Clipping-Noise Guided Sign-Selection for PAR Reduction in OFDM Systems

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***Abstract*—The peak-to-average power ratio (PAR) of orthogonal frequency division multiplexing (OFDM) systems can be reduced by using an optimal set of subcarrier signs. However, this sign selection is a hard discrete optimization problem. We therefore consider the use of the clipping noise, generated when the OFDM signal is clipped at a given threshold level, to find a good set of signs. The key idea of clipping-noise guided sign-selection (CGS) is to iteratively flip the signs of those subcarriers with high levels of clipping noise. In each iteration, the key task is to determine the number and locations of such subcarriers. We develop suitable criteria for this task and derive CGS algorithms that can handle both unitary (e.g., M-ary phase shift keying) and nonunitary (e.g., M-ary quadrature amplitude modulation) signal constellations. The simulation results show that the PAR reduction of CGS is about 1 dB larger than that of derandomization and tone reservation for a 256-subcarrier system, and is about 1–2 dB larger than that of partial transmit sequence (PTS) and selective mapping (SLM). CGS also removes the error floor due to nonlinear amplifiers.**

***Index Terms*—Clipping-noise guided, orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAR) reduction, sign-selection.**

I. Introduction

THE high peak-to-average power ratio (PAR) in orthogonal frequency division multiplexing (OFDM) systems requires the high power amplifier (HPA) with a large linear range that is inefficiently used [1]. If the linear range of HPA is not sufficient, the large PAR leads to in-band distortion and out-of-band radiation [1]. The previously proposed PAR-reduction techniques have included clipping and filtering [2]–[6] which may increase the bit error rate (BER); tone reservation [7]–[9], which lowers the throughput and may offer limited PAR reduction gain if reserved tones are on the edge of the OFDM frequency band; and distortionless coding methods based on block codes, convolutional codes and Golay complementary sequences [10]–[14], which, result in low coding rates for a large number of subcarriers. A comprehensive tutorial review of PAR reduction techniques can be found in [15].

Probabilistic PAR reduction techniques are especially suited for OFDM systems with a large number of (≥ 64) subcarriers [16]–[24]. Selected mapping (SLM) and partial transmit sequences (PTS) [16]–[21] are such techniques that obtain moderate PAR reduction with limited complexity. A more general technique that includes both SLM and PTS as special cases is the sign-selection method [23], [24]. This method selects the subcarrier signs to significantly reduce the PAR. However, since there are 2^N different sign patterns, where N is the number of subcarriers, optimal sign selection is a hard discrete optimization problem with the complexity of $\mathcal{O}(2^N)$. In [24], a deterministic sign-selection algorithm based on derandomization is developed. This algorithm iteratively optimizes each sign by minimizing the probability that the PAR is larger than a preset threshold. Although this algorithm limits the PAR to $\mathcal{O}(\log N)$, it requires $N-1$ iterations per OFDM symbol, and each iteration involves the computation of a large number of hyperbolic functions, leading to high complexity.

These probabilistic PAR reduction algorithms are computationally complex for large PAR reduction. In this paper, we propose two clipping-noise guided sign-selection algorithms. These algorithms use a new clipper model to obtain clipping noise from the time-domain OFDM signal, and then flip the signs of those subcarriers with high levels of clipping noise. In developing these algorithms, a number of clipper models were tested. For example, we found that the conventional soft limiter [25] does not provide a sufficient performance; the new clipper model obtains larger PAR reductions than the soft limiter. The difference between the new clipper and the soft limiter is that the clipping noise generated by the new clipper contains the entire samples of large peaks higher than a predefined threshold, while that generated by the soft limiter contains only fragments of these large peaks.

This paper considers PAR reduction based on clipping-noise guided sign-selection. The key idea of clipping-noise guided sign-selection (CGS) is to iteratively flip the signs of those subcarriers with high levels of clipping noise. In each iteration, the key task is to determine the number and locations of such subcarriers. We develop suitable criteria for this task and derive two CGS algorithms that can handle both unitary (e.g., M-ary phase shift keying) and nonunitary (e.g., M-ary quadrature amplitude modulation) signal constellations. The simulation results show that the PAR reduction of CGS is about 1 dB larger than that of derandomization and tone reservation for a 256-subcarrier system, and is about 1–2 dB larger than that of PTS and SLM. CGS also removes the error floor due to nonlinear HPAs.

This paper is organized as follows. Section II characterizes the OFDM System and reviews the sign-selection technique. The clipping-noise guided sign-selection algorithms are proposed in Section III. In Section IV, the proposed algorithms are compared with SLM, PTS, and the derandomization algorithm. Section V concludes this paper.

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II. Sign-Selection Technique for PAR Reduction

A. Characterization of OFDM System

The time-domain OFDM signal may be expressed as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi kt/T}, \quad 0 \le t \le T \tag{1}$$

where T is the OFDM symbol period, and $X_k, k = 0, \ldots, N-1$, are typically chosen from an M-ary PSK or QAM constellation. Without loss of generality, we set $E\left\{|X_k|^2\right\} = 1$. We call $\mathbf{X} = [X_0, \ldots, X_{N-1}]$ an OFDM block. In practice, JN samples of $x(t)$ are efficiently computed by an inverse discrete Fourier transform (IDFT)[1]

$$\begin{aligned} x_n &= \text{IDFT}[\mathbf{X}] \\ &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi(nk/JN)}, \quad n = 0, \ldots, JN-1 \end{aligned} \tag{2}$$

where J is the oversampling factor. The PAR may be defined as

$$\xi = \frac{\max_{t\in[0,T)} |x(t)|^2}{P_{av}} \tag{3}$$

where $P_{\text{av}} = E\left\{|x(t)|^2\right\}$ is the average power, and $P_{\text{av}} = E\left\{|X_k|^2\right\} = 1$. The PAR may also be computed by using the discrete samples x_n similar to (3), and is approximately equal to ξ when $J \ge 4$ [26], [27].

B. Sign-Selection Technique

The sign of each subcarrier symbol X_k is denoted by the variable $s_k \in \{+1, -1\}$. A set of signs must be chosen to reduce the PAR. The original OFDM block $\mathbf{X}$ is thus replaced by

$$\hat{\mathbf{X}} = [s_0 X_0, \ldots, s_{N-1} X_{N-1}] \tag{4}$$

and the discrete-time transmit signal is given by

$$\hat{x}_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} s_k X_k e^{j2\pi(nk/JN)}, \quad n = 0, \ldots, JN-1. \tag{5}$$

Note that the choice of s_k does not affect the average power of $\hat{x}_n$, which is the same as that of x_n. The PAR reduction problem is therefore equivalent to minimizing the maximum peak of the amplitude of $\hat{x}_n$. Consequently, the PAR reduction problem is reformulated as

$$\begin{aligned} &\min_{\mathbf{s}} \quad \max_{n=0,\ldots,JN-1} \left| \sum_{k=0}^{N-1} s_k X_k e^{j2\pi(nk/JN)} \right| \\ &\text{subject to:} \quad \mathbf{s} \in \{1, -1\}^N \end{aligned} \tag{6}$$

where $\mathbf{s} = [s_0, \ldots, s_{N-1}]$, and $\{1, -1\}^N$ is the set of N-dimensional binary vectors. The optimal solution $\mathbf{s}^{(\text{opt})}$ may be sent to the receiver as side information for the correct detection of input modulation symbols. Alternatively, one can eliminate explicit side information by sacrificing one bit per subcarrier. For example, with 64QAM, five data bits and one sign bit per constellation point can be transmitted [23]. Note that the throughput loss may be avoided by using the hexagonal constellation [28], [29] or tone-injection. The proposed sign-selection algorithms can be easily applied to these cases.

