SPECTRUM SHARING IN WIRELESS NETWORKS: A QoS-AWARE SECONDARY MULTICAST APPROACH WITH WORST USER PERFORMANCE OPTIMIZATION

Khoa T. Phan†, *Sergiy A. Vorobyov*†, *Nicholas D. Sidiropoulos*‡, *and Chintha Tellambura*†

†Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB, CANADA; Email: khoa, vorobyov, chintha@ece.ualberta.ca

‡ Department of Electronic and Computer Engineering, Technical University of Crete, Chania, GREECE; Email: nikos@telecom.tuc.gr

ABSTRACT

Secondary spectrum usage has the potential to considerably increase spectrum utilization. In this paper, quality-of-service (QoS)-aware spectrum underlay is considered. A multi-antenna access point transmits common information to a set of secondary receivers, by means of appropriate beamforming; the objective is to maximize the multicast beamforming rate (or, equivalently, minimize the signal to noise ratio) for the secondary receivers, while explicitly limiting interference to primary receivers. The formulation incorporates the pragmatic case of inaccurate channel state information (CSI) for both primary and secondary users. Simulation examples illustrate the effectiveness of the proposed approach.

1. INTRODUCTION

While almost all usable frequencies have already been licensed, actual measurements indicate that portions of licensed spectrum remain unused for large amounts of time, space, and frequency [1]. One of the approaches allowing for improved bandwidth efficiency is the introduction of secondary spectrum licensing. Usually, the secondary users first listen to the environment, then decide to transmit if there exist channels that are not currently used by primary users, in so-called *spectrum overlay* mode [2].

In our work, we investigate the spectrum sharing problem from the *spectrum underlay* perspective [2]. The concept of *'interference temperature'* has been introduced in [3], and it indicates the allowable interference level at the primary receivers. Through the use of beamforming and power control techniques, the interference to the primary network can be effectively controlled. Therefore, even when the primary users are operating, the network of secondary users is able to exchange information continuously without any need for channel sensing. In traditional cellular systems, the aforementioned techniques are well-known and are used to control co-channel interference [4]- [6]. Note that in [4], the authors consider the transmission of independent information to each of the downlink users, while the multicast scenario is considered in [5], [7], [8]. In the context of cognitive networks, the transmit power control and dynamic spectrum management problem has been initiated in [3]. In [9], two iterative algorithms for jointly optimal power control and beamforming have been proposed.

In this paper, we consider a secondary downlink multicast network, where the secondary access point (AP) is equipped with an antenna array and the objective is to transmit a *common* data stream to all secondary users. The AP uses transmit beamforming to direct signal power towards all secondary users while limiting interference to primary users. In this scenario, the design of the transmit beamformer is formulated as an optimization problem. The objective is to maximize the multicast beamforming rate for the secondary network while ensuring that the interference power at every primary receiver is under a certain threshold. Maximizing the (common) multicast beamforming rate for the secondary system is equivalent to maximizing the minimum received SNR over all secondary users [5]. Both cases of perfect and imperfect CSI knowledge are considered. The proposed problem formulations contain the one in [5] as a special case; the latter is NP-hard, which implies that the former are NP-hard as well. Approximate solutions, however, can be efficiently generated using semidefinite relaxation (SDR) techniques.

2. SYSTEM MODEL

The network which consists of several secondary users in the presence of multiple primary transmitter-receiver links is considered. An example of such network can be the temporary deployment of a secondary wireless local area net-

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work (WLAN) in the area of an existing primary WLAN. The particular scenario considered here is one in which the secondary WLAN AP transmits common information to all secondary users. The secondary AP (or base station) is equipped with M antennas while each of N secondary and K primary users has single antenna. Since the primary and secondary networks coexist, the operation of the latter must not cause excessive interference to the former. This can be accomplished in two ways. One is to severely limit the total transmission power of the secondary AP, which will limit the interference to any primary receiver irrespective of the associated coupling channel vector, by virtue of the Cauchy-Schwart inequality. The drawback of this approach is that it is typically over-constrains the transmission power and thus the spectral efficiency of the secondary network. A more appealing alternative for the secondary AP is to estimate the channel vectors between its antenna array and the primary receivers and use beamforming techniques. If the primary system operates in a time-division duplex mode, this can be accomplished by monitoring primary transmissions in the reverse link. Otherwise, blind beamforming techniques could be employed. Alternatively, the primary system could cooperate (under a 'sublet' agreement) with the secondary system to pass along channel estimates (see also [9], [10] and references therein) - albeit this is far less appealing from a practical standpoint. In a nutshell, although perfect cannel state information (CSI) will not be available in the considered scenario, very accurate CSI can be obtained in certain, for example fixed wireless or low-mobility cases. Either way, (approximate or partials/statistical) knowledge of the primary channel vectors enables (approximate) spatial nulling to protect the primary receivers while directing higher power towards the secondary receivers - thereby increasing the transmission rate for the secondary system.

