

# Improved OFDMA Uplink Frequency Offset Estimation via Cooperative Relaying: AF or DcF?

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**Abstract**—This paper evaluates the performance improvement in Orthogonal Frequency-Division Multiplexing Access (OFDMA) uplink frequency offset estimation achieved with cooperative relaying. The transmission of each source node (node  $S$ ) can be improved by optimizing the diversity gain through exploiting the cooperation of the other nodes (cooperative relays), and the relays can operate in either the amplify-and-forward (AF) or decode-and-compensation-and-forward (DcF) mode. One or more than one geographically closely located mobile nodes comprise a cooperative group (CG), and the nodes of the same CG cooperate with each other. In each transmission, the role of the relay is to “help” the source node transmit its training sequence, or, in other words, the relay creates a parallel route between the source node and the destination terminal to improve the reliability of the transmission of  $S$ . In the proposed cooperative scheme, the total power used to transmit each training sequence, including that consumed in node  $S$  and the relay, is kept constant. Based on the interference analysis, the Signal-to-Interference-plus-Noise Ratio (SINR) in both the relay and the destination terminal are derived. The AF mode’s cooperative scheme always outperforms that of the DcF mode in terms of frequency offset estimation accuracy due to the estimation error propagation in the latter.

## I. INTRODUCTION

In orthogonal Frequency-Division Multiplexing Access (OFDMA), each user employs a different set of orthogonal sub-carriers to transmit data simultaneously. The use of orthogonal sub-carrier sets for different users eliminates multiple access interference (MAI) under perfect conditions. OFDMA, therefore, has been or is being considered for various wireless systems [1].

Frequency synchronization issues for OFDMA have been widely researched [1]. However, all these algorithms perform frequency offset estimation based on a point-to-point connection, and the performance may be degraded due to signal fading. To improve the reliability of the signal transmission, multiple links connecting the source node and the destination terminal can be created to transmit an identical symbol independently. Some combining schemes, e.g., equal gain combining (EGC) and maximal ratio combining (MRC), can be used to effectively combine the received signals through different links. One way to realize a spatial diversity gain is to use “cooperative diversity” (see [2], [3]). Three time-division multiple-access-based cooperative schemes are proposed in [2], where one relay is used to improve the transmit quality from node  $S$  (the source terminal) to node  $D$  (the destination

terminal), and the relay can operate in either the amplify-and-forward (AF) or decode-and-forward (DF) mode. A performance comparison of the AF and DF modes is evaluated in [3], which suggests choosing geometrically closed users as partners to reduce the probability of inter-user outage.

Unfortunately, none of these studies has considered the frequency offset. Since diversity combiners require co-phased signals, the bit error rate (BER) performance of the cooperative transmission may be degraded, because the phase rotations due to the frequency offsets in different links are different. In [4], cooperative OFDMA uplink transmission is studied, and the performance degradation in terms of the cooperative transmission rate due to the frequency offset is discussed. A space-time cooperative OFDM (CO-OFDM) system is proposed in [5], where a time-domain periodic training sequence is used to perform both synchronization and channel estimation.

In this paper, a cooperative scheme is proposed to improve the performance of any conventional training/pilot-based OFDMA uplink frequency offset estimation algorithm, but the exact training/pilot design is beyond the scope of this paper. Without loss of generality, perfect time synchronization is also assumed in this paper. In the proposed cooperative scheme, the source node (node  $S$ ) and its relays comprise a cooperative group (CG), and the relays can operate in either the AF or the decode-and-compensation-and-forward (DcF) mode. The DcF in this paper is slightly different from the DF proposed in [2]: if the relay operates in the DcF mode, the relay should first estimate the frequency offset between the source node and itself, and then demodulate and decode the received training sequence. After doing so, the relay will use the estimation result to pre-compensate for the frequency offset in the decoded training sequence and re-transmit this re-generated training sequence to the destination terminal. The proposed cooperative transmission is performed within two time slots. In the first time slot,  $S$  transmits its training sequence to its relays and  $D$ , and the chosen relay (the relay with the best  $S \rightarrow R$  channel will be chosen as the relay) re-transmits the training sequence to  $D$  in the second time slot. We first evaluate the Signal-to-Interference-plus-Noise Ratio (SINR) in both  $D$  and  $R$ , and then analyze the variance error of the frequency offset estimation in a given SINR.

The remainder of this paper is organized as follows. Section II proposes the cooperative OFDMA uplink signal model.

