Transactions Letters

Quasi-Orthogonal STBC with Minimum Decoding Complexity: Performance Analysis, Optimal Signal Transformations, and Antenna Selection Diversity

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***Abstract*—This letter presents a new method to directly analyze and optimize symbol error rate (SER) performance of minimum decoding complexity (MDC) ABBA space-time block codes based on a tight union bound on SER. Additionally, a new signal transformation for rectangular quadrature amplitude modulation is proposed to provide better performance than the existing ones with lower encoding/decoding complexities. It is also shown that MDC-ABBA codes achieve full-diversity with antenna selection and limited feedback.**

***Index Terms*—Quasi-orthogonal space-time block codes, ABBA codes, performance analysis.**

I. Introduction

ABBA codes [1], a class of space-time block codes (STBC), have been proposed to increase the code rate of orthogonal STBC (OSTBC) [2]. Recently, Yuen *et al.* [3] have shown that ABBA codes enable pairwise real-symbol (PWRS) decoding, which is the *minimum decoding complexity* (MDC) achievable by non-orthogonal STBC. This fact makes MDC-ABBA codes attractive for practical applications.

To design MDC-ABBA codes with full-diversity, conventional signals, like quadrature amplitude modulation (QAM) or phase-shift keying (PSK), need to be transformed [3], [4]. The authors in [3], [4] employ the coding gain metric [5] to derive the optimal signal transformations for QAM and 8PSK. However, this method provides no guarantee for minimizing the symbol error rate (SER).

In this letter, we present a novel approach, based on a tight union bound on SER, to directly analyze and optimize SER performance of MDC-ABBA codes. We first show how to adapt the signal transformations proposed in [3], [4] to the newly proposed decoder of MDC-ABBA codes in [6]. The exact symbol pairwise error probability (SPEP) and the union bound on the SER are derived. For all the examined constellations, the union bound is only 0.1 dB from the simulated SER when SER $< 10^{-2}$. The union bound can thus be used to precisely predict the performance of MDC-ABBA codes and, moreover, to optimize the signal transformations for any constellation. Furthermore, for rectangular QAM (QAM-R), we propose a new signal transformation combining signal rotation and power allocation. Our new method performs better and has lower encoding/decoding complexities than that proposed in [4]. We also show that MDC-ABBA codes achieve full diversity in the systems with antenna selection and limited feedback.

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II. Preliminaries

We consider the transmission of ABBA codes over a quasi-static Rayleigh flat fading channel with M transmit (Tx) and N receive (Rx) antennas. Let $\mathcal{H} = [h_{ij}]_{\substack{i=1,\dots,M\\ j=1,\dots,N}}$ be the channel matrix. The channel gains are assumed to be uncorrelated and known perfectly at the receiver, but not at the transmitter.

Let A_k and B_k $(k = 1, 2, \cdots, K)$ be the $t \times m$ basis matrices of an OSTBC $\mathcal{O}_m$ with rate $\mathsf{R}_{\mathcal{O}_m}$. Two blocks of data, each of K symbols, are mapped into two code matrices $\mathcal{A}$ and $\mathcal{B}$ of $\mathcal{O}_m$ as $\mathcal{A} = \sum_{k=1}^{K}(s_k A_k + s_k^* B_k)$, $\mathcal{B} = \sum_{k=1}^{K}(s_{k+K} A_k + s_{k+K}^* B_k)$[1], where s_k is the data symbol with unit average energy. The ABBA codes for $M = 2m$ Tx antennas are constructed from $\mathcal{O}_m$ as $\mathcal{Q}_M = \begin{bmatrix}\mathcal{A} & \mathcal{B}\\ \mathcal{B} & \mathcal{A}\end{bmatrix}$.

Let Q be a transmitted ABBA code matrix, the Rx signal vector over antenna n is $\boldsymbol{y}_n = \sqrt{\frac{\rho\kappa}{2}}Q\boldsymbol{h}_n + \boldsymbol{w}_n$, where $\boldsymbol{h}_n$ is the nth column of $\mathcal{H}$, $\boldsymbol{w}_n$ is the noise vector with independent identically distributed (i.i.d.) entries $\sim \mathcal{CN}(0,1)$, $\kappa = \frac{1}{m\mathsf{R}_{\mathcal{O}_m}}$, ρ is the average receive signal-to-noise ratio (SNR).