Equation (6) describes a combinatorial optimization problem over the N dimensional binary space $\{1, -1\}^N$. Let $s_0 \equiv 1$ without loss of generality. Since the size of the search space is 2^{N-1}, which grows exponentially with N, one has to resort to suboptimal solutions. For example, the previously developed SLM and PTS can be identified as special cases of (6). The SLM method [17] uses a codebook of sign sequences $\mathcal{C}$ containing K sign sequences $\{\mathbf{s}^{(1)}, \ldots, \mathbf{s}^{(K)}\}$. For a given OFDM block, a sign sequence $\mathbf{s}^{(i)}$ leading to the lowest PAR is selected from $\mathcal{C}$. This sign sequence is clearly a suboptimal solution of (6). The PTS method [21] partitions an OFDM block to G subgroups, assigns an optimal sign to each subgroup, and hence reduces the size of the solution space from 2^{N-1} to 2^{G-1}. The PTS solution is then a suboptimal solution of (6).

C. Peak Cancelation

In order to derive our algorithms, it is necessary to rewrite (4) as

$$\hat{\mathbf{X}} = \mathbf{X} - \mathbf{C} \tag{7}$$

where $\mathbf{C} = [\alpha_0 X_0, \ldots, \alpha_{N-1} X_{N-1}]$ is the peak-canceling vector, and

$$\alpha_k = \begin{cases} 0, & s_k = 1 \\ 2 & s_k = -1. \end{cases}$$

The discrete-time transmit signal in (5) may now be expressed as

$$\begin{aligned} \hat{x}_n &= \text{IDFT}[\hat{\mathbf{X}}] = x_n - c_n \\ &= x_n - \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \alpha_k X_k e^{j2\pi(nk/JN)} \end{aligned} \tag{8}$$

in which c_n can be viewed as a peak-canceling signal. Note that only negative signs ($s_k = -1$) contribute to the peak-canceling signal. In Section III, we propose two suboptimal sign-selection algorithms, which utilize the strength of the clipping noise on each subcarrier to find the signs that should be negative.

[1] In this paper, all IDFT/DFT operations are of size JN. Moreover, for the ease of subcarrier index notation, we use the zero-padding scheme to calculate x_n; i.e., the IDFT operation is applied to the extended vector

$$\mathbf{X}_{\text{ext}} = [X_0, \ldots, X_{N-1}, \underbrace{0, \ldots, 0}_{(J-1)N \text{ zeros}}].$$

Note that, in practice, the zero-insertion scheme is widely used to calculate x_n, where the IDFT operation is applied to the extended vector

$$\mathbf{X}_{\text{ext}} = [X_0, \ldots, X_{N/2-1}, \underbrace{0, \ldots, 0}_{(J-1)N \text{ zeros}}, X_{N/2}, \ldots, X_{N-1}].$$

However, statistically, our algorithm obtains the same PAR reduction performance in both oversampling schemes.

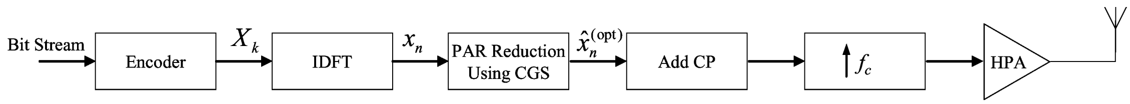

Fig. 1. OFDM system with clipping-noise guided sign-selection.

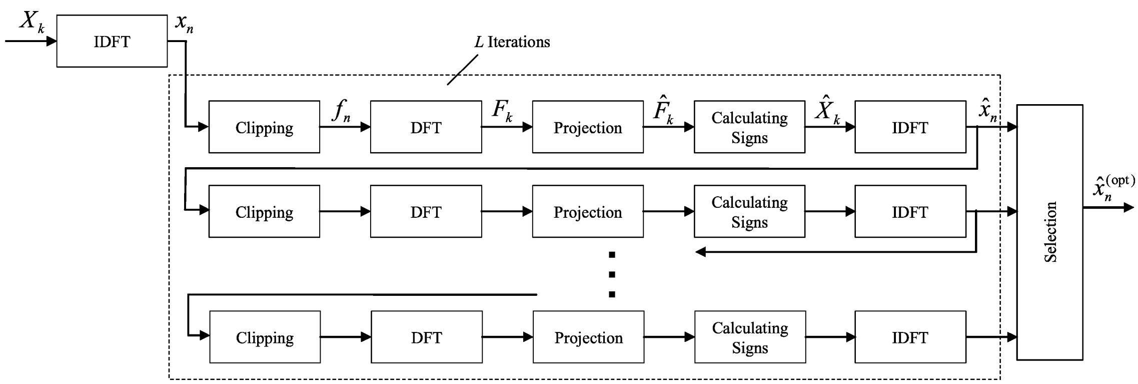

Fig. 2. Clipping-noise guided sign-selection algorithm.

III. Clipping-Noise Guided Sign-Selection Algorithm

The OFDM system with CGS is shown in Fig. 1. The input bitstream is first mapped to data symbols X_k by using a signal constellation (or even multiple constellations if bit-loading is employed). Each block of N data symbols (called an OFDM block) is fed into an IDFT block to generate the signal x_n. The CGS block finds an optimal time-domain OFDM signal $\hat{x}_n^{(\mathrm{opt})}$ with the lowest PAR. With the Cyclic Prefix (CP) appended to $\hat{x}_n^{(\mathrm{opt})}$, the time-domain OFDM signal is modulated to the carrier frequency, amplified by the HPA, and transmitted through the antenna.

The CGS block is expanded in Fig. 2, and the IDFT block is also included to facilitate the complexity calculation discussed later. In each iteration, the new clipper process samples x_n and outputs the clipping noise f_n. The DFT of the clipping noise F_k, $k=0,\ldots,N-1$, are used to calculate an index set of negative signs, $\mathcal{S}=\{k:\alpha_k=2\}$, that *may* reduce the peaks of x_n below threshold A. A candidate OFDM block is thus generated and is used as the input of the next iteration. After L iterations, the candidate OFDM block with the smallest PAR is selected for transmission.