Let $\mathbf{h}_i$, $\mathbf{g}_k$ denote the $M \times 1$ complex vectors which model the channel gains from M transmit antennas to the secondary user i, $i = 1, \ldots, N$ and to the receiver of the primary link k, $k = 1, \ldots, K$, respectively. Also let $\mathbf{w}$ denote the beamforming weight vector applied to the transmit antenna elements. If the transmitted signal is zero-mean and white with unit variance, and the noise at ith receiver is zero-mean and white with variance σ_i^2, then the received SNR of the ith user can be expressed as

$$\mathrm{SNR}_i = \frac{|\mathbf{w}^H \mathbf{h}_i|^2}{\sigma_i^2}. \quad (1)$$

Note that for the sake of simplicity, we do not consider the interference caused by primary users. As long as the secondary receivers know the interference level, our model can be easily extended to include this information. The interference power to the receiver of the primary link k is given by $|\mathbf{w}^H \mathbf{g}_k|^2$, $k = 1, \ldots, K$. We should note here that (slow rate) reverse link communications from N users to the AP, for example, for the purpose of channel estimation, may also cause interference to the primary users. Here, we only consider the interference caused by the downlink transmission from the AP.

3. BEAMFORMING IN SECONDARY WIRELESS NETWORKS

3.1. Beamforming with Perfect CSI

The case of perfect CSI is first considered. Although perfect CSI is typically not available in the considered scenario, very accurate CSI can be obtained in certain (fixed wireless, or low-mobility) cases. Moreover, the case of perfect CSI will serve as a stepping stone towards developing more realistic robust beamforming designs for the case of inaccurate CSI, as we will see in the sequel. With perfect CSI, the beamformer design problem can be written as

$$\max_{\mathbf{w}} \quad \min_{i=1,\ldots,N} \left\{ \frac{|\mathbf{w}^H \mathbf{h}_i|^2}{\sigma_i^2} \right\} \quad (2a)$$

$$\text{subject to} \quad \|\mathbf{w}\|_2^2 \le P \quad (2b)$$

$$|\mathbf{w}^H \mathbf{g}_k|^2 \le \eta_0, \ k = 1, \ldots, K \quad (2c)$$

where $\|\cdot\|_2^2$ denotes the Euclidean norm of a vector. The constraint (2b) on the transmit power is essential due to the power limitation at the transmitter, while the constraints (2c) keep the interference to the primary users below some threshold η_0. It can be observed that *one* of the constraints in the problem (2a)-(2c) should be satisfied with equality at optimality. Otherwise, the optimal beamforming weight vector $\mathbf{w}$ could always be scaled up, and thus, improving the objective function. Moreover, we should also note that the optimization problem (2a)-(2c) is always feasible no matter how many primary links are present.

Introducing a new variable t, the optimization problem (2a)-(2c) can be equivalently rewritten as

$$\min_{\mathbf{w},\, t \ge 0} \quad -t \quad (3a)$$

$$\text{subject to} \quad \frac{|\mathbf{w}^H \mathbf{h}_i|^2}{\sigma_i^2} \ge t, \ i = 1, \ldots, N \quad (3b)$$

$$\|\mathbf{w}\|_2^2 \le P \quad (3c)$$

$$|\mathbf{w}^H \mathbf{g}_k|^2 \le \eta_0, \ k = 1, \ldots, K. \quad (3d)$$

It is easy to check that the constraints (3c)-(3d) are convex on $\mathbf{w}$ and t. However, the constraints (3b) are nonlinear and *nonconvex* on $\mathbf{w}$ and t. Moreover, the problem (3a)-(3d) belongs to the class of semi-infinite nonconvex quadratic programming problems. It is well known that a general nonconvex quadratically constrained quadratic programming (QCQP) problem is NP-hard and, therefore, intractable. The particular problem above contains the one in [5] as a special case, and is therefore NP-hard. Fortunately, efficient approximate solutions can be generated using semidefinite relaxation. Note that in [5], the constraint on total transmission power must be met with equality; this is not the case for (3a)-(3d), due to the presence of the primary interference constraints.