The equivalent Tx-Rx signals for a data vector $\boldsymbol{s}_k = [s_k\ s_{k+K}]^\mathsf{T}$ of ABBA codes is presented for 1 Rx antenna in [6, (11)]. For multiple Rx antennas, this signal relation can

[1]Superscript * denotes conjugate operation. From now on, superscripts $^\mathsf{T}$ and † stand for matrix transpose and transpose conjugate. The $n \times n$ identity and all-zero matrices are denoted by $\boldsymbol{I}_n$ and $\boldsymbol{0}_n$, respectively. The diagonal matrix with elements of vector $\boldsymbol{x}$ on the main diagonal is denoted by $\mathrm{diag}(\boldsymbol{x})$. Kronecker product is denoted by $\otimes$. A mean-m and variance-σ^2 circularly complex Gaussian random variable is written by $\mathcal{CN}(m, \sigma^2)$. $\Re(X)$ and $\Im(X)$ denote the real and imaginary parts of a matrix X, respectively.

be generalized as follows.

$$\underbrace{\sum_{n=1}^{N}(E_{kn}^{\dagger}\boldsymbol{y}_n + F_{kn}^{\mathsf{T}}\boldsymbol{y}_n^*)}_{\hat{\boldsymbol{y}}_k} = \sqrt{\frac{\rho\kappa}{2}}Z\boldsymbol{s}_k + \underbrace{\sum_{n=1}^{N}(E_{kn}^{\dagger}\boldsymbol{w}_n + F_{kn}^{\mathsf{T}}\boldsymbol{w}_n^*)}_{\hat{\boldsymbol{w}}_k} \quad (1)$$

where $E_{kn} = \begin{bmatrix}\boldsymbol{e}_{k1,n} & \boldsymbol{e}_{k2,n}\end{bmatrix}$ $(k = 1, \ldots, K)$, $\boldsymbol{e}_{ki,n} = \left(A_k \otimes \Pi^{i-1}\right)\boldsymbol{h}_n$ $(i = 1, 2)$, $F_{kn} = \begin{bmatrix}\boldsymbol{f}_{k1,n} & \boldsymbol{f}_{k2,n}\end{bmatrix}$, $\boldsymbol{f}_{ki,n} = \left(B_k \otimes \Pi^{1-i}\right)\boldsymbol{h}_n$, $\Pi = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$, $Z = \sum_{j=1}^{N}\sum_{i=1}^{m} H_{i,j}^{\dagger}H_{i,j}$, and $H_{i,j} = \begin{bmatrix}h_{i,j} & h_{i+m,j}\\ h_{i+m,j} & h_{i,j}\end{bmatrix}$.

Note that the noise vector $\hat{\boldsymbol{w}}_k$ is correlated with covariance matrix $V = \mathbb{E}[\hat{\boldsymbol{w}}_k\hat{\boldsymbol{w}}_k^{\dagger}] = Z \neq \boldsymbol{I}_M$. Let $\hat{H} = Z^{\frac{1}{2}}$. Thus this correlated noise vector can be whitened by a matrix $\hat{H}^{-1} = Z^{-\frac{1}{2}}$ [7].

Since Z is real, the real and imaginary parts of the two sides of (1) can be decoupled as

$$\underbrace{\begin{bmatrix}\Re(\hat{\boldsymbol{y}}_k)\\ \Im(\hat{\boldsymbol{y}}_k)\end{bmatrix}}_{\bar{\boldsymbol{y}}_k} = \sqrt{\rho\kappa/2}\underbrace{\begin{bmatrix}Z & \boldsymbol{0}_2\\ \boldsymbol{0}_2 & Z\end{bmatrix}}_{\widetilde{H}}\begin{bmatrix}\Re(\boldsymbol{s}_k)\\ \Im(\boldsymbol{s}_k)\end{bmatrix} + \underbrace{\begin{bmatrix}\Re(\hat{\boldsymbol{w}}_k)\\ \Im(\hat{\boldsymbol{w}}_k)\end{bmatrix}}_{\bar{\boldsymbol{w}}_k}. \quad (2)$$

The detection of vectors $\Re(\boldsymbol{s}_k)$ or $\Im(\boldsymbol{s}_k)$ in (2) can be decoupled and involves only 2 real symbols. Therefore, MDC-ABBA codes are single-symbol decodable. In order to achieve full-diversity, signal transformation may be required for the data vectors $\Re(\boldsymbol{s}_k)$ and $\Im(\boldsymbol{s}_k)$ before sending them to the channels. We next adapt the existing signal transformations in [3], [4] to our new decoding framework.