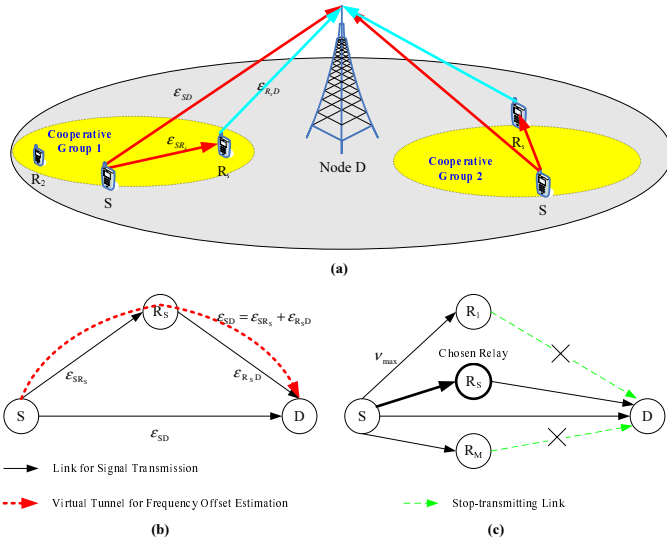


Fig. 1. Cooperative frequency offset estimation in OFDMA uplink transmission (a) Cooperative Group organization (b) Virtual tunnel created by cooperative relay (c) Relay selection in cooperative OFDMA uplink transmission.

The frequency offset estimation in the proposed cooperative scheme is analyzed in Section III, and the numerical results are given in Section IV. Finally, Section V concludes the paper.

*Notation:*  $(\cdot)^H$  denotes the conjugate transpose of a matrix. The imaginary unit is  $j = \sqrt{-1}$ . A circularly symmetric complex Gaussian variable with mean  $a$  and variance  $\sigma^2$  is denoted by  $z \sim \mathcal{CN}(a, \sigma^2)$ .  $\mathbf{x}[i]$  represents the  $i$ -th element of vector  $\mathbf{x}$ .  $[\mathbf{A}]_{ij}$  represents the  $ij$ -th element of matrix  $\mathbf{A}$ .  $\mathbb{E}\{x\}$  and  $\text{Var}\{x\}$  denote the mean and the variance  $x$ , respectively.

## II. COOPERATIVE OFDMA UPLINK SIGNAL MODEL

In the OFDMA uplink, each subcarrier of each user is modulated by using complex data symbols from a signal constellation, e.g., phase-shift keying (PSK) or quadrature amplitude modulation (QAM). The total number of subcarriers is assumed to be  $N$ , and  $M$  users are accessing the base station. Each user is assumed to be allocated  $N_u$  unique subcarriers, where  $MN_u \leq N$ . A cyclic-prefix (CP) is inserted in each OFDMA symbol to mitigate the inter-symbol-interference (ISI), and the power consumption in CP is assumed to be negligible compared to the power consumed in the data-carrying part of the symbol.

In a conventional non-cooperative transmission, each user transmits its signal to the base station by using its own subcarriers, and the subcarriers allocated to the other users will be modulated by using NULL signals to maintain an orthogonality among the different users. If one user can transmit its own signal by using its own subcarriers and simultaneously transmit signals for the other users by modulating these users' subcarriers with their signals, the reliability of the each user's transmission will be improved. Without loss

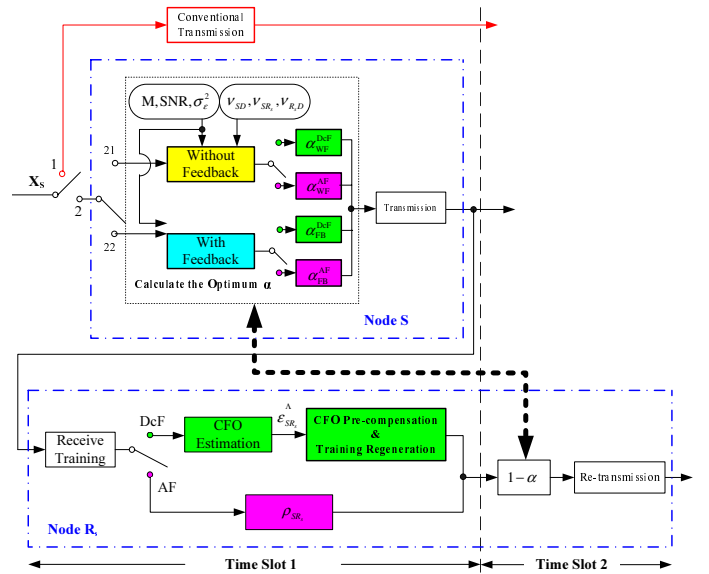


Fig. 2. Adaptive cooperation in OFDMA uplink frequency offset estimation.

