Robust OFDMA Uplink Synchronization by Exploiting the Variance of Carrier Frequency Offsets

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***Abstract*—In this paper, the uplink frequency offset estimation for orthogonal frequency-division multiplexing access (OFDMA) is discussed. We consider a general subcarrier allocation scheme where each user subcarrier group need not be contiguous. For an OFDMA uplink, we model the frequency offset for each user as an independent and identically distributed (i.i.d.) random variable with mean zero and variance σ_ϵ^2. An analysis of multiple access interference (MAI) is performed, and the Cramer–Rao lower bound (CRLB) for the estimation of variance of each user is derived. The signal-to-interference-plus-noise ratio (SINR) is derived as a function of the variance of the frequency offset. The variance of the frequency offset estimation error is lower bounded as a function of the signal-to-noise ratio (SNR). Successive interference cancellation (SIC) and iterative frequency offset estimation are considered. An estimate of the variance of the frequency offset is derived as a function of SINR and SNR. An estimate of the range of frequency offsets is derived using the assumption of uniformly distributed frequency offsets. Based on this estimate of the range of frequency offsets, the accuracy of any existing algorithm can be improved. Thus, new versions of the SIC-based frequency offset estimation and differential estimation algorithms are derived. Extensive simulation results are provided for a 16-user, 256-subcarrier OFDMA system over a multipath fading channel.**

***Index Terms*—Frequency offset estimation, multiple access interference (MAI), orthogonal frequency-division multiplexing access (OFDMA), synchronization.**

I. Introduction

In ORTHOGONAL frequency-division multiplexing access (OFDMA), each user employs a different set of orthogonal subcarriers to simultaneously transmit data. The use of orthogonal subcarrier sets for different users eliminates multiple access interference (MAI) under perfect conditions. OFDMA, therefore, has been or is being considered for various wireless systems [1]–[4]. Frequency synchronization is critically important for the OFDMA uplink. With imperfect synchronization, intercarrier interference (ICI) is generated due to the loss of subcarrier orthogonality, contributing both cochannel and interchannel interference. Moreover, the uplink frequency offset estimation for such systems is a multiple-parameter estimation problem and, hence, is much more difficult than the downlink case.

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The synchronization issues for OFDMA have been widely researched [1]–[8]. Reference [1] discusses a cyclic prefix (CP)-based synchronization for a multiuser OFDMA system. Timing and carrier frequency offset synchronization algorithms for generalized asynchronous and quasi-synchronous OFDMA systems using null subcarriers and subcarrier hopping are developed in [5]. A frequency offset compensation method is developed in [6]. The high-resolution blind frequency offset estimator proposed in [7] is suitable for both block-based and interleaved subcarrier allocation; however, it requires multiple OFDMA blocks per estimation. A reliable one-block OFDMA uplink synchronization algorithm is proposed in [3]. An uplink timing and frequency synchronization method for an OFDMA system is proposed in [2]. A high-performance maximum-likelihood (ML) algorithm for both synchronization and channel estimation for an OFDMA uplink transmission is studied in [8], and the complexity is reduced by employing an alternating projection method (this method simplifies the problem of multidimensional optimization into several 1-D optimization problems). An iterative time and frequency synchronization scheme using the space-alternating generalized expectation-maximization algorithm for interleaved OFDMA uplink systems is proposed in [4]. The MAI cancellation in an OFDMA system is discussed in [9]. A conventional estimator, such as in [10], is considered as a candidate for frequency offset estimation in [9].

The performance of such algorithms is a function of the received signal-to-interference-plus-noise ratio (SINR). When the frequency offsets of all the active users are large, each user signal is subject to heavy MAI, and therefore, the frequency offset estimation for each user will be considerably degraded. This degradation can be 10 dB or more for some systems in the high signal-to-noise ratio (SNR) region.

This paper studies the OFDMA uplink frequency offset estimation. Unlike [1] and [5], we consider a general subcarrier allocation scheme where each user subcarrier group need not be contiguous (this helps to exploit the channel diversity). For an OFDMA uplink, the frequency offset for each user is assumed to be an independent and identically distributed (i.i.d.) random variable (RV) with mean zero and variance σ_ϵ^2. An analysis of MAI is performed, and the Cramer–Rao lower bound (CRLB) for the estimation variance of each user is derived. SINR is derived as a function of the variance of the frequency offset. The variance of the frequency offset estimation error is lower bounded as a function of the SNR. Successive interference cancellation (SIC) and iterative frequency offset estimation are considered. An estimate of the variance of the frequency offset

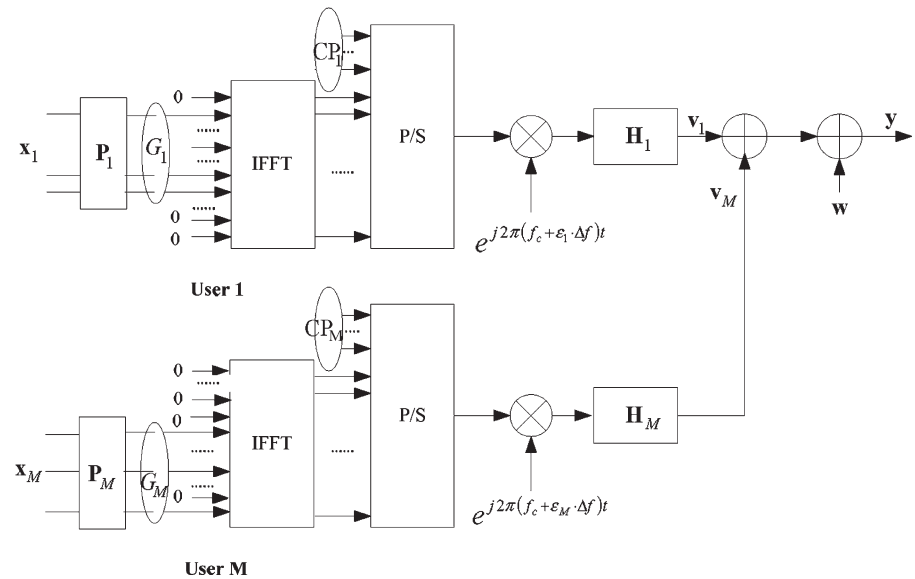

Fig. 1. Systematic structure of OFDMA uplink transmission in the presence of frequency offset.

is derived as a function of SINR and SNR. An estimate of the range of frequency offsets is derived using the assumption of uniformly distributed frequency offsets. Based on this estimate of the range of frequency offsets, the accuracy of any existing algorithm can be improved. Thus, new versions of the SIC-based frequency offset estimation and differential estimation algorithms are derived. Extensive simulation results are provided for a 16-user 256-subcarrier OFDMA system over a multipath fading channel.

This paper is organized as follows. Section II proposes the OFDMA uplink signal model. The MAI due to frequency offsets is analyzed in Section III. Section IV discusses the iterative frequency offset estimators, and Section V analyzes the performance improvement in the conventional algorithms by exploiting the variance of frequency offsets. Section VI analyzes the proposed frequency offset estimators and presents simulation results. Section VII concludes this paper.

Notation: $(\cdot)^T$ and $(\cdot)^H$ are the transpose and the complex conjugate transpose, respectively. $\odot$ is the Hadamard–Schur product of matrices. The imaginary unit is $\jmath = \sqrt{-1}$. $(\cdot)^*$ denotes the complex conjugate. A real Gaussian variable with mean a and variance σ^2 is denoted by $x \sim \mathcal{N}(a, \sigma^2)$. A circularly symmetric complex Gaussian variable with mean m and variance σ_z^2 is represented as $z \sim \mathcal{CN}(m, \sigma_z^2)$. $\mathbf{x}[i]$ is the ith element of the vector $\mathbf{x}$, and $\|\mathbf{x}\|_2^2 = \sum_i |\mathbf{x}[i]|^2$. $\mathbf{I}_N$ and $\mathbf{O}_N$ are the $N \times N$ identity matrix and the $N \times N$ all-zero matrix, respectively. $\mathbf{0}_N$ is the $N \times 1$ all-zero vector. $\mathbb{E}\{x\}$ and $\mathrm{Var}\{x\}$ are the mean and variance, respectively, of x.

II. OFDMA Uplink Signal Model

As in [2], the quasi-synchronized transmission of user signals is considered in this paper. The base station first performs uplink time and frequency offset estimation, and then, the estimates are sent through a downlink control channel to the users to help their time and frequency adjustment. Perfect timing synchronization is assumed, so only the uplink frequency offset estimation is discussed in this paper. Note that the OFDMA uplink frequency offset estimation can be subdivided into two phases, i.e., acquisition and tracking. When a user starts accessing a base station, its instantaneous frequency offset may be very large, and an acquisition algorithm is needed to estimate and correct this large frequency offset. Channel estimation will also be performed after acquisition, which is beyond the scope of this paper. After acquisition, the residual frequency offset of this user will be well within a finite range, e.g., within ± 0.5 subcarrier spacing, and this user will run in the tracking phase. Since the channel side information (CSI) is available at the tracking phase, pilot/training-based algorithms can be performed to estimate the frequency offset with high accuracy. In this paper, only the frequency offset tracking phase is considered, and the CSI of each user is assumed to be available at the base station.

An OFDMA uplink transmission system with M users is depicted in Fig. 1, where Δf represents the subcarrier bandwidth. The total number of subcarriers is N. An $N \times 1$ vector $\underline{\mathbf{x}}_k(n)$ represents the nth block of the frequency-domain symbols sent by the kth user, where $k \in \{1, 2, \ldots, M\}$. In the following sections, the temporal index n will be omitted for brevity. The ith entry of $\underline{\mathbf{x}}_k$ (i.e., $\underline{\mathbf{x}}_k[i]$) is nonzero if and only if the ith subcarrier is allocated to the kth user, where $i \in \{1, 2, \ldots, N\}$. In OFDMA, the $\underline{\mathbf{x}}_k[i]$ for each (k, i) is a complex data symbol of a signal constellation, e.g., M-ary phase shift keying (PSK) or M-ary quadrature amplitude modulation. G_k represents the subcarrier group allocated to the kth user (the elements of G_k are indexes of all those subcarriers), and $\mathcal{N}_k$ represents the cardinality of G_k. Note that $\bigcap_{k \neq l} G_k G_l = \emptyset$ and $\bigcup_{k=1}^{M} G_k \subseteq \{0, 1, \ldots, N-1\}$. $\underline{\mathbf{x}}_k$ can also be simplified into a $\mathcal{N}_k \times 1$ vector $\mathbf{x}_k$ by deleting all the zero entries of $\underline{\mathbf{x}}_k$ and keeping the nonzero entries unchanged.

