\hat{\boldsymbol{\nu}}_k - \boldsymbol{\nu}_k^\perp\right\|_2^2\right\}, \\ &\text{s.t. trace}\{\boldsymbol{\Xi}_k^H \boldsymbol{\Xi}_k\} = \frac{E_p}{M}.\end{aligned} \tag{3}$$

For the sake of brevity, the extension express of $\mathbb{E}\left\{\left\|\hat{\boldsymbol{\nu}}_k - \boldsymbol{\nu}_k^\perp\right\|_2^2\right\}$ is not shown here. We find that, if a group of pilots can be found to satisfy $\boldsymbol{\Xi}_k^\dagger \boldsymbol{\Xi}_{l\neq k} = \mathbf{O}_N$ for each $0 \leq k, l \neq k \leq N-1$, the minimum MSE $(\hat{\boldsymbol{\nu}}_k)$ can be achieved by minimizing $\text{trace}\left\{(\boldsymbol{\Xi}_k^H \boldsymbol{\Xi}_k)^{-1}\right\}$, which requires that $\boldsymbol{\Xi}_k^H \boldsymbol{\Xi}_k$ has only one non-zero element, i.e.,

$$\boldsymbol{\Xi}_k^H \boldsymbol{\Xi}_k = \text{diag}\left\{\mathbf{0}_{\vartheta_k}^T, E_p/M, \mathbf{0}_{N-\vartheta_k-1}^T\right\}, \tag{4}$$

where ϑ_k represents the index of the non-zero diagonal element of $\boldsymbol{\Xi}_k^H \boldsymbol{\Xi}_k$. Note that $\boldsymbol{\Xi}_k^\dagger \boldsymbol{\Xi}_{l\neq k} = \mathbf{O}_N$ requires $\vartheta_k \neq \vartheta_{l\neq k}$, and, the minimum MSE $(\hat{\boldsymbol{\nu}}_k)$ can be given by

$$\min_{\boldsymbol{\Xi}_k} \text{MSE}\left(\hat{\boldsymbol{\nu}}_k\right) = \frac{\frac{E_d}{\mathcal{N} - \mathcal{N}_p} + \sigma_w^2}{\frac{E_p}{M}}, \tag{5}$$

where $E_d/(\mathcal{N} - \mathcal{N}_p)$ represents the interference contributed by the data subcarriers. If only pilots are transmitted, the Cramer-Rao Lower Bound (CRLB) is obtained as $\text{MSE}\left(\hat{\boldsymbol{\nu}}_k\right) \geq \frac{M\sigma_w^2}{E_p}$. From (5), we can see that $\text{MSE}\left(\hat{\boldsymbol{\nu}}_k\right)$ is independent of the actual frequency offset and k. However, this finding does not mean that $\text{Var}\left\{\hat{\varepsilon}_k\right\}$ is identical for each user k. In the next subsection, we will show that the performance of $\hat{\varepsilon}_k$ depends on ϑ_k, and a larger ϑ_k implies a smaller $\text{Var}\left\{\hat{\varepsilon}_k\right\}$.

B. Estimation of ε_k based on $\hat{\boldsymbol{\nu}}_k$

The analysis in Section III-A shows that, if the pilots can be designed to satisfy (4), $\text{MSE}\left(\hat{\boldsymbol{\nu}}_k\right)$ can be minimized and, $\hat{\boldsymbol{\nu}}_k$ in (2) can be rewritten as

$$\hat{\boldsymbol{\nu}}_k = \underbrace{\left[\mathbf{0}_{\vartheta_k}^T, e^{\frac{j2\pi\vartheta_k\varepsilon_k}{N}}, \mathbf{0}_{N-\vartheta_k-1}^T\right]^T}_{\boldsymbol{\nu}_k^\perp} + \left[\mathbf{0}_{\vartheta_k}^T, \xi_k, \mathbf{0}_{N-\vartheta_k-1}^T\right]^T, \tag{6}$$

