# LINEAR PRECODERS FOR OSTBC MIMO SYSTEMS WITH CORRELATED RAYLEIGH FADING CHANNELS BASED ON CONVEX OPTIMIZATION

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## ABSTRACT

In this paper, we present a precoder design framework and a computationally simple precoding technique for OSTBC based MIMO wireless systems with both transmit and receive correlations for the case of Rayleigh fading. It is assumed that the correlation among receive antennas is independent of the correlation among transmit antennas (and vice versa). The transmit and receive correlation matrices are assumed to be available at the transmitter, while the instantaneous channel state information (CSI) is unknown. The proposed precoder minimizes the upper bound on the symbol error rate (SER). Our main contribution consists of developing a convex formulation for originally non-convex problem of SER minimization for precoder design. Additionally, it can be shown that previously known solutions for some special cases of precoder design naturally follow from our more general results. Numerical simulations illustrate the improved performance of the proposed precoders in terms of the output SER.

## 1. INTRODUCTION

Orthogonal space-time block codes (OSTBCs) have been developed in order to exploit the advantages of multiple-input multiple-output (MIMO) systems such as diversity gain [1], [2]. OSTBCs have received much interest because of simple, symbol-by-symbol maximum likelihood (ML) detection [2]. It is important to note that OSTBCs are designed for the case no channel state information (CSI) at the transmitter. However, if some degree of CSI is available at the transmitter, the performance of OSTBC MIMO systems can be further improved. For example, precoding techniques can be combined with OSTBCs to adapt to the current channel conditions without changing the fixed structures of the transmitter and the receiver [3]– [11]. However, these and many other papers focus on transmit antenna correlations, but with little emphasis on receive antenna correlations [3]– [11].

Traditional MIMO precoding techniques aim at improving the system performance using full CSI at transmitter [4]– [6]. Reference [5] shows that a number of different precoder design criteria can be cast as a weighted minimum mean square error (MSE) criterion with a proper choice of weight coefficients. CSI at transmitter can be obtained if time-division duplex (TDD) mode is employed, but availability of such knowledge at the transmitter can significantly consume the bandwidth of the system in frequency-division duplex (FDD) systems. Hence, it is reasonable to assume that the transmitter has only partial channel knowledge such as transmit and receive correlation matrices. These statistics vary at a much slower rate than the instantaneous channel and can be obtained reliably at the transmitter [12].

Therefore, this paper considers the problem of MIMO precoder design using only the knowledge of transmit and receive antenna correlations. Specifically, we show that the problem of OSTBC precoder design based on minimization of the exact SER over a jointly transmit-receive correlated Rayleigh fading MIMO channel can be transformed into a convex optimization problem. This is a significant observation because (1) there is a unique globally optimal precoder and (2) wide variety of computational algorithms are available for convex problems. We also show that the optimal precoder has a special structure that allows the number of optimization variables to be reduced significantly. We also identify several cases where closed-form designs are feasible.

## 2. SYSTEM MODEL

The received signal for a communication system that employs an OSTBC and a linear precoder at the transmitter may be written as

$$\mathbf{Y} = \sqrt{\frac{E_s}{N_\mathrm{t}}}\mathbf{HFC} + \mathbf{Z} \tag{1}$$

where $N_\mathrm{t}$ and $N_\mathrm{r}$ are the numbers of transmit and receive antennas, respectively, $E_s$ is the total transmitted power, $\mathbf{Y}$ is the $N_\mathrm{r} \times T$ received signal matrix, $\mathbf{C}$ is the $N_\mathrm{t} \times T$ OSTBC matrix, $\mathbf{F}$ is the $N_\mathrm{t} \times N_\mathrm{t}$ precoding matrix, $\mathbf{H}$ is the $N_\mathrm{r} \times N_\mathrm{t}$ channel matrix, and $\mathbf{Z}$ is the $N_\mathrm{t} \times T$ additive white Gaussian noise (AWGN) matrix with zero-mean and $N_0$-variance entries (independent and identically distributed - i.i.d.). The receiver uses the ML decoder.