The time-domain transmit vector of the kth user is given by $\bar{\mathbf{s}}_k = \mathbf{F}\underline{\mathbf{x}}_k = \mathbf{F}_k \mathbf{x}_k$, where $\mathbf{F}$ denotes the $N \times N$ *inverse discrete Fourier transform* (IDFT) matrix with $[\mathbf{F}]_{nk} = (1/\sqrt{N}) e^{(\jmath 2\pi nk/N)}$ for $0 \leq n, k \leq N-1$, and $\mathbf{F}_k$ is an

$N \times \mathcal{N}_k$ IDFT matrix that is specified by G_k ($\mathbf{F}_k$ can be generated from $\mathbf{F}$ by deleting all the columns with the column indexes not belonging to G_k). A length N_g CP is used to mitigate the intersymbol interference.

The discrete-time channel impulse response of the kth user is $\mathbf{h}_k = [h_k(0), h_k(1), \ldots, h_k(L_k - 1)]^T$, where L_k is the maximum delay of the kth user. The frequency-domain channel attenuation matrix of the kth user is given by $\mathbf{H}_k = \mathrm{diag}\{H_k^i : i \in G_k\}$, where $H_k^i = \sum_{n=0}^{L_k-1} h_k(n)e^{-(\jmath 2\pi ni/N)}$. The received signal at the base station can be represented as

$$\mathbf{y} = \sum_{k=1}^{M} \mathbf{y}_k = \sum_{k=1}^{M} \left(\mathbf{E}_k \underbrace{\mathbf{F}_k \mathbf{H}_k \mathbf{\Phi}_k \mathbf{x}_k}_{\mathbf{s}_k} + \mathbf{w}_k \right) \tag{1}$$

where $\mathbf{\Phi}_k = \mathrm{diag}\{\sqrt{P_i} : i \in G_k\}$, with P_i representing the power allocation to the ith subcarrier, and $\mathbf{E}_k$ is given by $\mathbf{E}_k = \mathrm{diag}\{e^{\jmath\psi_k}, e^{\jmath((2\pi\varepsilon_k/N)+\psi_k)}, \ldots, e^{\jmath((2\pi\varepsilon_k \times (N-1)/N)+\psi_k)}\}$ with ψ_k and ε_k representing the initial phase and the normalized frequency offset (the frequency offset normalized to the subcarrier spacing) of the kth user, respectively. $\mathbf{w}_k$ in (1) is a vector of additive white Gaussian noise (AWGN) with $\mathbf{w}_k[i] \sim \mathcal{CN}(0, \sigma_w^2)$.

Let $\mathbf{s}_k$ be the training sequence transmitted by the kth user. The received signal $\mathbf{y}$ is a complex Gaussian random vector with the following probability density function (pdf) [11]:

$$f(\mathbf{y}) = \frac{1}{(\pi)^N \det[\mathbf{C}]} \exp\left\{-(\mathbf{y}-\mathbf{s})^H \mathbf{C}^{-1}(\mathbf{y}-\mathbf{s})\right\} \tag{2}$$

where $\mathbf{s} = \sum_{k=1}^{M} \mathbf{s}_k$, and $\mathbf{C} = \mathbb{E}\{(\mathbf{y}-\mathbf{s})(\mathbf{y}-\mathbf{s})^H\}$. The frequency offsets of different users are assumed to be i.i.d. RVs with zero mean and variance σ_ϵ^2 (not necessarily Gaussian). For an unbiased frequency offset estimator $\hat{\varepsilon}_k$, the CRLB is given by $\mathrm{Var}\{\hat{\varepsilon}_k|\mathbf{y}\} \geq [\mathbf{\Lambda}_M^{-1}]_{kk}$, where the klth element of the Fisher information matrix (FIM) $\mathbf{\Lambda}_M$ is $[\mathbf{\Lambda}_M]_{kl} = \mathrm{trace}(\mathbf{C}^{-1}(\partial\mathbf{C}/\partial\varepsilon_k)\mathbf{C}^{-1}(\partial\mathbf{C}/\partial\varepsilon_l))$, as given in [12].

In a multiple-access system, the unknown parameters of interfering users can be treated as nuisance parameters, which will degrade the estimation accuracy of the parameters of interest. Regarding OFDMA uplink frequency offset estimation, the following result holds.

Lemma 1—Estimation Error Increases: The OFDMA uplink frequency offset estimation error for each user will not reduce as the number of interfering users increases.

Proof: For an OFDMA uplink transmission with a total of $(M-1)$ users accessing a base station, the received vector is defined as

$$\tilde{\mathbf{y}} = \sum_{k=1}^{M-1} \mathbf{y}_k = \sum_{k=1}^{M-1} \left(\mathbf{E}_k \underbrace{\mathbf{F}_k \mathbf{H}_k \mathbf{\Phi}_k \mathbf{x}_k}_{\mathbf{s}_k} + \mathbf{w}_k \right). \tag{3}$$

The covariance matrix of $\tilde{\mathbf{y}}$ is represented as $\tilde{\mathbf{C}}$. Define the current FIM as $\mathbf{\Lambda}_{M-1}$, where $[\mathbf{\Lambda}_{M-1}]_{kl} = \mathrm{trace}(\tilde{\mathbf{C}}^{-1}(\partial\tilde{\mathbf{C}}/\partial\varepsilon_k)\tilde{\mathbf{C}}^{-1}(\partial\tilde{\mathbf{C}}/\partial\varepsilon_l))$. If a new user (the Mth user) is accessing the base station, the receive vector and its covariance matrix are $\mathbf{y}$ and $\mathbf{C}$, respectively, and the new FIM is

$$\mathbf{\Lambda}_M = \begin{bmatrix} \mathbf{\Lambda}_{M-1} & \mathbf{b} \\ \mathbf{b}^H & [\mathbf{\Lambda}_M]_{MM} \end{bmatrix} \tag{4}$$

where $\mathbf{b}$ is a $(M-1) \times 1$ vector with $\mathbf{b}[k] = \mathrm{trace}(\mathbf{C}^{-1}(\partial\mathbf{C}/\partial\varepsilon_k)\mathbf{C}^{-1}(\partial\mathbf{C}/\partial\varepsilon_M))$, $1 \leq k \leq M-1$, and $[\mathbf{\Lambda}_M]_{MM} = \mathrm{trace}(\mathbf{C}^{-1}(\partial\mathbf{C}/\partial\varepsilon_M)\mathbf{C}^{-1}(\partial\mathbf{C}/\partial\varepsilon_M))$. The inverse of $\mathbf{\Lambda}_M$ can be represented as

$$\mathbf{\Lambda}_M^{-1} = \begin{bmatrix} \mathbf{\Lambda}_{M-1}^{-1} & \mathbf{0} \\ \mathbf{0}^H & 0 \end{bmatrix} + \begin{bmatrix} -\mathbf{\Lambda}_{M-1}^{-1}\mathbf{b} \\ 1 \end{bmatrix} \cdot \left([\mathbf{\Lambda}_M]_{MM} - \mathbf{b}^H\mathbf{\Lambda}_{M-1}^{-1}\mathbf{b}\right)^{-1} \cdot \begin{bmatrix} -\mathbf{b}^H\mathbf{\Lambda}_{M-1}^{-1} & 1 \end{bmatrix}. \tag{5}$$

By using $\mathbf{\Lambda}_{M|M-1}^{-1}$ to represent the northwestern $(M-1) \times (M-1)$ submatrix of $\mathbf{\Lambda}_M^{-1}$, we have

$$\mathbf{\Lambda}_{M|M-1}^{-1} = \mathbf{\Lambda}_{M-1}^{-1} + \left([\mathbf{\Lambda}_M]_{MM} - \mathbf{b}^H\mathbf{\Lambda}_{M-1}^{-1}\mathbf{b}\right)^{-1} \cdot \mathbf{\Lambda}_{M-1}^{-1}\mathbf{b}\mathbf{b}^H\mathbf{\Lambda}_{M-1}^{-1} \tag{6}$$

where $[\mathbf{\Lambda}_M^{-1}]_{MM} = ([\mathbf{\Lambda}_M]_{MM} - \mathbf{b}^H\mathbf{\Lambda}_{M-1}^{-1}\mathbf{b})^{-1} > 0$. Since $\mathbf{\Lambda}_{M-1}^{-1}\mathbf{b}\mathbf{b}^H\mathbf{\Lambda}_{M-1}^{-1}$ is a nonnegative definite matrix, we have

$$\begin{aligned}\left[\mathbf{\Lambda}_{M|M-1}^{-1}\right]_{ii} &= \left[\mathbf{\Lambda}_{M-1}^{-1}\right]_{ii} + \left([\mathbf{\Lambda}_M]_{MM} - \mathbf{b}^H\mathbf{\Lambda}_{M-1}^{-1}\mathbf{b}\right)^{-1} \cdot \left[\mathbf{\Lambda}_{M-1}^{-1}\mathbf{b}\mathbf{b}^H\mathbf{\Lambda}_{M-1}^{-1}\right]_{ii} \\ &\geq \left[\mathbf{\Lambda}_{M-1}^{-1}\right]_{ii}, \quad 1 \leq i \leq M-1. \end{aligned} \tag{7}$$

■

Lemma 1 indicates that the MAI increases if the number of users accessing a base station increases. For an unbiased estimator for the kth user, the CRLB is related to $[\mathbf{\Lambda}_M^{-1}]_{kk}$, and the following result holds.

Lemma 2—Inequality of FIM: In OFDMA uplink frequency offset estimation, $[\mathbf{\Lambda}_M^{-1}]_{kk} \geq [\mathbf{\Lambda}_M]_{kk}^{-1}$ for each k.