where

$$\xi_k = \frac{M}{E_p} \sum_{m=1}^{M} \tilde{\mathbf{x}}_k^{pH} \mathbf{f}_{\vartheta_k}^* \mathbf{f}_{\vartheta_k}^T \tilde{\mathbf{x}}_k^d e^{\frac{j2\pi\vartheta_k\varepsilon_m}{N}} + \frac{M}{E_p} \tilde{\mathbf{x}}_k^{pH} \mathbf{f}_{\vartheta_k}^* \mathbf{w}[\vartheta_k]$$

with $\xi_k \sim \mathcal{CN}\left(0, 2\sigma_\xi^2\right)$, and $\sigma_\xi^2 = \dfrac{M}{2E_p}\left(\dfrac{E_d}{\mathcal{N}-\mathcal{N}_p} + \sigma_w^2\right)$. Based on (6), ε_k can be estimated as

$$\hat{\varepsilon}_k = \frac{N \cdot \arg\left\{\hat{\boldsymbol{\nu}}_k\left[\vartheta_k\right]\right\}}{2\pi\vartheta_k} = \frac{N \cdot \arg\left\{e^{\frac{\jmath 2\pi\vartheta_k\varepsilon_k}{N}} + \xi_k\right\}}{2\pi\vartheta_k}. \quad (7)$$

We know that (7) is conditionally unbiased, and at a high signal-to-interference-noise ratio (SINR), the variance error of $\hat{\varepsilon}_k$ can be approximated as

$$\mathrm{Var}\left\{\hat{\varepsilon}_k\right\} \cong \frac{MN^2}{8\pi^2\vartheta_k^2 E_p}\left(\frac{E_d}{\mathcal{N}-\mathcal{N}_p} + \sigma_w^2\right). \quad (8)$$

(8) implies that $\mathrm{Var}\left\{\hat{\varepsilon}_k\right\}$ is a monotonically decreasing function of ϑ_k, and the minimum variance error is achieved if $\vartheta_k = N-1$. Unfortunately, $\boldsymbol{\Xi}_k^\dagger \boldsymbol{\Xi}_{l\neq k} = \mathbf{O}_N$ requires $\vartheta_k \neq \vartheta_{l\neq k}$, i.e., when multiple users perform uplink frequency offset estimation simultaneously, only one user can achieve the minimum frequency offset estimation variance error.

C. Optimal Pilot Design for Frequency Offset Estimation

In order to find the optimal pilots that satisfy (4), we reperent the frequency-domain indexes of the pilots for the k-th user as $(\theta_{k,1}, \ldots, \theta_{k,\mathcal{N}_p})$, where $0 \le \theta_{k,1} < \cdots < \theta_{k,\mathcal{N}_p} \le N-1$. By resolving (4), the optimal pilots is given by

$$\begin{aligned} &\tilde{\mathbf{x}}_k^p[\theta_{k,i}] = \frac{E_p}{M\mathcal{N}_p} e^{\frac{2\pi\theta_{k,i}\vartheta_k}{N}}; \\ &\text{s.t.} \quad 1 \le k \le M,\ 1 \le i \le \mathcal{N}_p, M\mathcal{N}_p \le N; \\ &\qquad \left(\theta_{k,2} - \theta_{k,1}\right)_N = \cdots \left(\theta_{k,\mathcal{N}_p} - \theta_{k,\mathcal{N}_p-1}\right)_N \\ &\qquad\qquad = \left(\theta_{k,1} - \theta_{k,\mathcal{N}_p}\right)_N. \end{aligned} \quad (9)$$

The $\tilde{\mathbf{x}}_k^p$ are constant-modulus. As $\tilde{\mathbf{x}}_k^p = \mathbf{H}_k \mathbf{x}_k^p$, in a frequency-selective fading channel, $\mathbf{x}_k^p$ are not constant-modulus and should be designed based on the estimation of $\mathbf{H}_k$. In the time variant wireless channels, $\mathbf{x}_k^p$ should be changed adaptively according to the current channel attenuation.