The proposed CGS algorithms are based on the new clipper model. Given a predefined threshold A, the clipping noise in this case is

$$f_n=\begin{cases} x_n & |x_n|\geq A\\ 0 & |x_n|<A.\end{cases} \tag{9}$$

Note that this clipper is different from the conventional soft limiter used in the literature, which generates clipping noise as [25]:

$$f_n=\begin{cases} x_n-Ae^{j\phi_n} & |x_n|>A\\ 0 & |x_n|\leq A\end{cases} \tag{10}$$

where ϕ_n is the phase of x_n. While both the clipper models generate clipping noise when x_n's have peaks, the new model (9) contains the *entire* samples of large peaks higher than A, whereas the soft limiter (10) contains only fragments of the samples which exceed A. Although both models can work with our proposed algorithms, note that (9) need less computations than (10).[2] Moreover, the simulations show that the new model leads to larger PAR reduction for nonunitary constellation input than the soft limiter.

In this section, we first develop two criteria for selecting signs by using the level of clipping noise. We first propose the clipping-noise guided sign-selection algorithm for unitary constellations (e.g., PSK), where $|X_k|\equiv 1$, and extend it to nonunitary constellations (e.g., QAM).

A. Sign-Selection Criteria

Recall that only the subcarriers with the negative signs contribute to the peak-canceling signal. Finding the index set of negative signs $\mathcal{S}$ requires two criteria to determine the number of negative signs (which is the size of $\mathcal{S}$) and to select the elements of $\mathcal{S}$. Using the relationship between the peak-canceled samples and the original samples (8), we have

$$|\hat{x}_n|\geq|x_n|-\frac{2}{\sqrt{N}}\sum_{k\in\mathcal{S}}|X_k|.$$

Thus, a necessary condition for limiting $|\hat{x}_n|$ smaller than A is that $\mathcal{S}$ must satisfy

$$\frac{2}{\sqrt{N}}\sum_{k\in\mathcal{S}}|X_k|\geq|x_n|_{\max}-A. \tag{11}$$

On the other hand, since

$$|\hat{x}_n|\leq|x_n|+\frac{2}{\sqrt{N}}\sum_{k\in\mathcal{S}}|X_k|$$

a larger size of $\mathcal{S}$ increases the chance of obtaining a "bad" candidate OFDM block with a large PAR. Therefore, the size of $\mathcal{S}$ is determined as follows:

1) Criterion 1 (Size of $\mathcal{S}$): The $\mathcal{S}$ that *may* limit $|\hat{x}_n|$ to no larger than A must have the minimum size and satisfy (11).

[2]Compared to (9), (10) requires additional phase angle computation, two real multiplications and subtraction per clipped sample.

Selecting the elements of $\mathcal{S}$ depends on the clipping noise spectrum

$$F_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{JN-1} f_n e^{-j2\pi(nk/JN)}, \quad k = 0, \ldots, N-1 \tag{12}$$

where the clipping noise f_n is given in (9). Projecting F_k to X_k, we have

$$\hat{F}_k = D_k X_k, \quad k = 0, \ldots, N-1$$

where

$$D_k = \Re\left[\frac{F_k X_k^*}{|X_k|^2}\right] \tag{13}$$

and $\Re[x]$ represents the real part of x, and $(\cdot)^*$ represents the complex conjugate.

A peak-canceling signal may be obtained by taking the IDFT of $\hat{F}_k$ as

$$\hat{f}_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \hat{F}_k e^{j2\pi(nk/JN)} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} D_k X_k e^{j2\pi(nk/JN)}. \tag{14}$$

If a large peak of f_n, or equivalently x_n, occurs at n_0, then, $\hat{f}_n$ also has a large peak at n_0 with the same direction of f_{n_0}. Moreover, due to peak regrowth [6], usually $\left|\hat{f}_{n_0}\right| < |f_{n_0}|$. Therefore, $\hat{f}_n$ may be scaled by the optimum factor $\beta^{(\mathrm{opt})} > 0$ to further reduce the peaks of x_n. The modified discrete samples of the OFDM signal are

$$\bar{x}_n = x_n - \beta^{(\mathrm{opt})} \hat{f}_n = x_n - \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \beta^{(\mathrm{opt})} D_k X_k e^{j2\pi(nk/JN)}. \tag{15}$$

A comparison of (8) and (15) suggests that α_k can be obtained by rounding some $\beta^{(\mathrm{opt})} D_k$ to 2 and rounding others to 0. The resulting modified discrete samples $\hat{x}_n$ may have larger peaks than A. However, by minimizing the rounding error, the peaks of $\hat{x}_n$ may still be lower than $|x_n|_{\max}$. The mean squared rounding error is upper-bounded as

$$\begin{aligned}\varepsilon &= E\left\{|\hat{x}_n - \bar{x}_n|^2\right\} \\ &= \frac{1}{JN} \sum_n \left| \frac{1}{\sqrt{N}} \sum_{k\in\mathcal{S}} \left(2X_k - \beta^{(\mathrm{opt})} D_k X_k\right) e^{j2\pi(nk/JN)} + \frac{1}{\sqrt{N}} \sum_{k\notin\mathcal{S}} \beta^{(\mathrm{opt})} D_k X_k e^{j2\pi(nk/JN)} \right|^2 \\ &\le \frac{1}{N} \left(\sum_{k\in\mathcal{S}} \left|(2 - \beta^{(\mathrm{opt})} D_k) X_k\right| + \sum_{k\notin\mathcal{S}} \left|\beta^{(\mathrm{opt})} D_k X_k\right| \right)^2. \end{aligned} \tag{16}$$

The upper bound on ε is determined by the term inside $(\cdot)^2$ and suggests the following criterion.

2) Criterion 2 (Elements of $\mathcal{S}$): Define the maximum rounding error as

$$\varepsilon_{\max} = \sum_{k\in\mathcal{S}} \left|(2 - \beta^{(\mathrm{opt})} D_k) X_k\right| + \sum_{k\notin\mathcal{S}} \left|\beta^{(\mathrm{opt})} D_k X_k\right|. \tag{17}$$

Then, $\mathcal{S}$ must be generated by rounding $\beta^{(\mathrm{opt})} D_k$ to 2 or 0 such that $\varepsilon_{\max}$ is minimized.

B. Clipping-Noise Guided Sign-Selection for Unitary Constellations

Substituting $|X_k| \equiv 1$ into (11), the size of $\mathcal{S}$ can be calculated as

$$I = \left\lceil \frac{\sqrt{N}(|x_n|_{\max} - A)}{2} \right\rceil \tag{18}$$

where $\lceil x \rceil$ represent the smallest integer greater than x. In unitary modulations, minimizing $\varepsilon_{\max}$ involves D_k only.

Theorem 1: For unitary modulations, $\varepsilon_{\max}$ is minimized if $D_k \ge D_m$ for all $k \in \mathcal{S}$ and $m \notin \mathcal{S}$.