Proof: Without loss of generality, suppose that the number of accessed users is M, and that the frequency offsets of the different users are i.i.d. RVs. From [12, p. 1352], for a Hermitian positive definite matrix $\mathbf{\Lambda}_M$, the inequality

$$\left[\mathbf{\Lambda}_M^{-1}\right]_{kk} = \left[\mathbf{I}_M \odot \mathbf{\Lambda}_M^{-1}\right]_{kk} \geq \left[\mathbf{I}_M \odot \mathbf{\Lambda}_M\right]_{kk}^{-1} = [\mathbf{\Lambda}_M]_{kk}^{-1} \tag{8}$$

is always satisfied for each $1 \leq k \leq M$. ■

In the following sections, $\mathbf{\Lambda}$ is used to denote the FIM when the number of users accessing a base station is not specified. By using Lemma 2, the CRLB for the kth user is derived as

$$\begin{aligned}\mathrm{Var}\{\hat{\varepsilon_k}|\mathbf{y}\} &\geq [\mathbf{\Lambda}^{-1}]_{kk} \geq [\mathbf{\Lambda}]_{kk}^{-1} \\ &= \frac{1}{\sum_{i=1}^{\mathcal{N}_k} \frac{\left(\frac{\partial\lambda_{k,i}}{\partial\varepsilon_k}\right)^2}{(\lambda_{k,i}+z_{k,i})^2} + \sum_{l\neq k}\sum_{j=1}^{\mathcal{N}_l} \frac{\left(\frac{\partial z_{l,j}}{\partial\varepsilon_k}\right)^2}{(\lambda_{l,j}+z_{l,j})^2}} \\ &= \frac{1}{\alpha_k \cdot \mathrm{SNR}_k^2} + \frac{1}{\beta_k \cdot \mathrm{SNR}_k} + \frac{1}{\varpi_k \cdot \mathrm{SIR}_k} \end{aligned} \tag{9}$$

where $\lambda_{k,i}$ and $z_{k,i}$ are defined in Appendix A, and α_k, β_k, and ϖ_k are training-sequence-dependent coefficients for the kth user. For a length N training sequence with $N \gg 1$, $\alpha_k \gg 1$, $\beta_k \gg 1$, and $\varpi_k \gg 1$. The first two terms in (9) relate to the SNR of the kth user and the third term to the signal-to-interference ratio (SIR) of the kth user.

III. Interference Analysis in OFDMA Systems

Consider a system with a total of M users accessing a base station. The analysis of the MAI due to the frequency offsets is based on the following assumptions.

1) $\bigcap_{m\neq n} G_m G_n = \emptyset$, and $\bigcup_m G_m \subseteq \{0,1,\ldots,N-1\}$, where $m,n \in \{1,2,\ldots,M\}$.
2) $\mathcal{N}_m \ll N, 1 \leq m \leq M$.
3) ε_m for each m is an i.i.d. RV with mean zero and variance σ_ϵ^2.

A. Interference Reduction by Using Preprojector Method

The CRLB for an OFDMA uplink synchronization is derived in (9) and is related to multiple variables. To analyze the received SINR of the kth user, define a projection matrix $\mathbf{P}_k = \mathbf{F}_k(\mathbf{F}_k^H\mathbf{F}_k)^{-1}\mathbf{F}_k^H = \mathbf{F}_k\mathbf{F}_k^H$, and the signal transmitted by the user k is demodulated as

$$\begin{aligned}\mathbf{r}_k &= \mathbf{P}_k\mathbf{y}\\ &= \mathbf{P}_k\sum_l \underbrace{\mathbf{E}_l\,\underbrace{\mathbf{F}_l\mathbf{H}_l\boldsymbol{\Phi}_l\mathbf{x}_l}_{\mathbf{s}_l}}_{\mathbf{v}_l} + \mathbf{P}_k\mathbf{w}\\ &= \mathbf{P}_k\mathbf{v}_k + \mathbf{P}_k\sum_{l\neq k}\mathbf{v}_l + \mathbf{P}_k\mathbf{w}\\ &= \tilde{\mathbf{s}}_k + \underbrace{\boldsymbol{\Upsilon}_{k;k} + \boldsymbol{\Upsilon}_{l\neq k;k}}_{\boldsymbol{\Upsilon}_k} + \tilde{\mathbf{w}}_k \end{aligned} \tag{10}$$

where $\mathbf{P}_k\mathbf{v}_k = \tilde{\mathbf{s}}_k + \boldsymbol{\Upsilon}_{k;k}$, with $\tilde{\mathbf{s}}_k$ representing the useful part of the kth user signal and $\boldsymbol{\Upsilon}_{k;k}$ representing the ICI due to ε_k; $\boldsymbol{\Upsilon}_{l\neq k;k} = \mathbf{P}_k\sum_{l\neq k}\mathbf{v}_l$ represents the interference from interfering users (i.e., MAI); and $\tilde{\mathbf{w}}_k$ is the AWGN added to the signal space of the kth user. In the following analysis, $\boldsymbol{\Upsilon}_k = \boldsymbol{\Upsilon}_{k;k} + \boldsymbol{\Upsilon}_{l\neq k;k}$ is used to represent the interference on the kth user (ICI + MAI). By using $\mathbf{P}_k$, the multidimensional estimation problem is reduced to several single-parameter estimation problems.

From (9) and (10), the CRLB of the kth user is

$$\mathrm{Var}\{\hat{\varepsilon_k}|\mathbf{r}_k\} \geq \frac{1}{\mathrm{trace}\left(\tilde{\mathbf{C}}_k^{-1}\frac{\partial\tilde{\mathbf{C}}_k}{\partial\varepsilon_k}\tilde{\mathbf{C}}_k^{-1}\frac{\partial\tilde{\mathbf{C}}_k}{\partial\varepsilon_k}\right)} \tag{11}$$

where

$$\tilde{\mathbf{C}}_k = \mathbb{E}\left\{(\mathbf{r}_k-\mathbf{s}_k)(\mathbf{r}_k-\mathbf{s}_k)^H\right\} \cong \left(\mathbf{I}_N - \Sigma_k^2\right)\mathbf{C}_k \tag{12}$$

with $\Sigma_k = \mathrm{diag}\{0, -(\jmath 2\pi\varepsilon_k/N),\ldots,-(\jmath 2\pi\varepsilon_k\times(N-1)/N)\}$.

Noting that

$$\mathrm{trace}\left(\tilde{\mathbf{C}}_k^{-1}\frac{\partial\tilde{\mathbf{C}}_k}{\partial\varepsilon_k}\tilde{\mathbf{C}}_k^{-1}\frac{\partial\tilde{\mathbf{C}}_k}{\partial\varepsilon_k}\right) \leq \mathrm{trace}\left(\mathbf{C}_k^{-1}\frac{\partial\mathbf{C}_k}{\partial\varepsilon_k}\mathbf{C}_k^{-1}\frac{\partial\mathbf{C}_k}{\partial\varepsilon_k}\right) - \frac{8\pi^2(N-1)(2N-1)\varepsilon_k^2}{3N^3} \tag{13}$$

(11) can be approximated as

$$\begin{aligned}&\mathrm{Var}\{\hat{\varepsilon_k}|\mathbf{r}_k\}\\ &\geq \frac{1}{\mathrm{trace}\left(\mathbf{C}_k^{-1}\frac{\partial\mathbf{C}_k}{\partial\varepsilon_k}\mathbf{C}_k^{-1}\frac{\partial\mathbf{C}_k}{\partial\varepsilon_k}\right) - \frac{8\pi^2(N-1)(2N-1)\varepsilon_k^2}{3N^3}}\\ &> \frac{1}{\sum_{i=1}^{\mathcal{N}_k}\frac{\left(\frac{\partial\lambda_{k,i}}{\partial\varepsilon_k}\right)^2}{(\lambda_{k,i}+z_{k,i})^2}}. \end{aligned} \tag{14}$$

B. SINR Analysis

From (10), the receive SINR of user k can be represented as

$$\begin{aligned}\mathrm{SINR}_k &\triangleq \frac{\mathbb{E}\left\{\|\tilde{\mathbf{s}}_k\|_2^2\right\}}{\mathbb{E}\left\{\|\boldsymbol{\Upsilon}_k\|_2^2\right\} + \mathbb{E}\left\{\|\tilde{\mathbf{w}}_k\|_2^2\right\}}\\ &= \frac{\mathrm{SNR}_k}{\frac{\pi^2\sigma_\epsilon^2\mathrm{SNR}_k}{3}+1}\cdot\left(1-\frac{\pi^2\sigma_\epsilon^2}{3}+\frac{\pi^4\sigma_\epsilon^4}{20}\right) \end{aligned} \tag{15}$$

as derived in Appendix B. Since $(1/\mathrm{SINR}_k) = (1/\mathrm{SNR}_k) + (1/\mathrm{SIR}_k)$ always holds, SIR_k can be represented as $\mathrm{SIR}_k = (3/\pi^2\sigma_\epsilon^2)\cdot(1-(\pi^2\sigma_\epsilon^2/3)+(\pi^4\sigma_\epsilon^4/20))$, and the conditional variance derived in (14) is lower bounded as

$$\mathrm{Var}\{\hat{\varepsilon_k}|\mathbf{r}_k\} \geq \frac{1}{\alpha_k\cdot\mathrm{SNR}_k^2} + \frac{1}{\beta_k\cdot\mathrm{SNR}_k} + \frac{\pi^2\sigma_\epsilon^2}{3\varpi_k\left(1-\frac{\pi^2\sigma_\epsilon^2}{3}+\frac{\pi^4\sigma_\epsilon^4}{20}\right)}. \tag{16}$$

At a high SNR, the conditional variance becomes $\lim_{\mathrm{SNR}_k\to\infty}\mathrm{Var}\{\hat{\varepsilon_k}|\mathbf{r}_k\} \geq (\pi^2\sigma_\epsilon^2/3\varpi_k(1-(\pi^2\sigma_\epsilon^2/3)+(\pi^4\sigma_\epsilon^4/20)))$.

The SINR reduction due to the MAI is shown in Fig. 2. A considerable MAI is added to the user of interest due to the frequency offsets of the interfering users, and a larger σ_ϵ^2 implies a higher MAI. For example, for an SNR of 30 dB, when $\sigma_\epsilon^2 = 10^{-3}$, the received SINR is about 13.6 dB. If σ_ϵ^2 is increased to 10^{-2}, the received SINR is reduced to only about 4.55 dB.

C. Frequency Offset Analysis

The frequency offsets in an OFDMA system are usually generated by the mismatch between transmit and receive oscillators and/or the Doppler shift due to user mobility. By using a SIC method, e.g., that proposed in [9], the MAI can be iteratively

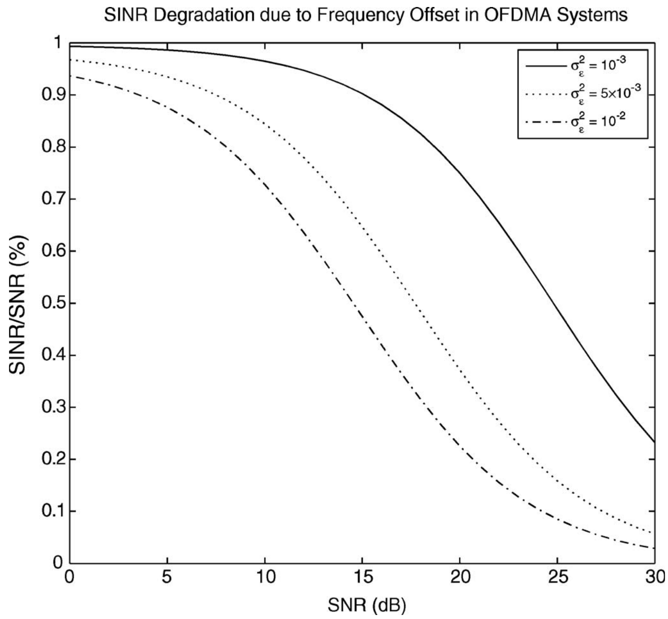

Fig. 2. SINR reduction due to the frequency offset in OFDMA systems.

