Performance Analysis of Transmit and Receive Antenna Selection with space-time coding

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***Abstract*—This paper analyzes the performance of multiple-input multiple-output (MIMO) systems with transmit and receive antenna selection (T-RAS). The average bit error rate (BER), average symbol error rata (SER), outage probability and ergodic capacity are derived by utilizing the characteristic function (CF) of the joint output signal-to-noise ratios (SNR). Our approach can be used over not only independent but also arbitrary correlated Rayleigh, Nakagami-m and Rician fading channels. Simulation results are provided to validate our numerical calculations. We also illustrate the effect of antenna array configuration and the operating environment (fading, angular spread, mean angle-of-arrival(AOA), mean angle-of-departure (AOD)) on the average BER performance.**

I. Introduction

Multiple-input multiple-output (MIMO) systems with multiple antennas at transmit and receiver ends of a wireless link exploit spatial diversity to mitigate the impact of fading [1]. Although the use of multiple antennas at the transmitter and the receiver improves the overall system performance, the cost is concomitantly increased because each active transmit/recieve antenna pair requires a dedicated radio frequency chain, along with an increase in signal processing complexity [2]. A promising low-complexity solution that has received much attention in the wireless community is the selection of a subset of all available transmit/receive antenna pairs. Types of antenna selection are transmit antenna selection (TAS), receive antenna selection (RAS) and transmit and receive antenna selection (T-RAS).

Previously, both RAS and TAS have been extensively investigated. In particular, RAS (also known as selection diversity or selection combining) has been researched for several decades and RAS performance in various channel/correlation models have been comprehensively treated. Among many others, [3] analyzed three-branch selection combining over arbitrarily correlated Rayleigh fading channels, theoretical analysis for generalized selection combining receiver with nonidentical fading is presented in [4], and other contributions include [5]–[8]. In [9], performance of TAS is analyzed for selecting one antenna at the transmitter. The symbol error rate (SER) of TAS is derived in [10].

In performance analysis of antenna selection, one needs the statistical distribution of the maximum of a set of branch signal-to-noise ratios (SNRs). If these are statistically independent, then the distribution of the maximum is readily derivable. This is actually the case for both TAS and RAS if the actual channel gains are independent. Consequently, many analytical studies focus on independent fading channels. However, with T-RAS, the branch SNRs are not independent even if the fading channels are independent. Although order statistics is a well-established branch of statistics, there is surprisingly few analytical results are available on order statistics of correlated random variables [11]. For this reason, the analysis is often made tractable by constraining antenna selection at either the transmitter or the receiver – but not both together [12].

To the best of our knowledge, only one paper has analyzed T-RAS to date. Cai and Giannakis [13] analyze error rate performance for selecting one transmit antenna and arbitrary number of receive antennas in independent Rayleigh fading. Thus, a general analysis of joint selection of L_t out of N_t transmit antennas and L_r out of N_r receive antennas remains unsolved. The difficulty lies in the fact that in TAS or RAS, order statistics of independent fading gains can be employed to get the probability density function (pdf) of the output SNR. However, even in the case of independent fading, in T-RAS, order statistics of independent fading gains are not applicable.

In this paper [1], we thus present a framework of performance analysis for the general T-RAS for an arbitrary number of transmit and receive antennas. We leverage the analytical framework introduced for arbitrary correlated multi-branch selection combining [14]. Although the present work is exclusively concerned with Rayleigh flat-fading MIMO channels, our analysis can be extended to more complicated models such as Nakagami-m or Rician fading channels.

II. System and Channel Model

We consider a MIMO system with N_t total transmit antennas and N_r total receive antennas. The slow-fading channel gains are available at the receiver. The receiver selects a subset of L_t transmit antennas and L_r receive antennas. The selection information is conveyed to the transmit side via a feedback link. Orthogonal space-time block code (OSTBC) signal matrices are sent of the subset of selected transmit antennas.