III. Analyzing the Existing Signal Transformations

1. *Signal rotation proposed by Yuen, Guan, and Tjhung (YGT)* [3]:

Let the input symbols be $d_k = a_k + \mathrm{j}\, b_k, (k = 1, \ldots, 2K)$; they are drawn from a constellation $\mathcal{S}$. In [3], the transmitted symbols $s_k = p_k + \mathrm{j}\, q_k$ are generated as follows: $\Re(\boldsymbol{s}_k) = \begin{bmatrix}p_k & p_{k+K}\end{bmatrix}^{\mathsf{T}} = R\begin{bmatrix}a_k & b_k\end{bmatrix}^{\mathsf{T}}$ and $\Im(\boldsymbol{s}_k) = \begin{bmatrix}q_k & q_{k+K}\end{bmatrix}^{\mathsf{T}} = R\begin{bmatrix}a_{k+K} & b_{k+K}\end{bmatrix}^{\mathsf{T}}$, where $R = \begin{bmatrix}\cos(\alpha) & \sin(\alpha)\\ \sin(\alpha) & -\cos(\alpha)\end{bmatrix}$. The rotation angle for QAM is $\alpha = \frac{1}{2}\arctan(\frac{1}{2}) = 13.2825°$.

2. *Signal transformation proposed by Wang, Wang, and Xia (WWX)* [4]:

In general, we can jointly transform the input data vector $\hat{\boldsymbol{c}}_k = \begin{bmatrix}a_k & b_k & a_{k+K} & b_{k+K}\end{bmatrix}^{\mathsf{T}}$ by a real transformation $\mathcal{R}$ to generate transmitted symbols p_k, q_k, p_{k+K}, and q_{k+K} as

$$\begin{aligned}\begin{bmatrix}\Re(\boldsymbol{s}_k)^{\mathsf{T}} & \Im(\boldsymbol{s}_k)^{\mathsf{T}}\end{bmatrix}^{\mathsf{T}} &= \begin{bmatrix}p_k & p_{k+K} & q_k & q_{k+K}\end{bmatrix}^{\mathsf{T}}\\ &= \mathcal{R}\begin{bmatrix}a_k & b_k & a_{k+K} & b_{k+K}\end{bmatrix}^{\mathsf{T}}. \quad (3)\end{aligned}$$

Wang *et al.* [4] present three transformations $\mathcal{R}$, which are, however, permutation-equivalent. We thus consider only the first case with transformation: $\begin{bmatrix}p_k & q_k & p_{k+K} & q_{k+K}\end{bmatrix}^{\mathsf{T}} = \mathcal{R}_W\hat{\boldsymbol{c}}_k$, where $\mathcal{R}_W = \begin{bmatrix}U_1 & U_2\\ U_1R_1 & U_2R_2\end{bmatrix}$, and U_1, U_2, R_1, R_2 are 2×2 real matrices, $R_1^2 = R_2^2 = \boldsymbol{I}_2$. Compared with (3), the symbols p_{k+K} and q_k are permuted such that $\begin{bmatrix}\Re(\boldsymbol{s}_k)^{\mathsf{T}} & \Im(\boldsymbol{s}_k)^{\mathsf{T}}\end{bmatrix}^{\mathsf{T}} = \begin{bmatrix}p_k & p_{k+K} & q_k & q_{k+K}\end{bmatrix}^{\mathsf{T}} = \pi\begin{bmatrix}p_k & q_k & p_{k+K} & q_{k+K}\end{bmatrix}^{\mathsf{T}} = \pi\mathcal{R}_W\hat{\boldsymbol{c}}_k$, where π is a square permutation matrix to permute rows 2 and 3 of data vector $\hat{\boldsymbol{c}}_k$. Substituting $\pi\mathcal{R}_W$ into (2), we have

$$\bar{\boldsymbol{y}}_k = \sqrt{\rho\kappa/2}(\widetilde{H}\pi\mathcal{R}_W)\hat{\boldsymbol{c}}_k + \bar{\boldsymbol{w}}_k. \quad (4)$$

The matrix $\widetilde{H}\pi\mathcal{R}_W$ in (4) is not block-diagonal; thus PWRS decoding seems to be impossible. However, by multiplying both sides of (4) with $(\pi\mathcal{R}_W)^{\mathsf{T}}$, the matrix $(\pi\mathcal{R}_W)^{\mathsf{T}}\widetilde{H}\pi\mathcal{R}_W$ becomes block diagonal again; then PWRS decoding is possible.