D. Effect of Imperfect Channel Knowledge on Frequency Offset Estimation

In the proposed frequency offset estimator, perfect channel knowledge is assumed in (2). However, in real systems, an estimation error always occurs in the channel estimation, and this error causes the performance loss of the proposed frequency offset estimator. We assume that $\hat{\mathbf{H}}_k = \mathbf{H}_k + \Delta\mathbf{H}_k$, where $\Delta\mathbf{H}_k$ represents the estimation error of $\mathbf{H}_k$. The elements of $\Delta\mathbf{H}_k$ are i.i.d. RVs and $[\Delta\mathbf{H}_k]_{ii} \sim \mathcal{CN}\left(0, \sigma_{\Delta H}^2\right)$, where $\sigma_{\Delta H}^2 \triangleq \dfrac{\mathbb{E}\left\{\mathrm{trace}\left\{\Delta\mathbf{H}_k^H \Delta\mathbf{H}_k\right\}\right\}}{\mathbb{E}\left\{\mathrm{trace}\left\{\mathbf{H}_k^H \mathbf{H}_k\right\}\right\}}$. We also define

$$\hat{\boldsymbol{\Xi}}_k = \mathrm{diag}\left\{\mathbf{F}\hat{\mathbf{H}}_k \mathbf{x}_k^p\right\} = \underbrace{\mathrm{diag}\left\{\mathbf{F}\mathbf{H}_k \mathbf{x}_k^p\right\}}_{\boldsymbol{\Xi}_k} + \underbrace{\mathrm{diag}\left\{\mathbf{F}\boldsymbol{\Delta}\mathbf{H}_k \mathbf{x}_k^p\right\}}_{\Delta\boldsymbol{\Xi}_k}. \quad (10)$$

Based on (10), ν_k is actually estimated as

$$\hat{\boldsymbol{\nu}}_k = \hat{\boldsymbol{\Xi}}_k^\dagger \mathbf{y} = \boldsymbol{\nu}_k^\perp - \hat{\boldsymbol{\Xi}}_k^\dagger \Delta\boldsymbol{\Xi}_k \boldsymbol{\nu}_k + \sum_{m=1}^{M} \hat{\boldsymbol{\Xi}}_k^\dagger \boldsymbol{\Omega}_m \boldsymbol{\nu}_m + \bar{\boldsymbol{\Xi}}_k^\dagger \mathbf{w}. \quad (11)$$

Given a high SINR, the variance error of the frequency offset estimation is approximated as

$$\mathrm{Var}\left\{\hat{\varepsilon}_k\right\} \cong \frac{N^2}{8\pi^2\vartheta_k^2}\left[\frac{M}{E_p}\left(\frac{E_d}{\mathcal{N}_k - \mathcal{N}_p} + \sigma_w^2\right) + \frac{\sigma_{\Delta H}^2}{N}\right]. \quad (12)$$

In (12), $\dfrac{\sigma_{\Delta H}^2}{N}$ represents the influence of channel estimation errors, which is much smaller than that contributed by the data vectors. Therefore, the proposed frequency offset estimation is robust to the channel estimation errors.

IV. Optimal Pilot Design for LS Channel Estimation

In this section, an LS channel estimator will be proposed to estimation CSI, which is needed for the proposed frequency offset estimation. After taking the *Discrete Fourier Transform* (DFT) of the received vector, the output can be given by

$$\begin{aligned} \tilde{\mathbf{r}} = \mathbf{F}^H\mathbf{y} &= \sum_{k=1}^{M} \underbrace{\sqrt{N}\mathbf{E}_k^{\mathrm{cir}} \mathbf{X}_k^p \mathbf{F}_{L_{max}}^H}_{\mathbf{P}_k\ (N\times L_{max})} \tilde{\mathbf{h}}_k \\ &+ \sum_{k=1}^{M} \underbrace{\sqrt{N}\mathbf{E}_k^{\mathrm{cir}} \mathbf{X}_k^d \mathbf{F}_{L_{max}}^H}_{\mathbf{D}_k\ (N\times L_{max})} \tilde{\mathbf{h}}_k + \underbrace{\mathbf{F}^H\mathbf{w}}_{\boldsymbol{\eta}\ (N\times 1)} = \mathbf{P}\mathbf{h} + \mathbf{D}\mathbf{h} + \boldsymbol{\eta}, \end{aligned} \quad (13)$$