The effects of transmit and receive correlations for a Rayleigh flat fading channel can be reasonably modeled as [12]

$$\mathbf{H} = \mathbf{R}_\mathrm{r}^{1/2}\tilde{\mathbf{H}}\mathbf{R}_\mathrm{t}^{1/2} \tag{2}$$

where $\mathbf{R}_\mathrm{t}$ and $\mathbf{R}_\mathrm{r}$ are the transmit and receive side correlation matrices of sizes $N_\mathrm{t} \times N_\mathrm{t}$ and $N_\mathrm{r} \times N_\mathrm{r}$, respectively, and $\tilde{\mathbf{H}}$ consists of i.i.d. zero-mean and unit-variance elements. It is assumed in the model (2) that the correlation among receive antennas is independent of the correlation among transmit antennas (and vice versa).

With the use of OSTBCs, the ML decoder can be simplified to a symbol-by-symbol decoder of the following form [2]

$$\tilde{s}_q = \sqrt{\frac{E_s}{N_\mathrm{t}}}\left(\frac{1}{K}\|\mathbf{HF}\|_F^2\right)s_q + \nu_q, \quad q = 1, ..., Q \tag{3}$$

where $s_q, q = 1, ..., Q$ are the complex information-bearing symbols prior to space-time encoding; $\nu_q \sim \mathcal{CN}\left(0, \frac{1}{K}\|\mathbf{HF}\|_F^2 N_0\right)$

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stands for circularly symmetric complex Gaussian zero-mean variable with variance $\frac{1}{K}\|\mathbf{HF}\|_F^2 N_0$.

Using (3) the effective instantaneous signal-to-noise ratio (SNR) per symbol can be expressed as

$$\gamma = \frac{|\tilde{s}_q|^2}{|\nu_q|^2} = \frac{E_s}{KN_{\mathrm{t}}N_0}\alpha \tag{4}$$

where

$$\alpha = \|\mathbf{HF}\|_F^2 \tag{5}$$

is the Frobenius norm of the effective channel $\mathbf{HF}$.

## 3. PRECODER DESIGN PROBLEM FORMULATION

For brevity, we limit our discussion to M-PAM only. Our results can be straightforwardly extended for M-QAM and M-PSK modulations as well. Given the receive instantaneous SNR $\gamma$, the SER of M-PAM can be evaluated as [13]

$$\mathrm{SER}_{\mathrm{PAM}}^{\gamma} = \frac{2}{\pi}\frac{M-1}{M}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{g_{\mathrm{PAM}}\gamma}{\sin^2\phi}\right) d\phi \tag{6}$$

where $g_{\mathrm{PAM}} = 3/(M^2-1)$. The average SER is thus given by

$$\mathrm{SER}_{\mathrm{PAM}} = \int_0^{\infty} \mathrm{SER}_{\mathrm{PAM}}^{\gamma} f_\gamma(\gamma)\, d\gamma \tag{7}$$

where $f_\gamma(\gamma)$ is the probability density function (pdf) of the SNR $\gamma$.

Using (4) and (6), we can rewrite (7) as

$$\mathrm{SER}_{\mathrm{PAM}} = \frac{2}{\pi}\frac{M-1}{M}\int_0^{\infty}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{\tilde{g}\alpha}{\sin^2\phi}\right) f_\alpha(\alpha)\, d\phi\, d\alpha \tag{8}$$

where $\tilde{g} = g_{\mathrm{PAM}}E_s/(KN_{\mathrm{t}}N_0)$, and $f_\alpha(\alpha)$ stands for the pdf of random variable $\alpha$ in (5).

Using the moment generating function approach, the exact SER can be expressed as

$$\mathrm{SER}_{\mathrm{PAM}} = \frac{2}{\pi}\frac{M-1}{M}\int_0^{\frac{\pi}{2}} \det\left(\mathbf{I} + \frac{\tilde{g}}{\sin^2\phi}\mathbf{R}_{\mathrm{r}} \otimes \left(\mathbf{F}\mathbf{F}^H\mathbf{R}_{\mathrm{t}}\right)\right)^{-1} d\phi. \tag{9}$$

where $\otimes$, $\mathbf{I}$ denote the Kronecker product, the identity matrix of appropriate size respectively. The SER (9) is a function of the precoding matrix $\mathbf{F}$. Therefore, the precoder design problem can be formulated as the following SER minimization problem

$$\min_{\tilde{\mathbf{F}}} \mathrm{SER}_{\mathrm{PAM}} \quad \text{subject to} \quad \mathrm{Tr}(\tilde{\mathbf{F}}) = 1, \quad \tilde{\mathbf{F}} \succcurlyeq \mathbf{0} \tag{10}$$

where $\tilde{\mathbf{F}} = \mathbf{F}\mathbf{F}^H$ is a new optimization variable, and the constraint $\mathrm{Tr}(\tilde{\mathbf{F}}) = 1$ guarantees that the average transmitted power is constant. Note that a similar formulation has been taken in [15]. Here, we show that the problem (10) is in fact convex. The following theorem is in order.