[1] *Notation*: Bold symbols denote matrices or vectors. $(\cdot)^T$ and $(\cdot)^H$ denotes transpose and complex conjugate transpose respectively. $\mathbb{E}\{x\}$ denotes the expectation of x. $\mathbf{A}_{m,n}$ is the (m,n)th entry of the matrix $\mathbf{A}$. $\det(\mathbf{A})$ and $\text{vec}(\mathbf{A})$ denote the determinant and vectorization of the matrix $\mathbf{A}$. $\|\mathbf{A}\|_F^2$ is the Frobenius norm of $\mathbf{A}$. $\mathbf{A} \otimes \mathbf{B}$ denotes the Kronecker product of the matrices $\mathbf{A}$ and $\mathbf{B}$. A circularly complex Gaussian variable z with mean μ and variance σ^2 is denoted by $z \sim \mathcal{CN}(\mu, \sigma^2)$. And $\jmath = \sqrt{-1}$.

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Let the vector of complex information-bearing symbols be $\mathbf{s} = [s_1, s_2, \ldots, s_Q]^T$ of size $Q \times 1$. Different signal constellations such as M-ary pulse amplitude modulation (M-PAM), M-ary quadrature amplitude modulation (M-QAM), or M-ary phase shift keying (M-PSK) can be used with the normalization $\mathbb{E}(|s_i|^2) = 1$. The input vector is mapped into an OSTBC signal matrix $\mathbf{X}(\mathbf{s})$ of size $N_t \times T$ with the following properties [1]:

- All the elements in $\mathbf{X}(\mathbf{s})$ are linear functions of $s_1, s_2, \ldots, s_Q$ and their complex conjugates.
- $\mathbf{X}(\mathbf{s})^H \mathbf{X}(\mathbf{s}) = \|\mathbf{s}\|_F^2 \mathbf{I}$ for all $\mathbf{s} \in \mathbb{C}^Q$.

Since T symbol periods are necessary to transmit Q symbols, the symbol rate R_s of the OSTBC is defined as $R_s = Q/T$. OSTBC provide maximal diversity order in a fading channel and are amenable to low-complexity maximum likelihood decoding. In this case, maximizing the channel Frobenius norm maximizes SNR as well as minimizes the instantaneous probability of error [12].

Let $\mathbf{H}$ be the $N_r \times N_t$ channel matrix. The envelop of the each element $h_{k,l}$ $(1 \leq k \leq N_r, 1 \leq l \leq N_t)$ of $\mathbf{H}$ will follow either the Rayleigh, Nakagami-m, or Rician fading model. We consider the cases of both $h_{k,l}$ are independent and are correlated with each other. Correlation may caused by realistic conditions, such as, insufficient angular spread induced by the scattering environment or closely spaced antenna elements.

Let the actual $L_r \times L_t$ transmission matrix be $\widetilde{\mathbf{H}}$, the received signals can be expressed by

$$\mathbf{Y} = \sqrt{\frac{E_s}{L_t}} \widetilde{\mathbf{H}} \mathbf{X} + \mathbf{N}, \qquad (1)$$

where E_s is the energy of the transmitted symbol, $\mathbf{Y}$ is the $L_r \times T$ received signal matrix, $\mathbf{X}$ represents the $L_t \times T$ transmitted signal matrix and $\mathbf{T}$ is the block symbol periods. The elements of $\mathbf{N}$ are identical independent distributed (i.i.d.) Gaussian random variables $\sim \mathcal{CN}(0, \sigma^2)$, where σ^2 is the noise variance.

To maximize the total received signal power we are to select the subset of transmit and receive antennas that yields the largest instantaneous output SNR. There are $N = \binom{N_t}{L_t} \cdot \binom{N_r}{L_r}$ possible selections of transmit and receive antennas. Let $\widetilde{\mathbf{H}}_s$ $(1 \leq s \leq N)$ be the N channel sub-matrix correspond to the N possible antenna subsets. Define $||\widetilde{\mathbf{H}}_s||_F^2 = \sum_{m=1}^{L_r} \sum_{n=1}^{L_t} |h_{m,n}|^2$, where $h_{m,n}$ is the (m,n)th element of $\widetilde{\mathbf{H}}_s$ and $1 \leq m \leq L_t, 1 \leq n \leq L_r$. Using OSTBC, the instantaneous output SNR for each antenna subset can be given by