where $\mathbf{E}_k^{\mathrm{cir}} = \mathbf{F}^H\mathbf{E}_k\mathbf{F}$, $\mathbf{F}_{L_{max}}$ is the first L_{max} rows of $\mathbf{F}$, $\mathbf{X}_k^d = \mathrm{diag}\{\mathbf{x}_k^d\}$, $\mathbf{X}_k^p = \mathrm{diag}\{\mathbf{x}_k^p\}$ with $\mathbf{x}_k^d$ and $\mathbf{x}_k^p$ being the $N \times 1$ data and pilot vectors of the k-th user. In (13), $\mathbf{P} = [\mathbf{P}_1, \cdots, \mathbf{P}_M]$, $\mathbf{D} = [\mathbf{D}_1, \cdots, \mathbf{D}_M]$, and $\mathbf{h} = \left[\tilde{\mathbf{h}}_1^T, \cdots, \tilde{\mathbf{h}}_M^T\right]^T$.

A. LS Channel Estimation

When performing LS channel estimation, $\mathbf{P}$ should be full-column rank, which requires that $N \ge ML_{max}$. Since the frequency offsets can be estimated with negligible errors, $\mathbf{P}$ instead of the estimation of $\mathbf{P}$, is used in the channel estimation. By defining $\mathbf{P}^\dagger = \left(\mathbf{P}^H\mathbf{P}\right)^{-1}\mathbf{P}^H$, the LS estimation of $\mathbf{h}$ is given by

$$\hat{\mathbf{h}}_{\mathrm{LS}} = \mathbf{P}^\dagger\tilde{\mathbf{r}} = \mathbf{h} + \mathbf{P}^\dagger\mathbf{D}\mathbf{h} + \mathbf{P}^\dagger\boldsymbol{\eta}, \quad (14)$$

where $\mathbf{P}^\dagger\mathbf{D}\mathbf{h}$ is an interference contributed by the data vectors. An conditionally unbiased estimator estimator can be achieved if and only if $\mathbb{E}\left\{\mathbf{P}^\dagger\mathbf{D}\mathbf{h}\right\} = \mathbf{0}_{ML_{max}\times 1}$. The MSE of $\hat{\mathbf{h}}_{\mathrm{LS}}$ can be given by

$$\begin{aligned} \mathrm{MSE}\left(\hat{\mathbf{h}}_{\mathrm{LS}}\right) &= \frac{1}{ML_{max}}\mathbb{E}\left\{\left\|\hat{\mathbf{h}}_{\mathrm{LS}} - \mathbf{h}\right\|_2^2\right\} \\ &= \frac{\mathrm{trace}\left\{\mathbf{V}^H\left(\mathbf{P}^H\mathbf{P}\right)^{-2}\mathbf{V}\boldsymbol{\Phi}\right\}}{ML_{max}} + \frac{\sigma_w^2\mathrm{trace}\left\{\left(\mathbf{P}^H\mathbf{P}\right)^{-1}\right\}}{ML_{max}}, \end{aligned} \quad (15)$$

where $\boldsymbol{\Phi} = \mathbb{E}\{\mathbf{h}\mathbf{h}^{\mathbf{H}}\}$, $\mathbf{V} = \mathbf{P}^H\mathbf{D}$ and $\mathbf{V}$ represents the power spread of the data subcarriers to the signal space of the pilots.

B. The Optimal Pilot Design and Placement

Before designing the optimal pilots, we first analyze $\mathbf{P}^H\mathbf{P}$:

$$\mathbf{P}^H\mathbf{P} = \begin{bmatrix} \mathbf{G}_{1,1} & \dots & \mathbf{G}_{1,M} \\ \vdots & \ddots & \vdots \\ \mathbf{G}_{M,1} & \dots & \mathbf{G}_{M,M} \end{bmatrix}, \tag{16}$$

where $\mathbf{G}_{k,i} = N\mathbf{F}_{L_{max}}\mathbf{X}_k^{pH}\mathbf{E}_{k,i}^{\text{cir}}\mathbf{X}_i^p\mathbf{F}_{L_{max}}^H$ and $\mathbf{E}_{k,i}^{\text{cir}} = \mathbf{F}^H\mathbf{E}_i^H\mathbf{E}_k\mathbf{F}$. To minimize the MSE$(\hat{\mathbf{h}}_{\text{LS}})$, the following conditions should be satisfied simultaneously:

Presupposition:

1. $\mathbf{G}_{k,k} = \dfrac{E_p}{M}\mathbf{I}_{L_{max}}$, $1 \le k \le M$.
2. $\mathbf{G}_{k,i\neq k} = \mathbf{O}_{L_{max}}$ for each $(k, i \neq k)$.