of generality, we use node  $S$  to represent the source node and  $R_k, k \in \{1, \dots, M\}$ , to represent the  $k$ -th relay of  $S$ , where  $M \leq \mathcal{M} - 1$  is the total number of relays of  $S$  in one transmission. OFDMA uplink transmission with cooperative relays is shown in Fig. 1(a). In different transmissions,  $M$  may be different.

For each  $S$  and  $D$ ,  $\varepsilon_{SD} = \varepsilon_{SR_s} + \varepsilon_{R_s D}$  is satisfied for the chosen relay  $R_s$ , where  $\varepsilon_{ab}$  represents the frequency offset between nodes  $a$  and  $b$ . When the wireless link  $S \rightarrow D$  suffers a deep fading, a virtual tunnel can be created by using a relay  $R_s$  to improve the reliability of the transmission of  $S$ , as shown in Fig.1(b). If  $R_s$  can estimate  $\varepsilon_{SR_s}$  with a high accuracy based on a received training sequence, it can re-generate the training sequence by multiplying the  $k$ -th sample of the training sequence with  $e^{\frac{j2\pi k \hat{\varepsilon}_{SR_s}}{N}}$  ( $\hat{\varepsilon}_{SR_s}$  is the estimated  $\varepsilon_{SR_s}$ ), and then sends the resulting training sequence to  $D$ . Since the frequency offsets of the received training sequences transmitted through the links of both  $S \rightarrow D$  and  $S \rightarrow R_s \rightarrow D$  are identical (i.e., equal to  $\varepsilon_{SD}$ ), node  $D$  does not care which link the received training sequence is transmitted from.

Cooperative transmission can be performed within two time slots. The time slot is conceptually interchangeable with the training sequence. In the first time slot, each source node transmits its training sequence to all its relays and the node  $D$ . In the second time slot, the chosen relay will forward the received/decoded training sequence of  $S$  to  $D$ . The forward operation depends on which forward mode is applied. If the AF mode is applied, the relay simply amplifies and re-transmits the received training sequence, including the additive noise, to  $D$ ; if the DcF mode is applied, the chosen relay first decodes the received symbols and estimates the frequency offset between  $S$  and itself. After doing so, the relay will

use the estimated frequency offset to pre-compensate for the frequency offset between  $S$  and itself and then re-transmit the re-generated training sequence to  $D$ .

#### A. The First Time Slot

In the first time slot, the received signal at node  $D$  and relay  $R_k$  can be represented as

$$\mathbf{Y}_{D,1} = \mathbf{E}_{SD} \mathbf{F}_S \mathbf{H}_{SD} \Phi_{S,1} \mathbf{X}_S + \sum_{R_s \neq S} \mathbf{E}_{R_k D} \mathbf{F}_{R_k} \mathbf{H}_{R_k D} \Phi_{R_k,1} \mathbf{X}_{R_k} + \mathbf{W}_{D,1} \quad (1)$$

and

$$\mathbf{Y}_{R_k,1} = \mathbf{E}_{SR_k} \mathbf{F}_S \mathbf{H}_{SR_k} \Phi_{S,1} \mathbf{X}_S + \sum_{R_l \neq S, R_k} \mathbf{E}_{R_l R_k} \mathbf{F}_{R_l} \mathbf{H}_{R_l R_k} \Phi_{R_l,1} \mathbf{X}_{R_l} + \mathbf{W}_{R_k}, \quad (2)$$

respectively, where  $\mathbf{Y}_{D,1}$  and  $\mathbf{Y}_{R_k,1}$  are  $N \times T$  matrices.  $\mathbf{E}_{ab}$  is an  $N \times N$  diagonal matrix with

$$\mathbf{E}_{ab} = \text{diag} \left\{ e^{j\psi_{ab}}, e^{j\left(\frac{2\pi\epsilon_{ab}}{N} + \psi_{ab}\right)}, \dots, e^{j\left(\frac{2\pi\epsilon_{ab}(N-1)}{N} + \psi_{ab}\right)} \right\}, \quad (3)$$