$$\gamma_j = \frac{E_s}{N_0 R_s L_t} \|\widetilde{\mathbf{H}}_s\|_F^2, \qquad (2)$$

where R_s is the symbol rate (symbol/s), N_0 is the one-side power spectral density of the white Gaussian noise. The receiver will select antenna subset with maximum instantaneous output SNR and the output SNR of T-RAS is equal to

$$\gamma = \max\{\gamma_1, \cdots, \gamma_N\}. \qquad (3)$$

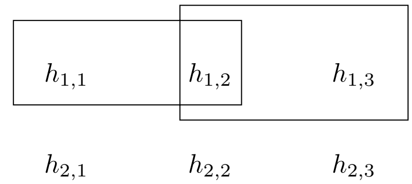

Fig. 1. Two possible antenna selection subsets

When the transmitter side receives the selection information, the selected transmit antennas are connected to the available L_t RF chains and the actual transmission occurs with $\widetilde{\mathbf{H}}$.

III. The CF of T-RAS

Different from TAS and RAS, T-RAS is not amenable to the theory of order statistics under the assumption of independent random variables. We give an illustrative example.

Example 1: Consider an MIMO system with 3 available transmit antennas and 2 available receive antennas on independent fading channels and the channel matrix is given in Fig. 1. In TAS analysis, the output SNR corresponding to each transmit antenna is the transmit SNR multiplied by the square norm of each column of the channel matrix $\mathbf{H}$, i.e., as for the first transmitter, $\gamma_1 = (|h_{1,1}|^2 + |h_{2,1}|^2)E_s/N_0$. Arrange the γ_i in descend order and denote them by $\gamma_{(1)} \geq \gamma_{(2)} \geq \gamma_{(3)}$. If $\gamma_1, \gamma_2, \gamma_3$ are identical independent distributed (i.i.d.), the joint or individual pdf of $\gamma_{(i)}$ can be given by the theory of order statistics [11]. The SER or BER can be derived based on the known pdf of $\gamma_{(i)}$. On the other hand, the analysis of RAS requires the distribution of sorted output SNRs, the statistics of which can also be obtained by the same order statistics theory.

Now, in T-RAS generalized scheme, if 2 transmit antennas and 1 receive antenna are to be selected, there are $\binom{3}{2} \cdot \binom{2}{1} = 6$ different choices of antenna subsets. The submatrices of two possible antenna subsets are shown in fig. 1. The left rectangle corresponds to the 1st and 2nd transmit antennas and the 1st receive antennas being selected and the output SNR is given by $\gamma_1 = (|h_{1,1}|^2 + |h_{1,2}|^2)E_s/2N_0R_s$. The right rectangle corresponds to selecting the 2nd and 3rd transmit antennas and the 1st receive antennas, where the output SNR is given by $\gamma_2 = (|h_{1,2}|^2 + |h_{1,3}|^2)E_s/2N_0R_s$. The other γ_j $(3 \leq j \leq 6)$ are defined similarly. We can see that even if the channels $h_{k,i}$ are independent with each other, γ_1 and γ_2 are no longer independent since there exists a common term $h_{1,2}$. Thus, the case of joint antenna selection does not satisfy the condition for the theory of order statistics.

For correlated channels, determination of the statistics of γ_j will become even more complicated since the correlation between different γ_j will be caused by the common terms as well as the original correlation.

To solve this problem, we use Zhang's [14] analytical framework suggested in a multi-branch selection combining problem, which expresses the joint pdf of γ_j as multiple Fourier transform of its characteristic function (CF). The cumulative distribution function (cdf) and pdf of the maximum