Let the frequency domain indexes of the pilots for the k-th user be $(\theta_{k,1}, \cdots, \theta_{k,\mathcal{N}_p})$, the condition 1 of **Presupposition** requires $\left|[\mathbf{X}_m^p]_{\theta_{k,z}\theta_{k,z}}\right|^2 = \dfrac{E_p}{M\mathcal{N}_p}$. Condition 2 of **Presupposition** can be rewritten as $\mathbf{F}_{L_{max}}\mathbf{X}_k^{pH}\mathbf{E}_{k,i}^{\text{cir}}\mathbf{X}_n^p\mathbf{F}_{L_{max}}^H = \mathbf{O}_{L_{max}}$, which requires the following conditions to be satisfied in the pilots design and placements:

$$\begin{aligned} &\arg\{\mathbf{x}_k^p[\theta_{k,z}]\} = \frac{2\pi\theta_{k,z}K_p(k-1)}{N}; \\ &\text{s.t.}\quad ML_{max} \le \mathcal{N}_p \le N, M\mathcal{N}_p \le N, \frac{N}{\mathcal{N}_p} = \text{integer}; \\ &\qquad K_p \ge L_{max}, \ \frac{\theta_{k,z}K_p(k-i)}{N} \neq \text{integer}; \\ &\qquad (\theta_{k,2}\cdot l - \theta_{k,1}\cdot l)_N = \cdots \left(\theta_{k,\mathcal{N}_p}\cdot l - \theta_{k,\mathcal{N}_p-1}\cdot l\right)_N \\ &\qquad\quad = \left(\theta_{k,1}\cdot l - \theta_{k,\mathcal{N}_p}\cdot l\right)_N, l = 1, 2, \cdots, L_{max}-1; \end{aligned} \tag{17}$$

From (9) and (17), we can see that optimal pilots should be uniformly distributed in the frequency-domain for both frequency offset estimation and channel estimation.

C. Effect of Imperfect Frequency Offset Estimation on Channel Estimation

The pilot design and placement requirements in (17) are based on the perfect frequency offset knowledge, which is not always abtainable in real systems. By using $\wp_k$ to represent the estimation error of ε_k with $\wp_k \sim \mathcal{N}(0, \sigma_\wp^2)$, $\mathbf{E}_k$ can be estimated as $\hat{\mathbf{E}}_k \cong \mathbf{E}_k + \Delta\mathbf{E}_k$, where $\Delta\mathbf{E}_k = \jmath\wp_k\mathbf{\Pi}\mathbf{E}_k$, and $\mathbf{\Pi} = \text{diag}\left\{0, \dfrac{2\pi}{N}, \cdots, \dfrac{2\pi\times(N-1)}{N}\right\}$.

By using $\hat{\mathbf{E}}_k$ instead of $\mathbf{E}_k$ and using $\hat{\mathbf{P}}$ instead of $\mathbf{P}$ in (13), the LS channel estimator can be expressed by $\hat{\mathbf{h}}_{\text{LS}} = \hat{\mathbf{P}}^\dagger\tilde{\mathbf{r}}$. The MSE of the LS estimator in the presence of the frequency offset estimation errors can be expressed as

$$\begin{aligned} \text{MSE}\left(\hat{\mathbf{h}}_{\text{LS}}\right) = &\frac{M^2\text{trace}\left\{\mathbf{J}\mathbf{J}^H\mathbf{\Phi}\right\}}{L_{max}E_p^2}\cdot\sigma_\wp^2 \\ &+ \frac{M^2\text{trace}\left\{\mathbf{V}^H\mathbf{V}\mathbf{\Phi}\right\}}{L_{max}E_p^2} + \frac{M\sigma_w^2}{E_p}, \end{aligned} \tag{18}$$

where $\mathbf{J} = \text{diag}\{\mathbf{J}_1, \cdots, \mathbf{J}_M\}$ and

$$\mathbf{J}_i = N\mathbf{F}_{L_{max}}\mathbf{X}_i^{pH}\mathbf{F}^H\mathbf{\Pi}\mathbf{F}\mathbf{X}_i^p\mathbf{F}_{L_{max}}^H, 1 \le i \le M.$$