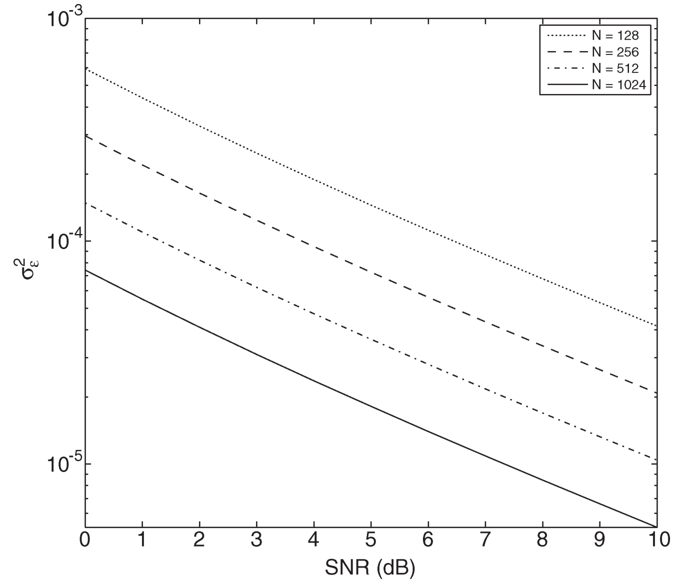

Fig. 3. Minimum variance of the frequency offset in a noisy OFDMA uplink transmission.

reduced, and at the $(n+1)$th iteration $(n \geq 1)$, $\mathbf{r}_k$ can be represented as

$$\mathbf{r}_k^{(n+1)} = \tilde{\mathbf{s}}_k^{(n+1)} + \boldsymbol{\Upsilon}_{k;k}^{(n+1)} + \boldsymbol{\Upsilon}_{l<k}^{(n+1)} + \boldsymbol{\Upsilon}_{l>k}^{(n)} + \tilde{\mathbf{w}}_k^{(n+1)} \tag{17}$$

where the superscript $^{(n)}$ denotes the vector value at the nth iteration, and

$$\tilde{\mathbf{s}}_k^{(n+1)} = \mathbf{P}_k \boldsymbol{\Delta}_k^{(n+1)} \mathbf{s}_k \tag{18a}$$

$$\boldsymbol{\Upsilon}_{k;k}^{(n+1)} = \mathbf{P}_k \left(\mathbf{I}_N - \boldsymbol{\Delta}_k^{(n+1)} \right) \mathbf{s}_k \tag{18b}$$

$$\boldsymbol{\Delta}_k^{(n+1)} = \mathrm{diag}\left\{ 1, e^{\frac{\jmath 2\pi e_k^{(n+1)}}{N}}, \ldots, e^{\frac{\jmath 2\pi e_k^{(n+1)} \times (N-1)}{N}} \right\} \tag{18c}$$

$$e_k^{(n+1)} = \varepsilon_k - \hat{\varepsilon}_k^{(n+1)}. \tag{18d}$$

As n increases, (17) converges to its steady state as $\mathbf{r}_k = \tilde{\mathbf{s}}_k + \boldsymbol{\Upsilon}_{k;k} + \boldsymbol{\Upsilon}_{l\neq k} + \tilde{\mathbf{w}}_k$.

Although the MAI can be reduced by using the SIC-based method, it can never be totally eliminated in a noisy environment. In a synchronized OFDMA system (frequency offset acquisition has been performed, and most of the frequency offset has been compensated for), the frequency offset of each user is limited with a finite range so that

$$\mathrm{Var}\{\hat{\varepsilon_k} | \mathbf{r}_k\} = \sigma_\epsilon^2 > \frac{1}{\alpha_k \cdot \mathrm{SNR}_k^2} + \frac{1}{\beta_k \cdot \mathrm{SNR}_k} + \frac{\pi^2 \sigma_\epsilon^2}{3\varpi_k}. \tag{19}$$

By resolving (19), σ_ϵ^2 can be lower bounded as

$$\sigma_\epsilon^2 > \frac{3\varpi_k}{3\varpi_k - \pi^2} \cdot \left(\frac{1}{\alpha_k \cdot \mathrm{SNR}_k^2} + \frac{1}{\beta_k \cdot \mathrm{SNR}_k} \right) \tag{20}$$

and as SNR_k increases, $\lim_{\mathrm{SNR}_k \to \infty} \sigma_\epsilon^2 = 0$. Note that the distribution of frequency offsets is not specified here because any distribution with mean zero and variance σ_ϵ^2 satisfies (19) and (20). For example, without loss of generality, we can assume that $\varepsilon_k \sim \mathcal{N}(0, \sigma_\epsilon^2)$. However, if we assume that ε_k is a uniformly distributed RV with the distribution range $(-\epsilon, \epsilon)$, where $\epsilon > 0$, we have

$$\epsilon = \sqrt{3}\sigma_\epsilon > \sqrt{\frac{9\varpi_k}{3\varpi_k - \pi^2} \cdot \left(\frac{1}{\alpha_k \cdot \mathrm{SNR}_k^2} + \frac{1}{\beta_k \cdot \mathrm{SNR}_k} \right)}. \tag{21}$$

Now, σ_ϵ^2 will be analyzed in real systems with a finite SNR. If the training sequence proposed in [10] is used by each user, we have $\alpha_k = \varpi_k = 4\pi^2 N$ and $\beta_k = 2\pi^2 N$ for each k. Fig. 3 shows σ_ϵ^2 as a function of SNR and N. When the SNR and N are large enough, σ_ϵ^2 is negligible. For example, when $\mathrm{SNR}_k = 10$ dB, σ_ϵ^2 is only about 4×10^{-5} with $N = 128$, and it will reduce to 5×10^{-6} when $N = 1024$.

IV. Iterative Frequency Offset Estimation

In an OFDMA uplink, the frequency offset estimation is a multidimensional estimation problem, and the frequency offset vector $\mathcal{E} = [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_M]^T$ can be estimated based on $\mathbf{y}$. This section will introduce two iterative algorithms for the frequency offset estimation.

A. ML Estimation

Based on the received training sequence $\mathbf{y}$, an ML estimator of $\mathcal{E}$ is given by

$$\begin{aligned} \hat{\mathcal{E}}_{\mathrm{ML}} &= \arg\min_{\hat{\mathcal{E}}} \left\| \mathbf{y} - \sum_l \underbrace{\hat{\mathbf{E}}_l \mathbf{F}_l \mathbf{H}_l \boldsymbol{\Phi}_l}_{\mathbf{A}_l} \mathbf{x}_l \right\|_2^2 \\ &= \arg\min_{\hat{\mathcal{E}}} \left(\mathbf{y} - \sum_l \mathbf{A}_l \mathbf{x}_l \right)^H \left(\mathbf{y} - \sum_l \mathbf{A}_l \mathbf{x}_l \right) \end{aligned} \tag{22}$$

where

$$\hat{\mathbf{E}}_k = \mathrm{diag}\{e^{\jmath\psi_k}, e^{\jmath((2\pi\hat{\varepsilon}_k/N)+\psi_k)}, \ldots, e^{\jmath((2\pi\hat{\varepsilon}_k\times(N-1)/N)+\psi_k)}\}$$

and

$$\hat{\mathcal{E}} = [\hat{\varepsilon}_1, \hat{\varepsilon}_2, \ldots, \hat{\varepsilon}_M]^T$$

By taking the partial derivative of $(\mathbf{y} - \sum_l \mathbf{A}_l\mathbf{x}_l)^H(\mathbf{y} - \sum_l \mathbf{A}_l\mathbf{x}_l)$ with respect to each l and setting each equation to zero, $\mathbf{x}_l$ can be estimated as

$$\hat{\mathbf{x}}_l = \left(\mathbf{A}_l^H\mathbf{A}_l\right)^{-1}\mathbf{A}_l^H\mathbf{y} - \sum_{k\neq l}\left(\mathbf{A}_l^H\mathbf{A}_l\right)^{-1}\mathbf{A}_l^H\tilde{\mathbf{P}}_k\mathbf{y} \tag{23}$$

where $1 \leq k, l \leq M$, and $\tilde{\mathbf{P}}_k = \mathbf{A}_k(\mathbf{A}_k^H\mathbf{A}_k)^{-1}\mathbf{A}_k^H$. By replacing $\mathbf{x}_l$ in (22) by using (23), (22) can be simplified as

$$\begin{aligned}\hat{\mathcal{E}}_{\mathrm{ML}} &= \arg\min_{\hat{\mathcal{E}}}\left\|\mathbf{y} - \sum_l\left(\tilde{\mathbf{P}}_l\mathbf{y} - \sum_{k\neq l}\tilde{\mathbf{P}}_l\tilde{\mathbf{P}}_k\mathbf{y}\right)\right\|_2^2\\ &= \arg\min_{\hat{\mathcal{E}}}\left\|\mathbf{y} - \sum_l\tilde{\mathbf{P}}_l\mathbf{y} + \sum_{k\neq l}\tilde{\mathbf{P}}_l\tilde{\mathbf{P}}_k\mathbf{y}\right\|_2^2. \end{aligned}\tag{24}$$

Since a multidimensional parameter estimator is inefficient, a multidimensional estimation is usually decomposed into multiple 1-D estimators to reduce the complexity [8], [13]. The frequency offset of each user can be iteratively estimated by using (24), and this algorithm will converge to its steady state after several iterations. However, $\tilde{\mathbf{P}}_l$ for each l should be calculated at each iteration, and the matrix inverse operation in $\tilde{\mathbf{P}}_l$ complicates $\hat{\mathcal{E}}_{\mathrm{ML}}$. In real systems, pilots/training sequences are usually used to estimate the frequency offsets. The MAI can be eliminated by using a SIC-based algorithm, and the estimation performance of the SIC-based algorithm is as good as that of (24).

B. SIC-Based Algorithm by Using Known Pilots/Training Sequences

Suppose that the number of users that access a base station is M, and $\mathbf{H}_1\mathbf{x}_1, \mathbf{H}_2\mathbf{x}_2, \ldots, \mathbf{H}_M\mathbf{x}_M$ are assumed to be perfectly known at the base station (this requires perfect CSI knowledge). Note that because the channel estimates will degrade in the presence of the frequency offset, perfect CSI knowledge is not available. However, CSI can be estimated in the presence of the frequency offset by exploiting the received $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_M\}$. For example, joint channel and frequency offset estimators are proposed in [14], providing the CSI with a high accuracy.