where  $\psi_{ab}$  and  $\epsilon_{ab}$  representing the initial phase and normalized carrier frequency offset (the frequency offset normalized to one subcarrier bandwidth) between nodes  $a$  and  $b$ , respectively. Without loss of generality, we assume that  $\psi_{ab} = 0$ . We also assume that the oscillators of the mobile nodes should be calibrated with the oscillator of the base station and, therefore, that each  $\epsilon_{aD}$ ,  $a \in \{S, R_1, \dots, R_M\}$ , can be approximated as an independent and identically distributed (i.i.d.) random variable (RV) with mean zero and variance  $\sigma_\epsilon^2$ .  $\mathbf{F}_z$  is an  $N \times N_u$  matrix, which denotes the Inverse Discrete Fourier Transform (IDFT) matrix for the  $z$ -th user. In this paper, we use a set  $G_z$  to represent the subcarriers allocated to node  $z$ .  $\mathbf{F}_z$  can be generated from the  $N \times N$  IDFT matrix  $\mathbf{F}$  with  $[\mathbf{F}]_{mn} = \frac{1}{\sqrt{N}} e^{j\frac{2\pi mn}{N}}$ ,  $0 \leq m, n \leq N-1$ , by deleting all the columns with the column indexes not belonging to  $G_z$ , where  $N$  is the Discrete Fourier Transform (DFT) length.  $\Phi_{z,1} = \text{diag} \left\{ \sqrt{P_{z,1,i}} : i \in G_z, z \in \{S, R_1, \dots, R_M\} \right\}$  is the  $N_u \times N_u$  diagonal matrix with each diagonal entry representing the transmit power of one subcarrier of node  $z$  in the first time slot. We assume that  $\alpha N_u \bar{P}$  is allocated to node  $S$  in the first time slot, and in the second time slot, the chosen relay uses the remaining power, i.e.,  $(1-\alpha)N_u \bar{P}$ , to re-transmit the received training sequence, where  $\bar{P}$  represents the average power of each subcarrier, and  $0 < \alpha < 1$ .  $\mathbf{X}_z = [\mathbf{x}_{z,1}, \dots, \mathbf{x}_{z,T}]$  represents the transmit matrix of node  $z$ , which is an  $N_u \times T$  matrix ( $T = 1, 2, \dots$ ), and without loss of generality, we assume that  $[\mathbf{X}_z]_{mn} \sim \mathcal{CN}(0, 1)$ .  $\mathbf{W}_{R_k}$  and  $\mathbf{W}_{D,1}$  are  $N_u \times T$  matrices of additive white Gaussian noise (AWGN) with  $\{\mathbf{W}_{R_k}[m], \mathbf{W}_{D,1}[m]\} \sim \mathcal{CN}(0, \sigma_w^2)$ .  $\mathbf{H}_{zb} = \text{diag} \left\{ H_{zb}^{(i)} \right\}$ ,  $i \in G_z, z \in \{S, R_1, \dots, R_M\}, b \in \{D, R_1, \dots, R_M\}$ , with  $H_{zb}^{(i)}$  denoting the channel attenuation between nodes  $z$  and  $b$  at the  $i$ -th subcarrier. We make the following assumptions

for channel attenuations:  $H_{zb}^{(i)} \sim \mathcal{CN}(0, 1)$  for each  $b \neq z, D$ , and  $H_{zD}^{(i)} \sim \mathcal{CN}(0, \mathcal{L}_u)$  for each  $z$ , where  $\mathcal{L}_u$  is the large-scale fading coefficient with  $\mathcal{L}_u < 1$ .

Based on the interference analysis in [6], the effective SINR at nodes  $D$  and  $R_k$  is given by

$$\gamma_{SD,1} = \frac{\alpha \bar{P} \beta_1 \cdot \nu_{SD}}{\frac{\mathcal{L}_u \pi^2 N_u \sigma_\epsilon^2 \alpha \bar{P}}{3} + N_u \sigma_w^2}, \quad (4a)$$

$$\gamma_{SR_k,1} = \frac{\alpha \bar{P} \beta_2 \cdot \nu_{SR_k}}{\frac{2\pi^2 N_u \sigma_\epsilon^2 \alpha \bar{P}}{3} + N_u \sigma_w^2}, \quad (4b)$$

respectively, where  $\nu_{Sz} = \text{trace} \left\{ \mathbf{H}_{Sz} \mathbf{H}_{Sz}^H \right\}$ ,  $z \in \{R_k, D\}$ ,  $\beta_1 = 1 - \frac{\pi^2 \sigma_\epsilon^2}{3} + \frac{\pi^4 \sigma_\epsilon^4}{20}$  and  $\beta_2 = 1 - \frac{2\pi^2 \sigma_\epsilon^2}{3} + \frac{\pi^4 \sigma_\epsilon^4}{5}$ .