Proof: We first can show that the product $\pi^{\mathsf{T}}\widetilde{H}\pi = \begin{bmatrix}z_1\boldsymbol{I}_2 & z_2\boldsymbol{I}_2\\ z_2\boldsymbol{I}_2 & z_1\boldsymbol{I}_2\end{bmatrix}$, where z_1 and z_2 are the elements of Z such that $Z = \begin{bmatrix}z_1 & z_2\\ z_2 & z_1\end{bmatrix}$. Then, $\bar{H} = \widehat{\mathcal{R}}_W^{\mathsf{T}}\widetilde{H}\widehat{\mathcal{R}}_W = \mathcal{R}_W^{\mathsf{T}}\pi^{\mathsf{T}}\widetilde{H}\pi\mathcal{R}_W = \begin{bmatrix}X_1 & \boldsymbol{0}_2\\ \boldsymbol{0}_2 & X_2\end{bmatrix}$, where $X_i = z_1U_i^{\mathsf{T}}U_i + z_2R_i^{\mathsf{T}}U_i^{\mathsf{T}}U_i + z_2U_i^{\mathsf{T}}U_iR_i + z_1R_1^{\mathsf{T}}U_i^{\mathsf{T}}U_iR_i$ $(i = 1, 2)$. ∎

We have some comparisons on the signal transformations by Yuen *et al.* and Wang *et al.* as follows.

- Complexity: The 4×4 WWX transformation $\mathcal{R}_W$ has higher encoding complexity compared with the 2×2 YGT rotation R. Also, the multiplication of $(\pi\mathcal{R}_W)^{\mathsf{T}}$ and $\bar{\boldsymbol{y}}_k$ in (4) increases the decoding complexity of WWX transformation, compared with that of YGT rotation.
- Performance: For square QAM (QAM-S), the WWX transformation in [4, Theorem 2] provides no SNR gain compared with the YGT rotation [3]. The transformation in [4, Theorem 3] performs better with rectangular QAM (QAM-R) at the cost of higher encoding/decoding complexities.

We next optimize the performance of MDC-ABBA codes based on the union bound on SER. Only the signal rotation of Yuen *et al.* is considered due to its mathematical convenience. More importantly, we will show that by combining power allocation and signal rotation, better performance for QAM-R can be achieved than by using the transformation in [4, Theorem 3], however, with less complexity.

IV. Optimal Signal Transformations

A. Exact Symbol Pairwise Error Probability

We first derive the exact SPEP. With noise whitening and signal rotation, we can rewrite (2) as

$$\hat{H}^{-1}\Re(\hat{\boldsymbol{y}}_k) = \sqrt{\rho\kappa/2}\hat{H}R\begin{bmatrix}a_k & b_k\end{bmatrix}^{\mathsf{T}} + \hat{H}^{-1}\Re(\hat{\boldsymbol{w}}_k). \quad (5)$$

and another equation can be similarly written for the data vector $\begin{bmatrix}a_{k+K} & b_{k+K}\end{bmatrix}$ [6].

Since $\hat{H}^{-1}\Re(\hat{\boldsymbol{w}}_k)$ and $\hat{H}^{-1}\Im(\hat{\boldsymbol{w}}_k)$ are real random Gaussian vectors with i.i.d. entries (zero-mean and variance $N_0 = 1/2$), the information vectors $\begin{bmatrix}a_k & b_k\end{bmatrix}^{\mathsf{T}}$ and $\begin{bmatrix}a_{k+K} & b_{k+K}\end{bmatrix}^{\mathsf{T}}$ $(k = 1, 2, \ldots, K)$ experience the same channels; they are subject to the same error probability. We thus can consider the error probability of one of the two vectors only; the subscript of symbols can be omitted for short. Furthermore, the pairwise error probability of each vector is also the SPEP.

Consider two distinct symbols $d = a + \mathrm{j}\, b$ and $\hat{d} = \hat{a} + \mathrm{j}\,\hat{b}$. Let $\delta_1 = a - \hat{a}, \delta_2 = b - \hat{b}, \Delta = [\delta_1\ \delta_2]^{\mathsf{T}}$, the conditional SPEP of d and $\hat{d}$ can be expressed by the Gaussian Q-function as [8]

$$P(d \to \hat{d}|\hat{H}) = Q\left(\sqrt{\rho\kappa|\hat{H}R\Delta|^2/8N_0}\right). \quad (6)$$