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SNR γ among all possible γ_j are given as [14, eq. (8)] and [14, eq. (9)]

$$F_\gamma(\gamma) = \frac{1}{(2\pi)^N} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(\mathbf{t}) \prod_{k=1}^{N} \frac{1 - e^{-\jmath t_k \gamma}}{\jmath t_k} d\mathbf{t} \quad (4)$$

where $\mathbf{t} = (t_1, \cdots, t_N)$ and

$$\begin{aligned} f_\gamma(\gamma) = & \frac{1}{(2\pi)^N} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \phi(\mathbf{t}) \prod_{k=1}^{N} (\jmath t_k)^{-1} \\ & \times \sum_{l=1}^{N} (-1)^{l+1} \sum_{b_1+\cdots+b_N=l} \frac{\jmath T_N}{\exp(\jmath\gamma T_N)} d\mathbf{t}, \end{aligned} \quad (5)$$

where $N = \binom{N_t}{L_t}\binom{N_r}{L_r}$ and for brevity, we denote $T_N = b_1 t_1 + \cdots + b_N t_N$ and $b_1, \cdots, b_N$ are binary variables that take the values of 0 or 1. In (4), $\phi(\mathbf{t})$ is the joint CF for γ_j, which solely depends on the channel environment and is independent of the modulation scheme. However, in our situation the determination of CF becomes more complicated. We will derive the CF for various fading environments commonly encountered in practice.

The joint CF of the N possible output SNRs γ_j is defined

$$\phi(\mathbf{t}) = \mathbb{E}\left\{e^{\jmath t_1 \gamma_1 + \cdots + \jmath t_N \gamma_N}\right\}, \quad (6)$$

where γ_j $(1 \le j \le N)$ are the output SNRs of each possible antenna selection defined in (2). The main difficulty in evaluating (6) is that γ_j is not corresponding to one single fading channel but related to the $L_t \times L_r$ summation of the square norm of the elements of the jth channel matrix $\widetilde{\mathbf{H}}_s$. Thus, the CF for the output SNR γ_j in [14] is not suitable for the joint antenna selection problem.

In order to overcome this problem, we substitute (2) into (6) and obtain

$$\phi(\mathbf{t}) = \mathbb{E}\left\{e^{a\jmath\left(|h_{1,1}|^2 \sum_{k=1}^{N_c} t_{1k} + \cdots + |h_{N_r,N_t}|^2 \sum_{k=1}^{N_c} t_{(N_r N_t)k}\right)}\right\}, \quad (7)$$

where $a = \frac{E_s}{N_0 L_t R_s}, N_c = \binom{N_t - 1}{L_t - 1}\binom{N_r - 1}{L_r - 1}$ and $\sum_{k=1}^{N_c} t_{ik}$ is a linear function of t_i $(1 \le i \le N)$ in (6) determined by the arrangement order of different γ_j. The CF can be obtained by evaluating (7) with respect to $|h_{k,i}|^2$. By defining the vector $\mathbf{h}$ as

$$\mathbf{h} = \text{vec}[\mathbf{H}], \quad (8)$$

the exponential in (7) can be rewritten as the Hermitian quadratic form of $\mathbf{h}$ as

$$\phi(\mathbf{t}) = \mathbb{E}\left\{e^{\mathbf{h}^H \mathbf{Q} \mathbf{h}}\right\}, \quad (9)$$

where $\mathbf{Q}$ is the diagonal matrix with the diagonal elements be coefficient of $|h_{k,i}|^2$, i.e.,

$$\mathbf{Q} = \text{diag}\left\{a\jmath \sum_{k=1}^{N_c} t_{1k}, \cdots, a\jmath \sum_{k=1}^{N_c} t_{(N_r N_t)k}\right\}. \quad (10)$$

Notice that the form of $\mathbf{Q}$ depends on the number of the selected and available transmit and receive antennas. We will give an example later to illustrate how to obtain $\mathbf{Q}$.

A. Rayleigh fading case

The Rayleigh fading is frequently used to model the statistics of signals transmitted through radio channels. For correlated Rayleigh fading channels, the channel vector $\mathbf{h}$ follows the complex Gaussian distribution, $\mathbf{h} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$, where $\mathbf{R}$ is the covariance matrix defined by

$$\mathbf{R} = \frac{1}{2}\mathbb{E}[\ (\mathbf{h} - E\{\mathbf{h}\})(\mathbf{h} - E\{\mathbf{h}\})^H]. \quad (11)$$

The exponential in (7) can be looked as the CF of Hermitian quadratic forms in $\mathbf{h}$. From [15] [16, eq.(B-3-14)], the joint CF of output γ_j can be derived as

$$\phi(\mathbf{t}) = \det(\mathbf{I} - \mathbf{\Psi}\mathbf{Q})^{-\mathbf{1}}. \quad (12)$$

We notice that the joint CF can also be achieved by redefining the diagonal matrix $\mathbf{S}$ to the diagonal matrix $\mathbf{Q}$. We allow correlation to be evaluated in terms of the transmit and receive antenna spacing, mean angle-of-arrival (AOA), mean angle-of-departure (AOD) and transmit and receive angular spread.