#### B. The Second Time Slot

In the second time slot, one of the  $M$  candidates, i.e.,  $R_1, \dots, R_M$ , will be chosen as the relay of  $S$ . In this paper, we assume that each mobile node knows the channels within its own CG.  $R_s$  is chosen as the relay of  $S$  if

$$R_s = \arg \max_{R_1, \dots, R_M} \{\nu_{SR_1}, \dots, \nu_{SR_M}\}, \quad (5)$$

as shown in Fig. 1(c). Either the AF or DcF mode can be operated in the relay. If the DcF mode is operated,  $\epsilon_{SR_s}$  should be estimated at the relay, and the estimation result, i.e.,  $\hat{\epsilon}_{SR_s}$ , should be used to re-generate the training sequence. The estimation error in  $\hat{\epsilon}_{SR_s}$  will be propagated to the estimation result in the link  $S \rightarrow R_s \rightarrow D$ . Based on (5), the variance error in  $\hat{\epsilon}_{SR_s}$ , i.e.,  $\mathbb{E} \left\{ |\hat{\epsilon}_{SR_s} - \epsilon_{SR_s}|^2 \right\}$ , can be minimized. From [7, page 41] we know that each  $\nu_{SR_k}$  is a central *chi-square* RV with  $2N_u$  degrees of freedom. The expectation of  $\nu_{\max} = \max \{\nu_{SR_1}, \dots, \nu_{SR_M}\}$  is  $\bar{\nu}_{\max} = \mathbb{E} \{\nu_{\max}\}$ . Since  $\bar{\nu}_{\max}$  is a monotonically increasing function of  $M$ , a higher  $\bar{\nu}_{\max}$  can be achieved for a larger  $M$ .

1) *AF Mode*: If the relay operates in the AF mode, it simply re-transmits the received training sequence to the node  $D$ . The received training sequence at node  $D$  in the second time slot is

$$\mathbf{Y}_{D,2}^{\text{AF}} = \rho_{SR_s} \mathbf{E}_{R_s D} \mathbf{F}_S \mathbf{H}_{R_s D} \Phi_{R_s,2} \mathbf{F}_S^H \mathbf{Y}_{R_s,1} + \mathbf{T}_{R_s,2}^{\text{AF}} + \mathbf{W}_{D,2}, \quad (6)$$

where  $\Phi_{R_s,2} = \text{diag} \left\{ \sqrt{P_{R_s,2,i}} : i \in G_S \right\}$  represents the power consumed in  $R_s$  for re-transmission in the second time slot with  $P_{R_s,2,i} = (1-\alpha)\bar{P}$  for each  $i$ , and  $\mathbf{T}_{R_s,2}^{\text{AF}}$  represents the transmission of nodes other than  $R_s$  in the second time slot.  $\rho_{SR_s} = (\alpha \bar{\nu}_{\max} \bar{P} / N_u + \sigma_w^2)^{-\frac{1}{2}}$  represents the amplifying coefficients at node  $R_s$  when it re-transmits the received training sequence in the second time slot. From [4], the received SINR is given by

$$\gamma_2^{\text{AF}} = \frac{\mathcal{L}_u \alpha (1-\alpha) \bar{P}^2 \beta_1 \beta_2 \bar{\nu}_{\max} \nu_{R_s D}}{2\alpha \eta_\epsilon + N_u \sigma_w^2 + ((1-\alpha)\eta_\epsilon + N_u \sigma_w^2) \xi_{SR_s}}, \quad (7)$$

where  $\xi_{SR_s} = \mathcal{L}_u \alpha \bar{P} \beta_2 \bar{\nu}_{\max} + 2\alpha \eta_\epsilon + N_u \sigma_w^2$  and  $\eta_\epsilon = \mathcal{L}_u \bar{P} \pi^2 N_u \sigma_\epsilon^2 / 3$ .