Based on the joint pdf of $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_M\}$ and $\mathbf{y}$, $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_M\}$ can be estimated as

$$\begin{aligned}&\{\hat{\mathbf{v}}_1, \hat{\mathbf{v}}_2, \ldots, \hat{\mathbf{v}}_M\}\\ &\quad= \arg\max_{\hat{\mathbf{v}}_M}\{\ln f(\hat{\mathbf{v}}_M|\hat{\mathbf{v}}_{M-1}, \ldots, \hat{\mathbf{v}}_1; \mathbf{y})\}\\ &\qquad+ \arg\max_{\{\hat{\mathbf{v}}_1,\ldots,\hat{\mathbf{v}}_{M-1}\}}\{\ln f(\hat{\mathbf{v}}_{M-1}, \ldots, \hat{\mathbf{v}}_1; \mathbf{y})\}\\ &\quad= \sum_{m=1}^{M}\arg\max_{\hat{\mathbf{v}}_m}\left\{\ln f\left(\hat{\mathbf{v}}_m|\mathbf{y} - \sum_{l=1}^{M-1}\hat{\mathbf{v}}_l\right)\right\} + \ln f(\mathbf{y})\\ &\quad= \sum_{m=1}^{M}\arg\max_{\hat{\mathbf{v}}_m}\left\|\hat{\mathbf{v}}_m^H\left(\mathbf{y} - \sum_{l=1}^{M-1}\hat{\mathbf{v}}_l\right)\right\|_2^2. \end{aligned}\tag{25}$$

User $(l+1)$ estimates $\mathbf{v}_{l+1}$ based on $(\mathbf{y} - \sum_{m=1}^{l}\hat{\mathbf{v}}_m)$, which can be performed after the synchronization of the previous l users, where $l \leq M-1$. There is a substantial tradeoff between the estimation accuracy and the number of iterations in (25), and, in each iteration, the estimation accuracy of $\hat{\mathbf{v}}_m$ depends on that of $\hat{\mathbf{v}}_{l\neq m}$. Note that, usually, there is no closed-form solution for (25), and the frequency offsets that maximize (25) should be searched in a given range to optimize the cost function. This search range must be sufficiently large to reduce the probability that the actual offset is outside it.

V. Improving Estimators by Exploiting the Frequency Offset Variance in OFDMA Uplink

In an OFDMA uplink frequency offset estimation, the performances of the conventional estimators are sensitive to SINR (see [10], [15], and [16]). In Section III, the frequency offset of the mth user ε_m is assumed to be an i.i.d. RV with mean zero and variance σ_ϵ^2. If σ_ϵ^2 is known *a priori*, it can be used to improve the estimation accuracy.

Note that CSI is critical in some pilot-based frequency offset estimation algorithms, and the variance of the frequency offsets cannot be correctly estimated without CSI. The enhanced frequency offset estimation in the single-user scenario with CSI is discussed in [17], where the frequency offset estimation by using pilot/training symbols, null subcarriers, or their combination is analyzed.

The OFDMA receiver structure for an uplink frequency offset estimation is shown in Fig. 4. The SNR/SINR estimates for each user are used to estimate σ_ϵ^2, and the estimate is used to improve the frequency offset estimation. In (15), SINR is derived as a function of σ_ϵ^2, and this function is invertible. Therefore, using SNR_k and SINR_k, an estimate of σ_ϵ^2 is given by

$$\begin{aligned}\hat{\sigma}_\epsilon^2 =& \frac{10\mathrm{SNR}_k(\mathrm{SINR}_k+1)}{3\pi^2\mathrm{SNR}_k}\\ &-\frac{10\sqrt{\mathrm{SNR}_k^2(\mathrm{SINR}_k+1)^2 - \frac{9}{5}\mathrm{SNR}_k(\mathrm{SNR}_k - \mathrm{SINR}_k)}}{3\pi^2\mathrm{SNR}_k}. \end{aligned}\tag{26}$$

Note that an estimate of σ_ϵ^2 is not sufficient to specify the actual distribution of frequency offsets. However, if σ_ϵ^2 is available, some reasonable distributions can be assumed to improve the frequency offset estimation. For example, if ε_k is a uniformly distributed RV with the distribution range $(-\epsilon_k, \epsilon_k)$, the maximum deviation is given by

$$\hat{\epsilon}_k = \sqrt{3}\hat{\sigma}_\epsilon. \tag{27}$$

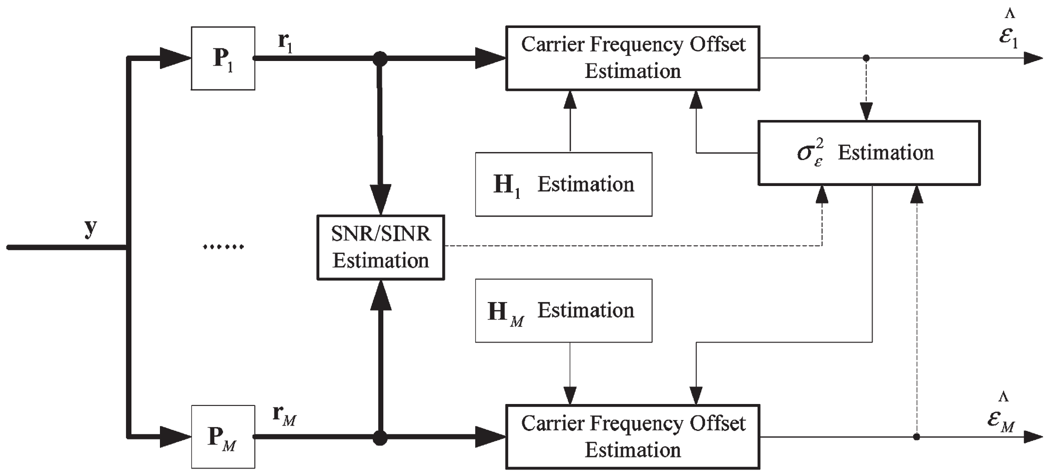

Fig. 4. Receiver structure for OFDMA uplink frequency offset estimation.

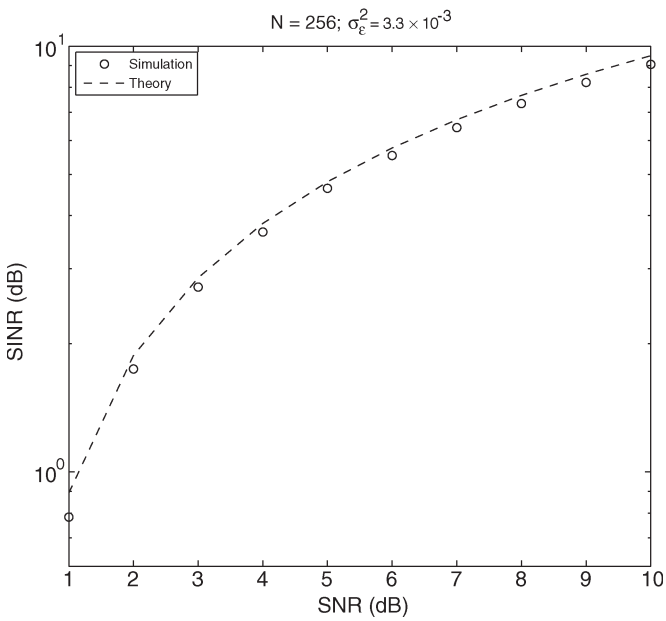

Fig. 5. SINR as a function of SNR and σ_ϵ^2 in OFDMA uplink.

Several high-quality SNR estimators have been proposed in [18]–[20]. For example, by modulating each subcarrier of each user with a PSK signal, a well-known nondata-aided estimator proposed in [18] can be used to estimate SNR_k as

$$\hat{\mathrm{SNR}}_k = \frac{\sqrt{2m_2^2 - m_4}}{m_2 - \sqrt{2m_2^2 - m_4}} \tag{28}$$

where $m_2 = (1/\mathcal{N}_k)(\mathbf{F}_k^H\mathbf{y})^H(\mathbf{F}_k^H\mathbf{y})$, and $m_4 = (1/\mathcal{N}_k)\sum_{i=0}^{\mathcal{N}_k-1}((\mathbf{F}_k^H\mathbf{y})[i])^4$. When $\mathcal{N}_k$ is large enough, the performance of (28) is independent of the frequency offset.

The SINR of user k, i.e., SINR_k, can be estimated as

$$\mathrm{SINR}_k \cong \frac{\left|\left(\mathbf{F}_k^H\mathbf{y}\right)^H\mathbf{x}_k\right|^2}{\left\|\mathbf{F}_k^H\mathbf{y}\right\|_2^2 - \left|\left(\mathbf{F}_k^H\mathbf{y}\right)^H\mathbf{x}_k\right|^2} \tag{29}$$

where $\mathbf{x}_k$ represents the training sequence transmitted by the kth user. Fig. 5 illustrates the SINR estimation accuracy with $N = 256$ and $\sigma_\epsilon^2 = 3.3 \times 10^{-3}$. The simulation results show that the SINR for each user can be accurately estimated based on (29). For an OFDMA uplink transmission with M users accessing a base station, σ_ϵ^2 can be represented as

$$\sigma_\epsilon^2 = \max\left\{\hat{\sigma}_{\epsilon_1}^2, \hat{\sigma}_{\epsilon_2}^2, \ldots, \hat{\sigma}_{\epsilon_M}^2, \frac{1}{M}\sum_{i=1}^{M}\hat{\varepsilon}_i^2\right\} \tag{30}$$

where $\hat{\varepsilon}_k$ represents the currently estimated ε_k, and $\hat{\sigma}_{\epsilon_i}^2$ represents the estimated σ_ϵ^2 for the ith user. If uniformly distributed RVs with the distribution range $(-\epsilon, \epsilon)$ are used to approximate the actual distribution of the frequency offsets, ϵ can be represented as

$$\epsilon = \max\{\sqrt{3}\hat{\sigma}_{\epsilon_1}, \sqrt{3}\hat{\sigma}_{\epsilon_2}, \ldots, \sqrt{3}\hat{\sigma}_{\epsilon_M}, |\hat{\varepsilon}_1|, |\hat{\varepsilon}_2|, \ldots, |\hat{\varepsilon}_M|\}. \tag{31}$$

A. Performance Improvement in the SIC-Based Algorithm Using the Variance of the Frequency Offsets

Using the training sequences, an OFDMA uplink frequency offset estimation can be performed via the SIC-based method, as given in (25). The performance of (25) can be improved by exploiting the variance of the frequency offset of each user [the same goes for (24)]. The idea is to search ε_k for each k subject to $\mathrm{Var}\{\varepsilon_k\} = \sigma_\epsilon^2$ to optimize the cost function, as given by Algorithm 1, which is an improved version of (25). The search complexity can be minimized by using a probability distribution with the smaller range given the variance. For example, if ε_k is assumed uniformly distributed with variance σ_ϵ^2, then the search interval is $(-\sqrt{3}\sigma_\epsilon, \sqrt{3}\sigma_\epsilon)$. However, if a Gaussian $\varepsilon_k \sim \mathcal{N}(0, \sigma_\epsilon^2)$ is assumed, the search range increases to an infinite area. Because of this fact, the use of the uniform distribution to model the frequency offset is preferable. In the following sections, the uniform distribution is shown to be a good approximation to the actual distribution. The performance degradation due to the mismatch between the uniform distribution and the actual distribution will also be analyzed.