V. Joint Frequency Offsets and Channel Estimation

The CSI is assumed to be available for the porposed frequency offset estimation, vice versa. The frequency offset and channel information, therefore, should be estimated jointly, which is shown in Fig. 1.

At the beginning, the receiver should be switched to "1" to perform the initial frequency offset estimation. Since the receiver has not performed channel estimation, a constant-envelope pilot that satisfies (9) should be used by each user, i.e., taking $\mathbf{x}_k^p$ instead of $\tilde{\mathbf{x}}_k^p$. Two consecutive blocks of the pilots, $\mathbf{x}_k^p(1)$ and $\mathbf{x}_k^p(2)$, which are specified by $\vartheta_{k,1}$ and $\vartheta_{k,2}$, are transmitted by the k-th user. Note that $\vartheta_{k,1}$ is not necessarily identical to $\vartheta_{k,2}$. We assume that the channel does not change during a two-symbol period. Let $\hat{\boldsymbol{\nu}}_{k,1}$ and $\hat{\boldsymbol{\nu}}_{k,2}$ denote the output vectors of the demodulation (6). The initial frequency offset estimation can then be performed as

$$\hat{\varepsilon}_k^{\text{Ini}} = \frac{N\cdot\arg\{\hat{\boldsymbol{\nu}}_{k,2}[\vartheta_{k,2}]\times(\hat{\boldsymbol{\nu}}_{k,1}[\vartheta_{k,1}]^*)\}}{2\pi(N+\vartheta_{k,2}-\vartheta_{k,1}-1)}. \tag{19}$$

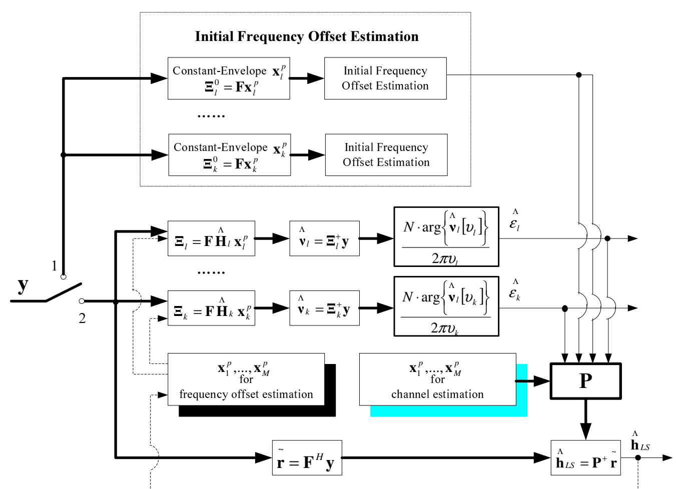

Fig. 1. Joint frequency offsets and channel estimation.

Then, the receiver should be switched to "2" to perform the channel estimation and the frequency offset tracking. We first generate the optimal pilots for the channel estimation based on (17). The frequency offset estimation results are used to generate the matrix $\mathbf{P}$. The channel attenuation can be estimated with a high accuracy by using the proposed estimator, and this estimation result will be used to adaptively optimize the pilots for the frequency offset estimation presented in (9).

VI. Numerical Results

In our simulation, quasi-static OFDMA wireless channels are assumed and path gains $h_k(l) = e^{-l}$ are considered with $L_{max} = 4$. The normalized pilot-to-noise ratio (NPNR), defined as NPNR $= \dfrac{E_p}{M\cdot\mathcal{N}_p\cdot\sigma_w^2}$, is used instead of SNR on each pilot subcarrier. The average power of data is specified

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as a rate to the power of pilots, i.e., $\dfrac{E_d}{\mathcal{N}-\mathcal{N}_p} = \rho \cdot \dfrac{E_p}{\mathcal{N}_p}$, where ρ is a positive and real number.