2) *DcF Mode*: If the relay operates in the DcF mode, the relay should first demodulate and decode the received training sequence, and then re-generate this training sequence and re-transmit it in the second time slot. Here, we assume that the relay node can always demodulate and decode the received training sequence correctly. After  $R_s$  estimating  $\varepsilon_{SR_s}$ ; i.e.,  $\hat{\varepsilon}_{SR_s,1} = \varepsilon_{SR_s} + e_{SR_s,1}$  ( $e_{SR_s,1}$  represents the estimation error of  $\hat{\varepsilon}_{SR_s,1}$ ), we should use  $\hat{\varepsilon}_{SR_s,1}$  to pre-compensate for the frequency offset between  $S$  and  $R_s$  to re-generate the training sequence and re-transmit this sequence. The received training sequence at node  $D$  in the second time slot is

$$\mathbf{Y}_{D,2}^{\text{DcF}} = \mathbf{E}_{R_s D} \mathbf{F}_S \mathbf{H}_{R_s D} \Phi_{R_s,2} \hat{\mathbf{E}}_{SR_s} \mathbf{X}_S + \mathbf{T}_{R_s,2}^{\text{DcF}} + \mathbf{W}_{D,2}, \quad (8)$$

where  $\mathbf{T}_{R_s,2}^{\text{DcF}}$  represents the transmission of nodes other than  $R_s$ , and  $\hat{\mathbf{E}}_{SR_s} = \text{diag} \left\{ 1, e^{\frac{j2\pi\varepsilon_{SR_s,1}}{N}}, \dots, e^{\frac{j2\pi\varepsilon_{SR_s,1}(N-1)}{N}} \right\}$ . The average SINR in node  $D$  is

$$\gamma_2^{\text{DcF}} = \frac{(1-\alpha)\bar{P}\beta_1 \cdot \nu_{R_s D}}{(1-\alpha)\eta_\epsilon + N_u \sigma_w^2}. \quad (9)$$

### III. FREQUENCY OFFSET ESTIMATION IN THE COOPERATIVE SCHEME

In the first time slot,  $\varepsilon_{SD}$  is estimated as  $\hat{\varepsilon}_{SD,1} = \varepsilon_{SD} + e_{SD,1}$ , where  $e_{SD,1}$  represents the estimation error. From [8], [9] we know that for an unbiased estimator  $\hat{\varepsilon}_{SD,1}$ , the Cramer-Rao Lower Bound (CRLB) can be achieved as

$$\text{Var} \left\{ e_{SD,1} \middle| \nu_{SD} \right\} \geq \frac{1}{\mathcal{A}_T \cdot \gamma_{SD,1}}, \quad (10)$$

where  $\mathcal{A}_T$  is a positive coefficient specified by the structure of the training sequence  $\mathbf{X}_S$ . Similarly, the frequency offset estimation at  $R_s$  is  $\hat{\varepsilon}_{SR_s,1} = \varepsilon_{SR_s} + e_{SR_s,1}$ . For an unbiased estimator, the CRLB is

$$\text{Var} \left\{ e_{SR_s,1} \middle| \nu_{SR_s} \right\} \geq \frac{1}{\mathcal{A}_T \cdot \gamma_{SR_s,1}}. \quad (11)$$

The frequency offset estimation in the second time slot can be represented as  $\hat{\varepsilon}_{SR_s D,2} = \varepsilon_{SD} + e_{SR_s D,2}$ . The CRLB for the AF and the DcF modes are

$$\text{Var} \left\{ e_{SR_s D,2} \middle| \nu_{SR_s}, \nu_{R_s D}; \text{AF} \right\} \geq \frac{1}{\mathcal{A}_T \cdot \gamma_2^{\text{AF}}}, \quad (12a)$$

$$\text{Var} \left\{ e_{SR_s D,2} \middle| \nu_{SR_s}, \nu_{R_s D}; \text{DcF} \right\} \geq \frac{1}{\mathcal{A}_T} \left( \frac{1}{\gamma_{SR_s,1}} + \frac{1}{\gamma_2^{\text{DcF}}} \right). \quad (12b)$$