THEOREM 1: The optimization problem (10) is convex on $\tilde{\mathbf{F}} \succcurlyeq \mathbf{0}$.

PROOF: The proof will be shown in a journal version of this paper.

The problem (10) can be solved, for example, using gradient descend method. The complexity of the gradient descend method applied to the problem (10) is mostly due to the gradient computation for the SER function (9). Finally, the optimal precoding matrix $\mathbf{F}_{\mathrm{opt}}$ can be found using the solution of the problem (10) as $\mathbf{F}_{\mathrm{opt}} = \tilde{\mathbf{F}}_{\mathrm{opt}}^{1/2}$.

## 4. LOW COMPLEXITY LINEAR PRECODERS FOR OSTBC MIMO SYSTEMS

Consider the following eigenvalue decomposition (EVD) of the correlation matrices $\mathbf{R}_{\mathrm{t}}$ and $\mathbf{R}_{\mathrm{r}}$:

$$\mathbf{R}_{\mathrm{t}} = \mathbf{U}_{\mathrm{t}}\mathbf{\Sigma}_{\mathrm{t}}\mathbf{U}_{\mathrm{t}}^H \tag{11}$$

$$\mathbf{R}_{\mathrm{r}} = \mathbf{U}_{\mathrm{r}}\mathbf{\Sigma}_{\mathrm{r}}\mathbf{U}_{\mathrm{r}}^H \tag{12}$$

where $\mathbf{U}_{\mathrm{t}}$ and $\mathbf{U}_{\mathrm{r}}$ are the matrices of eigenvectors, and $\mathbf{\Sigma}_{\mathrm{t}}$ and $\mathbf{\Sigma}_{\mathrm{r}}$ are the diagonal eigenvalue matrices of the transmit and receive correlation matrices, respectively.

Using algebraic manipulations, we can show that (10) is equivalent to the following problem

$$\begin{aligned} &\min_{\bar{\mathbf{F}}} \int_0^{\frac{\pi}{2}} \det\left(\mathbf{I} + \frac{\tilde{g}}{\sin^2\phi}\mathbf{\Sigma}_{\mathrm{r}} \otimes \left(\mathbf{\Sigma}_{\mathrm{t}}^{1/2}\bar{\mathbf{F}}\mathbf{\Sigma}_{\mathrm{t}}^{1/2}\right)\right)^{-1} d\phi \\ &\text{subject to} \quad \mathrm{Tr}(\bar{\mathbf{F}}) = 1, \quad \bar{\mathbf{F}} \succcurlyeq \mathbf{0} \end{aligned} \tag{13}$$

where we have introduced the the new matrix variable $\bar{\mathbf{F}} = \mathbf{U}_{\mathrm{t}}^H\tilde{\mathbf{F}}\mathbf{U}_{\mathrm{t}}$. We note that $\mathrm{Tr}(\bar{\mathbf{F}}) = \mathrm{Tr}(\mathbf{U}_{\mathrm{t}}^H\tilde{\mathbf{F}}\mathbf{U}_{\mathrm{t}}) = \mathrm{Tr}(\mathbf{U}_{\mathrm{t}}\mathbf{U}_{\mathrm{t}}^H\tilde{\mathbf{F}}) = \mathrm{Tr}(\tilde{\mathbf{F}}) = 1$ because the matrix $\mathbf{U}_{\mathrm{t}}$ is unitary.

THEOREM 2: An optimal matrix $\bar{\mathbf{F}}$ which minimizes the objective function in (13) is diagonal.

PROOF: We will make use of the Hadamard inequality [14], which states that for any positive definite matrix $\mathbf{A} = [a_{ij}]$ of size $n \times n$, $\det(\mathbf{A}) \geq \prod_{i=1}^n a_{ii}$ where equality holds if and only if $\mathbf{A}$ is diagonal; equivalently, $\det(\mathbf{A}^{-1}) \geq \prod_{i=1}^n \frac{1}{a_{ii}}$.