The impact of channel estimation errors on the performance of the frequency offset estimation is shown in Fig. 2, where σ_H^2 denotes the variance of channel estimation error. When $\rho = 0.1$, there is about 2 dB performance improvement as σ_H^2 decreases from 10^{-1} to 10^{-2}. However, the performance improvement is negligible from $\sigma_H^2 = 10^{-2}$ to a perfect channel estimation. As the accuracy of $\sigma_H^2 = 10^{-2}$ is easy to achieve, the performance loss in frequency offset estimation due to the channel estimation error is negligible. The performance of the frequency offset estimation can be improved considerably by decreasing ρ. When no data subcarrier is modulated, i.e. $\rho = 0$, the CRLB can nearly be achieved at a high NPNR.

The performance of the LS channel estimation with imperfect frequency offset estimation is also evaluated and shown in Fig. 3, with the variance of frequency offset estimation error denoted as $\sigma_\wp^2$. Compared to reducing the frequency offset estimation errors, reducing the average power of data subcarriers contributes a negligible performance improvement. Keeping $\sigma_\wp^2$ to be 10^{-2} unchanged, there is about 0.5 dB performance improvement that can be achieved by decreasing ρ from 0.5 to 0.1. For a given $\rho = 0.5$, the performance improvement about 4 dB can be achieved by decreasing $\sigma_\wp^2$ from 10^{-3} to 10^{-4}. A performance floor will always appear and the CRLB cannot be achieved at a high NPNR.

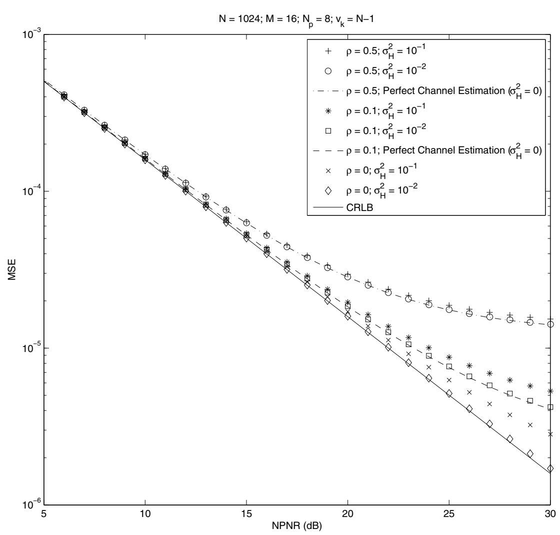

Fig. 2. MSE of frequency offset estimation with imperfect channel estimation.

VII. Conclusions

In this paper, we discussed the optimal pilots design and placement for the frequency offset and channel estimation in an OFDMA uplink transmission. A frequency offset estimation can be performed by adaptively modulating the pilots for each user based on the current channel estimation results. By

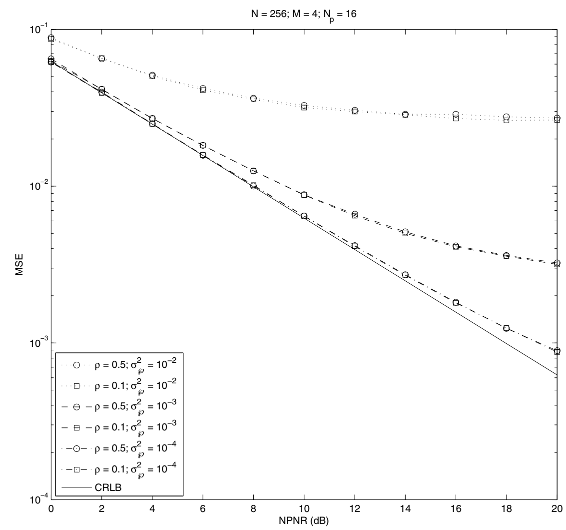

Fig. 3. MSE of LS channel estimation in the presence of frequency offset estimation errors.

employing a joint frequency offset and channel estimation scheme, the frequency offset estimation result can be fed back to the transmitter to adaptively optimize the channel estimation, and vice versa.

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