By using MRC, the minimum variance of  $e_{SD}$  in a two-time-slot period can be achieved as

$$\text{Var} \left\{ e_{SD} \middle| \text{AF} \right\} \geq \frac{1}{\mathcal{A}_T (\gamma_{SD,1} + \gamma_2^{\text{AF}})}, \quad (13a)$$

$$\text{Var} \left\{ e_{SD} \middle| \text{DcF} \right\} \geq \frac{1}{\mathcal{A}_T \left( \gamma_{SD,1} + \frac{\gamma_{SR_s,1} \gamma_2^{\text{DcF}}}{\gamma_{SR_s,1} + \gamma_2^{\text{DcF}}} \right)}. \quad (13b)$$

#### A. Without CSI Feedback from the Base Station

In this case, the optimal  $\alpha$  should minimize the expected variance error as

$$\alpha_{\text{WF}}^{\text{AF}} = \arg \min_{0 < \alpha < 1} \mathbb{E}_\nu \left\{ \frac{1}{\mathcal{A}_T (\gamma_{SD,1} + \gamma_2^{\text{AF}})} \right\} \quad (14a)$$

$$\alpha_{\text{WF}}^{\text{DcF}} = \arg \min_{0 < \alpha < 1} \mathbb{E}_\nu \left\{ \frac{1}{\mathcal{A}_T \left( \gamma_{SD,1} + \frac{\gamma_{SR_s,1} \gamma_2^{\text{DcF}}}{\gamma_{SR_s,1} + \gamma_2^{\text{DcF}}} \right)} \right\} \quad (14b)$$

for the AF and DcF modes, respectively, where the expectation is performed with respect to  $(\nu_{SD}, \nu_{SR_s}, \nu_{R_s D})$ .

#### B. With CSI Feedback from the Base Station

If the base station feedbacks the current CSI to the mobile nodes, for a given  $(\nu_{SD}, \nu_{SR_s}, \nu_{R_s D})$ , the adaptively optimized  $\alpha$  is given by

$$\alpha_{\text{FB}}^{\text{AF}} = \arg \min_{0 < \alpha < 1} \left\{ \frac{1}{\mathcal{A}_T (\gamma_{SD,1} + \gamma_2^{\text{AF}})} \right\} \quad (15a)$$

$$\alpha_{\text{FB}}^{\text{DcF}} = \arg \min_{0 < \alpha < 1} \left\{ \frac{1}{\mathcal{A}_T \left( \gamma_{SD,1} + \frac{\gamma_{SR_s,1} \gamma_2^{\text{DcF}}}{\gamma_{SR_s,1} + \gamma_2^{\text{DcF}}} \right)} \right\} \quad (15b)$$

for the AF and DcF modes, respectively.

#### C. Adaptively Switch Between Cooperative and Conventional Non-Cooperative Transmissions

A hybrid cooperative scheme can be performed to adaptively optimize the transmission, as shown in Fig. 2. In this scheme, if an  $\alpha$  ( $0 < \alpha < 1$ ) can be found to make the cooperative transmission outperform the conventional transmission, the transmitter should be switched to “2” to perform the cooperative transmission; if not, the transmitter can be switched to “1” to perform the conventional transmission. Since the base station may either feedback CSI to the mobile nodes or not, the cooperative transmission may be performed in two cases: (1) if the base station does not feedback CSI, the second switch should be switched to “21” to perform “Without Feedback” cooperation; and (2) if the base station feedbacks the CSI to the mobile nodes, the second switch should be switched to “22” to perform “With Feedback” cooperation.

### IV. NUMERICAL RESULTS

In our simulation, an OFDMA uplink transmission with a DFT length of 1024 is considered. A length-64 CP is padded to the front of each symbol or training sequence, and the length of the CP is assumed to be larger than the maximum multipath delay of the wireless channel. Eight subcarriers are allocated to each user. The algorithm proposed in [8] is used in this paper.

Fig. 3 compares the variance errors of the proposed cooperative scheme and that of the conventional estimation as functions of  $\alpha$ , where we assume that SNR=20 dB and  $M = 16$ . When  $\alpha$  is small, the proposed cooperative scheme may achieve a higher variance error than that achieved in the conventional estimation. For each  $\alpha$ , the AF mode always