Proof: By substituting $|X_k| \equiv 1$ into (17)

$$\varepsilon_{\max} = \sum_{k\in\mathcal{S}} \left|2 - \beta^{(\mathrm{opt})} D_k\right| + \sum_{k\notin\mathcal{S}} \left|\beta^{(\mathrm{opt})} D_k\right|.$$

Let $\mathcal{S} = \{k_1, k_2, \ldots, k_I\}$ where $D_k \ge D_m$ for any $k \in \mathcal{S}$ and $m \notin \mathcal{S}$. Without loss of generality, let $\mathcal{S}' = \{k_1, \ldots, k_{I-1}, m\}$, $m \notin \mathcal{S}$. Then

$$\varepsilon_{\max} - \varepsilon'_{\max} = \left|2 - \beta^{(\mathrm{opt})} D_{k_I}\right| + \left|\beta^{(\mathrm{opt})} D_m\right| - \left|2 - \beta^{(\mathrm{opt})} D_m\right| - \left|\beta^{(\mathrm{opt})} D_{k_I}\right|$$

where $\varepsilon_{\max}$ and $\varepsilon'_{\max}$ are the maximum rounding errors associated with $\mathcal{S}$ and $\mathcal{S}'$, respectively. By discussing the signs of $\beta^{(\mathrm{opt})} D_{k_I}$, $\beta^{(\mathrm{opt})} D_m$, $\left(2 - \beta^{(\mathrm{opt})} D_{k_I}\right)$ and $\left(2 - \beta^{(\mathrm{opt})} D_m\right)$, we have $\varepsilon_{\max} - \varepsilon'_{\max} \le 0$. Since $m \notin \mathcal{S}$ is arbitrary, Theorem 1 is proved. ■

Note that $\beta^{(\mathrm{opt})}$ is irrelevant to the decision about $\mathcal{S}$ in the minimization of $\varepsilon_{\max}$ for unitary modulations. In other words, we may substitute $\beta^{(\mathrm{opt})}$ with an arbitrary positive number without making a wrong decision about $\mathcal{S}$. However, for nonunitary modulations, $\beta^{(\mathrm{opt})}$ must be found before $\mathcal{S}$ is chosen. We will discuss this topic in Section III-C.

Also note that $\varepsilon_{\max} - \varepsilon'_{\max} = 0$ when $\beta^{(\mathrm{opt})} D_{k_I} \ge \beta^{(\mathrm{opt})} D_m \ge 2$ or $D_m \le D_{k_I} \le 0$. Thus, we are free to include[3] any k to $\mathcal{S}$ if $\beta^{(\mathrm{opt})} D_k \ge 2$ and to exclude any m from $\mathcal{S}$ if $D_m \le 0$.

The largest I samples of D_k must be rounded to 2 and the others to 0. Our algorithm is then summarized as follows.

[3]We do not exploit this freedom in Algorithm 1 since we do not calculate $\beta^{(\mathrm{opt})}$.

Algorithm 1:

1) Choose a magnitude threshold A, and the number of iterations L.

Runtime:

1) For each $\mathbf{X}$, calculate x_n using **(2)**. Note that $J \geq 4$ is required. Let $\eta = |x_n|_{\max}$.

2) If $\eta > A$, let the iteration number $l = 1$ and go to Step 3; otherwise, transmit x_n and terminate.

3) Set the index set of negative signs empty $\mathcal{S} = \emptyset$; calculate f_n by using **(9)**.

4) Calculate D_k by using **(13)**. If all $D_k \leq 0$, go to Step 8; otherwise,

5) Calculate I by using **(18)**; generate $\mathcal{S}$ by rounding the largest I samples of D_k to 2 and rounding the other D_k to 0.

6) Calculate $\hat{\mathbf{X}}$ and $\hat{x}_n$ by using **(7)** and **(8)**, respectively. If $|\hat{x}_n|_{\max} < \eta$, let $\eta = |\hat{x}_n|_{\max}$, and store $\hat{x}_n$ as $\hat{x}_n^{(\mathrm{opt})}$.

7) If $l = L$ go to Step 8; otherwise, increase l by one, let $\mathbf{X} = \hat{\mathbf{X}}$ and $x_n = \hat{x}_n$, and go to Step 3.

8) Transmit $\hat{x}_n^{(\mathrm{opt})}$.

The complexity of this algorithm can be upper bounded as two fast Fourier transforms (FFTs) per iteration. The detailed complexity analysis is given in the Appendix.

C. Clipping Guided Sign-Selection Algorithm for Nonunitary Constellations

Let $\bar{d} = E\{|X_k|\}$. The size of $\mathcal{S}$ can be calculated from (11) as

$$I = \left\lceil \frac{\sqrt{N}(|x_n|_{\max} - A)}{2\bar{d}} \right\rceil. \tag{19}$$

For square M-ary QAM constellations, we have

$$\bar{d} = \frac{4}{M}\sqrt{\frac{6}{M-1}} \sum_{m=1}^{\sqrt{M}/2} \sum_{n=1}^{\sqrt{M}/2} \sqrt{(m-0.5)^2 + (n-0.5)^2}. \tag{20}$$

In nonunitary modulations, minimizing $\varepsilon_{\max}$ requires knowing $\beta^{(\mathrm{opt})}$. By using the adaptive-scaling algorithm [30], $\beta^{(\mathrm{opt})}$ is found by minimizing the out-of-range power P, i.e.

$$\beta^{(\mathrm{opt})} = \arg\min_{\beta} P \tag{21}$$

where

$$P = \sum_{|\bar{x}_n| > A} (|\bar{x}_n| - A)^2$$

is the total power of those $|\bar{x}_n| > A$, and $\bar{x}_n$ is the peak-reduced signal calculated in (15). Equation (21) is equivalent to [30]

$$\beta^{(\mathrm{opt})} = \arg\min_{\beta} \sum_{n \in \mathcal{S}_p} |f_n - \beta \hat{f}_n|^2 \tag{22}$$

where $\mathcal{S}_p = \{n : |x_n| > A, |x_n| > |x_{n-1}|, \text{ and } |x_n| \geq |x_{n+1}|\}$ is the index set of the peaks of f_n. By solving (22), we have [30]

$$\beta^{(\mathrm{opt})} = \frac{\Re\left[\sum_{n \in \mathcal{S}_p} f_n \hat{f}_n^*\right]}{\sum_{n \in \mathcal{S}_p} |\hat{f}_n|^2}. \tag{23}$$

Since $|X_k|$ is not a constant, the minimization of $\varepsilon_{\max}$ for nonunitary constellations depends on both $\beta^{(\mathrm{opt})} D_k$ and $|X_k|$. To illustrate the relationship between $\beta^{(\mathrm{opt})} D_k$, $|X_k|$ and $\varepsilon_{\max}$, we consider the following example.

1) Example 1: Suppose we have chosen $I - 1$ elements of $\mathcal{S}$, and the last element will be selected from $\beta^{(\mathrm{opt})} D_0$ and $\beta^{(\mathrm{opt})} D_1$, where $\beta^{(\mathrm{opt})} D_0 = 1.1$ and $\beta^{(\mathrm{opt})} D_1 = 1.04$.

Case I: X_k are 4QAM symbols.