From Section II, we can show that $\hat{H}$ is a 2×2 real circulant matrix [6]. Hence $\hat{H}^\dagger\hat{H} = \hat{H}\hat{H} = \hat{H}^2 = Z$. We can use eigenvalue decomposition for $H_{i,j}$ so that $H_{i,j} = F_2^\dagger\Lambda_{i,j}F_2$, where F_2 is a 2×2 discrete Fourier transform matrix, $\Lambda_{i,j} = \mathrm{diag}(\lambda_{i,j,1}, \lambda_{i,j,2})$ and $[\lambda_{i,j,1}\ \lambda_{i,j,2}]^\mathsf{T} = F_2[h_{i,j}\ h_{i+M/2,j}]^\mathsf{T}$. Since $h_{i,j}$ and $h_{i+M/2,j}$ are i.i.d. $\sim \mathcal{CN}(0,1)$, so are $\lambda_{i,j,1}$ and $\lambda_{i,j,2}$. Thus, $Z = \sum_{j=1}^{N}\sum_{i=1}^{M/2} F_2\,\mathrm{diag}(|\lambda_{i,j,1}|^2, |\lambda_{i,j,2}|^2)F_2$.

Let $x \triangleq |\hat{H}R\Delta|^2 = (R\Delta)^\dagger\hat{H}^\dagger\hat{H}(R\Delta)$, one has $x = \sum_{j=1}^{N}\sum_{i=1}^{M/2}\left[\beta_1^2|\lambda_{i,j,1}|^2 + \beta_2^2|\lambda_{i,j,2}|^2\right]$, where $[\beta_1\ \ \beta_2]^\mathsf{T} = F_2R\Delta$. We can apply the Craig's formula [9] to derive the conditional SPEP in (6).

$$P(d \to \hat{d}|\hat{H}) = \frac{1}{\pi}\int_0^{\pi/2}\prod_{j=1}^{N}\prod_{i=1}^{M/2}\exp\left(-\frac{\rho\kappa\left(\beta_1^2|\lambda_{i,j,1}|^2 + \beta_2^2|\lambda_{i,j,2}|^2\right)}{4\sin^2\theta}\right)d\theta. \quad (7)$$

Since $\lambda_{i,j,1}$ and $\lambda_{i,j,2}$ are i.i.d $\sim \mathcal{CN}(0,1)$, we can apply a method based on the moment generation function (MGF) [8] to obtain the unconditional SPEP in the following:

$$P(d \to \hat{d}) = \frac{1}{\pi}\int_0^{\pi/2}\left[\left(1+\frac{\rho\kappa\beta_1^2}{4\sin^2\theta}\right)\left(1+\frac{\rho\kappa\beta_2^2}{4\sin^2\theta}\right)\right]^{-MN/2}d\theta. \quad (8)$$

A closed form of SPEP without integration can be derived, Only the final result is given below for brevity.

$$P(d \to \hat{d}) = \sum_{i=0}^{\widehat{L}-1}\left(u_i\epsilon_1^i\epsilon_2^{\widehat{L}}\mathcal{M}_{1,i} + v_i\epsilon_1^{\widehat{L}}\epsilon_2^i\mathcal{M}_{2,i}\right). \quad (9)$$

where $\widehat{L} = MN/2$, and

$$u_i = \frac{(-1)^i\widehat{L}(\widehat{L}+1)\ldots(\widehat{L}+i-1)}{i!(\epsilon_2-\epsilon_1)^{\widehat{L}+i}}, \quad (10a)$$

$$v_i = \frac{(-1)^i\widehat{L}(\widehat{L}+1)\ldots(\widehat{L}+i-1)}{i!(\epsilon_1-\epsilon_2)^{\widehat{L}+i}}, \quad (10b)$$

$$\epsilon_k = \frac{4}{\rho\kappa\beta_k^2} \qquad (k=1,2), \quad (10c)$$

$$\mathcal{M}_{k,i} = \left(\frac{1-\eta_k}{2}\right)^{\widehat{L}-i}\sum_{l=0}^{\widehat{L}-i-1}\binom{\widehat{L}-i-1+l}{l}\left(\frac{1+\eta_k}{2}\right)^l, \quad (10d)$$

$$\eta_k = \sqrt{1/(1+\epsilon_k)} \qquad (k=1,2). \quad (10e)$$

B. Optimal Signal Rotations Based on tight SER Union Bound

Assume that d_i and d_j, $i,j = 1,\ldots,L$, are signals drawn from a constellation $\mathcal{S}$ of size L. From the SPEP in (8), we can find the union bound on SER of constellation $\mathcal{S}$ with MDC-ABBA codes as

$$P_u(\mathcal{S}) = \frac{2}{L}\sum_{i=1}^{L-1}\sum_{j=i+1}^{L}P(d_i \to d_j). \quad (11)$$