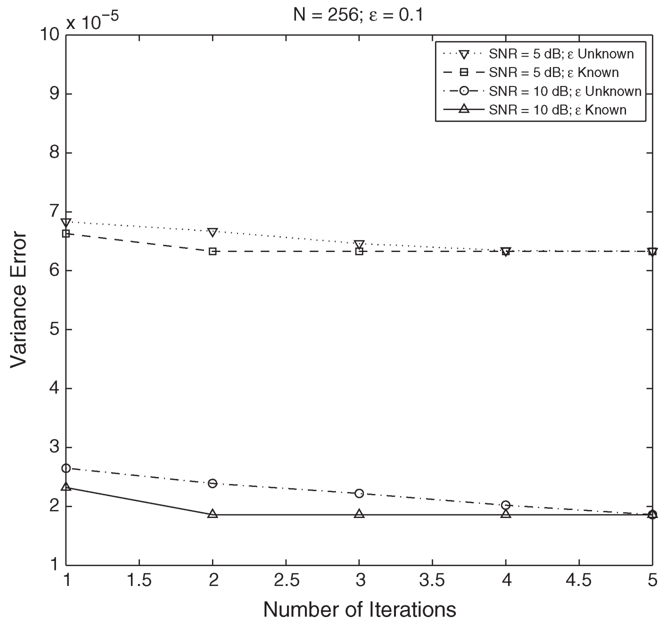

Fig. 6. SIC-based frequency offset estimation in OFDMA uplink transmission.

Algorithm 1: SIC-Based Estimator With ϵ Knowledge
Iteration 0:
for $m = 1$ to M
$\hat{\mathbf{v}}_m^{(0)} = \mathbf{0}_N; \mathbf{y}_m^{(0)} = \mathbf{0}_N; \mathbf{k}_m^{(0)} = \mathbf{0}_N;$
end
end ***Iteration*** 0;
⋮
Iteration $(i+1)$:
for $m = 1$ to M
$\mathbf{k}_m^{(i+1)} = \mathbf{y} - \sum_{l=1}^{m-1} \mathbf{y}_l^{(i+1)} - \sum_{l=m+1}^{k} \mathbf{y}_l^{(i)};$
$\hat{\mathbf{v}}_m^{(i+1)} = \arg\max_{-\epsilon<\hat{\varepsilon}_m<\epsilon} \|\hat{\mathbf{v}}_m^H \mathbf{k}_m^{(i+1)}\|_2^2;$
$\mathbf{y}_m^{(i+1)} = \hat{\mathbf{v}}_m^{(i+1)};$
end
end ***Iteration*** $(i+1)$;
⋮
end Algorithm 1.

Fig. 6 compares the performance of the SIC-based estimators with and without the variance estimate. The total number of subcarriers is 256, and the number of users is 16. Perfect channel and initial phase estimation is assumed. The estimators with/without the variance estimate reach the same accuracy, albeit at different convergence rates. Equation (25) without the variance estimate converges to its steady state in five iterations, but Algorithm 1 converges in two iterations.

B. Performance Improvement in Conventional Differential Algorithms by Exploiting the Variance of the Frequency Offset

The SIC-based algorithms can perform the OFDMA uplink frequency offset estimation with high accuracy. However, channel and initial phase estimation is required, which complicates the use of the algorithm in a high-mobility environment. Differential algorithms are proposed in many classical frequency offset estimators, where a training sequence comprising two or more identical replicas is usually used [10], [15], [16], [21]. Their performance is independent of the initial phases, and if the channel does not change during the training sequence period, the frequency offsets can be reliably estimated without the CSI knowledge.

Some original differential algorithms for orthogonal frequency-division multiplexing systems, e.g., [10] and [15], can also be used for an OFDMA uplink transmission, and the least-square (LS) or ML principle can be applied for each user to perform the frequency offset estimation. Assuming that M users access a base station, with e_k representing the frequency offset estimation error of the kth user $(1 \le k \le M)$, the frequency offset estimation vector can be represented as

$$\tilde{\mathcal{E}}_{\mathrm{LS}} = [\hat{\varepsilon}_1, \ldots, \hat{\varepsilon}_M]^T = \underbrace{[\varepsilon_1, \ldots, \varepsilon_M]^T}_{\mathcal{E}} + \underbrace{[e_1, \ldots, e_M]^T}_{\mathbf{e}} \quad (32)$$

where $\hat{\varepsilon}_k = \arg\min_{\hat{\varepsilon}_k} \|\mathbf{P}_k \mathbf{y} - \hat{\mathbf{E}}_k \mathbf{s}_k\|_2^2$. For the conditionally unbiased estimations, $\mathbb{E}\{\mathbf{e}\} = \mathbf{0}_M$, and the variance matrix of $\mathbf{e}$ is given by $\mathrm{Var}\{\tilde{\mathcal{E}}_{\mathrm{LS}}\} = \mathbb{E}\{\mathbf{e}\mathbf{e}^H\} = \mathbf{C}_e$.

The MAI due to the frequency offsets will degrade the estimation accuracy of (32), and this performance loss partially comes from our lack of knowledge of the frequency offset variance. Under the assumption of uniform distribution for the frequency offsets, (32) can be improved as $\tilde{\mathcal{E}}_\epsilon = [\hat{\varepsilon}_{\epsilon,1}, \ldots, \hat{\varepsilon}_{\epsilon,M}]^T$, where $\hat{\varepsilon}_{\epsilon,k} = \arg\min_{-\epsilon<\hat{\varepsilon}_k<\epsilon} \|\mathbf{P}_k \mathbf{y} - \hat{\mathbf{E}}_k \mathbf{s}_k\|_2^2$ for each k. From [22], the covariance matrix of the estimation error of $\tilde{\mathcal{E}}_\epsilon$ is lower bounded by $\widetilde{\mathbf{M}} = (\mathbf{C}_{\mathcal{E}\mathcal{E}}^{-1} + \mathbf{C}_e^{-1})^{-1}$, where $\mathbf{C}_{\mathcal{E}\mathcal{E}} = \sigma_\epsilon^2 \mathbf{I}_M$. If $\mathbb{E}\{e_k e_{l \ne k}\} = 0$ is satisfied for each k, $\widetilde{\mathbf{M}}$ is reduced to

$$\widetilde{\mathbf{M}} = \mathrm{diag}\left\{\frac{1}{\tilde{\sigma}_1^2}, \frac{1}{\tilde{\sigma}_2^2}, \ldots, \frac{1}{\tilde{\sigma}_M^2}\right\} \quad (33)$$

where $\tilde{\sigma}_i^2 = (1/\sigma_\epsilon^2) + (1/\mathrm{Var}\{e_i\})$.

TABLE I
SUBCARRIER ALLOCATION IN OFDMA UPLINK TRANSMISSION
(BANDWIDTH = 10 MHz, DFT LENGTH = 256, CP = 16)

Number of Users	16
Number of Subcarriers Allocated	224
Number of Guard Band Subcarriers	32
Subcarrier Allocation	Interleaved
Normalized CFO	Uniformly distributed in (-ϵ,ϵ)
Training Sequence Used	Like in [10]
Maximum Multipath Delays (μs)	0.8
Initial Phase	Uniformly distributed in (0,2π)

VI. NUMERICAL RESULTS

A wireless OFDMA system with a bandwidth of 10 MHz and 256 subcarriers is considered, and a length-16 CP is used. Quaternary PSK modulation is used in all subcarriers, and the interleaved subcarrier allocation is performed among different users to exploit the frequency diversity. A multipath Rayleigh fading channel is considered for each user, as shown in Table I.

The estimator proposed in [10] (a differential frequency offset estimator based on a training sequence comprising two identical replicas) is used as an example to illustrate the

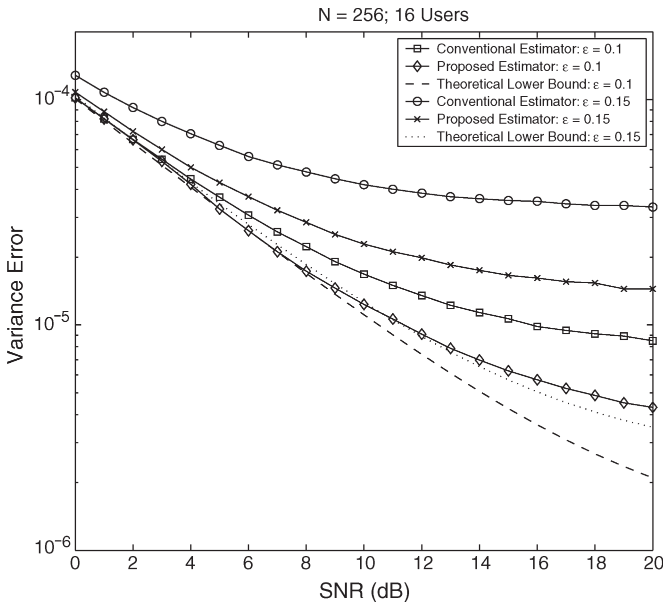

Fig. 7. Performance comparison between conventional and the proposed frequency offset estimators when all the accessed users are running at the tracking phase.