Since $D_0 > D_1$, $\varepsilon_{\max}$ is minimized when $\alpha_0 = 2$ and $\alpha_1 = 0$.

Case II: X_k are 16QAM symbols, and $X_0 = 0.5 + 0.5j$, $X_1 = 1.5 + 1.5j$.

It is easy to verify that $\varepsilon_{\max} = 2.84$ if $\alpha_0 = 2$ and $\alpha_1 = 0$. However, $\varepsilon_{\max} = 2.81$ when $\alpha_0 = 0$ and $\alpha_1 = 2$. Thus, the optimum sign-selection is $\alpha_0 = 0$ and $\alpha_1 = 2$ although $D_0 > D_1$. □

In general, we have the following theorem for minimizing $\varepsilon_{\max}$ in nonunitary cases.

Theorem 2: In each iteration, let $\mathcal{S}$ have the size I that is calculated in (19). Define

$$T_k = \left|\left(2 - \beta^{(\mathrm{opt})} D_k\right) X_k\right| - \left|\beta^{(\mathrm{opt})} D_k X_k\right|. \tag{24}$$

Then, for nonunitary modulations, $\varepsilon_{\max}$ is minimized if

$$T_k \leq T_m \text{ for all } k \in \mathcal{S} \text{ and } m \notin \mathcal{S}. \tag{25}$$

Proof: Suppose $\mathcal{S}$ satisfies (25). Let $\mathcal{S} = \mathcal{S}_1 \cup \mathcal{S}_2$ and $\mathcal{S}_1 \cap \mathcal{S}_2 = \emptyset$. We may form another set $\mathcal{S}'$ by replacing $\mathcal{S}_2$ with $\mathcal{S}_3$ where $\mathcal{S}_3 \cap \mathcal{S} = \emptyset$. Since $\mathcal{S}$ and $\mathcal{S}'$ have the same size, $\mathcal{S}_2$ and $\mathcal{S}_3$ also have the same size. In the following, we show that $\mathcal{S}$ leads to a smaller maximum rounding error than $\mathcal{S}'$.

Let $\mathcal{S}_4$ contains the indices that are not in $\mathcal{S}_1$, $\mathcal{S}_2$ and $\mathcal{S}_3$, i.e.

$$\bigcup_{i=1}^{4} \mathcal{S}_i = \mathcal{N}$$

where $\mathcal{N} = [0, \ldots, N-1]$ is the OFDM index set and

$$\mathcal{S}_i \cap \mathcal{S}_k = \emptyset, \text{ for any } i \neq k.$$

The maximum rounding error caused by $\mathcal{S}$ is the given by

$$\varepsilon_{\max} = \sum_{k\in\mathcal{S}_1\cup\mathcal{S}_2} \left|\left(2-\beta^{(\mathrm{opt})}D_k\right)X_k\right| + \sum_{k\in\mathcal{S}_3\cup\mathcal{S}_4} \left|\beta^{(\mathrm{opt})}D_kX_k\right|$$

and the maximum rounding error caused by $\mathcal{S}'$ is

$$\varepsilon'_{\max} = \sum_{k\in\mathcal{S}_1\cup\mathcal{S}_3} \left|\left(2-\beta^{(\mathrm{opt})}D_k\right)X_k\right| + \sum_{k\in\mathcal{S}_2\cup\mathcal{S}_4} \left|\beta^{(\mathrm{opt})}D_kX_k\right|.$$

Therefore, we have

$$\begin{aligned}\varepsilon_{\max} - \varepsilon'_{\max} &= \sum_{k\in\mathcal{S}_2} \left|\left(2-\beta^{(\mathrm{opt})}D_k\right)X_k\right| + \sum_{k\in\mathcal{S}_3}\left|\beta^{(\mathrm{opt})}D_kX_k\right| \\ &\quad - \sum_{k\in\mathcal{S}_3} \left|\left(2-\beta^{(\mathrm{opt})}D_k\right)X_k\right| - \sum_{k\in\mathcal{S}_2}\left|\beta^{(\mathrm{opt})}D_kX_k\right| \\ &= \sum_{k\in\mathcal{S}_2}\left(\left|\left(2-\beta^{(\mathrm{opt})}D_k\right)X_k\right| - \left|\beta^{(\mathrm{opt})}D_kX_k\right|\right) \\ &\quad - \sum_{k\in\mathcal{S}_3}\left(\left|\left(2-\beta^{(\mathrm{opt})}D_k\right)X_k\right| - \left|\beta^{(\mathrm{opt})}D_kX_k\right|\right) \\ &= \sum_{k\in\mathcal{S}_2} T_k - \sum_{k\in\mathcal{S}_3} T_k \le 0.\end{aligned}$$

Since $\mathcal{S}_2$ and $\mathcal{S}_3$ are arbitrarily selected, $\mathcal{S}$ minimizes the maximum rounding error. ■

Our algorithm for nonunitary modulations can be modified from Algorithm 1 as follows, where the omitted parts are the same as those in Algorithm 1:

Algorithm 2:

• • •

5) Find $\mathcal{S}$ as follows.

a) Calculate I, $\beta^{(\mathrm{opt})}$ and T_k using **(19)**, **(23)**, and **(24)**, respectively.

b) Find the smallest I samples of T_k, and denote them as $T_{k_1},\ldots,T_{k_I}$; set $\alpha_i = 2$ for $i = k_1,\ldots,k_I$, and set other α_i to 0.

• • •

Since calculating $\beta^{(\mathrm{opt})}$ involves an FFT operation (for calculating $\hat{f}_n$), the complexity of this algorithm is upper-bounded as three FFTs per iteration.

Remark 1 (Adaptively Calculating the Size of $\mathcal{S}$): We may directly use (11) to find the size of $\mathcal{S}$. That is, in each iteration, we start from an empty set $\mathcal{S} = \emptyset$ and flip the signs one-by-one. The condition in (11) is tested every time a sign is flipped. When (11) is satisfied, we generate the $\hat{\mathbf{X}}$ and go on to the next iteration.

Note that the size of $\mathcal{S}$ now depends on the choice of its elements. Then, the condition of Theorem 2 is not satisfied. Although in this case (25) does not ensure the minimization of $\varepsilon_{\max}$, the resulting $\varepsilon_{\max}$ is still small. Moreover, an adaptive size of $\mathcal{S}$ more precisely meets Criterion 1. Thus, adaptively calculating the size of $\mathcal{S}$ would not degrade the PAR reduction performance. In fact, the simulations show that the PAR reduction performance is improved by using an adaptive size of $\mathcal{S}$.

Remark 2 (Simplification): The complexity of Algorithm 2 can be simplified to two FFTs per iteration by using the mean of $\beta^{(\mathrm{opt})}$, which is calculated at the initialization stage by using the level crossing theory [31], and is used for all OFDM blocks.