Let $\mathbf{A} = \mathbf{I} + \frac{\tilde{g}}{\sin^2\phi}\mathbf{\Sigma}_{\mathrm{r}} \otimes \left(\mathbf{\Sigma}_{\mathrm{t}}^{1/2}\bar{\mathbf{F}}\mathbf{\Sigma}_{\mathrm{t}}^{1/2}\right)$. According to the Hadamard inequality, $\det(\mathbf{A})^{-1}$ is minimized if and only if the matrix $\mathbf{A}$ is diagonal. Since the matrices $\mathbf{\Sigma}_{\mathrm{t}}$ and $\mathbf{\Sigma}_{\mathrm{r}}$ are diagonal, the matrix $\mathbf{A}$ is diagonal if and only if the matrix $\bar{\mathbf{F}}$ is also diagonal. Finally, we note that the integral in (13) can be seen as a sum of terms for each of which $\mathbf{A}$ is diagonal if and only if $\bar{\mathbf{F}}$ is diagonal. This completes the proof. □

Note that a similar diagonal property has been shown, for example, in [3], [15], but the above proof is simpler and more direct.

Using Theorem 2 and replacing the diagonal elements of $\bar{\mathbf{F}}$ by a more familiar notation, that is, $\mathbf{x} = \mathrm{diag}(\bar{\mathbf{F}}) \in \mathcal{R}^{N_{\mathrm{t}}}$, (13) can be simplified to the following optimization problem

$$\begin{aligned} &\min_{\mathbf{x}} \int_0^{\frac{\pi}{2}} \prod_{i=1}^{N_{\mathrm{r}}}\prod_{j=1}^{N_{\mathrm{t}}} \left(1 + \frac{\tilde{g}}{\sin^2\phi}[\Sigma_{\mathrm{r}}]_i[\Sigma_{\mathrm{t}}]_j x_j\right)^{-1} d\phi \\ &\text{subject to} \sum_{j=1}^{N_{\mathrm{t}}} x_j = 1,\ x_j \geq 0,\ j = 1, \ldots, N_{\mathrm{t}} \end{aligned} \tag{14}$$

where $[\Sigma]_j$ denotes the $j$th diagonal element of the diagonal matrix $\mathbf{\Sigma}$, and $x_j$ stands for the $j$th element of the vector $\mathbf{x}$. Note that (14) is mathematically equivalent to (10). However, the optimization vector variable in (14) has a significantly smaller dimension than that of the optimization matrix variable in (10).

We now consider an upper bound to the integral in (14) by setting $\phi = \pi/2$. The resulting optimization problem is

$$\min_{\mathbf{x}} \prod_{i=1}^{N_{\mathrm{r}}} \prod_{j=1}^{N_{\mathrm{t}}} (1 + \tilde{g}[\Sigma_{\mathrm{r}}]_i [\Sigma_{\mathrm{t}}]_j x_j)^{-1}$$

$$\text{subject to } \sum_{j=1}^{N_{\mathrm{t}}} x_j = 1,\ x_j \geq 0,\ j = 1, \ldots, N_{\mathrm{t}}. \quad (15)$$

Experimental evidence (see Fig. 1) shows that the performance gap between (14) and (15) is negligible. Therefore, we focus on optimally solving (15) by reformulating it as a convex problem.

Making use of the properties of the logarithmic function that $\log(x)$ is monotonic increasing for nonnegative $x$, and $\log(x^{-1}) = -\log(x)$, we can recast (15) as

$$\min_{\mathbf{x}} \sum_{j=1}^{N_{\mathrm{t}}} \sum_{i=1}^{N_{\mathrm{r}}} -\log\big(1 + \tilde{g}[\Sigma_{\mathrm{r}}]_i [\Sigma_{\mathrm{t}}]_j x_j\big)$$

$$\text{subject to } \sum_{j=1}^{N_{\mathrm{t}}} x_j = 1,\ x_j \geq 0,\ j = 1, \ldots, N_{\mathrm{t}}. \quad (16)$$

It is easy to prove the following theorem.

THEOREM 3: The optimization problem (16) is convex on the variable $\mathbf{x}$, $0 \leq x_j \leq 1$, $\forall j = 1, \ldots, N_{\mathrm{t}}$.