TABLE II
SUBCARRIER ALLOCATION IN OFDMA UPLINK TRANSMISSION WITH NEW ACCESSED USERS (BANDWIDTH = 10 MHz, DFT LENGTH = 256, CP = 16)

Number of Users in Tracking Phase	16
Number of Subcarriers Allocated	224
Number of Guard Band Subcarriers	32
Subcarrier Allocation	Interleaved
Normalized CFO in Tracking Phase	Uniformly distributed in $(-\epsilon,\epsilon)$
Number of New Accessed Userss	1; 2; 3; 4
Normalized CFO of New User	Uniformly distributed in (-1.6,1.6)
Training Sequence Used	Like in [10]
Maximum Multipath Delays (μs)	0.8
Initial Phase	Uniformly distributed in $(0,2\pi)$

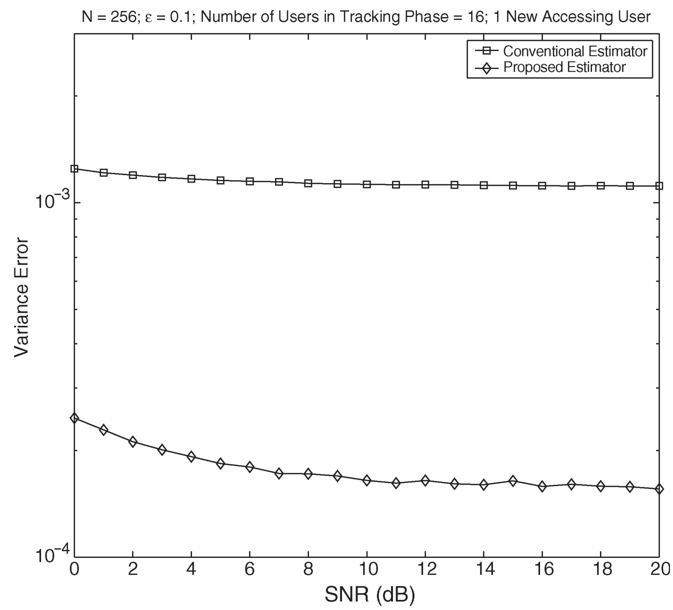

Fig. 8. Performance comparison between the conventional and the proposed estimators when there is one new user accessing the base station.

performance improvement with knowledge of the frequency offset variance. Note that the performance of any conventional algorithm can be improved by exploiting this knowledge.

Fig. 7 shows the performance improvement in the conventional estimations with (31) or without (31). In this simulation, all the users are assumed to run at the frequency offset tracking phase, and a larger ϵ implies a higher estimation error in either the conventional estimator (that without using the frequency offset variance) or the proposed one (that with the frequency offset variance). A performance floor always appears at a high SNR in either the conventional estimator or the proposed one due to the irreducible MAI. The estimation accuracy in the conventional estimator can be improved by exploiting the knowledge of the variance, and this performance improvement becomes larger as the SNR increases. For example, when $\epsilon = 0.1$, the performance improvement of an algorithm with variance knowledge over one without can be up to 3 dB at a high SNR.

In an OFDMA uplink, the users may dynamically access or depart from a base station. A new accessing user may have an instantaneously large frequency offset, causing a heavy MAI to its frequency-domain neighbors. Table II shows a scenario of the base station, where 16 current users are running at the tracking phase, and this simulation considers the case that only one new user is starting to access the base station. The new user has an initial frequency offset of 1.6 times the subcarrier spacing. Note that the integer frequency offset in the new user should be estimated and corrected by using an acquisition algorithm, e.g., as in [10] and [23], but the use of such an algorithm is beyond the scope of this paper.

This section only illustrates the performance degradation of the frequency offset estimation in tracking users due to the large frequency offset of the new accessing user (Fig. 8). A considerable performance degradation appears in the conventional estimator (without variance knowledge); the proposed estimator with variance knowledge is more robust for combating the instantaneously large MAI. For example, at an SNR of 10 dB, the variance error in the conventional estimator is about 1.13×10^{-3}, and this error can be reduced to about 1.65×10^{-4} in the proposed estimator.

A much heavier MAI is generated if there are multiple new users simultaneously accessing the base station, as shown in Fig. 9, where the simulation parameters are also defined in Table II. This simulation keeps SNR $= 10$ dB unchanged, and one to four new users access the base station. In this case, the instant interference on the existing users is mainly contributed by these new users. The proposed estimator considerably outperforms the conventional estimator. For example, with two new users accessing the base station, the variance error for the conventional estimator is about 4.16×10^{-3}, and that for the proposed estimator is about 2.9×10^{-3}. For the case of four new users, the variance error for the conventional estimator increases to about 4.7×10^{-3}, and that for the proposed estimator is 3.08×10^{-3}.

Thus far, the frequency offsets for different users were assumed to be uniformly distributed RVs. In real systems, the actual distribution may not be uniform, and, hence, a uniformly distributed RV mismatches the distribution of ε_k. However, the exact distribution may be well approximated by a uniform

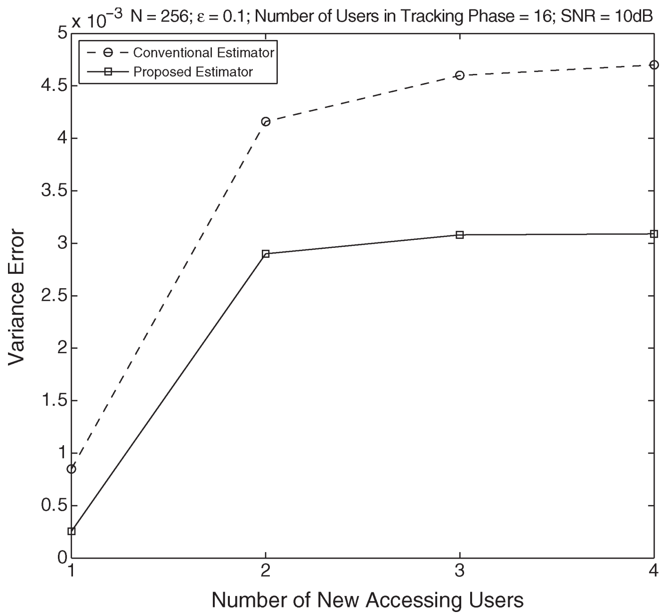

Fig. 9. Performance comparison between the conventional and the proposed estimators when there are multiple new users simultaneously accessing the base station.

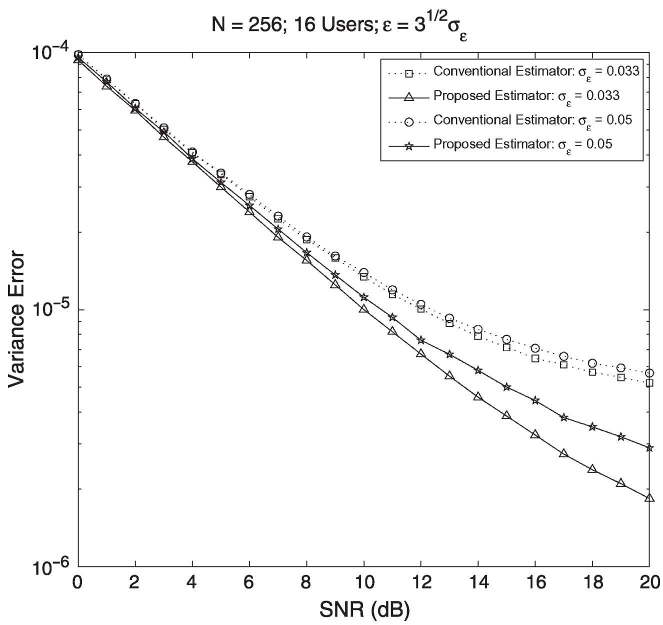

Fig. 10. Performance improvement in the proposed estimator over the conventional estimator when the actual frequency offsets are approximated as uniformly distributed RVs.

distribution. For example, assuming that the actual distribution is Gaussian with $\varepsilon_k \sim \mathcal{N}(0, \sigma_\epsilon^2)$, the probability of ε_k being outside of the range $(-\sqrt{3}\sigma_\epsilon, \sqrt{3}\sigma_\epsilon)$ is $P\{|\varepsilon_k| > \sqrt{3}\sigma_\epsilon\} \cong 0.0833$, i.e., most of the realization of ε_k falls into the range $(-\sqrt{3}\sigma_\epsilon, \sqrt{3}\sigma_\epsilon)$. Since a uniformly distributed RV with a range $(-\sqrt{3}\sigma_\epsilon, \sqrt{3}\sigma_\epsilon)$ can also result in a variance of σ_ϵ^2, we can set $\epsilon = \sqrt{3}\sigma_\epsilon$ and use this uniform distribution to approximate the actual distribution. For OFDMA uplink frequency offset estimation with the proposed estimator, σ_ϵ is first estimated by using (30), and then set $\hat{\epsilon} = \sqrt{3}\hat{\sigma}_\epsilon$ [or directly use (31)] to approximate ε_k as a uniformly distributed RV in $(-\hat{\epsilon}, \hat{\epsilon})$. Fig. 10 compares the performance of the conventional estimator with or without variance knowledge when all the user frequency offsets are i.i.d. Gaussian-distributed RVs. The simulation results show that $\varepsilon_k \sim \mathcal{N}(0, \sigma_\epsilon^2)$ can be well approximated as a uniformly distributed RV with $\epsilon = \sqrt{3}\sigma_\epsilon$, and that at a high SNR, the prior variance knowledge can provide a performance improvement of about 3 dB.

VII. CONCLUSION

This paper has discussed the OFDMA uplink frequency offset estimation, and the performance degradation due to MAI has been analyzed. The variance of the frequency offset of each user may be estimated through MAI analysis. This estimate of the variance of frequency offsets helps to improve the robustness and accuracy of the frequency offset estimation. For an OFDMA uplink with all the accessed users running at the tracking phase, the frequency offset of each user can be assumed to be an i.i.d. RV with zero mean and a variance of σ_ϵ^2, and a uniformly distributed RV can be used to approximate the actual distribution of the frequency offset. The simulation results prove the validity of this uniform approximation.