TABLE I
OPTIMAL ROTATION ANGLES OF POPULAR CONSTELLATIONS

Signal	Optimal α	Signal	Optimal α
4QAM	14.382°	8QAM-S	12.268°
4TRI	31.155°	8QAM-R	13.166°
8PSK	5.915°, 39.085°	8QAM-SR	31.964°
8APSK	33.472°	16PSK	24.883°, 42.617°
8TRI-a	30.284°	16TRI	0°
8TRI-b	0°	16QAM-S	13.195°

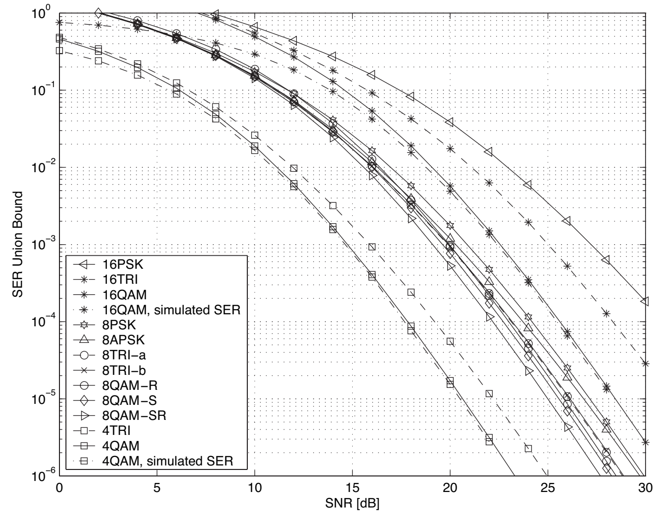

Fig. 1. Simulated SER of 4- and 16-QAM, and SER union bound of 4-, 8-, 16-ary constellations, 4 Tx/1 Rx antennas.

The SER union bounds of 4- and 16QAM with YGT rotation ($\alpha = 13.2825°$) are plotted in Fig. 1. The union bound is only about 0.1 dB apart from the simulated SER when SER $< 10^{-2}$. Therefore, the union bound can be used to accurately predict the SER performance of MDC-ABBA codes and, furthermore, to optimize the signal rotation R.

We run computer search to find the optimal rotations, which minimize the SER union bound, for popular 4-, 8- and 16-ary constellations. Their geometrical shapes can be found in [10], [11]. Note that in [11, Fig. 2], the labels of square 8QAM (8QAM-S) and 8QAM-R should be swapped. The rotation angle is searched in the range $[0°, 45°]$ (with increment of $0.001°$), because if α is an optimal angle, the following angles are also optimal $-\alpha, 90°\pm\alpha, 180°\pm\alpha, 270°\pm\alpha$. The SNR is chosen such that the SER of corresponding optimal rotation angle is about 10^{-8}. At such low SER, the optimal rotation angles also yield full-diversity MDC-ABBA codes. The results are summarized in Table I.

The SER union bounds of several 4-, 8- and 16-ary constellations are illustrated in Fig. 1. Signal constellations carved from the lattice of equilateral triangle (TRI signals) have the best minimum Euclidean distance and perform well compared with QAM when they are used for fading channels [10]. However, combined with MDC-ABBA codes, QAM signals always perform much better than the others.

The new optimal rotation angles for QAM (square or rectangular) constellations are very close to the proposed angle $\alpha = 13.2825$ by minimizing CPEP [3]. Therefore, the SNR gains in these cases are negligible compared to the results of [3] and [4, Theorem 2]. Nevertheless, we are able to find the optimal rotations for PSK and TRI, and to compare their performances with QAM[2]. We next present a new approach, which is applicable to QAM-R, to find the best transformation so that the MDC-ABBA codes perform better but have lower complexity than that proposed in [4, Theorem 3].

C. Optimal Signal Rotations with Power Allocations

For QAM-R, the average powers of the real and imaginary parts of the signal points are unbalanced. We may change the power allocation to the real and imaginary parts of QAM-R signals to a get better overall SER. In particular, the real and imaginary parts of QAM-R signals are scaled by constants μ_1 and μ_2, respectively, before they are rotated. Thus, if the input constellation is $\mathcal{S} = \{d \mid d = a + \mathrm{j}\, b, a, b \in \mathbb{R}\}$, the constellation with new power allocation is $\bar{\mathcal{S}} = \{\bar{d} \mid \bar{d} = \mu_1 a + \mathrm{j}\, \mu_2 b; a, b \in \mathbb{R}\}$. The average energy of the constellation must be kept unchanged. Therefore, μ_1 and μ_2 are mutually dependent. For example, for 8QAM-R with signal points $\{(\pm 3 \pm \mathrm{j}, \pm 1 \pm \mathrm{j})/\sqrt{48}\}$, μ_1 and μ_2 are constrained as $5\mu_1^2 + \mu_2^2 = 6$. We thus have two parameters μ_1 (or μ_2) and α to be optimized.