As an example, consider a MIMO wireless system with two receive antennas. In this case, the receive correlation matrix has two eigenvalues which we denote as $[\Sigma_{\mathrm{r}}]_1$ and $[\Sigma_{\mathrm{r}}]_2$.

The globally optimal solution to the problem (16) in the case of two receiving antennas can be found by solving the following system of Karush-Kuhn-Tucker (KKT) conditions

$$-\frac{1}{x_j + \tilde{g}_{1j}} - \frac{1}{x_j + \tilde{g}_{2j}} + \mu = 0,\ j = 1, \ldots N_{\mathrm{t}} \quad (17)$$

$$\mu \left( \sum_{j=1}^{N_{\mathrm{t}}} x_j - 1 \right) = 0,\ \alpha_j x_j = 0,\ j = 1, \ldots N_{\mathrm{t}} \quad (18)$$

where $\tilde{g}_{1j} = 1/(\tilde{g}[\Sigma_{\mathrm{r}}]_1 [\Sigma_{\mathrm{t}}]_j)$ and $\tilde{g}_{2j} = 1/(\tilde{g}[\Sigma_{\mathrm{r}}]_2 [\Sigma_{\mathrm{t}}]_j)$. Introducing a new variable $\bar{\mu} = \frac{1}{\mu}$, we can write (17) as a second order equation

$$x_j^2 + \Big(\tilde{g}_{1j} + \tilde{g}_{2j} - 2\bar{\mu}\Big) x_j + \tilde{g}_{1j}\tilde{g}_{2j} - \bar{\mu}(\tilde{g}_{1j} + \tilde{g}_{2j}) = 0,\ j = 1, \ldots N_{\mathrm{t}} \quad (19)$$

which can be easily solved as

$$x_j = \frac{1}{2}\Big(-\tilde{g}_{1j} - \tilde{g}_{2j} + 2\bar{\mu} + \sqrt{(\tilde{g}_{1j} - \tilde{g}_{2j})^2 + 4\bar{\mu}^2}\Big), \forall j. \quad (20)$$

Then, $\bar{\mu}$ in (20) is chosen such that it satisfies the condition

$$\sum_j x_j = \frac{1}{2} \sum_j \Big(-\tilde{g}_{1j} - \tilde{g}_{2j} + 2\bar{\mu} + \sqrt{(\tilde{g}_{1j} - \tilde{g}_{2j})^2 + 4\bar{\mu}^2}\Big) = 1. \quad (21)$$

In general, (21) cannot be solved in closed form for the variable $\bar{\mu}$. However, a simple binary search can find $\bar{\mu}$. Note that this approach can also handle MIMO systems with more than two receive antennas, provided that the constant receive correlation model holds. Since the receive correlation matrix in this case has only two distinct eigenvalues, it is equivalent to the two receive antenna case. Moreover, it can be shown that in the case of no receive correlation the precoder in (16) boils down to the well-known water-filling precoder [16].

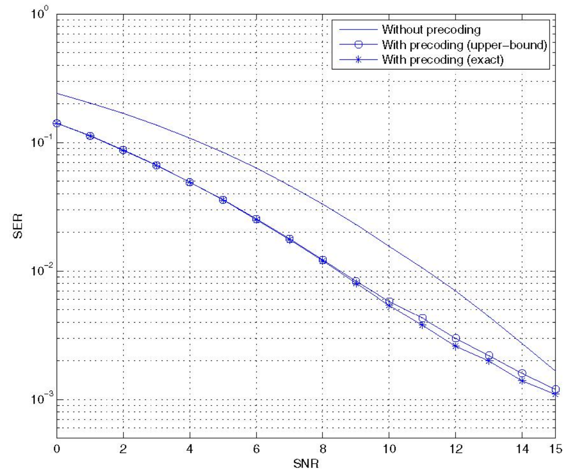


**Fig. 1**. The SER performance of Alamouti coded MIMO with transmit correlation only $\rho_{\mathrm{t}} = 0.6$

## 5. SIMULATION RESULTS

An OSTBC MIMO wireless system with 2 transmit and 2 receive antennas is considered. The Alamouti code and 4-PAM constellation are applied. The total available power at the transmitter is equal to $N_{\mathrm{t}}$, that is, $E_s = 2$. We use $\Sigma_t = \begin{bmatrix} 1 & \rho_t \\ \rho_t & 1 \end{bmatrix}$ and $\Sigma_r = \begin{bmatrix} 1 & \rho_r \\ \rho_r & 1 \end{bmatrix}$ and a Rayleigh fading channel.