Using the fact that $\mathbf{h}_i^H \mathbf{w}\mathbf{w}^H \mathbf{h}_i = \text{trace}(\mathbf{w}\mathbf{w}^H \mathbf{h}_i \mathbf{h}_i^H)$, the optimization problem (3a)-(3d) can be rewritten as follows

$$\min_{\mathbf{w},\, t\geq 0} \quad -t \tag{4a}$$

$$\text{subject to} \quad \text{trace}(\mathbf{w}\mathbf{w}^H \mathbf{H}_i) \geq t,\ i = 1, \ldots, N \tag{4b}$$

$$\text{trace}(\mathbf{w}\mathbf{w}^H) \leq P \tag{4c}$$

$$\text{trace}(\mathbf{w}\mathbf{w}^H \mathbf{G}_k) \leq \eta_0,\ k = 1, \ldots, K \tag{4d}$$

where trace$(\cdot)$ denotes the trace of a matrix, and the new notations $\mathbf{H}_i = \frac{1}{\sigma_i^2}\mathbf{h}_i\mathbf{h}_i^H,\ i = 1, \ldots, N$, $\mathbf{G}_k = \mathbf{g}_k\mathbf{g}_k^H,\ k = 1, \ldots, K$ are introduced.

Introducing also a new variable $\mathbf{X} = \mathbf{w}\mathbf{w}^H$ with $\mathbf{X}$ being symmetric positive semi-definite matrix, i.e., $\mathbf{X} \succcurlyeq \mathbf{0}$, the problem (4a)-(4d) can be equivalently reformulated as

$$\min_{\mathbf{X},\, t\geq 0} \quad -t \tag{5a}$$

$$\text{subject to:} \quad \text{trace}(\mathbf{X}\mathbf{H}_i) \geq t,\ i = 1, \ldots, N \tag{5b}$$

$$\text{trace}(\mathbf{X}) \leq P \tag{5c}$$

$$\text{trace}(\mathbf{X}\mathbf{G}_k) \leq \eta_0,\ k = 1, \ldots, K \tag{5d}$$

$$\mathbf{X} \succcurlyeq \mathbf{0},\ \text{rank}(\mathbf{X}) = 1. \tag{5e}$$

The objective function and the trace constraints in (5a)-(5e) are linear and, hence, convex on $\mathbf{X}$ and t. While the set of symmetric positive semidefinite matrices is convex, the rank constraint rank$(\mathbf{X}) = 1$ is not. However, by dropping the rank-one constraint, we obtain a SDP problem, which can be efficiently solved using interior point methods, at a complexity cost that is at most $\mathcal{O}((N+K+2+M^2)^{3.5})$ [5].

3.1.1. *Randomization*

Let $\mathbf{X}_{\text{opt}}$ denote the optimal solution to the problem (5a)-(5e). In order to recover the beamforming vector $\mathbf{w}_{\text{opt}}$ form the matrix $\mathbf{X}_{\text{opt}}$ the following randomization procedure is proposed. If the matrix $\mathbf{X}_{\text{opt}}$ is rank-one, then the optimal weight vector can be recovered from $\mathbf{X}_{\text{opt}}$ straightforwardly, using the principal eigenvector corresponding to the only non-zero eigenvalue. However, because of the SDR step, $\mathbf{X}_{\text{opt}}$ will not be rank-one in general, and the so-called *randomization* has to be used. Various randomization techniques have been developed so far, see [11], [12] and references therein. A common idea of these techniques in application to our problem is to generate a set of L candidate vectors $\{\tilde{\mathbf{w}}_{\text{cand},l}\}_{l=1}^{L}$ using $\mathbf{X}_{\text{opt}}$ and choose the best solution from these candidate vectors.