We now illustrate how to achieve the diagonal matrix $\mathbf{Q}$ in (12). There are two steps. First, define the N output SNR γ_j corresponded to the N possible antenna subset selections. Second, substitute the γ_js into the general CF function in (6). Then, the diagonal elements of $\mathbf{Q}$ corresponds to the coefficient of the $|h_{k,i}|^2$. We use the example illustrated before to show how to determine the matrix $\mathbf{Q}$ and thereby derive the CF for T-RAS.

Let us consider selecting 2 out of 3 transmit antennas and 1 out of 2 receive antennas. The number of possible antenna selection is $\binom{3}{2}\binom{2}{2} = 6$. If we arrange the 6 term of $\{\gamma_1, \cdots, \gamma_6\}$ as

$$\begin{aligned} \gamma_1 &= a(|h_{1,1}|^2 + |h_{1,2}|^2), \quad \gamma_2 = a(|h_{1,1}|^2 + |h_{1,3}|^2), \\ \gamma_3 &= a(|h_{1,2}|^2 + |h_{1,3}|^2), \quad \gamma_4 = a(|h_{2,1}|^2 + |h_{2,2}|^2) \\ \gamma_5 &= a(|h_{2,1}|^2 + |h_{2,3}|^2), \quad \gamma_6 = a(|h_{2,2}|^2 + |h_{2,3}|^2). \end{aligned} \quad (13)$$

and substituting (13) into (6), the exponential in (6) becomes

$$\begin{aligned} \jmath \sum_{k=1}^{6} t_k \gamma_k = & a\{|h_{1,1}|^2 \jmath(t_1 + t_2) + |h_{1,2}|^2 \jmath(t_1 + t_3) + \\ & |h_{1,3}|^2 \jmath(t_2 + t_3) + |h_{2,1}|^2 \jmath(t_4 + t_5) \\ & + |h_{2,2}|^2 \jmath(t_4 + t_6) + |h_{2,3}|^2 \jmath(t_5 + t_6)\}. \end{aligned} \quad (14)$$

Thus, the diagonal matrix $\mathbf{Q}$ can be readily obtained. Note that, the position of the diagonal elements of $\mathbf{Q}$ are fixed by the position of $h_{k,l}$ in the channel vector $\mathbf{h}$. With the knowledge of $\mathbf{Q}$, the joint CF for the output SNR γ_j of T-RAS can be obtained by (12) with the known correlation matrix Ψ. When the channels are i.i.d., the joint CF can be expressed by

$$\begin{aligned} \phi(\mathbf{t}) = & \frac{1}{(1 - a(t_1 + t_2)\jmath)(1 - a(t_1 + t_3)\jmath)(1 - a(t_2 + t_3)\jmath)} \\ & \times \frac{1}{(1 - a(t_4 + t_5)\jmath)(1 - a(t_4 + t_6)\jmath)(1 - a(t_5 + t_6)\jmath)}. \end{aligned} \quad (15)$$

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IV. PERFORMANCE ANALYSIS

With the CF derived in Section III, the pdf of the output SNR γ of T-RAS can be obtained from (5), which can be further used to evaluate system performance. In this section, we derive the closed-form expressions for the average BER, average SER, outage performance and ergodic capacity of a class of modulations for MIMO systems with T-RAS.