Unless A is small, the clipping noise consists of a series of pulses [6]. In [6], we have proved that, if the clipping noise contains only one dominant pulse, the peak of $\hat{f}_n$ is proportional to that of f_n

$$\left|\hat{f}_n\right|_{\max} = \gamma\left|f_n\right|_{\max}$$

where γ is the constant of proportionality, whose value depends on the OFDM bandwidth and the clipping level. The mean of γ is

$$\bar{\gamma} = \frac{2\sqrt{2}}{\sqrt{3\pi}}\frac{1}{\frac{A}{\sigma}}$$

where $2\sigma^2$ is the average power of the OFDM signals. Since $\beta^{(\mathrm{opt})}\hat{f}_n$ must be close to f_n at the positions of most peaks of f_n, we may estimate the mean of $\beta^{(\mathrm{opt})}$ as

$$\bar{\beta} = \frac{1}{\bar{\gamma}} = \frac{\sqrt{3\pi}A}{2\sqrt{2}\sigma}. \tag{26}$$

Remark 3 (Choice of A): As is common with clipping-based PAR reduction algorithms, one has to perform experiments to choose a suitable clipping threshold A. In general, A should be small if a large PAR reduction is required, and A should be relatively large if low complexity is desired.

Remark 4 (Further Discussion): Our algorithm is effective for large N and for OFDM symbols with large PAR. This result can be intuitively explained as follows.

1) Since SLM with a small number of candidates can effectively reduce the PAR, when N is large, there are a large number of sign sequences that can effectively reduce the PAR. So the CGS algorithms too can pick one of these sequences.
2) The PAR reduction performance of the CGS algorithms is determined mainly by the errors in estimating the size of $\mathcal{S}$ and in rounding $\beta^{(\mathrm{opt})}D_k$ to 2 or 0. In Section IV, we will see that using the average size of $\mathcal{S}$ leads to smaller PAR reduction than using the adaptive size of $\mathcal{S}$ while using $\beta^{(\mathrm{opt})}$ or $\bar{\beta}$ gives virtually the same PAR reduction. Therefore, the accuracy of estimating the size of $\mathcal{S}$ plays a more important role in our algorithm than the minimization of the rounding error.
   a) Effect of the error of estimating the size of $\mathcal{S}$.
   If such an estimation error occurs, we have either rounded too many D_ks to 2 or rounded too many D_ks

to 0. In either case, the maximum degradation of peak reduction is

$$\varepsilon_d = \sum_{k \in \mathcal{S}_e} \frac{2|X_k|}{\sqrt{N}},$$

where $\mathcal{S}_e$ is the difference between the $\mathcal{S}$ we are using and the optimum $\mathcal{S}$. Thus, $\mathcal{S}_e$ is small when N is large.

b) Effect of the error of rounding $\beta^{(\mathrm{opt})} D_k$ to 2 or 0.
Ideally, a peak canceling signal is a series of pulses, which cancels large peaks in the OFDM signal without introducing any new peaks or increasing any small peaks. Such an ideal signal is not realizable because of the spectrum constraints on peak canceling signals.
In our algorithms, the rounding errors add new peaks to the OFDM signal. In OFDM signals with small PAR, most peaks have comparable magnitude. Thus, the new peaks introduced by the rounding errors are likely to fall on some existing peaks of the OFDM signal. Little PAR reduction can be obtained.
On the other hand, in OFDM signals with large PAR, since the number of large peaks is small [30], the liklihood that the new peaks introduced by the rounding errors will fall on the large peaks of the OFDM signal is also small. Thus, the large PAR can be effectively reduced.

IV. Simulation Results

In this section, we compare the CGS algorithms with SLM, PTS, derandomization, and tone reservation techniques for a 256-subcarrier OFDM system. Four times oversampling $(J = 4)$ is used in optimization and eight times oversampling $(J = 8)$ is used in calculating the PAR after an optimal sign sequence is selected. For CGS, the average $\beta^{(\mathrm{opt})}$, $\bar{\beta}$, and the adaptively calculated size of $\mathcal{S}$ are used (see Remarks 1 and 2). The adaptive scaling algorithm [30] with eight iterations is used for tone reservation, where randomly selected 1/6 subcarriers are reserved for PAR reduction. Thus, the amount of redundancy in CGS is the same as that in tone reservation and derandomization, but is higher than that in SLM and PTS. The complexity of the tone reservation technique is the same as that of CGS with eight iterations.

Fig. 3 compares the PAR reduction of these algorithms in terms of the PAR complementary cumulative distribution function (CCDF) $F(\xi_0) = Pr[\xi > \xi_0]$. Unit-energy 64QAM is used in this experiment. Thus, for SLM, PTS and tone reservation, each data symbol contains 6 data bits. For CGS and derandomization, each data symbol contains 5 data bits and one sign bit for PAR reduction. The PAR reduction of tone reservation is about 0.1 dB larger than CGS with eight iterations at $F(\xi_0) = 10^{-3}$, and is the same at $F(\xi_0) = 10^{-4}$ (about 5.9 dB). Tone reservation requires a 0.42-dB average power increase, leading to a BER increase, while CGS does not change the average power. In this experiment, the performance of tone reservation (in terms of PAR reduction, complexity, and average power) is roughly the same as that of CGS. However, note that in tone reservation, the reserved tones are randomly selected, giving rise to the best PAR

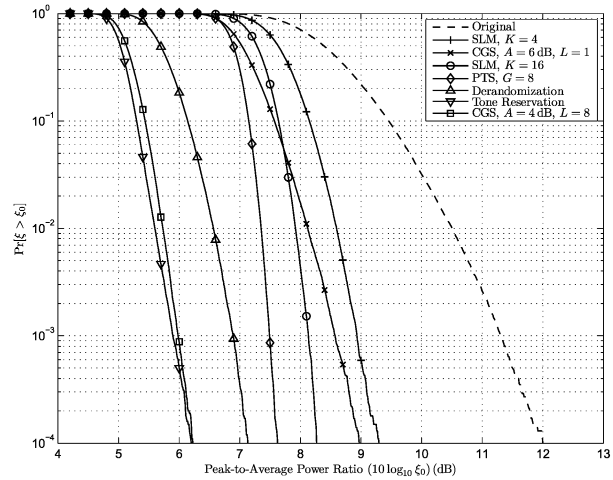

Fig. 3. PAR reduction comparison of CGS with $\bar{\beta}$ and adaptive size of $\mathcal{S}$, SLM, PTS, derandomization, and tone reservation, where $N = 256$, and 64QAM is used.

reduction performance. In practice, tone reservation is subject to certain constraints (e.g., reserved tones must be on the two ends of the OFDM band), and the PAR reduction is decreased.

An advantage of CGS is that its PAR reduction seems independent of the size of signal constellation. Thus, if a larger constellation is used, the redundancy of CGS is reduced. On the other hand, the PAR reduction of tone reservation replies on the amount of redundancy (i.e., the ratio between the number of reserved tones and N) [30]. Thus, if a large constellation is used, CGS obtains larger PAR reduction than tone reservation with the same amount of redundancy.