To obtain the candidate vectors, the eigen-decomposition of the matrix $\mathbf{X}_{\text{opt}}$ in the form $\mathbf{X}_{\text{opt}} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{U}^H$ is first computed. Then, the vector $\tilde{\mathbf{w}}_{\text{cand},l} = \mathbf{U}\boldsymbol{\Sigma}^{1/2}\mathbf{v}_l$ is selected as a candidate vector where $\mathbf{v}_l$ is uniformly distributed on the unit sphere. This ensures that the power constraint (2b) is satisfied, i.e.,

$$\|\tilde{\mathbf{w}}_{\text{cand},l}^H\|^2 = \text{trace}(\mathbf{X}_{\text{opt}}) \leq P \tag{6}$$

for any realization of $\mathbf{v}_l$. It is also necessary to check if the interference constraints (2c) are satisfied. If all K interference constraints are satisfied as inequalities, the objective (2a) can be increased by scaling the candidate beamforming vector $\tilde{\mathbf{w}}_{\text{cand},l}$ up by

$$\alpha = \min \left\{ \frac{P}{\|\tilde{\mathbf{w}}_{\text{cand},l}\|^2};\ \left.\frac{\eta_0}{|\tilde{\mathbf{w}}_{\text{cand},l}^H \mathbf{g}_k|^2}\right|_{k=1,\ldots,K} \right\} \geq 1. \tag{7}$$

If at least one of K interference constraints is not satisfied, the candidate beamforming vector $\tilde{\mathbf{w}}_{\text{cand},l}$ must be scaled down by

$$\beta = \min_{k=1,\ldots,K} \left\{ \frac{\eta_0}{|\tilde{\mathbf{w}}_{\text{cand},l}^H \mathbf{g}_k|^2} \right\} \leq 1 \tag{8}$$

The so obtained new scaled candidate beamforming vector always satisfies both the power constraint (2b) and the interference constraints (2c). Therefore, the sub-optimal beamforming vector is the new scaled candidate vector which yields the largest $\min_{i=1,\ldots,N} \left\{ \frac{|\tilde{\mathbf{w}}_{\text{cand},l}^H \mathbf{h}_i|^2}{\sigma_i^2} \right\}$ and, therefore, provides the maximum to the objective (2a).

3.2. Beamforming with Imperfect CSI

The previously considered assumption of perfect CSI is not always practical due to the time-varying nature of wireless propagation channels and the mobility of the users. Therefore, we propose an approach to robust beamforming design in the case of erroneous CSI which uses the concept of worst-case design (see, e.g., [6] and references therein). Specifically, assuming that all channel vectors are known with certain errors $\boldsymbol{\delta}$ and that these errors are all norm-bounded[1], that is, $\|\boldsymbol{\delta}\| \leq \varepsilon$ where the parameter ε is assumed to be known, the robust modification of the beamforming problem (2a)-(2c) can be written as

$$\max_{\mathbf{w}} \quad \min_{i=1,\ldots,N} \left\{ \min_{\|\boldsymbol{\delta}_i\|\leq\varepsilon} \frac{\left|\mathbf{w}^H(\mathbf{h}_i + \boldsymbol{\delta}_i)\right|^2}{\sigma_i^2} \right\} \tag{9a}$$

$$\text{subject to} \ \max_{\|\boldsymbol{\delta}_k\|\leq\varepsilon} \left|\mathbf{w}^H(\mathbf{g}_k + \boldsymbol{\delta}_k)\right|^2 \leq \eta_0, k = 1, \ldots, K \tag{9b}$$

$$\|\mathbf{w}\|_2^2 \leq P \tag{9c}$$

[1]Note that no statistical information about the channel error is required in this approach except for its norm upper-bound.

or, equivalently, as

$$\min_{\mathbf{w}, t \geq 0} \quad -t \tag{10a}$$

$$\text{subject to} \quad \min_{\|\boldsymbol{\delta}_i\| \leq \varepsilon} \frac{\left|\mathbf{w}^H(\mathbf{h}_i + \boldsymbol{\delta}_i)\right|^2}{\sigma_i^2} \geq t, i = 1, \ldots, N \tag{10b}$$

$$\max_{\|\boldsymbol{\delta}_k\| \leq \varepsilon} |\mathbf{w}^H(\mathbf{g}_k + \boldsymbol{\delta}_k)|^2 \leq \eta_0, k = 1, \ldots, K \tag{10c}$$

$$\|\mathbf{w}\|_2^2 \leq P. \tag{10d}$$

To simplify the problem, we modify the inequality constraints (10b)-(10c) using the approach similar to the one developed in [7] and [8]. Particularly, using the triangle inequality, we can write that

$$\left|\mathbf{w}^H(\mathbf{g}_k + \boldsymbol{\delta}_k)\right| \leq |\mathbf{w}^H \mathbf{g}_k| + |\mathbf{w}^H \boldsymbol{\delta}_k|. \tag{11}$$