A. BER Analysis

With the knowledge of the pdf of the output SNR γ, the average error rate can be obtained by

$$\overline{P}_e = \int_0^\infty P_e(\gamma) f_\gamma(\gamma) d\gamma, \tag{16}$$

where $P_e(\gamma)$ is the conditional error probability for either BER or SER given instantaneous output SNR γ and $f_\gamma(\gamma)$ is the pdf of the output SNR. Substituting (5) into (16) yields the average error probability expression

$$\overline{P}_e = \int_0^\infty \cdots \int_0^\infty \phi(\mathbf{t}) w(\mathbf{t}) d\mathbf{t} \tag{17}$$

with

$$\begin{aligned} w(\mathbf{t}) =& \frac{1}{(2\pi)^N} \int_{-\infty}^{\infty} P_e(\gamma) \prod_{k=1}^{N} (\jmath t_k)^{-1} \\ & \times \sum_{l=1}^{N} (-1)^{l+1} \sum_{b_1+\cdots+b_N=l} \frac{\jmath T_N}{\exp(\jmath\gamma T_N)} d\gamma. \end{aligned} \tag{18}$$

In (17), the first factor $\phi(t_1, \cdots, t_N)$ in the integrand is the joint CF of γ_j depending solely on the channel characters and the number of selected and available transmit and receive antennas. The second factor $w(\mathbf{t})$ is the weighting function, which depends only on the modulation scheme. Such a decomposition makes the analysis of error performance systematic. For a given modulation scheme operating over a specified environment, what we need is to determine these two factors and use (17) to obtain the average error probability.

Consider a MIMO system with M-ary square amplitude modulation (M-QAM) and Gray mapping. As discussed in [17], the conditional BER can be represented as a sum of $(\sqrt{M}-1)$ Q functions, expressed by

$$\overline{P}_{e|\text{BER}}(\gamma) = \sum_{i=1}^{\sqrt{M}-1} a_i Q(\sqrt{b_i \gamma}), \tag{19}$$

where the coefficients a_i and b_i depend on the constellation size M. The conditional BER of the binary phase shift keying (BPSK) and frequency shift keying (BFSK) can be looked as the special cases of (19) with $M = 4$, $a_1 = 1, b_1 = 2$ and $M = 4$, $a_1 = 1, b_1 = 1$, respectively. Inserting (19) into (18) yields (similar to [18, eq,(12)]),

$$\begin{aligned} w(\mathbf{t}) = & \sum_{i=1}^{\sqrt{M}-1} \frac{a_i}{2(2\pi)^N} \prod_{k=1}^{N} (\jmath t_k)^{-1} \\ & \times \left(1 + \sum_{l=1}^{N} (-1)^l \sum_{b_1+\cdots+b_N=l} \sqrt{\frac{b_i}{b_i + 2\jmath T_N}}\right). \end{aligned} \tag{20}$$

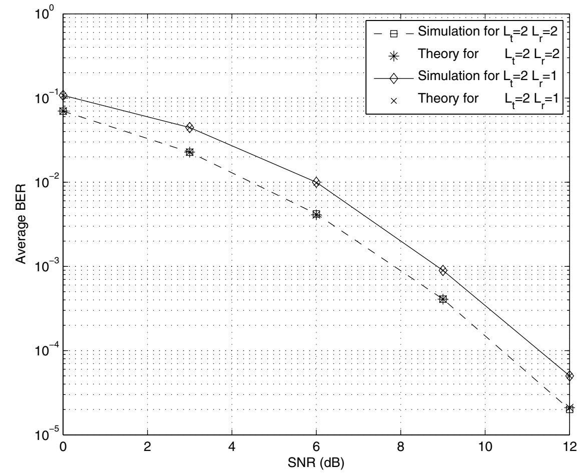

Fig. 2. Average BER versus transmit SNR over independent Rayleigh fading channels .

After substituting (20) into (17), the average BER of the MIMO systems with T-RAS and M-QAM, BPSK, or others can be numerically calculated.

B. SER Analysis

By defining $P_e(\gamma)$ in (16) as the conditional symbol error rate on the output SNR, we can derive the average SER. Here, we will use the moment generating function (MGF)-based approach to derive the average SER [19].