The best found parameter set (μ_1, μ_2, α) by computer search for 8- and 32QAM-R are $(0.9055, 1.3784, 0°)$ and $(0.8972, 1.3487, 1.954°)$, respectively. Using our new transformation, the frame error rate of MDC-ABBA codes with QAM-R is improved compared with YGT rotation, and also WWX transformation, however, with lower encoding/decoding complexities (see Fig. 2).

V. MDC-ABBA Codes with Antenna Selection and Limited Feedback

When a feedback channel exist between receiver and transmitter, transmit antenna selection (TAS) can be employed to provide significant SNR gains compared with the open loop STBC [12]. We thus investigate the performance of MDC-ABBA codes with TAS, with emphasis on the diversity order.

Let $\bar{\beta}_1 = \min(|\beta_1|, |\beta_2|), \bar{\beta}_2 = \max(|\beta_1|, |\beta_2|)$, we have $\sum_{j=1}^{N}\sum_{i=1}^{M/2}\left[\bar{\beta}_1^2\left(|\lambda_{i,j,1}|^2 + |\lambda_{i,j,2}|^2\right)\right] \leq x \leq \sum_{j=1}^{N}\sum_{i=1}^{M/2}\left[\bar{\beta}_2^2\left(|\lambda_{i,j,1}|^2 + |\lambda_{i,j,2}|^2\right)\right]$. Since $[\lambda_{i,j,1} \quad \lambda_{i,j,2}]^{\mathsf{T}} = F_2[h_{i,j} \quad h_{i+M/2,j}]^{\mathsf{T}}$, we get $|\lambda_{i,j,1}|^2 + |\lambda_{i,j,2}|^2 = |h_{i,j}|^2 + |h_{i+M/2,j}|^2$. Therefore $\bar{\beta}_1^2\|\mathcal{H}\|^2 \leq x \leq \bar{\beta}_2^2\|\mathcal{H}\|^2$ and the SPEP in (6) is bounded as

$$P(d \to \hat{d}|\,\mathcal{H}) \geq Q\left(\sqrt{\rho\kappa\bar{\beta}_2^2\|\,\mathcal{H}\,\|_{\mathsf{F}}^2/4}\right), \qquad (12a)$$

$$P(d \to \hat{d}|\,\mathcal{H}) \leq Q\left(\sqrt{\rho\kappa\bar{\beta}_1^2\|\,\mathcal{H}\,\|_{\mathsf{F}}^2/4}\right). \qquad (12b)$$

[2]One of reviewers has suggested to find the optimal signal rotation by simulation-based computer search, without knowing the SER union bound. Note that the simulation time to obtain SER $= 2.13 \times 10^{-5}$ (170 errors), for 8QAM-SR is about 1.62 hours, using Matlab Release 14, 2GHz microprocessor, Redhat Linux version 9 operating system. If the rotation angle $\alpha \in [0,\ 90°)$, with raw increment of $0.1°$, then 900 runs will be completed in more than 60 days. If we want to compare 10 different constellations, the brute-force search could spend 1 year and 8 months. While our search method takes only less than 5 minutes to find the best rotation angle for 8QAM-SR, however, with finer increment of $0.001°$. This comparison highlights the efficiency of our approach over the brute-force search.

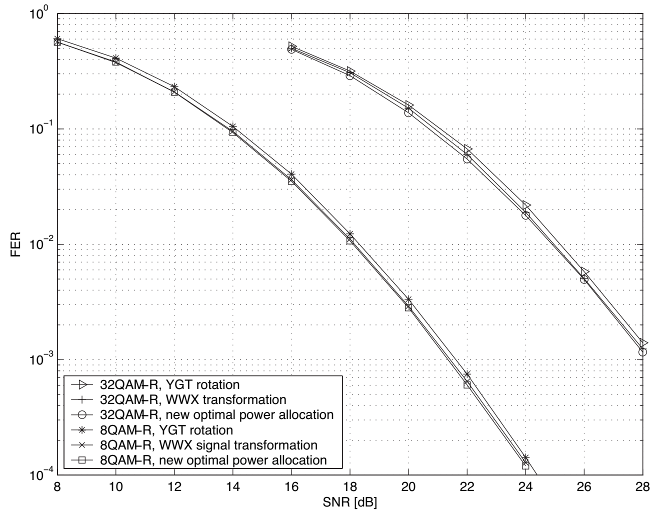

Fig. 2. Performances of MDC-ABBA codes with new optimal power allocation and existing signal transformations for QAM-R, 4 Tx/1 Rx antennas.