At $F(\xi_0) = 10^{-4}$, the PAR reduction of CGS with eight iterations is about 2.1 dB larger than that of SLM with 16 candidates, 1.4–dB larger than that of PTS with eight randomly partitioned subgroups, and 0.9 dB larger than that of the derandomization algorithm. In this experiment, both SLM and CGS require 16 FFTs per OFDM block.[4] PTS must test $2^7 = 128$ combinations.

CGS with one iteration obtains a 3.2-dB PAR reduction, a 0.3-dB gain over SLM with four candidates. In this experiment, CGS and SLM require two and four FFTs per OFDM block.

The average computation time of these algorithms is listed in Table I for a Pentium IV 3.40G computer with Matlab R14 Service Pack 2. The time complexity of CGS with eight iterations is only 89% of SLM with 16 candidates, 9.4% of PTS with eight subgroups, and 1.4% of the derandomization algorithm.

SLM, PTS, derandomization, and CGS are now compared in terms of the power spectrum density (PSD) of the output of the OFDM transmitter. As before, 64QAM is used. The peak-reduced OFDM signal is passed through a solid-state power amplifier (SSPA) [1]

$$y(t) = \frac{|x(t)|}{\left(1 + \left(\frac{|x(t)|}{C}\right)^{2p}\right)^{(1/2p)}} e^{j\phi(t)}$$

[4]This is a loose complexity upperbound for CGS. The actual complexity of CGS is lower than that of SLM (see Table I).

TABLE I
TIME COMPLEXITY ($N = 256$ AND 64QAM)

Algorithm	Average Computation Time(millisecond)
CGS, $A = 4\,\mathrm{dB}$, $L = 8$	4.9
SLM, $K = 16$	5.5
PTS, $G = 8$	52.1
Derandomization	362.0

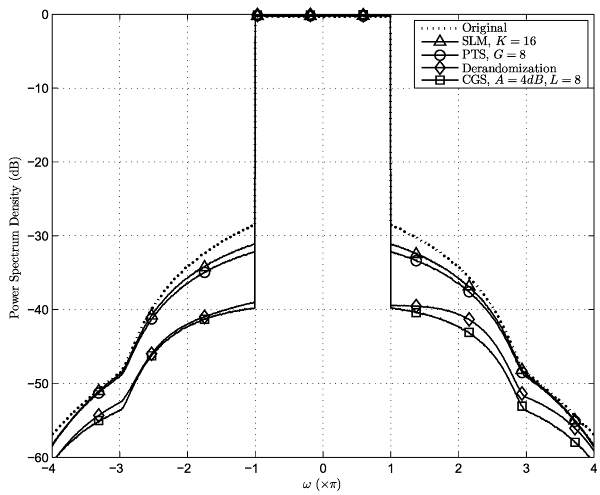

Fig. 4. Power spectrum density of SLM, PTS, derandomization, and CGS, where $N = 256$, and 64QAM is used.

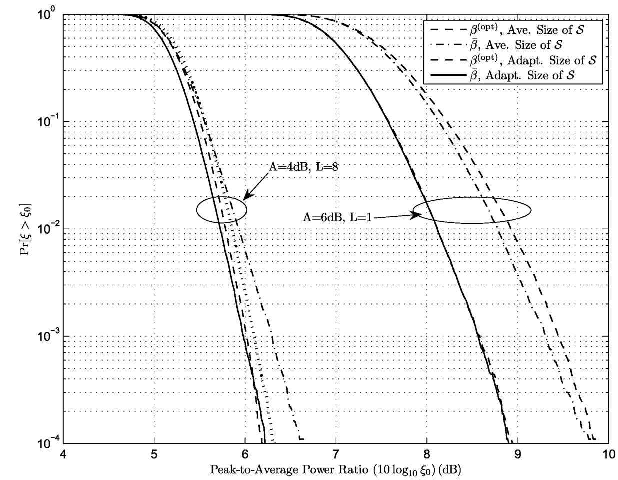

Fig. 6. PAR reduction comparison of different configurations of CGS, where $N = 256$, and 16QAM is used.

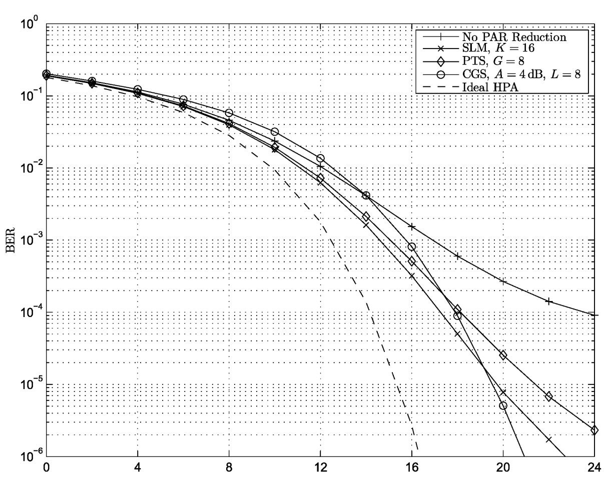

Fig. 5. BER of SLM, PTS and CGS, where $N = 256$, 64QAM is used for CGS and 32QAM is used for SLM and PTS.

where $x(t) = |x(t)|e^{j\phi(t)}$ is the input, and $y(t)$ is the output of SSPA. Usually, $p = 3$ for practical SSPA. In our simulations, we choose $C = 6$ dB. Fig. 4 shows the result. CGS with eight iterations leads to only -40 dB out of band radiation, which is 1 dB lower than derandomization, 8 dB lower than PTS with eight subgroups, 9 dB lower than SLM with 16 candidates, and 12 dB lower than that obtained without using any PAR reduction techniques, respectively.

Fig. 5 compares the BER performance of CGS, SLM, and PTS for a 256-subcarrier OFDM system. 64QAM is used for CGS where one bit per subcarrier is sacrificed for PAR reduction, and 32QAM is used for SLM and PTS. For SLM and PTS, we assume the receiver has perfect side information, and ignore the throughput loss due to side information. Thus, the throughput of CGS is the same as that of SLM and PTS. In this experiment, we choose a tight SSPA saturation point $C = 5\,\mathrm{dB}$. For reference purpose, the BER when the SSPA is used but no PAR reduction technique is applied to the OFDM signal, and the BER when an ideal HPA is used are also simulated.

Without PAR reduction, the received OFDM signal exhibits an error floor at the BER of 10^{-4}. The BER is reduced when PAR reduction techniques are applied. Although the BERs of SLM and PTS are smaller than the BER of CGS at the low signal-to-noise ratio (SNR) region,[5] the BER of CGS is smaller than those of SLM and PTS when $E_b/N_0 > 19\mathrm{dB}$. The error floor due to the nonlinearity of HPA when CGS is used is much smaller than that when SLM or PTS is used.