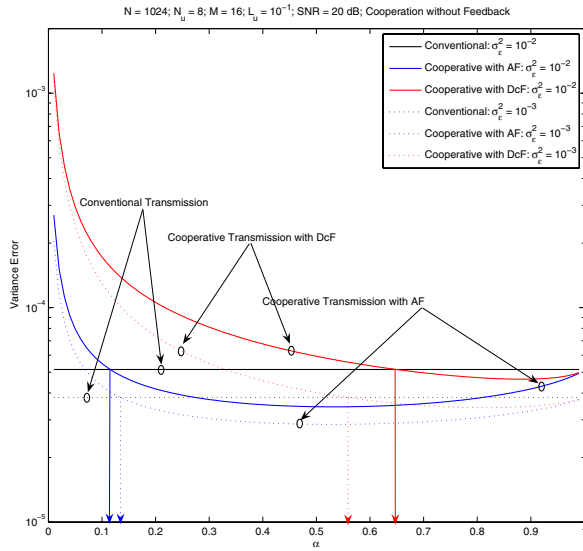


Fig. 3. Cooperative frequency offset estimation as a function of  $\alpha$  without feedback from the base station.

outperforms the DcF mode. From Fig. 3, the optimal  $\alpha$ , i.e.,  $\alpha_{WF}^{AF}$  and  $\alpha_{WF}^{DcF}$ , can easily be found. For example, when  $\sigma_\epsilon^2 = 10^{-2}$ , we have  $\alpha_{WF}^{AF} = 0.53$  (with a variance error  $2.85 \times 10^{-5}$ ) and  $\alpha_{WF}^{DcF} = 0.81$  (with a variance error  $3.42 \times 10^{-5}$ ).

When the base station feedbacks CSI to the mobile nodes,  $\alpha$  can be adaptively optimized to minimize the variance error in the proposed cooperative scheme. Fig. 4 evaluates the performance in the proposed cooperative scheme as a function of  $M$  when the base station feedbacks CSI to the mobile nodes. In this simulation, for each  $M$ , the variance errors achieved at  $\alpha_{FB}^{AF}$  (for the AF mode) and  $\alpha_{FB}^{DcF}$  (for the DcF mode) are evaluated. Like the performance advantage achieved by the scheme that without feedback from the base station, a performance advantage over the conventional scheme can be achieved in the cooperative scheme with feedback, and the AF mode will still outperform the DcF mode for each  $M$ . We can explain this finding as follows: In the interference-limited cooperative transmission, the interference due to the frequency offset in  $S \rightarrow R_s$  link is twice that of either the  $S \rightarrow D$  or  $R_s \rightarrow D$  link. If the relay operates in the DcF mode,  $R_s$  should estimate  $\epsilon_{SR_s}$ , and the estimation error will be accumulated and propagated to the final result. When the frequency offset is large, the error in  $R_s$  will dominate the overall variance error. However, this error propagation from  $R_s$  to  $D$  can be mitigated in the AF mode.

### V. CONCLUSION

This paper discussed performance improvement in OFDMA uplink frequency offset estimation by using cooperative transmission. In the proposed cooperative scheme, the accuracy of OFDMA uplink frequency offset estimation was improved considerably without increasing the total power consumption

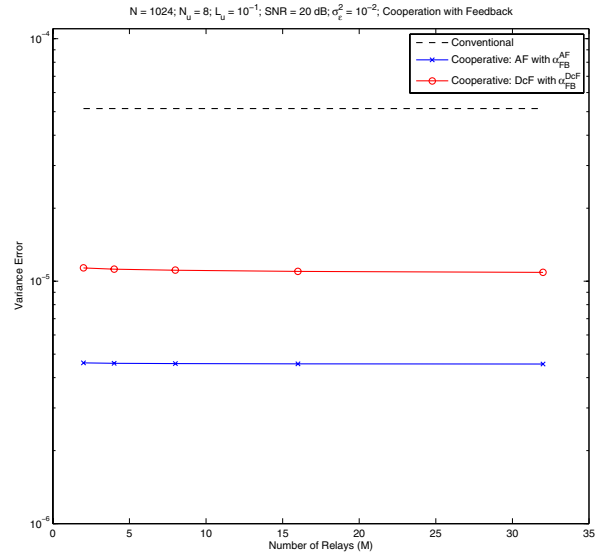


Fig. 4. Cooperative frequency offset estimation with feedback from the base station.

of each training sequence. The power allocation between the source node and the relay can be adaptively adjusted to optimize the cooperative scheme, provided that the base station feedbacks CSI to the mobile nodes. In the proposed cooperative scheme, an optimal  $\alpha$  can be found to achieve an advantage over the conventional transmission in terms of the frequency offset estimation error, and the estimation accuracy of the AF mode is always higher than that achieved by the DcF mode because of the estimation error propagation in the latter.

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