If $\bar{\beta}_1 \neq 0$ and $\bar{\beta}_2 \neq 0$ for all distinct pairs of symbols, the SPEP of MDC-ABBA codes in (12) is lower and upper-bounded by a (full-diversity) SPEP of some OSTBC transmitted over the same channel $\mathcal{H}$ with different SNR scales.

With TAS, M rows of $\mathcal{H}$ are chosen to formulate a selected channel $\bar{\mathcal{H}}$ so that Frobenius norm of $\bar{\mathcal{H}}$ is maximized. We now show that MDC-ABBA codes achieve full diversity with TAS and limited feedback. The concept of TAS with limited feedback [12] can be explained as follows. Choosing M out of M_t transmit antennas requires $b = \left\lceil \log_2 \binom{M_t}{M} \right\rceil$ feedback bits and b may be large. In some scenarios, it is required to keep b small. Therefore, instead of $\binom{M_t}{M}$ possible choices, the M antennas are selected from G predefined groups, such that $\lceil \log_2 G \rceil < b$, to reduce the number of feedback bits. This method is called limited feedback. Obviously, the selected M antennas may not be optimal with limited feedback; however, it is shown that OSTBC can achieve full transmit diversity M_t with limited feedback [12]. Therefore, MDC-ABBA codes also achieve full diversity with limited feedback.

We verify the diversity order of MDC-ABBA codes with TAS and limited feedback in Fig 3. In the simulations, 3 antennas are chosen out of 4 Tx antennas. The full feedback scheme requires 2-bit feedback. In the limited feedback system, there are only 2 choices to select 3 out of 4 antennas; thus only 1-bit feedback is needed. At high SNR, the BER curves of the open-loop and feedback schemes are parallel, which confirms that the limited and full feedback schemes achieve full diversity. Additionally, the limited feedback scheme gains 0.9 dB over the open-loop scheme at 10^{-4} BER.

VI. Conclusion

We have presented a new efficient method to directly analyze and optimize the SER performance of MDC-ABBA codes. Our approach relies on the exact SPEP, which is distinguished from the other methods with worst-case codeword PEP optimization. Additionally, a new signal transformation to improve the performance of rectangular QAM with less encoding/decoding complexity has been proposed. We have

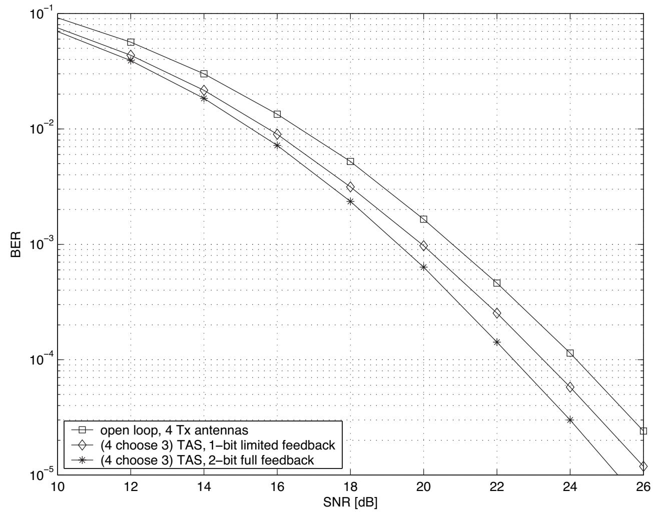

Fig. 3. Performance of MDC-ABBA codes with limited and full feedback, choose $M = 3$ transmit antennas from $M_t = 4$ antennas, and 1 receive antenna, 16QAM.

also shown that MDC-ABBA codes achieve full diversity with transmit antenna selection and limited feedback. Our results show that QAM signals have the best performance and, therefore, should be used with MDC-ABBA codes. Our approach to analyze SER of MDC-ABBA can be extended to investigate performance of other quasi-orthogonal STBC (QSTBC), for example, in [13]. Due to the space limit, performance comparison of MDC-ABBA codes and QSTBC are not presented in this letter. Nevertheless, interested readers can find some performance comparisons of MDC-ABBA and ABBA codes in [3], [6].

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