Fig. 6 compares Algorithm 2 to different configurations including $\bar{\beta}+$ average size of $\mathcal{S}$, $\bar{\beta}+$ adaptive size of $\mathcal{S}$, $\beta^{(\mathrm{opt})}+$ average size of $\mathcal{S}$, and $\beta^{(\mathrm{opt})}+$ adaptive size of $\mathcal{S}$. Two hundred fifty-six subcarriers and 16QAM symbols are used. The use of the adaptive size of $\mathcal{S}$ provides a larger PAR reduction than that obtained by the use of the average size of $\mathcal{S}$. On the other hand, using $\beta^{(\mathrm{opt})}$ leads to virtually the same PAR reduction as that obtained by using $\bar{\beta}$. Note that the complexity is three FFTs per iteration when $\beta^{(\mathrm{opt})}$ is used, but, two FFTs per iteration when $\bar{\beta}$ is used.

[5]This is because the channel noise is much larger than the nonlinear distortion of the SSPA at the low SNR region.

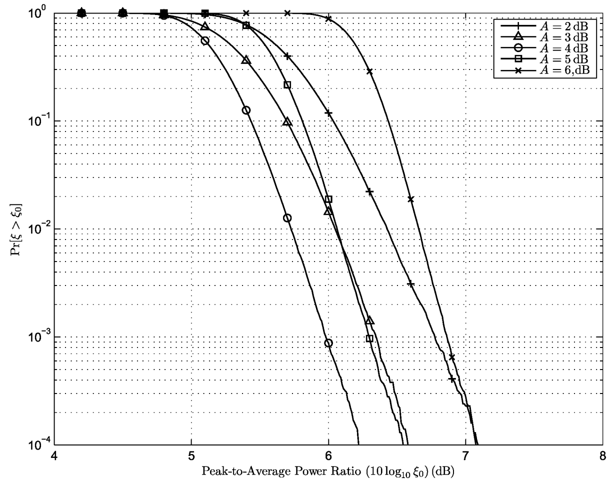

Fig. 7. PAR reduction comparison of CGS with $L = 8$ and different A, where $N = 256$, and 16QAM is used.

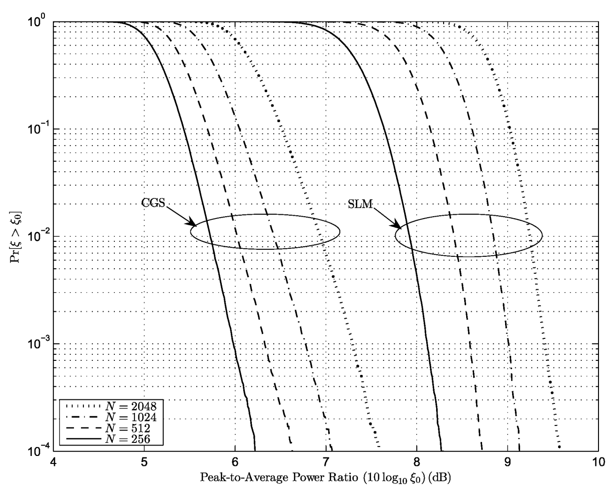

Fig. 8. PAR reduction comparison of CGS and SLM for 16QAM input and different N, where $K = 16$ for SLM, $L = 8$, and $A = 4$ dB for CGS.

Fig. 7 shows the performance of Algorithm 2 with fixed eight iterations and different clipping levels. Two hundred fifty-six subcarriers and 16QAM symbols are used, and the CGS configuration is $\bar{\beta}+$ adaptive size of $\mathcal{S}$. Note that $A = 4$ dB provides the largest PAR reduction. Other choices of A lead to performance degradation. However, since the largest performance degradation is only about 0.9 dB, the proposed CGS algorithm is not overly sensitive to the choice of A.

Fig. 8 compares CGS with $\bar{\beta}+$ adaptive size of $\mathcal{S}$ and SLM for OFDM systems with different number of subcarriers, where other parameters are same. When the number of subcarriers doubles, the PAR reduction of both algorithms decreases by about 0.4 dB.

V. Conclusion

Clipping-noise guided sign-selection for PAR reduction has been considered. We proposed criteria for choosing the number of signs that should be flipped and for choosing which subcarrier signs should be flipped. Using these two criteria, two CGS PAR reduction algorithms have been derived to handle unitary and nonunitary signal constellations. The simulation results show that the PAR reduction of CGS is about 1 dB larger than that of derandomization and tone reservation for a 256-subcarrier system, and is about 1–2 dB larger than that of PTS and SLM. CGS also removes the error floor due to the nonlinearity of the HPA.

Appendix

Complexity Analysis: We only analyze the complexity of Algorithm 1 only. Algorithm 2 can be analyzed similarly.

The complexity of our algorithm[6] is determined mainly by (8) and (12) while the complexity of other calculations involved in our algorithm is $\mathcal{O}(N)$ and is independent of J.

Equation (12) can be calculated by using an FFT. The average number of nonzero samples in f_n can be calculated as [30]

$$\bar{N}_f = JNe^{-A^2/(2\sigma^2)}$$

where $2\sigma^2 = 1$ is the mean power of the OFDM signal. $\bar{N}_f/N$ is usually small; e.g., $\bar{N}_f/N \approx 32\%$ for $A = 4$ dB and $J = 4$. Since most samples of f_n are 0, the complexity of (12) is much lower than that of a length-(JN) FFT.

Equation (8) also involves a small-size $\mathcal{S}$. The average size of $\mathcal{S}$ can be calculated as

$$E\{I\} = \int_A^\infty I(r)p(r)dr$$

where $I(r)$ is the size of $\mathcal{S}$ for $r = |x_n|_{\max}$

$$I(r) = \left\lceil \frac{\sqrt{N}(r-A)}{2\bar{d}} \right\rceil$$

and $\bar{d} = E\{|X_k|\}$. By using the approximate PAR cumulative distribution function (CDF) [32]

$$\Pr(\max|x_n| < r) \approx (1 - e^{-r^2})^{\alpha N}$$

where α is empirically obtained as 2.8, the probability density function (pdf) of PAR can be found as

$$p(r) = 2\alpha N r e^{-r^2}(1 - e^{-r^2})^{\alpha N - 1}.$$

Then

$$E\{I\} = \int_A^\infty \left\lceil \frac{\sqrt{N}(r-A)}{2\bar{d}} \right\rceil p(r)dr.$$

When $N = 256$ and $A = 4$ dB, $E\{I\} \approx 8.1$ for 4QAM and $E\{I\} \approx 8.6$ for 16QAM, where the simulated results are 7.9 and 8.4, respectively. If the adaptive size of $\mathcal{S}$ was used, $E\{I\}$ would be a little larger. For example, for $N = 256$, $A = 4$ dB and 16QAM symbol input, the mean of the adaptive size of $\mathcal{S}$ obtained by simulation is 12.6. In any case, the complexity of (8) is much less than that of a length-(JN) FFT.

To simplify the complexity comparison, we loosely upper bound the complexity of Algorithm 1 as two FFTs per itera-

[6]Here, the Runtime Step 1 is excluded because it is required in all OFDM systems with or without PAR reduction techniques.

tion. With similar analysis, it can be shown that the complexity upper-bounds of Algorithm 2 and its simplification are three and two FFTs per iteration, respectively.

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