Applying the Cauchy-Schwarz inequality, we also can write that

$$|\mathbf{w}^H \boldsymbol{\delta}_k| \leq \|\mathbf{w}\| \|\boldsymbol{\delta}_k\| \leq \varepsilon \|\mathbf{w}\| \tag{12}$$

where we have also used the fact that $\|\boldsymbol{\delta}_k\| \leq \varepsilon$. Hence,

$$\max_{\|\boldsymbol{\delta}_k\| \leq \varepsilon} |\mathbf{w}^H \boldsymbol{\delta}_k| = \varepsilon \|\mathbf{w}\|. \tag{13}$$

Substituting (13) into (11), we obtain

$$\max_{\|\boldsymbol{\delta}_k\| \leq \varepsilon} \left|\mathbf{w}^H(\mathbf{g}_k + \boldsymbol{\delta}_k)\right|^2 \leq \left(|\mathbf{w}^H \mathbf{g}_k| + \varepsilon \|\mathbf{w}\|\right)^2. \tag{14}$$

Expanding the right hand side of (14), we have

$$\begin{aligned}\left(|\mathbf{w}^H \mathbf{g}_k| + \varepsilon \|\mathbf{w}\|\right)^2 &= \left|\mathbf{w}^H \mathbf{g}_k\right|^2 + \varepsilon^2 \|\mathbf{w}\|^2 \\ &+ 2\varepsilon \|\mathbf{w}\| |\mathbf{w}^H \mathbf{g}_k| \\ &\leq \left|\mathbf{w}^H \mathbf{g}_k\right|^2 + \varepsilon^2 \|\mathbf{w}\|^2 \\ &+ 2\varepsilon \|\mathbf{w}\|^2 \|\mathbf{g}_k\| \\ &= \left|\mathbf{w}^H \mathbf{g}_k\right|^2 + \varepsilon(\varepsilon + 2\|\mathbf{g}_k\|)\|\mathbf{w}\|^2 \\ &= \mathbf{w}^H \tilde{\mathbf{G}}_k \mathbf{w} \end{aligned} \tag{15}$$

where the Cauchy-Schwarz inequality has been used again in the second line and the matrix $\tilde{\mathbf{A}}_i$ is computed as

$$\tilde{\mathbf{G}}_k = \mathbf{g}_k \mathbf{g}_k^H + \varepsilon\left(\varepsilon + 2\sqrt{\mathbf{g}_k^H \mathbf{g}_k}\right)\mathbf{I}. \tag{16}$$

Similarly, the left-hand side of the constraint (10b) can be upper bounded as follows

$$\min_{\|\boldsymbol{\delta}_k\| \leq \varepsilon} |\mathbf{w}^H(\mathbf{h}_i + \boldsymbol{\delta}_i)|^2 \leq \mathbf{w}^H \tilde{\mathbf{H}}_i \mathbf{w} \tag{17}$$

where the matrix $\tilde{\mathbf{H}}_i = \mathbf{h}_i \mathbf{h}_i^H + \varepsilon\left(\varepsilon - 2\sqrt{\mathbf{h}_i^H \mathbf{h}_i}\right)\mathbf{I}$, and the following triangle inequality has been used

$$|\mathbf{w}^H(\mathbf{h}_i + \boldsymbol{\delta}_i)| \geq |\mathbf{w}^H \mathbf{h}_i| - |\mathbf{w}^H \boldsymbol{\delta}_i|. \tag{18}$$

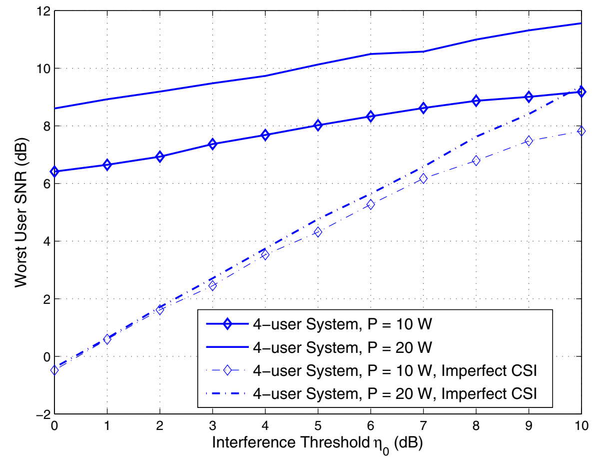

Fig. 1. Worst-user SNR versus interference threshold η_0.