APPENDIX A
ANALYSIS OF FIM

In this subsection, the superscript in $\mathbf{y}^M$, $\mathbf{C}^M$, and $\mathbf{s}^M$ will be omitted for brevity. The covariance matrix $\mathbf{C}$ can be rewritten as

$$\begin{aligned}\mathbf{C} &= \sum_k \mathbb{E}\left\{(\mathbf{y}_k - \mathbf{s}_k)(\mathbf{y}_k - \mathbf{s}_k)^H\right\} \\ &\quad + \underbrace{\sum_k \sum_{k \neq l} \underbrace{\mathbb{E}\left\{(\mathbf{y}_k - \mathbf{s}_k)(\mathbf{y}_l - \mathbf{s}_l)^H\right\}}_{\mathbf{Z}_{kl}}}_{\mathbf{Z}_k} \\ &= \sum_k (\mathbf{C}_k + \mathbf{Z}_k) \end{aligned} \tag{34}$$

where $\mathbf{C}_k = \mathbb{E}\{(\mathbf{y}_k - \mathbf{s}_k)(\mathbf{y}_k - \mathbf{s}_k)^H\}$, and $\mathbf{Z}_{kl} = \mathbb{E}\{(\mathbf{y}_k - \mathbf{s}_k)(\mathbf{y}_l - \mathbf{s}_l)^H\}$ is the MAI matrix of the kth user, which is contributed by the lth user. For the nonzero frequency offsets, $\sum_{k \neq l} \mathbf{Z}_{kl} \neq \mathbf{O}_N$. We can decompose $\mathbf{C}$ as

$$\mathbf{C} = \sum_k (\mathbf{C}_k + \mathbf{Z}_k) = \sum_k \mathbf{U}(\mathbf{D}_k + \tilde{\mathbf{D}}_k)\mathbf{U}^H \tag{35}$$

where $\mathbf{U}$ is an $N \times N$ unitary matrix, $\mathbf{D}_k = \mathrm{diag}\{0, \ldots, \lambda_{k,1}, \ldots, \lambda_{k,\mathcal{N}_k}, \ldots, 0\}$ with $\lambda_{k,i}$ representing the ith eigenvalue of $\mathbf{C}_k$, and $\tilde{\mathbf{D}}_k = \mathrm{diag}\{0, \ldots, z_{k,1}, \ldots, z_{k,\mathcal{N}_k}, \ldots, 0\}$, with $z_{k,i}$ representing the ith eigenvalue of $\mathbf{Z}_k$. The index of $z_{k,i}$ is identical to that of $\lambda_{k,i}$ for each i. Therefore, $\mathbf{C}^{-1}$ can be represented as $\mathbf{C}^{-1} = \sum_k \mathbf{U}(\mathbf{D}_k + \tilde{\mathbf{D}}_k)^{-1}\mathbf{U}^H$, where $(\mathbf{D}_k + \tilde{\mathbf{D}}_k)^{-1} = \mathrm{diag}\{0, \ldots, (\lambda_{k,1} + z_{k,1})^{-1}, \ldots, (\lambda_{k,\mathcal{N}_k} + z_{k,\mathcal{N}_k})^{-1}, \ldots, 0\}$. We also have $(\partial\mathbf{C}/\partial\varepsilon_k) = \mathbf{U} \cdot \mathbf{T}_{k,k} \cdot \mathbf{U}^H + \sum_{l \neq k} \mathbf{U} \cdot \mathbf{T}_{k,l} \cdot \mathbf{U}^H$, where $\mathbf{T}_{k,i} = \mathrm{diag}\{\ldots, (\partial(\lambda_{i,1} + z_{i,1})/\partial\varepsilon_k), \ldots, (\partial(\lambda_{i,\mathcal{N}_k} + z_{i,\mathcal{N}_k})/\partial\varepsilon_k), \ldots\}$.

$$\mathcal{I}_k(n) \cong \underbrace{\sum_{i\in G_k, i\neq n} ((-1)^{i-n}\sqrt{P_i}\mathbf{x}_k[i]H_k^i \pi/N \sin[(\pi(i-n)/N)]) \cdot}_{\eta_k(n)} \varepsilon_k$$

$$\mathcal{I}_{l\neq k}(n) \cong \sum_{l=1, l\neq k}^{M} \underbrace{\sum_{i\in G_l} ((-1)^{i-n}\sqrt{P_i}\mathbf{x}_l[i]H_l^i \pi/N \sin[(\pi(i-n)/N)]) \cdot}_{\eta_l(n)} \varepsilon_l$$

Note that $\mathbf{Z}_k$ is the MAI contributed by users other than k and is not a function of ε_k; therefore, $(\partial \mathbf{Z}_k/\partial \varepsilon_k) = \mathbf{O}_N$. $\mathbf{C}_l$ is also not a function of ε_k, so $(\partial \mathbf{C}_l/\partial \varepsilon_k) = \mathbf{O}_N$. From the above discussion, we have

$$\frac{\partial \mathbf{C}}{\partial \varepsilon_k} = \mathbf{U}\cdot \mathrm{diag}\left\{\ldots, \frac{\partial \lambda_{k,1}}{\partial \varepsilon_k}, \ldots, \frac{\partial \lambda_{k,\mathcal{N}_k}}{\partial \varepsilon_k}, \ldots\right\}\cdot \mathbf{U}^H + \sum_{l\neq k} \mathbf{U}\cdot \mathrm{diag}\left\{\ldots, \frac{\partial z_{l,1}}{\partial \varepsilon_k}, \ldots, \frac{\partial z_{l,\mathcal{N}_l}}{\partial \varepsilon_k}, \ldots\right\}\cdot \mathbf{U}^H. \quad (36)$$

When $l \neq k$, the klth element of FIM can be represented as

$$[\mathbf{\Lambda}]_{kl} = \sum_{i=1}^{\mathcal{N}_k} \frac{\frac{\partial \lambda_{k,i}}{\partial \varepsilon_k}\frac{\partial z_{k,i}}{\partial \varepsilon_l}}{(\lambda_{k,i}+z_{k,i})^2} + \sum_{j=1}^{\mathcal{N}_l} \frac{\frac{\partial \lambda_{l,j}}{\partial \varepsilon_l}\frac{\partial z_{l,j}}{\partial \varepsilon_k}}{(\lambda_{l,j}+z_{l,j})^2} + \sum_{n\neq k,l}\sum_{p=1}^{\mathcal{N}_n} \frac{\frac{\partial^2 z_{n,p}}{\partial \varepsilon_k \partial \varepsilon_l}}{(\lambda_{n,p}+z_{n,p})^2} \quad (37)$$

and the kkth element of FIM is given by

$$[\mathbf{\Lambda}]_{kk} = \sum_{i=1}^{\mathcal{N}_k} \frac{\left(\frac{\partial \lambda_{k,i}}{\partial \varepsilon_k}\right)^2}{(\lambda_{k,i}+z_{k,i})^2} + \sum_{l\neq k}\sum_{j=1}^{\mathcal{N}_l} \frac{\left(\frac{\partial z_{l,j}}{\partial \varepsilon_k}\right)^2}{(\lambda_{l,j}+z_{l,j})^2}. \quad (38)$$

Appendix B
SINR Analysis

Without loss of generality, we assume that M users accessed a base station, and the kth user is selected as the user of interest. The average SINR of user k is

$$\mathrm{SINR}_k = \frac{\mathbb{E}\left\{\sum_{n\in G_k}\left|\sqrt{P_n}\mathbf{x}_k[n]H_k^n \cdot \frac{\sin(\pi\varepsilon_k)}{N\sin\left(\frac{\pi\varepsilon_k}{N}\right)}\right|^2\right\}}{\mathbb{E}\left\{\sum_{n\in G_k}\left|\mathcal{I}_k(n)+\mathcal{I}_{l\neq k}(n)\right|^2\right\} + \mathbb{E}\left\{\left\|\mathbf{F}_k^H\mathbf{w}\right\|_2^2\right\}} \quad (39)$$

where

$$\mathcal{I}_k(n) = \sum_{i\in G_k, i\neq n} \frac{\sqrt{P_i}\mathbf{x}_k[i]H_k^i \sin[\pi(i-n+\varepsilon_k)]}{N\sin\left[\frac{\pi(i-n+\varepsilon_k)}{N}\right]} \quad (40)$$

$$\mathcal{I}_{l\neq k}(n) = \sum_{l=1, l\neq k}^{M}\sum_{i\in G_l} \frac{\sqrt{P_i}\mathbf{x}_l[i]H_l^i \sin[\pi(i-n+\varepsilon_l)]}{N\sin\left[\frac{\pi(i-n+\varepsilon_l)}{N}\right]}. \quad (41)$$

We also assume that $\mathbb{E}\{|\mathbf{x}_i[m]|^2\} = \sigma_x^2 = 1$, and $\sigma_w^2 = (\mathbb{E}\|\mathbf{F}_k^H\mathbf{w}[n]\|_2^2/\mathcal{N}_k) = (\mathbb{E}\|\mathbf{w}\|_2^2/N)$.

Note that $\sin[\pi(i-n+\varepsilon_l)] = (-1)^{i-n}\sin(\pi\varepsilon_l) \cong (-1)^{i-n}\pi\varepsilon_l$, and $\sin[(\pi(i-n+\varepsilon_l)/N)] \cong \sin[(\pi(i-n)/N)]$, when $\varepsilon_l \ll 1$. Therefore, we have $\mathcal{I}_k(n)$ and $\mathcal{I}_{l\neq k}(n)$, shown at the top of the page, and then, $\mathrm{SINR}_k = (\mathbb{E}\{\sum_{n\in G_k}|\sqrt{P_n}\mathbf{x}_k[n]H_k^n|^2\} \cdot \mathbb{E}\{|(\sin(\pi\varepsilon_k)/N\sin((\pi\varepsilon_k/N))|^2\} / \sum_{n\in G_k}\mathbb{E}\{|\eta_k(n)\varepsilon_k + \sum_{l=1,l\neq k}^{M}\eta_l(n)\varepsilon_l|^2\} + \mathbb{E}\{\|\mathbf{F}_k^H\mathbf{w}\|_2^2\})$. Note that $\eta_k(n)$, as well as each $\eta_{l\neq k}(n)$, is an RV with mean zero. If we assume that $\mathbb{E}\{\mathbf{x}_k[n]\mathbf{x}_l^*[m]\} = 0$ for $k \neq l$ or $n \neq m$, the average interference of the subcarrier n is

$$\mathbb{E}\left\{\left|\eta_k(n)\varepsilon_k + \sum_{l=1,l\neq k}^{M}\eta_l(n)\varepsilon_l\right|^2\right\} = \sum_{i\neq n}\frac{\pi^2\sigma_\epsilon^2\kappa_n}{N^2\sin^2\left[\frac{\pi(i-n)}{N}\right]} \quad (42)$$

where $\kappa_n = \mathbb{E}\{|\sqrt{P_i}H_m^i|^2\}$. When N is large enough and almost all the subcarriers are allocated, we have $\sum_{i\neq n}(1/N^2\sin^2[(\pi(i-n)/N)]) \cong \sum_{i\neq n}(1/\pi^2(i-n)^2) \cong (1/3)$, and

$$\mathrm{SINR}_k \cong \frac{\mathrm{SNR}_k}{\frac{\pi^2\sigma_\epsilon^2\mathrm{SNR}_k}{3}+1}\cdot\left(1-\frac{\pi^2\sigma_\epsilon^2}{3}+\frac{\pi^4\sigma_\epsilon^4}{20}\right) \quad (43)$$

where $\mathrm{SNR}_k = \mathbb{E}\{\kappa_n\}/\sigma_w^2$ represents the average SNR of the kth user, and σ_ϵ^2 is the variance of the frequency offsets.

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