From the pdf of γ in (5), the MGF of the output SNR can be derived as

$$\begin{aligned} M_\gamma(s) &= \int_0^\infty e^{-s\gamma} f_\gamma(\gamma) d\gamma \\ &= \frac{1}{(2\pi)^N} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \phi(\mathbf{t}) \prod_{k=1}^{N} (\jmath t_k)^{-1} \\ & \quad \times \sum_{l=1}^{N} (-1)^{l+1} \sum_{b_1+\cdots+b_N=l} \frac{\jmath T_N}{s + \jmath T_N} d\mathbf{t}. \end{aligned} \tag{21}$$

Using the MGF of γ, the SER of M-PSK can be calculated by

$$\overline{P}_{e|\text{MPSK}}(e) = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} M_\gamma \left(\frac{g_{\text{MPSK}}}{\sin^2\theta}\right) d\theta, \tag{22}$$

where $g_{\text{MPSK}} = \sin^2(\frac{\pi}{M})$, while the average SER of square M-QAM is

$$\begin{aligned} \overline{P}_{e|\text{MQAM}}(e) = & \left(1 - \frac{1}{\sqrt{M}}\right) \frac{4}{\pi} \int_0^{\frac{\pi}{2}} M_s \left(\frac{g_{\text{MQAM}}}{\sin^2\theta}\right) d\theta \\ & - \left(1 - \frac{1}{\sqrt{M}}\right)^2 \frac{4}{\pi} \int_0^{\frac{\pi}{4}} M_s \left(\frac{g_{\text{MQAM}}}{\sin^2\theta}\right) d\theta, \end{aligned} \tag{23}$$

where $g_{\text{MQAM}} = \dfrac{3}{2(M-1)}$.

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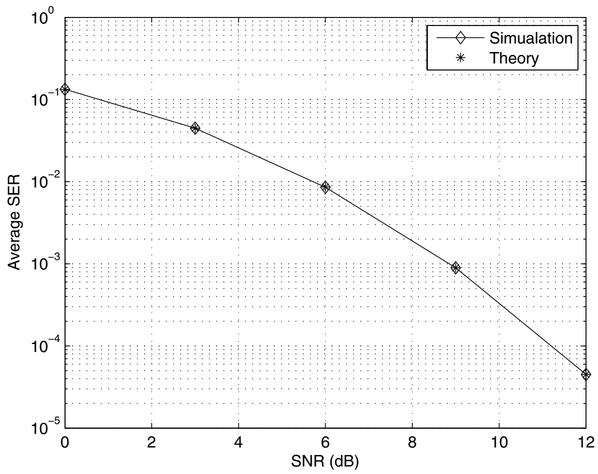

Fig. 3. Average SER versus transmit SNR over independent Rayleigh fading channels .

V. Numerical Results

4-QAM is used for all numerical examples. Fig. 2 depicts the simulation and theory results of the average BER in two generalized antenna selection MIMO systems over independent Rayleigh fading environments. The Alamouti space-time code is employed on the selected transmit antennas. One MIMO system chooses $L_t = 2$ out of 3 transmit antennas and $L_r = 2$ out of 2 receive antennas, and the other chooses $L_t = 2$ out of 3 transmit antennas and $L_r = 1$ out of 2 receive antennas. Integrals in (17) are approximated by using a truncated Riemann sum of equally-spaced sampling points distributed between -10 and 10. Extending the summation limits does not seem to improve the accuracy since the integrand is highly concentrated within the range $[-8,\ 8]$. In Fig. 2, the theory result of the both systems match extremely well with the simulation results. The system using both two receive antennas outperforms the MIMO with one receive antenna selected.

Fig. 3 demonstrates the theory result of the average SER in a generalized selection MIMO system where we choose $L_t = 2$ out of 3 transmit antennas and $L_r = 2$ out of 2 receive antennas. Simulation results are shown to be compared with the theoretical values. We see that the calculated values match extremely well with the simulation results.

VI. Conclusion

This paper presents a framework to analyze the performance of the MIMO systems with generalized transmit and receive antenna selection. The main difficulty is the correlation between the different subset antenna selections arising from antenna selection as well as the spatial correlation (if any). The problem can be conquered by expressing the pdf of the maximum output SNR as a function of the joint CF of all possible output SNRs. Thus, we derived several the pdf based performance measures, including the average BER, the average SER, the outage probability and ergodic capacity (the latter two are not shown for brevity). Numerical examples are given to illustrate the effect of antenna array configuration and the operating environment on the average BER performance through the correlation coefficient.

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