We consider the scenario of no receive correlation. The correlation coefficient between different transmit antennas is $\rho_{\mathrm{t}} = 0.6$. Fig. 2 displays the SER performance of the following transmission schemes: OSTBS based transmission without precoding, transmission with exact precoder (10), and transmission with precoder (16). The transmission schemes with the proposed precoders significantly outperform the scheme without any precoder. Particularly, the performance gain of the transmission scheme with the proposed precoding techniques over the no-precoding scheme is about 2.7 dB for an SER of $10^{-2}$. The performance of the precoding scheme (16), which is the solution to the upper bound problem to the original problem of exact SER minimization (10), is very close to the performance of the precoder (10). A very small performance difference between these two schemes can be visually noticed only in the SNR region [9, 15] dB. However, the precoder (16) has a significantly lower complexity than that of (10).

Fig. 3 shows the SER performance of the two schemes: without precoding and with precoder (16) for the case when the correlation coefficient between different transmit and receive antennas is equal to $\rho_{\mathrm{t}} = \rho_{\mathrm{r}} = 0.6$. The performance gain is about 2.3 dB for an SER of $2 \cdot 10^{-2}$. Fig. 4 display the SER performance of the two schemes versus the transmit correlation coefficient. The receive correlation in Fig. 4 is assumed to be constant and equal to $\rho_{\mathrm{r}} = 0.3$. Fig. 4 shows that the proposed precoding scheme effectively exploits the transmit correlation to improve the transmission quality.

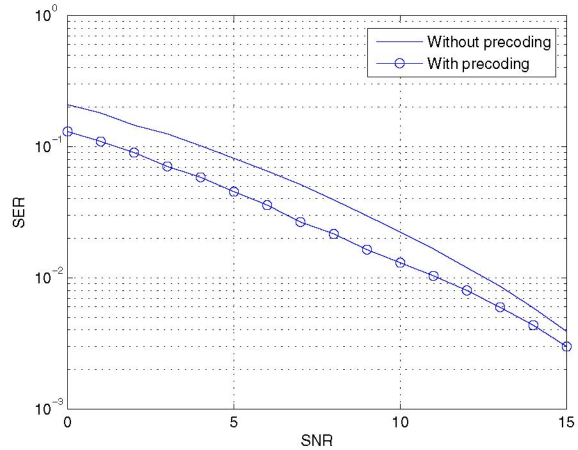


**Fig. 2**. The SER performance of Alamouti coded MIMO with transmit and receive correlations $\rho_\mathrm{r} = \rho_\mathrm{t} = 0.6$

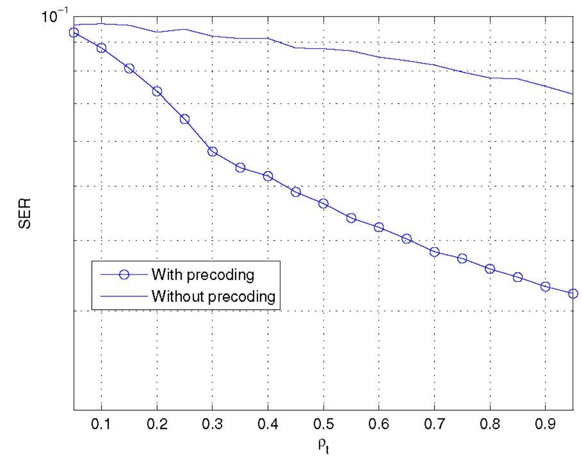


**Fig. 3**. The SER performance of Alamouti coded MIMO with constant receive correlation $\rho_\mathrm{r} = 0.3$, transmit correlation $\rho_\mathrm{t} = [0.05 : 0.95]$

## 6. CONCLUSIONS

Linear precoder for OSTBC based MIMO systems which requires only the knowledge of the transmit and receive correlation matrices at the transmitter is developed using convex optimization approach for the case of correlated Rayleigh fading channels. The connection to other precoders is shown. Numerical simulations illustrate the improved performance of the proposed precoders in terms of the output SER as compared to the system that does not use precoding.

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