The latter inequality holds true if $|\mathbf{w}^H \mathbf{h}_i| \geq |\mathbf{w}^H \boldsymbol{\delta}_i|$. It implies that $|\mathbf{w}^H \mathbf{h}_i| \geq \varepsilon \|\mathbf{w}\|$, i.e., the norm of the channel estimation error $\|\boldsymbol{\delta}_i\|$ is small enough.

Using the above results, we can modify the original problem (10a)-(10d), and then relax the modified problem as the following SDP problem

$$\min_{\mathbf{X},\, t \geq 0} \quad -t \tag{19a}$$

$$\text{subject to} \quad \operatorname{trace}(\mathbf{X}\tilde{\mathbf{H}}_i) \geq \sigma_i^2 t, \ i = 1, \ldots, N \tag{19b}$$

$$\operatorname{trace}(\mathbf{X}\tilde{\mathbf{G}}_k) \leq \eta_0, \ k = 1, \ldots, K \tag{19c}$$

$$\operatorname{trace}(\mathbf{X}) \leq P. \tag{19d}$$

Again, one can use randomization to generate candidate beamforming vectors in this case.

4. SIMULATION RESULTS

We consider two system configurations: The first configuration has a secondary network with 4-antenna AP and four users, the second configuration has a secondary network with 4-antenna AP and eight users. The standard independent identically distributed (i.i.d.) Rayleigh fading channel model is assumed with noise variance $\sigma_i^2 = 1, \forall i$. Only one primary link is considered and it is assumed that all users have the same QoS constraints. For the case of imperfect CSI, the error vector $\boldsymbol{\delta}$ is uniformly randomly generated in a sphere centered at zero with the radius $\varepsilon = 0.1$. Note that, in general, the radius ε depends on the accuracy of the channel estimation. Larger transmit power may provide better channel estimates.

Fig. 1 displays the max-min SNR versus the interference threshold η_0 for $P = 10, 20$ W. It can be seen that when interference tolerance increases, the performance improves

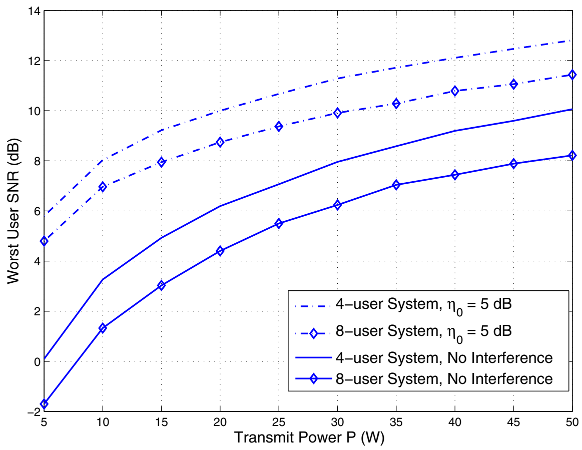

Fig. 2. Worst-user SNR versus transmit power P.

which is also the case when the transmit power increases. As expected, coarse CSI can substantially reduce the attainable beamforming gain.

Fig. 2 shows the SNR of the worst user against the transmit power P for two configurations with different interference thresholds. It is noticeable that the performance of the worst user in the 4-user network is better than that of its counterpart in the 8-user network. Furthermore, when the interference threshold becomes larger, the performance of the users in each network improves. Mathematically, it correspond to expanding the feasible set of the proposed beamforming problems, thus improving the objective function.

5. CONCLUSIONS

The multicast beamforming problem for secondary wireless networks has been addressed. The objective has been to optimize the performance of the secondary network under the constraint that interference to the primary network is below a certain tolerance level. Two problem formulations have been considered for situations of perfect and imperfect CSI. Although the proposed designs are nonconvex and NP-hard, a convex relaxation approach coupled with suitable randomization post-processing can provide approximate solutions at a moderate computational cost that is strictly bounded by a low-order polynomial. Our approach can also be applicable in conventional cellular systems when broadcasting to a number of receivers and at the same time protecting some specific 'directions' from interference.

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